Upgrading an Existing Capital Model – A Common Risk Driver Application
Predictive Analytics in Capital Modeling Working Party

Abstract: In this paper we apply a simple regression model to link performance of a D&O insurance line of business to the S&P 500 economic variable from an economic scenario generator (ESG). The regression structure is incorporated into an existing economic capital model. The distribution of the error term is constrained so that the final distribution of the D&O line is equivalent to the distribution previously used. We explore the impact this model change has on the existing correlation structures.

Keywords: ERM, regression, correlation, risk drivers

INTRODUCTION

The literature describes the benefits of using common risk drivers compared to the use of correlation matrices for inducing dependency relationships among risks in economic capital models. However, there is little guidance on how to calibrate the risk drivers, and still less guidance on how to introduce such linkage into an existing Economic Capital Model (ECM).

In this paper we develop a mathematical relationship between an economic variable and a line of business (LOB) in a company capital model. We then show a method of implementing that relationship, with a process that minimizes the impact on the existing LOB distribution while inducing correlation, as desired, between economic risk and the insurance risk.

The motivation for this paper is twofold. First, this is an interesting application of common risk drivers. Second, the process we demonstrate can be used to incorporate informative external variables or other sources into an existing economic capital model, with minimum disruption to the company’s existing ECM.

We provide a linked illustrative Excel workbook that shows our calculations and the Tables and Figures we present in this paper.
EXISTING CAPITAL MODEL FRAMEWORK

We address the following situation:

• The company has an ECM in place. Quarterly updates and reporting are established practices.
• Individual insurance LOBs are represented by defined risk distributions in the ECM.
• Investment risk is modeled using an economic scenario generator (ESG).
• A correlation matrix represents relationships between LOBs.
• There is no explicit correlation in the model between economic risk and insurance risk variables.

Historical results, industry research and management judgement all indicate that the risk distribution for the D&O LOB is influenced by economic conditions; in this case we represent this with a broad stock index.

The company would like to introduce an explicit relationship between the D&O LOB risk distribution and economic variables without changing the overall D&O LOB risk distribution. To illustrate our approach, we use the S&P index as the economic variable.

Expert Judgment Framework

The company experts believe, with support in recent historical data and anecdotal evidence, that D&O results are influenced by economic conditions. These economic conditions include stock market movements. The company has two prior expectations:

• Prior Expectation #1 (PE1): If the stock market performs worse than expected over the projected period, then underwriting (UW) results will be worse than planned.
• Prior Expectation #2 (PE2): If the stock market performs better than expected over the projected period, then, to a lesser degree than is the case in Prior

---

1 We note that while the D&O LOB risk distribution does not change, the total company risk distribution will change due to the introduction of correlation between investments and insurance risk. Moreover, changes to the D&O LOB risk distribution will change the effect of correlations between D&O and other LOBs, absent offsetting changes.

2 In Appendix 1 we discuss some of the alternatives we considered.
Upgrading an Existing Capital Model – A Common Risk Driver Application

Expectation #1, the UW results will also be worse than planned.\(^3\)

PE1 can be implemented with a linear relationship between the S&P error distribution and the loss ratio error distribution.

Combining PE1 and PE2 requires a more complicated relationship between the S&P error and the loss ratio error. That relationship exhibits a “turning point” from which both positive and negative deviations of the S&P index from expected produce increases in the LR above plan. In our example we use a quadratic relationship as a reasonably simple form meeting that requirement.

**Statistical Framework**

The table below shows notation we use in this paper. Variables with double dots, e.g. \( \ddot{x} \), refer to historical data. Unmodified variable names, e.g. \( x \), refer to values from distributions.

### Examples of Notation used in this document

| \( x_{err} \) | The distribution around an error variable one year in the future. |
| \( x_{err,i} \) | A simulated observation from \( x_{err} \) |
| \( \hat{x}_{err} \) | The observed distribution of the historical prediction errors of \( X \) |
| \( \hat{x}_t \) | A historical observation of the variable used to calculate historical errors. |
| \( \hat{x}_{err,t} \) | A historical observation of the error around the predicted historical variable. |
| \( k \) | Number of observations from historical dataset. \( k = \text{Max(year)} - \text{Min(year)} + 1 \) |
| \( N \) | Number of simulations run in model |
| \( E(X) \) | Expected value of \( X \) |
| \( SD(X) \) | Standard deviation of \( X \) |
| \( F_X(x) \) | Cumulative distribution function of \( X \); i.e. Probability that \( X \) is less than \( x \). |

**Definition of variables**

We apply that framework to the variables of interest in our work as follows:

- **x-based variables** refer to the explanatory variable – in this paper, the S&P index.
- **y-based variables** refer to the predicted variable – in this paper, the loss ratio.

