RISK AND RETURN FOR PROPERTY-CASUALTY INSURERS

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Introduction

Recent developments in the theory of finance have had a substantial impact upon the way in which returns to firms are viewed Such concepts as "efficient markets", "portfolio optimization' and 'systematic risk" were virtually unknown outside the academic community until the 1960's, but now have become a part of the financial analyst's day-to-day vocabulary The purpose of this paper is to determine relationships between risk and return for property-casualty insurers by applying modern concepts of finance, and to comment on the implications which result

To achieve this goal, a simple model of a property-casualty company is developed to show key accounting relationships, including the fundamental notion of return on surplus as a function of leveraged underwriting and investment income gains This model is then extended to treat the basic elements as random variables, from which the important risk-return results are derived The subsequent risk-return model illustrates the relationship between systematic and unsystematic risk explores the problem of finding an optimal balance between asset and underwriting portfolios, and applies efficient market criteria to find the expected underwriting profit margin under equilibrium conditions Applications are then discussed for the areas of product pricing, ruin theory, marketing, reinsurance, and regulation

Throughout the paper, financial concepts are introduced as needed, and explained in intuitive terms The reader should consult the References for more detailed explanation of these ideas, since it is beyond the scope (and intent) of this paper to develop the basic tools rigorously

Basic Model

To show the relationship among important variables we can establish a simple model of a property-casualty insurance company Certain assumptions will simplify the development (1) all expenses are treated as part of the losses (2) there are no taxes (3) the investment rate of return is linear through time These assumptions could be eliminated without significantly affecting the results but to do so would unnecessarily complicate the analysis

For notation, variables measuring point-in-time values are referenced by a subscript, and variables measuring flows through time contain parentheses We define the following variables

$W(t)$	Premium written during the period t to t+1 i e (t t+1)
$E(t)$	Premium earned during (t, t+1)
$IL(t)$	Losses incurred during (t t+1)
$PL(t)$	Losses paid during (t t+1)
$C(t)$	Mean cash flow during (t t+1)
LR_t	Loss reserve at time t
UR_t	Unearned premium reserve at time t
A_t	Total assets at time t
S_t	Shareholders' surplus at time t
$U(t)$	Underwriting income during (t t+1)
$I(t)$	Investment income during (t, t+1)

The following relationships can now be easily established

(1) $E(t) = bW(t) + (1-b)W(t-1)$

where $0 \geq b \geq 1$ More terms could be added if policies longer than one period were written

(2) $IL(t) = \ell E(t)$,

where ℓ is the loss ratio (assumed here to be constant through time)

(3) $PI(t) = a_0 IL(t) + a_1 IL(t-1) + \quad + a_m IL(t-m)$

where $\sum_{i=0}^{m} a_i = 1$, and the a_i indicate the claim payment pattern

(4) $C(t) = f_1 W(t) - f_2 PL(t)$,

where f_1 and f_2 represent the average duration for which these cash flows are exposed to the investment rate of return (note that $f_1 = b$, normally)

(5) $LR_t = (1-a_0)IL(t-1) + (1-a_0-a_1)IL(t-2) + \quad + a_m IL(t-m)$,

i e , the loss reserve equals the sum of unpaid losses from prior periods

(6) $UR_t = (1-b)W(t-1)$

Note that $b = \frac{1}{2}$ if premiums are written uniformly over $(t-1 \quad t)$

(7) $A_t = S_t + LR_t + UR_t$

or assets = liabilities

(8) $U(t) = E(t) - IL(t)$,

which is the normal definition of underwriting income

(9) $I(t) = R\left[A_t + C(t)\right]$,

where R is the investment rate of return and $A_t + C(t)$ represent mean assets during $(t, t+1)$

Since the only sources of increment to assets are cash flows and investment income, we have

$A_{t+1} - A_t = \Delta A_t = W(t) - PL(t) + I(t) = \Delta(S_t + LR_t + UR_t)$, or

$$(10) \quad W(t) - PL(t) + I(t) = \Delta S_t + \left[IL(t) - PL(t) \right] + \left[W(t) - E(t) \right]$$

since the change in loss reserve = incurred losses - paid losses and the change in unearned premium reserve = written premium - earned premium

