Upgrading an Existing Capital Model – A Common Risk Driver Application

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Abstract: In this paper we apply a simple regression model to link performance of a D&O insurance line of business to the S&P 500 economic variable from an economic scenario generator (ESG). The regression structure is incorporated into an existing economic capital model. The distribution of the error term is constrained so that the final distribution of the D&O line is equivalent to the distribution previously used. We explore the impact this model change has on the existing correlation structures.

Keywords: ERM, regression, correlation, risk drivers

INTRODUCTION

The literature describes the benefits of using common risk drivers compared to the use of correlation matrices for inducing dependency relationships among risks in economic capital models. However, there is little guidance on how to calibrate the risk drivers, and still less guidance on how to introduce such linkage into an existing Economic Capital Model (ECM).

In this paper we develop a mathematical relationship between an economic variable and a line of business (LOB) in a company capital model. We then show a method of implementing that relationship, with a process that minimizes the impact on the existing LOB distribution while inducing correlation, as desired, between economic risk and the insurance risk.

The motivation for this paper is twofold. First, this is an interesting application of common risk drivers. Second, the process we demonstrate can be used to incorporate informative external variables or other sources into an existing economic capital model, with minimum disruption to the company's existing ECM.

We provide a linked illustrative Excel workbook that shows our calculations and the Tables and Figures we present in this paper.

EXISTING CAPITAL MODEL FRAMEWORK

We address the following situation:

- The company has an ECM in place. Quarterly updates and reporting are established practices.
- Individual insurance LOBs are represented by defined risk distributions in the ECM.
- Investment risk is modeled using an economic scenario generator (ESG).
- A correlation matrix represents relationships between LOBs.
- There is no explicit correlation in the model between economic risk and insurance risk variables.

Historical results, industry research and management judgement all indicate that the risk distribution for the D&O LOB is influenced by economic conditions; in this case we represent this with a broad stock index.

The company would like to introduce an explicit relationship between the D&O LOB risk distribution and economic variables *without* changing the overall D&O LOB risk distribution[1]. To illustrate our approach, we use the S&P index as the economic variable.[2]

Expert Judgment Framework

The company experts believe, with support in recent historical data and anecdotal evidence, that D&O results are influenced by economic conditions. These economic conditions include stock market movements. The company has two prior expectations:

- Prior Expectation #1 (PE1): If the stock market performs worse than expected over the projected period, then underwriting (UW) results will be worse than planned.
- Prior Expectation #2 (PE2): If the stock market performs better than expected over the projected period, then, to a lesser degree than is the case in Prior

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[1] We note that while the D&O LOB risk distribution does not change, the total company risk distribution will change due to the introduction of correlation between investments and insurance risk. Moreover, changes to the D&O LOB risk distribution will change the effect of correlations between D&O and other LOBs, absent offsetting changes.

[2] In Appendix 1 we discuss some of the alternatives we considered.

Expectation #1, the UW results will also be worse than planned.[3]

PE1 can be implemented with a linear relationship between the S&P error distribution and the loss ratio error distribution.

Combining PE1 and PE2 requires a more complicated relationship between the S&P error and the loss ratio error. That relationship exhibits a "turning point" from which both positive and negative deviations of the S&P index from expected produce increases in the LR above plan. In our example we use a quadratic relationship as a reasonably simple form meeting that requirement.

Statistical Framework

The table below shows notation we use in this paper. Variables with double dots, e.g. $\ddot{x}$, refer to historical data. Unmodified variable names, e.g. x, refer to values from distributions.

Examples of Notation used in this document

x_{err}	The distribution around an error variable one year in the future.
$x_{err,i}$	A simulated observation from x_{err}
$\ddot{x}_{err}$	The observed distribution of the historical prediction errors of X
$\ddot{x}_t$	A historical observation of the variable used to calculate historical errors.
$\ddot{x}_{err,t}$	A historical observation of the error around the predicted historical variable.
k	Number of observations from historical dataset. k = Max(year) – Min(year) + 1
N	Number of simulations run in model
E(X)	Expected value of X
SD(X)	Standard deviation of X
$F_X(x)$	Cumulative distribution function of X; i.e. Probability that X is less than x.