---

\(^3\) PE1 has the obvious interpretation. PE2 is related to increased M&A activity or increased risk-taking activity, including M&A activity. Combined, the two Prior Expectations imply that predictable economic conditions produce the best UW results. Note also that ‘higher’ and ‘lower’ economic conditions are not the same as ‘up/down’, but rather are whether the trends in the market continue in the manner that are predicted when the planned LR is selected.
S&P Variables

The $x$-based variables we use in the paper are as follows:

\[ x_t = \text{S&P Index at the end of year } t \]

\[ \hat{x}_{err,t} = \frac{\text{S&P Index at the end of year } t}{E(\text{S&P Index at the end of year } t)} \]

For example, if at 12/31/2012 the expected value of the S&P index one year in the future, 12/31/2013, was 1,400, while the actual index value was 1,479, then

\[ \hat{x}_{err,2013} = \frac{1,479}{1,400} = 1.056. \]

In a company setting, the expected value of the S&P index would be obtained from an ESG. In this report we use the following simplified forecasting approach:

\[ E(\text{S&P Index at the end of year } t) = (x_{t-1} + d_{t-1}) \times (1 + r_{t-1}), \]

where,

\[ d_t = \text{dividend in year } t \]

\[ r_t = \text{1yr Treasury yield at end of year } t \]

and,

\[ \hat{x}_{err} = \text{Observed distribution of historical S&P prediction errors} \]

\[ x_{err} \text{ is the distribution of the error around the predicted level of the S&P index.} \]

Error in this paper is defined as the ratio of actual to expected.

\[ x_{err,t} = \frac{\text{S&P Index simulation } i}{\text{Avg}(\text{S&P Index simulations})} \]

Loss Ratio Variables

The $y$-based loss ratio variables we use in the paper have analogous definitions, as follows:

\[ y_t = \text{represents the observations of historical loss ratios.} \]

\[ \hat{y}_{err,t} = \frac{\text{Historical Loss Ratio for year } t}{\text{Planned Loss Ratio for year } t} \]

The historical loss ratio observations we use in this analysis are Schedule P industry accident year loss ratios for the Other Liability – Claims Made statutory line of business at the latest
Upgrading an Existing Capital Model – A Common Risk Driver Application

available maturity, up to 120 months developed for the most mature data points.

The planned loss ratio is the accident year loss ratio at 12 months; used as a proxy for the
planned loss ratio.

\[ \hat{y}_{err} = \text{Observed} \] distribution of historical Loss Ratio prediction errors

\[ y_{err} \] is the distribution of the error around the predicted ultimate loss ratio for the line of
business, that is, the ultimate loss ratio for accident year 2017.

Error in this paper is defined as the ratio of actual to expected, so

\[ y_{err,i} = \frac{\text{Loss Ratio simulation } i}{\text{Avg}(\text{Loss Ratio simulations})} \]

Original and Revised Loss Ratio Error Distribution

At this point we must introduce some notation to distinguish between our original loss
ratio error distribution and the revised one we are producing with this alternative model. The
goal is for the two to be as close as possible.

\[ y_{err}^{\text{Orig}} \] has a lognormal distribution LN(1, \sigma) and implicitly contains variability related
to economic conditions.

In our alternative model:

\[ y_{err}^{\text{Rev}} = y_{perr}^{\text{Rev}} * y_{lerr}^{\text{Rev}} \]

\[ y_{perr}^{\text{Rev}} \] is the distribution of the loss ratio predicted, based on the S&P index, versus
expected loss ratio, which will be defined later as a regression on \( x_{err} \).

\[ y_{lerr}^{\text{Rev}} \] is the variability in the loss ratio error that is independent of the S&P Index
volatility, or rather, \( x_{err} \). The distribution of \( y_{lerr}^{\text{Rev}} \) reflects the residual (multiplicative
basis) in \( y_{perr}^{\text{Rev}} \) versus \( y_{err}^{\text{Rev}} \).

Our goal is for \( y_{err}^{\text{Rev}} \) to be as close as possible to \( y_{err}^{\text{Orig}} \).