From (10) we get

$$(11) \quad \Delta S_t = S_{t+1} - S_t = I(t) + E(t) - IL(t) = I(t) + U(t)$$

Thus the increment to surplus can be directly separated into the two components of investment and underwriting income The <u>rate of return on surplus</u> is

$$R_s = (S_{t+1} - S_t)/S_t = I(t)/S_t + U(t)/S_t$$

$$= R \left[\text{mean assets} \right] /S_t + U(t)/S_t$$

$$= R \left[S_t + V_t \right] /S_t + U(t)/S_t, \quad (V_t = \text{mean reserves})$$

$$(12) \qquad = \underline{R + kvR + kU}$$

where $U = U(t)/W(t)$ or underwriting gain per unit of written premium $k = W(t)/S_t$ or the premium/surplus ratio and $v = V_t/W(t)$ or the reserve/premium ratio

Equation (12) is fundamental It shows that the return to surplus, or net worth, is levered by premiums (k) and by reserves (v) The first term R is the return on shareholders' funds which would exist even if no premium were written (k = 0) The second term kvR is the return on policyholders' funds which is a consequence of deferred payment of claims and prepayment of premium, it is levered by both v and k The third term kU is the underwriting gain, levered by k

A numerical example will help to illustrate the preceding concepts Let S_t = 100, W(t-2) = 160, W(t-1) = 200, W(t) = 240, R = 10, ℓ = 98, f_1 = b = 5 (premium written uniformly throughout the year), f_2 = 5 (claims paid uniformly), a_0 = 8, a_1 = 2 (80% of losses paid during the first year, 20% the second year)

Thus we get E(t) = 220, IL(t) = 215 6, IL(t-1) = 176 4, PL(t) = 207 76, C(t) =16 12, LR_t = 35 28 UR_t = 100, A_t = 235 28, and V_t = 151 4

Underwriting gain is U(t) = 220-215 6 = 4 4 and investment income is 10(235 28 +16 12) = 25 14 The change in surplus becomes 4 4 + 25 14 = 29 54 and consequently the return on surplus is 29 5% The levers are k = 240/100 = 2 4 and v = 151 4/240 = 63 and the unit underwriting gain U is 4 4/240 = 0183 The return on surplus can be expressed as in (12), in terms of three components

$$R_s = 10 + (2\ 4)(\ 63)(\ 1) + 2\ 4(\ 0183) = \underline{10} + \underline{151} + \underline{044} = \underline{295}$$

Risk and Return

The preceding analysis assumes that the values of the variables are known, but this is rarely true unless we view the results afterwards Assume now that the investment rate of return is a random variable, as is the underwriting gain per unit of premium Further, assume that we know the probability distribution of these variables so that means and variances can be computed Since R_s is a linear combination of R and U, it too will be a random variable Equation (12) may now be expressed as

$$(13) \quad \tilde{R}_s = (1+kv)\tilde{R} + k\tilde{U} = K\tilde{R} + k\tilde{U}$$

where the tildes denote random variables (the same variable without the tilde represents the expected value of the variable) The expectation and variance of $\tilde{R}_s$ are

$$(14) \quad E(\tilde{R}_s) = R_s = KE(\tilde{R}) + kE(\tilde{U}) = KR + kU,$$

$$(15) \quad V(\tilde{R}_s) = V(K\tilde{R}) + V(k\tilde{U}) + 2C(K\tilde{R},k\tilde{U})$$
$$= K^2V(\tilde{R}) + k^2V(\tilde{U}) + 2kKC(\tilde{R},\tilde{U})$$

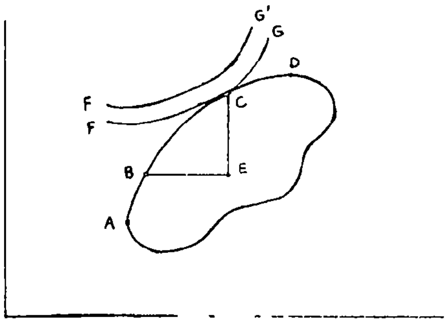
with $C(\tilde{R}\ \tilde{U})$ indicating the covariance between R and U

The variance (or its square root, the standard deviation) of the rate of return is the commonly used measure of the risk inherent in holding a security For the same expected return investors will normally prefer an asset with a lower variance of return, and conversely for the same variance investors will choose an asset with a higher return This behavior is called risk-aversion

When faced with several choices of expected value/variance of return pairs there may be no obvious selection However many candidates can be rejected

Figure 1

Expected Return



Variance

In figure 1 suppose the region bounded by ABCDA contains all the possible pairs of expected value/variance of return available Only the points along ABCD will be preferred since all interior points can be rejected For example, points B and C will always be chosen over point E, since C has a higher return than E, but with the same variance, and B has the same return as E, but with a lower variance The line ABCD is called the efficient frontier The exact choice of risk/return along ABCD will depend upon the investor's particular utility function, which can be described as a set of concentric curves such as FG and F'G', where the investor is indifferent to each risk/return combination along the curve