Definition of variables

We apply that framework to the variables of interest in our work as follows:

x-based variables refer to the explanatory variable – in this paper, the S&P index.

y-based variables refer to the predicted variable – in this paper, the loss ratio.

[3] PE1 has the obvious interpretation. PE2 is related to increased M&A activity or increased risk-taking activity, including M&A activity. Combined, the two Prior Expectations imply that predictable economic conditions produce the best UW results. Note also that 'higher' and 'lower' economic conditions are not the same as 'up/down', but rather are whether the trends in the market continue in the manner that are predicted when the planned LR is selected.

S&P Variables

The x-based variables we use in the paper are as follows:

$\ddot{x}_t$ = S&P Index at the end of year t

$$\ddot{x}_{err,t} = \frac{\text{S\&P Index at the end of year } t}{E(\text{S\&P Index at the end of year } t)}$$

For example, if at 12/31/2012 the expected value of the S&P index one year in the future, 12/31/2013, was 1,400, while the actual index value was 1,479, then

$\ddot{x}_{err,2013}$= 1,479/1,400 = 1.056.

In a company setting, the expected value of the S&P index would be obtained from an ESG. In this report we use the following simplified forecasting approach:

$$E(\text{S\&P Index at the end of year } t) = \left(\ddot{x}_{t-1} + d_{t-1}\right) * (1 + r_{t-1}),$$

where,

d_t = dividend in year t

r_t = 1yr Treasury yield at end of year t

and,

$\ddot{x}_{err}$=Observed distribution of historical S&P prediction errors

x_{err} is the distribution of the error around the predicted level of the S&P index.
Error in this paper is defined as the ratio of actual to expected.

$$x_{err,i} = \frac{\text{S\&P Index simulation i}}{Avg(\text{S\&P Index simulations})}$$

Loss Ratio Variables

The y-based loss ratio variables we use in the paper have analogous definitions, as follows:

$\ddot{y}_t$ represents the observations of historical loss ratios.

$$\ddot{y}_{err,t} = \frac{\text{Historical Loss Ratio for year } t}{\text{Planned Loss Ratio for year } t}$$

The historical loss ratio observations we use in this analysis are Schedule P industry accident year loss ratios for the Other Liability – Claims Made statutory line of business at the latest available maturity, up to 120 months developed for the most mature data points.

The planned loss ratio is the accident year loss ratio at 12 months; used as a proxy for the planned loss ratio.

$\ddot{y}_{err}$=Observed distribution of historical Loss Ratio prediction errors

y_{err} is the distribution of the error around the predicted ultimate loss ratio for the line of business, that is, the ultimate loss ratio for accident year 2017.

Error in this paper is defined as the ratio of actual to expected, so

$$y_{err,i} = \frac{\text{Loss Ratio simulation i}}{Avg(\text{Loss Ratio simulations})}$$

Original and Revised Loss Ratio Error Distribution

At this point we must introduce some notation to distinguish between our original loss ratio error distribution and the revised one we are producing with this alternative model. The goal is for the two to be as close as possible.

y_{err}^{Orig} has a lognormal distribution LN(1, sigma) and implicitly contains variability related to economic conditions.

In our alternative model:

$y_{err}^{Rev} = yP_{err}^{Rev} * yI_{err}^{Rev}$, where

yP_{err}^{Rev} is the distribution of the loss ratio predicted, based on the S&P index, versus expected loss ratio, which will be defined later as a regression on x_{err}.

yI_{err}^{Rev} is the variability in the loss ratio error that is independent of the S&P Index volatility, or rather, x_{err}. The distribution of yI_{err}^{Rev} reflects the residual (multiplicative basis) in yP_{err}^{Rev} versus y_{err}^{Rev}.

Our goal is for y_{err}^{Rev} to be as close as possible to y_{err}^{Orig}.