If \( y_{err}^{\text{Orig}} \) and \( y_{perr}^{\text{Rev}} \) and \( y_{lerr}^{\text{Rev}} \) were all lognormally distributed, then we could determine a
closed-form solution for \( y_{lerr}^{\text{Rev}} \). However, we want more flexibility in the choice of underlying
distributions of \( y_{err}^{\text{Orig}} \) and \( y_{lerr}^{\text{Rev}} \). Therefore, we take an approach that allows us to select any
appropriate regression model to represent the risk of the LOB explained by economic
variables and then define the distribution of the error term using a beta distribution for its
flexibility.
Upgrading an Existing Capital Model – A Common Risk Driver Application

Initial status

In the current capital model structure, the insurance risk distribution is derived from historical observations of ultimate loss ratios by accident year. There is no explicit assumed relationship between the D&O LOB and the S&P index.

In Table 1, below, the observed S&P Index in column 1 is from public sources. In practice, the predicted S&P Index in column 2 would be the average following year-end S&P index from the Economic Scenario Generator used in the company economic capital model. In this example, the S&P prediction is equal to the sum of the prior year’s S&P value and dividend inflated at the 1-year US Treasury rate (see formula above, in S&P Variables subsection).

<table>
<thead>
<tr>
<th>AY</th>
<th>S&amp;P Index</th>
<th>Loss Ratio (LR)</th>
<th>Observed Error</th>
<th>( \hat{x}_{err} )</th>
<th>( \hat{y}_{err} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Predicted</td>
<td>Observed</td>
<td>Planned</td>
<td>Observed</td>
<td></td>
</tr>
<tr>
<td>1990</td>
<td>359.7</td>
<td>328.8</td>
<td>60.3</td>
<td>62.4</td>
<td>0.914</td>
</tr>
<tr>
<td>1991</td>
<td>364.1</td>
<td>388.5</td>
<td>70.0</td>
<td>59.1</td>
<td>1.067</td>
</tr>
<tr>
<td>1992</td>
<td>471.2</td>
<td>435.6</td>
<td>71.7</td>
<td>65.7</td>
<td>1.044</td>
</tr>
<tr>
<td>1993</td>
<td>464.2</td>
<td>466.0</td>
<td>73.3</td>
<td>54.5</td>
<td>1.004</td>
</tr>
<tr>
<td>1994</td>
<td>495.9</td>
<td>455.2</td>
<td>76.4</td>
<td>60.1</td>
<td>0.918</td>
</tr>
<tr>
<td>1995</td>
<td>502.1</td>
<td>614.6</td>
<td>76.0</td>
<td>58.5</td>
<td>1.224</td>
</tr>
<tr>
<td>1996</td>
<td>660.9</td>
<td>743.3</td>
<td>73.9</td>
<td>58.0</td>
<td>1.125</td>
</tr>
<tr>
<td>1997</td>
<td>799.9</td>
<td>962.4</td>
<td>72.0</td>
<td>68.1</td>
<td>1.203</td>
</tr>
<tr>
<td>1998</td>
<td>1031.8</td>
<td>1190.1</td>
<td>66.7</td>
<td>86.8</td>
<td>1.153</td>
</tr>
<tr>
<td>1999</td>
<td>1260.9</td>
<td>1428.7</td>
<td>68.4</td>
<td>108.9</td>
<td>1.133</td>
</tr>
<tr>
<td>2000</td>
<td>1531.8</td>
<td>1330.9</td>
<td>71.1</td>
<td>110.2</td>
<td>0.869</td>
</tr>
<tr>
<td>2001</td>
<td>1418.9</td>
<td>1144.9</td>
<td>74.2</td>
<td>105.6</td>
<td>0.807</td>
</tr>
<tr>
<td>2002</td>
<td>1185.9</td>
<td>899.2</td>
<td>68.9</td>
<td>93.7</td>
<td>0.758</td>
</tr>
<tr>
<td>2003</td>
<td>927.3</td>
<td>1080.6</td>
<td>66.1</td>
<td>65.4</td>
<td>1.165</td>
</tr>
<tr>
<td>2004</td>
<td>1111.9</td>
<td>1199.2</td>
<td>65.9</td>
<td>45.4</td>
<td>1.079</td>
</tr>
<tr>
<td>2005</td>
<td>1225.2</td>
<td>1262.1</td>
<td>63.0</td>
<td>45.1</td>
<td>1.008</td>
</tr>
<tr>
<td>2006</td>
<td>1340.5</td>
<td>1416.4</td>
<td>64.3</td>
<td>49.0</td>
<td>1.057</td>
</tr>
<tr>
<td>2007</td>
<td>1513.4</td>
<td>1479.2</td>
<td>67.0</td>
<td>56.2</td>
<td>0.977</td>
</tr>
<tr>
<td>2008</td>
<td>1557.3</td>
<td>877.6</td>
<td>72.6</td>
<td>81.9</td>
<td>0.564</td>
</tr>
<tr>
<td>2009</td>
<td>909.3</td>
<td>1110.4</td>
<td>70.6</td>
<td>77.2</td>
<td>1.221</td>
</tr>
<tr>
<td>2010</td>
<td>1138.1</td>
<td>1241.5</td>
<td>70.3</td>
<td>73.5</td>
<td>1.091</td>
</tr>
<tr>
<td>2011</td>
<td>1267.9</td>
<td>1243.3</td>
<td>71.1</td>
<td>74.1</td>
<td>0.981</td>
</tr>
<tr>
<td>2012</td>
<td>1271.3</td>
<td>1422.3</td>
<td>69.5</td>
<td>74.2</td>
<td>1.119</td>
</tr>
<tr>
<td>2013</td>
<td>1455.9</td>
<td>1807.8</td>
<td>65.9</td>
<td>68.4</td>
<td>1.242</td>
</tr>
</tbody>
</table>