The optimal choice is determined by the lowest curve tangent to the efficient frontier In Figure 1 FC is the lowest indifference curve and is tangent to ABCD at point C which represents the optimal risk/return choice for this investor

Systematic Risk

According to traditional theory, the law of large numbers justifies the existence of insurance companies, who by insuring large numbers of exposures, are able to spread risk We can examine this principle in light of the risk/return model developed earlier

Let the firm be comprised of a single line of insurance with n identical exposures each having an underwriting gain of $\tilde{u}_i, i = 1, \ldots, N$ We will allow for covariance between exposures, and between individual exposures and the asset rate of return

Consequently we have $\tilde{U} = (\sum_i^N \tilde{u}_i)/N$ and

$$(16)\quad V(\tilde{U}) = (1/N^2)\Big[\sum^N V(\tilde{u}_i) + \sum_{i \neq j} C(\tilde{u}_i,\tilde{u}_j)\Big] = (1/N^2)\Big[NV(\tilde{u}_i) + (N^2-N)C(\tilde{u}_i,u_j)\Big]$$

$$= (1/N)\Big[V(\tilde{u}_i) - C(\tilde{u}_i,\tilde{u}_j)\Big] + C(\tilde{u}_i,\tilde{u}_j),\ \text{where } i \neq j,$$

$$(17)\quad C(\tilde{R},\tilde{U}) = C(\tilde{R}, \frac{1}{N}\sum^N u_i) = \frac{1}{N} C(\tilde{R},\sum^N u_i) = \frac{1}{N}\sum^N C(\tilde{R},\tilde{u}_i) = C(\tilde{R},\tilde{u}_i)$$

In the preceding, $V(\tilde{u}_i)$ and $C(\tilde{u}_i,\tilde{u}_j)$ indicate the variance and covariance of individual exposures Equation (15) then becomes

$$(18)\quad V(\tilde{R}_s) = K^2V(\tilde{R}) + 2kKC(\tilde{R}\ \tilde{u}_i) + k^2C(u_i,\tilde{u}_j) + \frac{k^2}{N}\Big[V(\tilde{u}_i) - C(\tilde{u}_i,\tilde{u}_j)\Big]$$

$$(19)\quad V(\tilde{R}_s) = K^2V(\tilde{R}) + 2kKC(\tilde{R},\tilde{u}_i) + k^2C(u_i,\tilde{u}_j)\ \text{as } N \to \infty$$

The variance given by (19) is called the <u>systematic risk</u> It cannot be reduced by increasing the number of exposures The last term of (18), however, can be reduced through the law of large numbers Thus a component of the underwriting risk can be reduced by diversification And if the covariance terms are zero, the underwriting risk $V(\tilde{u}_i)/N$ can be diversified to the point where the only return-on-surplus risk is the levered asset portfolio risk

Unfortunately the covariances for many lines of insurance are not negligible For example, over the long run we expect the amount of losses to be correlated with general price levels, which in turn are affected by stock market trends Thus underwriting gains will be related to the return on the insurer's assets, and consequently will be correlated among lines The following example will help illustrate the systematic risk concept

Suppose there are two possible outcomes for $\tilde{R}$ $R + b$ and $R - b$, each with probability ½ Assume that the distribution of $\tilde{u}_i$ outcomes is conditional upon the value of $\tilde{R}$ which occurs, according to

Value of R	$\text{Prob}\{\tilde{u}_i = u + a\}$	$\text{Prob}\{\tilde{u}_i = u - a\}$
$R + b$	p	$1 - p$
$R - b$	$1 - p$	p

where $0 \leq p \leq 1$

From the preceding, we get $E(\tilde{R}) = R$, $E(\tilde{u}_i) = u$, $V(\tilde{R}) = b^2$, $V(u_i) = a^2$, $C(\tilde{R},\tilde{u}_i) = (2p - 1)ab$ and $C(\tilde{u}_i,\tilde{u}_j) = (2p - 1)^2a^2$ Notice that if $p = 0$, $\tilde{R}$ and $\tilde{u}_i$, $\tilde{u}_i$ and $\tilde{u}_j$ are perfectly negatively correlated, if $p = ½$ they are uncorrelated and if $p = 1$ they are perfectly positively correlated

Let $u = -.05$, $a = 1$, $R = .08$, $b = .02$, $k = 2$ and $v = 1$. This implies $R_s = .14$. The following table provides values of $\sigma_s = \sqrt{V(\tilde{R}_s)}$, the standard deviation of return on surplus, for various values of N and p:

	p						
N	0	.20	.40	.50	.60	.80	1
1	1.94	1.96	1.99	2.00	2.01	2.04	2.06
10	1.94	1.25	.71	.64	.77	1.36	2.06
100	1.94	1.15	.39	.21	.50	1.27	2.06
1000	1.94	1.14	.34	.09	.46	1.26	2.06
∞	1.94	1.14	.34	.06	.46	1.26	2.06