If y_{err}^{Orig} and yI_{err}^{Rev} and yP_{err}^{Rev} were all lognormally distributed, then we could determine a closed-form solution for yI_{err}^{Rev}. However, we want more flexibility in the choice of underlying distributions of y_{err}^{Orig} and yP_{err}^{Rev}. Therefore, we take an approach that allows us to select any appropriate regression model to represent the risk of the LOB explained by economic variables and then define the distribution of the error term using a beta distribution for its flexibility.

Initial status

In the current capital model structure, the insurance risk distribution is derived from historical observations of ultimate loss ratios by accident year. There is no explicit assumed relationship between the D&O LOB and the S&P index.

In Table 1, below, the observed S&P Index in column 1 is from public sources. In practice, the predicted S&P Index in column 2 would be the average following year-end S&P index from the Economic Scenario Generator used in the company economic capital model. In this example, the S&P prediction is equal to the sum of the prior year's S&P value and dividend inflated at the 1-year US Treasury rate (see formula above, in S&P Variables subsection).

Table 1
Historical Data

AY	(1)	(2)	(3)	(4)	(5)	(6)
	S&P Index		Loss Ratio (LR)		Observed Error	
	Predicted	**Observed**	**Planned**	**Observed**	$\ddot{x}_{err}$	$\ddot{y}_{err}$
1990	359.7	328.8	60.3	62.4	0.914	1.035
1991	364.1	388.5	70.0	59.1	1.067	0.844
1992	417.2	435.6	71.7	65.7	1.044	0.917
1993	464.2	466.0	73.3	54.5	1.004	0.744
1994	495.9	455.2	76.4	60.1	0.918	0.787
1995	502.1	614.6	76.0	58.5	1.224	0.769
1996	660.9	743.3	73.9	58.0	1.125	0.784
1997	799.9	962.4	72.0	68.1	1.203	0.945
1998	1031.8	1190.1	66.7	86.8	1.153	1.302
1999	1260.9	1428.7	68.4	108.9	1.133	1.592
2000	1531.8	1330.9	71.1	110.2	0.869	1.549
2001	1418.9	1144.9	74.2	105.6	0.807	1.423
2002	1185.9	899.2	68.9	93.7	0.758	1.359
2003	927.3	1080.6	66.1	65.4	1.165	0.989
2004	1111.9	1199.2	65.9	45.4	1.079	0.689
2005	1252.2	1262.1	63.0	45.1	1.008	0.716
2006	1340.5	1416.4	64.3	49.0	1.057	0.763
2007	1513.4	1479.2	67.0	56.2	0.977	0.839
2008	1557.3	877.6	72.6	81.9	0.564	1.128
2009	909.3	1110.4	70.6	77.2	1.221	1.095
2010	1138.1	1241.5	70.3	73.5	1.091	1.045
2011	1267.9	1243.3	71.1	74.1	0.981	1.044
2012	1271.3	1422.3	69.5	74.2	1.119	1.068
2013	1455.9	1807.8	65.9	68.4	1.242	1.038

Mean	1.030	1.019
Std Dev	0.164	0.262

The loss ratio information in columns 3 and 4 is data from industry Schedule P data for the Other Liability (Claims Made) LOB for reporting years 1999-2016. Column 3 is the industry loss ratio developed to 120 months. Column 4 is the Schedule P accident year loss ratio at 12 months; used as a proxy for the planned loss ratio.

Columns 5 and 6 show actual versus expected results as ratios, column 5 = column 2/column 1 and column 6=column 4/column 3.