Mean                      1.030  1.019
Std Dev                   0.164  0.262
The loss ratio information in columns 3 and 4 is data from industry Schedule P data for the Other Liability (Claims Made) LOB for reporting years 1999-2016. Column 3 is the industry loss ratio developed to 120 months. Column 4 is the Schedule P accident year loss ratio at 12 months; used as a proxy for the planned loss ratio.

Columns 5 and 6 show actual versus expected results as ratios, column 5 = column 2/column 1 and column 6=column 4/column 3.

**Original Loss Ratio Error Distribution**

The historical data for the Loss Ratio error, Table 1 column 6, $\hat{y}_{err}$, is seen to have a mean of 1.019 and standard deviation 0.262. We used this information to parameterize $y_{err}^{orig}$, assuming a lognormal distribution with mean of 1 and standard deviation of 0.262, implies mu and sigma of 0.258 and -0.033.

**Predicted Loss Ratio Error Distribution**

In Figure 1 below, we compare the observed Loss Ratio error, Table 1 column 6, and the S&P index error, Table 1, column 5. In this Figure, we see that for 2000 and 2001 the S&P error indices ($\hat{x}$ values) are lower than expected, i.e., below the “1.0” line, and Loss Ratios errors ($\hat{y}$ values) are higher than expected, i.e., above the “1.0” line. This is consistent with Expectation #1. In general, we see LR errors and S&P index errors are on opposite sides of the “1.0” line, as expected.
We examine Expectation #2 in Figure 2, the scatter plot on the following page.

We consider four regions of S&P Error, i.e., S&P compared to expected S&P, as follows: 10% worse than expected (“≤0.9); between 10% worse than expected and expected, i.e., ‘bad’ but not too bad (≥0.9; ≤1.0); between expected and 10% better than expected, i.e., good but not too good (≥1.0; ≤1.1)” and more than 10% better than expected (>1.1). Table 2 below shows the LR error performance in each of those regions.

<table>
<thead>
<tr>
<th>S&amp;P Error Range</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>LR Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>≥1.0</td>
</tr>
<tr>
<td>&lt;0.9</td>
<td>0</td>
<td>4</td>
<td>4</td>
<td></td>
<td></td>
<td>0%</td>
</tr>
<tr>
<td>≥0.9; ≤1</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td></td>
<td></td>
<td>50%</td>
</tr>
<tr>
<td>≥1;≤1.1</td>
<td>6</td>
<td>1</td>
<td>7</td>
<td></td>
<td></td>
<td>86%</td>
</tr>
<tr>
<td>&gt;1.1</td>
<td>4</td>
<td>5</td>
<td>9</td>
<td></td>
<td></td>
<td>44%</td>
</tr>
</tbody>
</table>
Consistent with Expectation #1, the LR errors become increasingly favorable (0% over 1.0 to 86% over 1.0) in the first three regions. Consistent with Expectation #2, the LR errors become less favorable as the S&P error increases into the fourth region.