Notice that when there is perfect (positive or negative) correlation, increasing exposure will not reduce surplus risk, since in effect the insurer has only a <u>single exposure</u> because all gains and losses move together. Similarly, the surplus risk is minimum when there is no correlation between exposures and asset returns. Here the unsystematic risk diminishes rapidly with increasing exposure until the only remaining risk is due to the levered asset portfolio.

Optimal Asset and Underwriting Portfolios

The preceding section has shown how there is a limit to risk reduction through exposure increases. However, it may be possible to reduce risk (or increase return) by altering the composition of both the asset and underwriting portfolios. Let there be M lines of insurance (or individual exposures) with underwriting gains denoted by $\tilde{U}_i$ and characteristic reserve/premium ratios v_i. Let x_i be the proportion of total premium allocated to line i, so that $\sum_{i=1}^{M} x_i = 1$ and $0 \le x_i \le 1$. Similarly, let there be N assets invested, with returns represented by $\tilde{R}_j$, and y_j being the proportion of total assets being held in asset j, such that $\sum_{j=1}^{N} y_j = 1$ and $0 \le y_j \le 1$. Thus $K = 1 + k \sum_{i=1}^{M} x_i v_i$, $\tilde{U} = \sum_{i=1}^{M} x_i \tilde{U}_i$ and $\tilde{R} = \sum_{j=1}^{N} y_j \tilde{R}_j$, from which we have

(20) $$R_s = (1 + k \sum_{i=1}^{M} x_i v_i)(\sum_{j=1}^{N} y_j R_j) + k \sum_{i=1}^{M} x_i U_i$$

(21) $$V(\tilde{R}_s) = \sigma_s^2 = K^2 \sigma_R^2 + k^2 \sigma_U^2 + 2kK\sigma_{UR}, \text{ where}$$

$$\sigma_R^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} y_i y_j C(\tilde{R}_i, \tilde{R}_j),\ \sigma_U^2 = \sum_{i=1}^{M} \sum_{j=1}^{M} x_i x_j C(\tilde{U}_i, \tilde{U}_j),$$

$$\sigma_{UR} = \sum_{i=1}^{M} \sum_{j=1}^{N} x_i y_j C(\tilde{U}_i, \tilde{R}_j), \text{ and K as above.}$$

From (21) it is clear that the variance of return on surplus contains weighted covariances between the returns of all assets and all underwriting lines. However, for each feasible value of R_s there will be a single value of σ_s which is minimum, and hence optimal. The graph of these optimal values in the $R_s - \sigma_s$ plane forms an efficient frontier for the joint asset-underwriting portfolio. Note that this optimization problem is an extension of the classic investment portfolio problem of merely determining the composition of the asset portfolio; i.e., to optimize (R, σ_R).

Other authors have tackled the asset/underwriting optimization problem. Ferrarri [13] attempted to optimize only the (U, σ_U) choice, Kehane and Nye [16] tried to solve the joint problem, but ignored the reserve/premium levers $x_i v_i$ and their consequent interactions with the asset return mixture $y_j R_j$. Bachman and Lang [10] set up a two-asset, two-line problem with appropriate levers, but did not find the efficient frontier, and concentrated instead upon ruin probabilities.

The solution to the classic investment portfolio problem usually involves the techniques of Lagrangian multipliers and quadratic programming. Appendix I outlines a similar procedure for the more complex joint asset/underwriting problem.

It should be emphasized that the optimal (R_s, σ_s) pairs cannot generally be found by <u>separate determination</u> of (R, σ_R) and (U, σ_U) optimal sets. The following example will bear this out.

Suppose an insurer may choose between two assets and two lines with only four possible certain outcomes for the combination $(\tilde{R}, \tilde{U})$ depending upon the choice of asset and line:

Asset	Line	$\tilde{R}$	$\tilde{U}$
1	1	.05	.02
1	2	.04	.04
2	1	.06	0
2	2	.05	.02

In this case the optimal asset portfolio will contain all of Asset 2 since its return is in all cases superior to that of Asset 1. Similarly the optimal underwriting portfolio will consist of all Line 2.