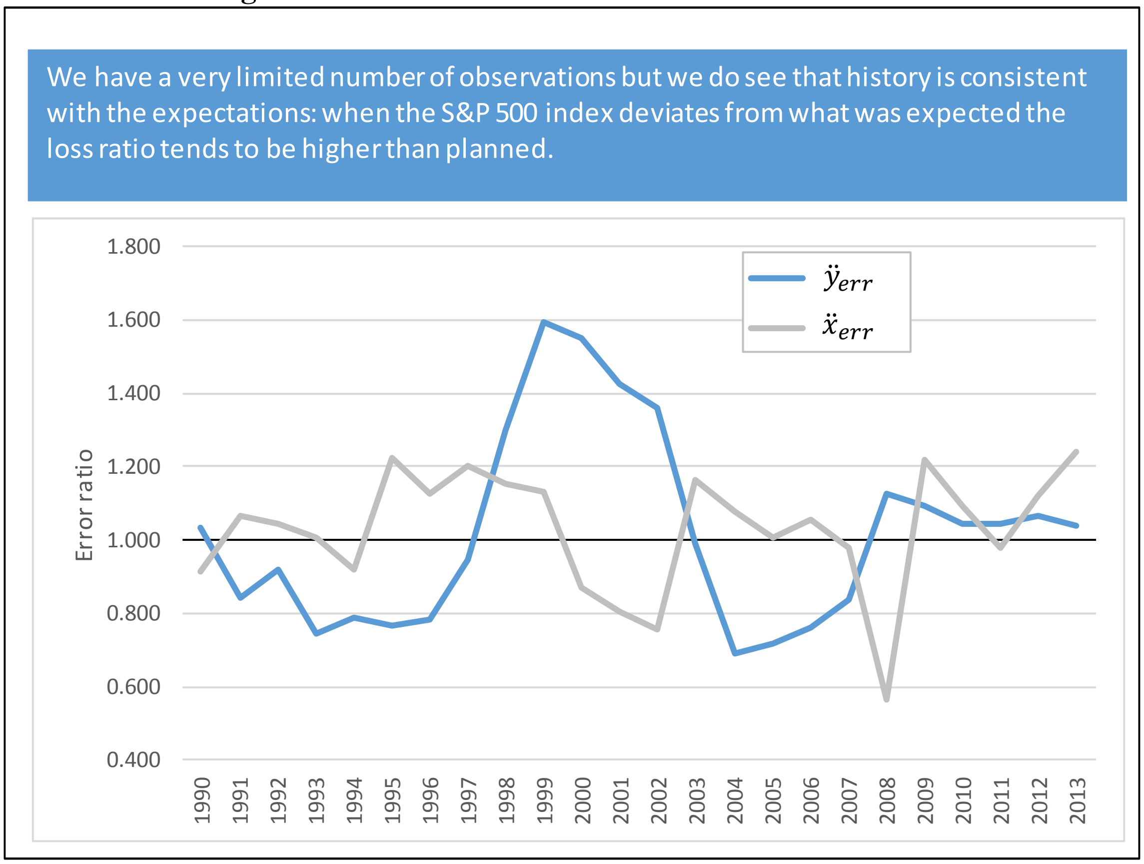
Original Loss Ratio Error Distribution

The historical data for the Loss Ratio error, Table 1 column 6, $\ddot{y}_{err}$, is seen to have a mean of 1.019 and standard deviation 0.262. We used this information to parameterize y_{err}^{Orig}, assuming a lognormal distribution with mean of 1 and standard deviation of 0.262, implies mu and sigma of 0.258 and -0.033.

Predicted Loss Ratio Error Distribution

In Figure 1 below, we compare the observed Loss Ratio error, Table 1 column 6, and the S&P index error, Table 1, column 5. In this Figure, we see that for 2000 and 2001 the S&P error indices ($\ddot{x}$ values) are lower than expected, i.e., below the "1.0" line, and Loss Ratios errors ($\ddot{y}$ values) are higher than expected, i.e., above the "1.0" line. This is consistent with Expectation #1. In general, we see LR errors and S&P index errors are on opposite sides of the "1.0" line, as expected.

Figure 1 – Loss Ratio Error vs S&P Index Error timeline



We examine Expectation #2 in Figure 2, the scatter plot on the following page.