But now let $k = 2$, with $v_1 = 1$ and $v_2 = 0$. The four possible $(\tilde{R},\tilde{U})$ combinations produce the following certain returns on surplus:

Asset	Line	$(1 + kv)\tilde{R} + k\tilde{U}$	$= \tilde{R}_s$
1	1	$(1 + 1\cdot 2).05 + 2(.02)$	$= .19$
1	2	$(1 + 0\cdot 2).04 + 2(.04)$	$= .12$
2	1	$(1 + 1\cdot 2).06 + 2(0)$	$= .18$
2	2	$(1 + 0\cdot 2).05 + 2(.02)$	$= .09$

Here the optimal asset/underwriting combination consists of all asset 1 and Line 1! The mixture of all Asset 2 and Line 2 in fact produces the worst return.

In the preceding illustration there is perfect correlation between $\tilde{R}$ and $\tilde{U}$, and the return $\tilde{R}_s$ is certain. Let us turn to another example where $\tilde{R}$ and $\tilde{U}$ are uncorrelated, and $\tilde{R}_s$ is random:

Let $U_1 = 0$, $v_1 = 1$, $V(\tilde{U}_1) = 0$, $U_2 = .02$, $v_2 = 0$, $V(\tilde{U}_2) = (.02)^2$,
$R_1 = .04$, $V(\tilde{R}_1) = 0$, $R_2 = .08$, $V(\tilde{R}_2) = (.04)^2$, and $k = 2$.

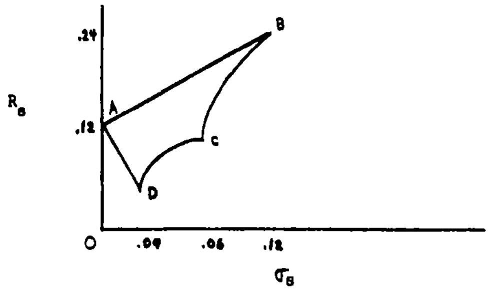
With x denoting the proportion of Line 1 (1-x for Line 2) and y the proportion of Asset 1 used, the above values produce, using equations (20) and (21):

(22) $R_s = .12 + .12x - .04y - .08xy$

(23) $\sigma_s^2 = .0016\left[(1-y)^2(1 + 2x)^2 + (1 - x)^2\right]$

Figure 2 depicts the feasible set of (R_s, σ_s) points obtained by letting x and y vary between 0 and 1:

<u>Figure 2</u>



The feasible set is bounded by <u>ABCDA</u>, with efficient frontier <u>AB</u> formed by $R_s = .12 + \sigma_s$, which occurs when x = 1. Point A represents (x = 1, y = 1) and point B represents (x = 1, y = 0), so the optimal combination of assets/lines has all of Line 1 and any mixture of Assets 1 and 2, the exact amount depending upon the insurer's utility preference.

From the values given in this example, the insurer might separately choose all of Line 2, since it has a higher expected underwriting return (but with a higher risk), and all of Asset 2, also with a higher return. Now because the insurer prefers expected return despite the corresponding risk, his optimal point along <u>AB</u> might be B, which uses all of Line 1 and all of Asset 2. Again, this example shows that separate optimal asset and underwriting portfolios will not necessarily produce the optimal risk-return combination for the return to surplus.

The cause of this apparent inconsistency is due to the v_i levers. If the reserve/premium ratio is large enough, an underwriting line can produce a superior return on surplus, through investment income, even if the underwriting gain is highly negative. And, as will be shown in the next section, the competitive underwriting return in an efficient market cannot be positive unless there is a correlation between the underwriting gain and the asset (stock) market as a whole.

Risk and Return in an Efficient Market

The preceding analysis has demonstrated how the individual property-casualty firm can optimize its own risk-return situation. What happens when all such firms, driven by the preferences of shareholders (policyholders for mutuals), try to optimize their asset/underwriting mixtures?

In financial theory, the equilibrium behavior of asset returns can be analyzed through the concept of an efficient market. The classical assumptions underlying this theory include (1) assets are infinitely divisible (2) information is free and available to everyone (3) there are no transactions costs (4) everyone pays the same price for an asset (5) no individual or firm can affect the price or availability of assets (6) the price of an asset fully reflects all of the available information concerning the asset.

Using the further assumption of the ability to borrow and lend any amount at a riskless rate of interest, we can determine that the optimal portfolio from an individual investor's viewpoint must be a linear combination of the riskless asset and the market portfolio of all assets (usually taken to mean the stock market). Any other portfolio would be suboptimal, since by adding other assets, more unsystematic risk can be diversified away. The risk-return relationship for such an efficient portfolio is given by

(24) $R_M = R_f + r_e \sigma_M$,

where R_M is the expected rate of return on the market portfolio, R_f is the riskless interest rate, r_e is the price of risk reduction for efficient portfolios, and σ_M is the standard deviation of the market portfolio rate of return.

From this relationship we can determine the risk-return relationships for individual securities which comprise the market portfolio:

$$(25)\qquad R_i = R_f + \frac{(R_M - R_f)\ C(\tilde{R}_i,\tilde{R}_M)}{\sigma_M^2}\ ,$$

where R_i is the expected rate of return on asset i. This relationship is called the capital asset pricing model.

The notion of covariance with the market rate of return lacks intuitive appeal, so if we define $b_i = C(\tilde{R}_i,\tilde{R}_M)/\sigma_M^2$ we get a measure of the volatility of the asset rate of return with respect to that of the market. In other words, each 1% change in the market rate will produce a b_i x 1% change in the expected return for asset i. The equilibrium expected rate of return for asset i now becomes

$$(26)\qquad R_i = R_f + b_i(R_m - R_f).$$

Now in an efficient market, the securities of property-casualty insurers will produce returns according to (26), since this relation holds for all firms. For an insurer, the relevant return is the gain on surplus, or R_s. Hence

(27) $R_s = R_f + b_s(R_m - R_f)$,

where $b_s = C(\tilde{R}_s, \tilde{R}_m)/\sigma_m^2$. Consequently

(28) $b_s = C(K\tilde{R} + k\tilde{U}, \tilde{R}_m)/\sigma_m^2 = Kb_R + kb_u$,

with $b_R = C(\tilde{R}, \tilde{R}_m)/\sigma_m^2$ and $b_u = C(\tilde{U}, \tilde{R}_m)/\sigma_m^2$.

Because the insurer's asset portfolio must also be in capital market equilibrium,

(29) $R = R_f + b_R(R_m - R_f)$.

Now we can develop the equilibrium expected underwriting gain. From the fundamental $R_s = KR + kU$, we have

(30) $kU = R_s - KR$

$$= R_s - \left[KR_f + b_s(R_m - R_f) - kb_u(R_m - R_f) \right],$$

from (28) and (29). Thus

(31) $kU = R_s - KR_f - b_s(R_m - R_f) + kb_u(R_m - R_f)$

$$= (1 - K)R_f + kb_u(R_m - R_f)$$

from (27). Finally, since $K = 1 + kv$,

(32) $U = -vR_f + b_u(R_m - R_f)$.

This important result shows that the equilibrium underwriting return is independent of both the insurer's asset portfolio composition and the level of premium written. In other words, U is not affected by R and k. And when $b_u = 0$, or the underwriting gain is uncorrelated with market returns, we have $U = -vR_f$, a negative value. In an efficient market this must be true, because if $U = -vR_f + c$, with $c > 0$, and the insurer holds all of its assets in a riskless security (e.g., Treasury bills) then

$$(33) \quad R_s = KR_f + k(-vR_f + c) = (1 + kv)R_f - kvR_f + kc = R_f + kc.$$

But this insurer has no systematic risk, and therefore its expected rate of return should equal the riskless rate R_f. Consequently, the market will force an adjustment through the underwriting margin U, driving it back down to $-vR_f$. In an efficient capital market there is no price for unsystematic risk, since an investor can merely diversify his holdings until it disappears.

A numerical example will help explain the previous development. Suppose we have a one-line insurer with $v = 1$, $k = 2$, and whose underwriting systematic risk is characterized by $b_u = .5$. In other words, a 10% change in the market rate of return R_m will on the average be accompanied by a 5% change in the underwriting return U. Further assume that the insurer's asset portfolio has $b_R = 1.5$. If the market rate of return is $R_m = .10$ and the riskless rate is $R_f = .05$, then the expected return on the asset portfolio is $R = .05 + 1.5(.10 - .05) = .125$. From (28) the total risk b_s is $3(1.5) + 2(.5) = 5.5$. This is a rather high degree of systematic risk whose price is determined by (27) as $R_s = .05 + 5.5(.05) = .325$, which also equals $KR + kU = 3(.125) + 2U$, yielding $U = -.025$. This value is also found from (32).

This insurer has a highly levered surplus risk due to k, b_R, and b_u. If the firm wishes to reduce its systematic risk b_s to a lower value, say $b_s = 2.5$, it can change k to .5 by reducing premium volume, or it can lower b_R to .5 by adjusting the investment portfolio. By switching its underwriting line to one with $b_u = -1$ (if such a line could be found) the insurer would also reduce b_s to 2.5, keeping $b_R = 1.5$ and $k = 2$. And, of course, a combination of all three methods could be used.