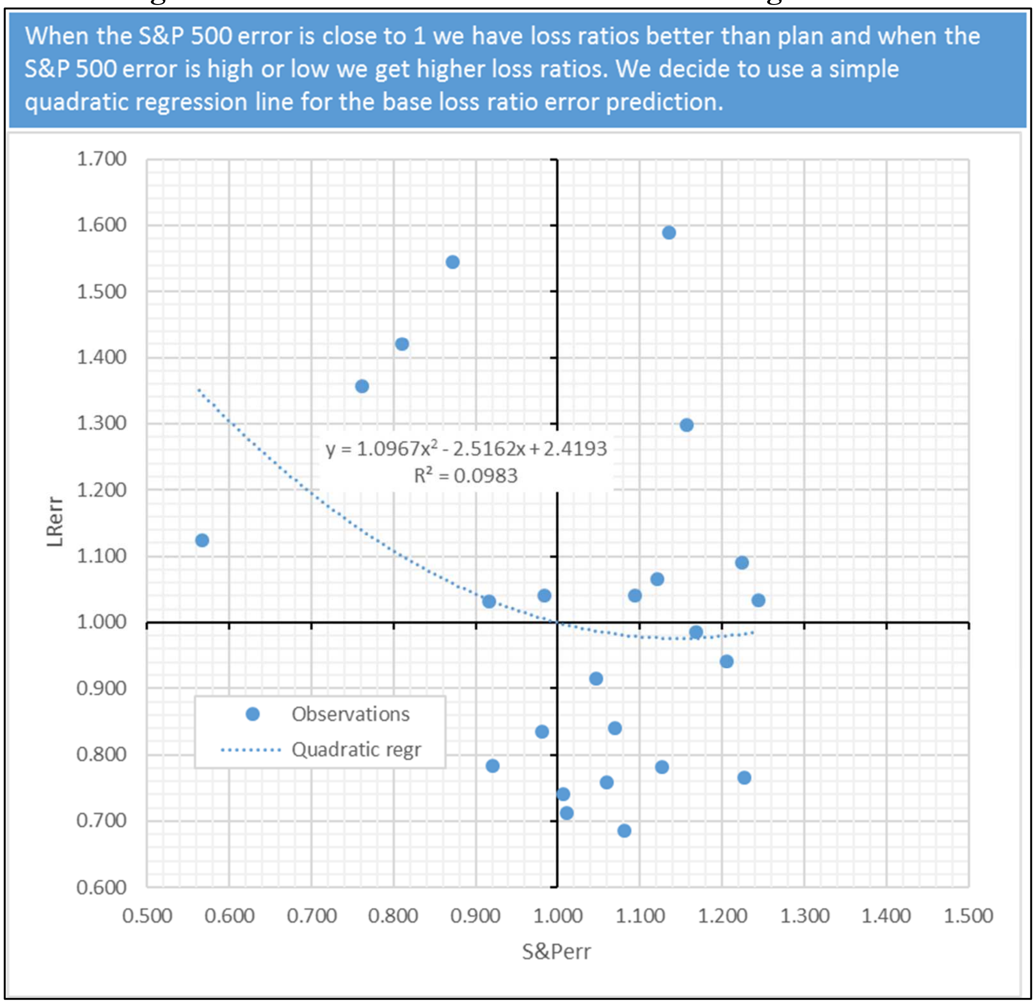
We consider four regions of S&P Error, i.e, S&P compared to expected S&P, as follows: 10% worse than expected ("<0.9); between 10% worse than expected and expected, i.e., 'bad' but not too bad (≥0.9; ≤1.0); between expected and 10% better than expected, i.e., good but not too good (≥1.0; ≤1.1)' and more than 10% better than expected (>1.1). Table 2 below shows the LR error performance in each of those regions

Table 2
LR Error Values by S&P Region

(1)	(2)	(3)	(4)	(5)
S&P Error Range	LR Error			
	<1.0	≥1.0	Total	% ≥1.0
<0.9	0	4	4	0%
≥.9; ≤1	2	2	4	50%
≥1;≤1.1	6	1	7	86%
>1.1	4	5	9	44%

Consistent with Expectation #1, the LR errors become increasingly favorable (0% over 1.0 to 86% over 1.0) in the first three regions. Consistent with Expectation #2, the LR errors become less favorable as the S&P error increases into the fourth region.