Although the capital asset pricing model, which produces the preceding results, is based upon rather strong assumptions regarding capital market behavior, its implications have held up quite well according to empirical evidence. For details, see Jensen [7], Fama [6] or the other references dealing with market efficiency. From these results we can expect that returns to property-casualty insurers behave correspondingly.

Applications

The preceding sections have examined the concepts of risk and return as applied to the special case of the property-casualty insurer (actually the model describes all non-life firms). The results have important implications for several different areas of application, as briefly outlined below:

1) Ruin Theory: The traditional treatment of ruin theory deals solely with variation due to claims, or in other words, underwriting gain/loss. However, this is only a part of the total risk to the insurer, since the firm is concerned with the possibility that surplus will be depleted. This implies that we would like to know the probability distribution of $\tilde{R}_s$, because when $\tilde{R}_s \leq -1$, the insurer becomes insolvent. Classical ruin theory considers only one component of surplus risk, underwriting gain, while ignoring the investment risk and the risk arising from covariance between investment and underwriting returns. For a large insurer, the systematic components of surplus risk may dominate the variation due to random claim fluctuations.

Another component of surplus risk is due to the uncertainty of loss reserves. The risk-return model developed in this paper has assumed implicitly that the loss reserve equals the actual value of unpaid losses. Appendix II shows how, when loss reserves equal the expected value of unpaid losses (which may fluctuate randomly), the variance of surplus return is increased. The notion of uncertain loss reserves is of great practical importance, and should be explored more fully in the context of the surplus risk-return model.

In any case, more attention should be focused upon the probability of insurer insolvency arising from sources other than pure claim fluctuations.

2) Product Pricing: Property-casualty insurers have traditionally ignored economic principles in an attempt to establish so-called "fair" underwriting profit margins. Over the long run, these profit margins can only be determined by competitive forces. Even in a less-than-efficient market, the prices of insurance products will be governed by their contribution to the overall risk of the insurer. Relationships such as (32) could be used as guidelines for setting profit margins, with more efficient insurers able to produce margins greater than that of the industry.

As shown in the analysis developed in this paper, investment income is an integral part of the total return to surplus, and through the reserve leverage, is closely related to underwriting gain. This interrelationship should be carefully considered when establishing a product's price.

3) Marketing: Portfolio theory, as extended to the joint asset/underwriting case, should prove useful to the insurer wishing to take advantage of a diversified mixture of business. By properly balancing the asset and underwriting portfolios, the insurer can produce an optimal surplus risk-return combination.

New products should be evaluated not only according to the return which they generate, but also by the risk which they create.

4) Reinsurance: The primary purpose of reinsurance is the reduction of risk. Since the relevant risk to the insurance firm is the surplus risk, we should be concerned with the cost of its reduction. Quota-share and other forms of pro-rata reinsurance basically reduce the premium/surplus ratio k and provide a proportional reduction in the insurer's systematic risk, without changing the other parameters of the insurance operation (v, U, R).

Stop-loss reinsurance, on the other hand, primarily affects the underwriting risk, since individual losses are truncated. Essentially, the variance of underwriting gain is reduced, while the net expected underwriting gain drops according to the reinsurer's own risk charge.

In terms of our property-casualty risk-return model, the use of reinsurance provides additional means of reducing surplus risk. Depending upon the cost and the nature of the risk reduction, the insurer may find reinsurance an appealing alternative to other means such as premium reduction, portfolio adjustments, or marketing realignments.

5) Regulation: One major area of insurance regulation concerns insurer solvency. This is the reason for current premium/surplus ratio requirements. However, these ratios should vary by line of insurance. In an efficient market the regulator would want monoline firms with different lines to have the same surplus risk

if their asset portfolios are identical. This implies that $k_1/k_2 = (v_2b_R + b_2)/(v_1b_R + b_1)$ where the subscripts distinguish the two companies, with b_1 and b_2 denoting the systematic underwriting risk for each insurer. Since b_R is a matter of choice for the insurer, the regulator can establish a standard, such as $b_m = 1$, the market systematic risk. Therefore the relative premium/surplus ratios by line of insurance can be determined as a function of the line characteristics v and b_u.

Another area of regulatory concern is rate fairness. Again, the basic result of Equation (32) can be used as a guideline for establishing competitive underwriting profit margins.

Summary

The usual treatment of return to property-casualty insurers ignores the risk inherent in their operating structure, or in some instances, considers only the risk of underwriting return. This paper has demonstrated, using simple models of the insurance firm and basic economic assumptions, that the risk of total surplus return is an important dimension, and should be considered jointly with the insurer's expected value of return.