Figure 2 - Loss Ratio Error vs S&P Index Error regression



With the concession that historical data is limited, we find that it is consistent with our underwriting Expectation #2. To apply our assumptions of Expectation #1 and Expectation #2, in Figure 2, we fit the data to a quadratic curve[4]:

$$yP_{err}^{Rev} = b_2\left(x_{err}\right)^2 + b_1 x_{err} + b_0 \qquad \text{(Regression coefficients found in Figure 2)}$$

[4] LINEST(LRerr,SPerr^{1,2}).

This fitted curve implies that the most favorable LR variance from expected arises when the S&P index error is about 15%. The expected LR variance becomes less favorable as the S&P index variation becomes more favorable beyond that level.

Fitting Error Distributions

At this stage we have specified yP_{err}^{Rev}, and we can leave this historical sample set to fit the distribution of the error term using simulated data consistent with our capital model. Remember, the goal of the error fit is to produce a final loss ratio error distribution, y_{err}^{Rev}, that very closely matches that of the original, y_{err}^{Orig}.

Table 3 column 2, shows the first 10 of 1,000 simulations of the S&P Index one year from the model valuation date, using the company Economic Scenario Generator. The mean value of the 1,000 simulations for the forecast year, Average (2), is 1,845.2. This is the S&P prediction in our model. Column 3 = Column 2 / Average (2). Then we calculate column 4, yP_{err}^{Rev}, using the quadratic relationship determined in the previous step:

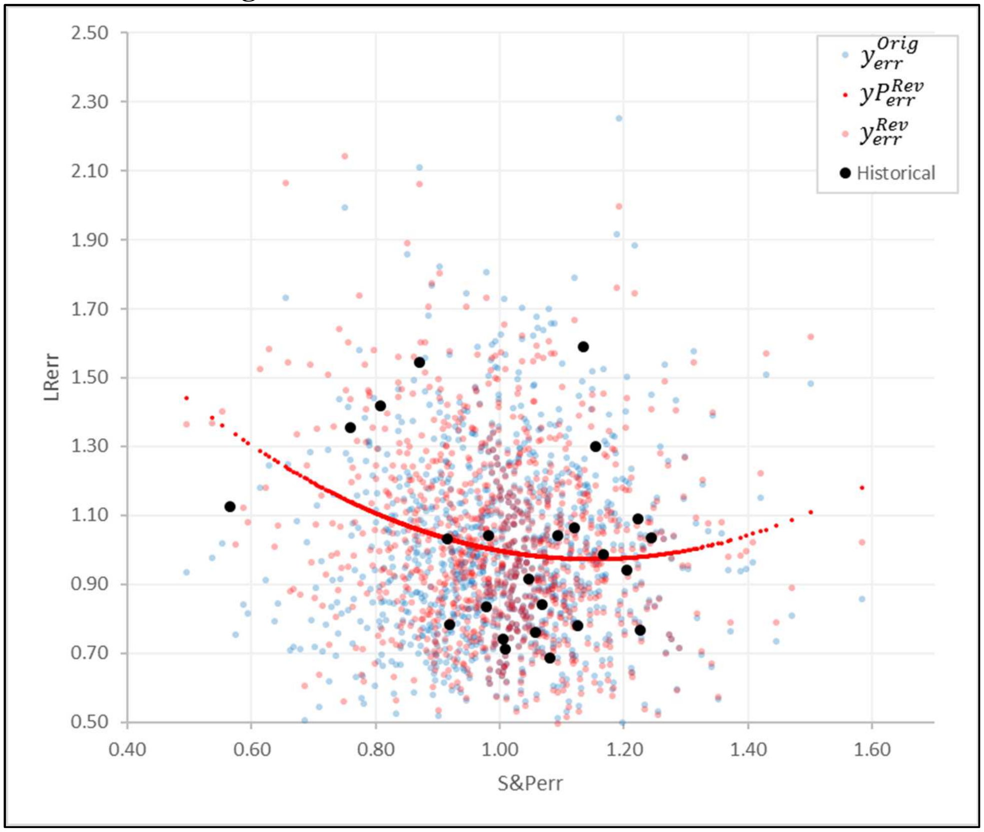
Column (4) = $b_0 + b_1$*(Column 3) + b_2*(Column 3)^2

Table 3
S&P Value from Company ESG and yP_{err}^{Rev} from Quadratic Model
(First 10 of 1000 simulations)

Simulated S&P Index and Error			
(1)	(2)	(3)	(4)
Sim	**S&P 500 Simulation**	x_{err}	yP_{err}^{Rev}
1	1291.6	0.70	1.20
2	1893.0	1.03	0.99
3	1638.8	0.89	1.05
4	2041.1	1.11	0.98
5	1652.2	0.90	1.05
6	1934.4	1.05	0.99
7	2022.5	1.10	0.98
8	1812.5	0.98	1.01
9	1819.4	0.99	1.00
10	1946.3	1.05	0.99

Figure 3, below, shows all the 1,000 simulated data points and the fitted quadratic relationship.

Figure 3 – Loss Ratio error vs S&P error



The next step is to calibrate yI_{err}^{Rev}.

As noted in the statistical framework, we assume that yI_{err}^{Rev} has a beta distribution. We select the beta distribution parameters to minimize the difference between y_{err}^{Rev} and y_{err}^{Orig} at selected percentiles. We do this using Excel Solver.[5]

Table 4 shows the fitted parameters for the beta distribution and the solver constraints used in fitting those parameters. Column 7 contains the solver constraints for the parameters in Column 6. The mean of the beta distribution is constrained to 1 so that the mean of y_{err}^{Rev} equals the mean of y_{err}^{Orig}.

[5] An analytical method to determine the parameters of the beta distribution requires numerical analysis. For simplicity's sake we used the excel solver.

Table 4
Beta Distribution Parameters and Solver Constraints

Final Values for Beta Distribution		
(5)	(6)	(7)
yI_{err}^{Rev} **Beta parameters**		**Solver constraints**
minY	0.21	0.0500
maxY	8.48	10.0000
alpha	8.80	0.1000
beta	83.32	0.1000
E[Impact]	1.000	1.0000
SD[Impact]	0.252	

The solver iteration that produces the values in Table 4 column 6 uses the values from Tables 4 and 5, column 8-14 as follows:

- Column (8) shows random values from a uniform distribution.
- Column (9) shows the observations $yI_{err,i}^{Rev}$, generated from a beta distribution with the parameters in (6) and the random variable values in (8).
- Column (10) = column (9) * $yP_{err,i}^{Rev}$, for i=1 to 1000, from Table 2, column 4. Column 10 is the new modeled y_{err}^{Rev} distribution.

Table 5
(First 10 of 1000 simulations)

Revised y_{err} distribution		
(8)	(9)	(10)
U	yI_{err}^{Rev}	y_{err}^{Rev}
0.1276	0.72	0.86
0.2136	0.79	0.79
0.7166	1.13	1.18
0.6149	1.05	1.03
0.0737	0.66	0.70
0.7246	1.13	1.12
0.7151	1.13	1.10
0.3371	0.88	0.88
0.5454	1.00	1.01
0.6134	1.05	1.03

We continue the calculation as follows:

- Column 11 shows the selected cumulative distribution function percentiles at which we compare y_{err}^{Rev} and the newly calculated y_{err}^{Orig} distribution.
- Column 12 is the value of y_{err}^{Orig} at the cumulative distribution probability level in column 11. In this example, the original model is lognormal, so these are the inverse cumulative lognormal values for the CDF levels in column 11. The method, however, does not require a parametric distribution for column 12.
- Column 13 shows the new $y_{err}^{Rev} = yP_{err}^{Rev} * yI_{err}^{Rev}$.
- Column 14 is the difference between column 12 and column 13, squared. The objective function is the sum of column 14. We determine the beta parameters in column 6 using Excel Solver to minimize the sum of column 14. You should be aware that there are an infinite set of beta parameters that will result in y_{err}^{Rev} fitting our needs and rerunning the solver multiple times will return a different set of parameters. That is, the beta parameters are unstable, but that does not affect the utility of the outcome as, in our tests, the distribution of yI_{err}^{Rev} is stable.