The results presented here are, of course, dependent upon the validity of the underlying assumptions, and should not necessarily be taken as truth. Since the strength of a model or theory lies in its ability to explain and predict behavior, much empirical work must be done in order to accurately apply the model results presented here. We eagerly await these future efforts.

Appendix I: The Joint Asset/Underwriting Portfolio Optimization Problem

Using Lagrangian multipliers, the basic problem becomes

(I-1) minimize $Z = -\lambda R + \sigma_s^2 + \lambda_1 \left(1 - \sum_{i=1}^{M} x_i\right) + \lambda_2\left(1 - \sum_{j=1}^{N} y_j\right)$

under constraints $\sum_{i=1}^{M} x_i = 1$, $\sum_{j=1}^{N} y_j = 1$ and

(I-2) $A_i \leq x_i \leq B_i$ for $i = 1,\ldots,M$ and $C_j \leq y_j \leq D_j$ for $j = 1,\ldots,N$.

We now find the following partial derivatives ($\partial K/\partial x_i = kv_i$) from equations (20) and (21):

$$\text{(I-3)}\quad \partial\sigma_s^2/\partial x_i = 2k^2 \sum_{j=1}^{M} x_j C(\tilde{U}_i,\tilde{U}_j) + 2kKv_i \sum_{h=1}^{N}\sum_{j=1}^{N} y_h y_j C(\tilde{R}_i,\tilde{R}_j)$$

$$+ 2kK \sum_{j=1}^{N} x_i y_j C(\tilde{U}_i,\tilde{R}_j) + 2k^2 v_i \sum_{h=1}^{M} \sum_{j=1}^{N} x_h y_j C(\tilde{U}_i,\tilde{R}_j)$$

$$\text{(I-4)}\quad \partial\sigma_s^2 / \partial y_j = 2K^2 \sum_{i=1}^{N} y_i C(\tilde{R}_i,\tilde{R}_j) + 2kK \sum_{i=1}^{M} x_i C(\tilde{U}_i,\tilde{R}_j)$$

$$\text{(I-5)}\quad \partial R_s/\partial x_i = kU_i + kv_i \sum_{j=1}^{N} y_j R_j$$

(I-6) $\partial R_s/\partial y_j = KR_j$ Thus

(I-7) $\partial Z/\partial x_i = -\lambda(\partial R_s/\partial x_i) + (\partial\sigma_s^2/\partial x_i) - \lambda_1 = 0$ for $i = 1,\ldots,M$

(I-8) $\partial Z/\partial y_j = -\lambda(\partial R_s/\partial y_j) + (\partial\sigma_s^2/\partial y_j) - \lambda_2 = 0$ for $j = 1,\ldots,N$

These M + N non-linear equations must now be solved subject to the M + N inequality constraints in (I-2).

Appendix II: <u>Effect of Uncertain Loss Reserve on Surplus Risk</u>

Assume that claims can only be paid in the current and succeeding periods. The loss reserve LR_t will therefore equal the paid losses in (t, t+1), which we treat as a random variable. In other words, $\tilde{LR}_t = \tilde{PL}(t)$. Incurred losses and investment return are also random. Premium is collected at the beginning of the period and losses are paid at the end of the period. Hence we get $C(t) = W(t)$ and $UR_t = 0$. Therefore

$$\text{(II-1)} \quad \tilde{I}(t) = \tilde{R}\left[A_t + W(t)\right] \quad \text{and} \quad \tilde{U}(t) = E(t) - \tilde{IL}(t)$$

$$\text{(II-2)} \quad \tilde{S}_{t+1} = \tilde{S}_t + \tilde{I}(t) + \tilde{U}(t) = A_t - \tilde{LR}_t + \tilde{I}(t) - \tilde{U}(t),$$

since the beginning assets A_t are fixed. We then have

$$\text{(II-3)} \quad V(\tilde{S}_{t+1}) = V(\tilde{LR}_t) + 2C(\tilde{LR}_t,\tilde{I}(t)) + 2C(\tilde{LR}_t,\tilde{U}(t)) + V\left[\tilde{I}(t) + \tilde{U}(t)\right]$$

If the loss reserve is known to equal paid losses, however, we get

$$\text{(II-4)} \quad V(\tilde{S}_{t+1}) = V\left[\tilde{I}(t) + \tilde{U}(t)\right].$$

Consequently the variance of surplus gain will be greater with the loss reserve as a random variable, provided that $V(\tilde{LR}_t)$ plus the covariance terms in (II-3) exceeds zero. In most cases this will be true, especially if there is no correlation between LR_t and $I(t)$ or $U(t)$.

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