Table 6
Minimizing Differences Between y_{err}^{Orig} and y_{err}^{Rev}

Fitting Beta Parameters			
(11)	(12)	(13)	(14)
Selected CDF levels	y_{err}^{Orig}	y_{err}^{Rev}	error
0.001	0.436	0.436	0.000
0.01	0.531	0.538	0.000
0.05	0.633	0.640	0.000
0.1	0.695	0.711	0.000
0.2	0.778	0.797	0.000
0.3	0.845	0.868	0.001
0.4	0.906	0.930	0.001
0.5	0.967	0.997	0.001
0.6	1.033	1.057	0.001
0.7	1.107	1.137	0.001
0.8	1.202	1.228	0.001
0.9	1.346	1.373	0.001
0.95	1.479	1.499	0.000
0.99	1.763	1.743	0.000
0.995	1.880	1.893	0.000
0.999	2.147	2.145	0.000
			Obj: 0.007

The quality of the fit between y_{err}^{Orig} and the newly constructed y_{err}^{Rev} is good, as evidenced by a comparison of columns 12 and 13.

Conclusions

We believe this is an interesting application of common risk drivers. Moreover, it is a demonstration of a process that can be used to incorporate informative external variables or other sources into an existing economic capital model with minimum disruption to the company's existing ECM.

Supplementary Material

We provide a linked illustrative Excel workbook that shows our calculations and the Tables and Figures we present in this paper.

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Appendix 1

In this Appendix we discuss three further issues:

1. Correlations to other LOBs
2. Alternative Models
3. Sensitivity Testing Extreme Values

Correlation to other LOBs

In the existing capital model, after separating the risk into components, yI_{err}^{Rev}, the component *not* related to the economic variable, is still part of the existing correlation matrix, but yP_{err}^{Rev}, the economic component, is not. Absent other adjustments, the separation reduces the correlation to other LOBs of business in ways that are not desirable.

There are two approaches to retaining the desired correlation between the D&O LOB and other LOBs. The complex approach is to update all LOBs with risk drivers. That may not be practical.

Instead, in practice, we increase the correlation factors in the copula between the D&O LOB, yI_{err}^{Rev}, and the other LOBs. That is, we increase the row/column in the matrix until the measured output correlations were consistent with what they were prior to the model changes.

In that way, we retain the pre-existing LOB correlation but we add explicit correlation between D&O loss ratios and investment risk.

Alternative Models

In this paper we used a quadratic regression model to relate LRs to the S&P Index.

We considered alternative structures and alternative variables that we outline here.

For structure, we chose quadratic rather than linear for the reasons described in the paper, even though, as noted, the data to support that decision in limited. We also considered a kernel function that uses nearest neighbors to estimate values at various simulated points. The results were interesting but given the limited historical data for calibration we chose the quadratic model.

For variables we considered the following:

- Unemployment
- Interest rates

- Changes in corporate bond spreads and yields,
- Change in average CEO salaries
- The number of securities class action lawsuits

We chose to consider only one variable because the historical data for calibration is limited.

For practical reasons, we also choose to consider only variables for which we have simulation forecast values from the ESG. The last two variables are not available in the ESG.

Unemployment, interest rates, corporate spreads and yields, and the S&P 500 index *are* variables simulated by the ESG. Plotting the relationships between the historical loss ratios and the index movements quickly revealed that many different relationships could have been used. None were clearly better than the S&P Index, particularly for use in an illustration such as the one presented in this paper. With more historical experience, it might have been clear that a different model was superior.

Sensitivity Testing Extreme Values

In this paper we did not demonstrate any tests for model validity at extreme S&P error points. That is, what would the predicted loss ratio be if the ESG produces an S&P error of 2.5, or 0.2? In practice these sorts of outcomes need to be tested for or your model may produce embarrassingly impossible outcomes in those extremes.