**Figure 2 - Loss Ratio Error vs S&P Index Error regression**

With the concession that historical data is limited, we find that it is consistent with our underwriting Expectation #2. To apply our assumptions of Expectation #1 and Expectation #2, in Figure 2, we fit the data to a quadratic curve:

\[ y_{err}^{Rev} = b_2(x_{err})^2 + b_1 x_{err} + b_0 \]  

(Regression coefficients found in Figure 2)

\[ ^4 \text{LINEST(LRerr,SPerr}^{1,2}). \]
Upgrading an Existing Capital Model – A Common Risk Driver Application

This fitted curve implies that the most favorable LR variance from expected arises when the S&P index error is about 15%. The expected LR variance becomes less favorable as the S&P index variation becomes more favorable beyond that level.

Fitting Error Distributions

At this stage we have specified \( y_{\text{err}}^{\text{Rev}} \), and we can leave this historical sample set to fit the distribution of the error term using simulated data consistent with our capital model. Remember, the goal of the error fit is to produce a final loss ratio error distribution, \( y_{\text{err}}^{\text{Rev}} \), that very closely matches that of the original, \( y_{\text{err}}^{\text{orig}} \).

Table 3 column 2, shows the first 10 of 1,000 simulations of the S&P Index one year from the model valuation date, using the company Economic Scenario Generator. The mean value of the 1,000 simulations for the forecast year, Average (2), is 1,845.2. This is the S&P prediction in our model. Column 3 = Column 2 / Average (2). Then we calculate column 4, \( y_{\text{err}}^{\text{Rev}} \), using the quadratic relationship determined in the previous step:

Column (4) = \( b_0 + b_1 \times \text{Column 3} + b_2 \times \text{Column 3}^2 \)

<table>
<thead>
<tr>
<th>Sim</th>
<th>S&amp;P 500 Simulation</th>
<th>( x_{\text{err}} )</th>
<th>( y_{\text{err}}^{\text{Rev}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1291.6</td>
<td>0.70</td>
<td>1.20</td>
</tr>
<tr>
<td>2</td>
<td>1893.0</td>
<td>1.03</td>
<td>0.99</td>
</tr>
<tr>
<td>3</td>
<td>1638.8</td>
<td>0.89</td>
<td>1.05</td>
</tr>
<tr>
<td>4</td>
<td>2041.1</td>
<td>1.11</td>
<td>0.98</td>
</tr>
<tr>
<td>5</td>
<td>1652.2</td>
<td>0.90</td>
<td>1.05</td>
</tr>
<tr>
<td>6</td>
<td>1934.4</td>
<td>1.05</td>
<td>0.99</td>
</tr>
<tr>
<td>7</td>
<td>2022.5</td>
<td>1.10</td>
<td>0.98</td>
</tr>
<tr>
<td>8</td>
<td>1812.5</td>
<td>0.98</td>
<td>1.01</td>
</tr>
<tr>
<td>9</td>
<td>1819.4</td>
<td>0.99</td>
<td>1.00</td>
</tr>
<tr>
<td>10</td>
<td>1946.3</td>
<td>1.05</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Figure 3, below, shows all the 1,000 simulated data points and the fitted quadratic relationship.
The next step is to calibrate $y_{\text{err}}^{\text{Rev}}$.

As noted in the statistical framework, we assume that $y_{\text{err}}^{\text{Rev}}$ has a beta distribution. We select the beta distribution parameters to minimize the difference between $y_{\text{err}}^{\text{Rev}}$ and $y_{\text{err}}^{\text{Orig}}$ at selected percentiles. We do this using Excel Solver.\footnote{An analytical method to determine the parameters of the beta distribution requires numerical analysis. For simplicity’s sake we used the excel solver.}

Table 4 shows the fitted parameters for the beta distribution and the solver constraints used in fitting those parameters. Column 7 contains the solver constraints for the parameters in Column 6. The mean of the beta distribution is constrained to 1 so that the mean of $y_{\text{err}}^{\text{Rev}}$ equals the mean of $y_{\text{err}}^{\text{Orig}}$. 
Upgrading an Existing Capital Model – A Common Risk Driver Application

Table 4
Beta Distribution Parameters and Solver Constraints

<table>
<thead>
<tr>
<th>Beta parameters</th>
<th>Solver constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{err}^{Rev}$</td>
<td></td>
</tr>
<tr>
<td>minY</td>
<td>0.21</td>
</tr>
<tr>
<td>maxY</td>
<td>8.48</td>
</tr>
<tr>
<td>alpha</td>
<td>8.80</td>
</tr>
<tr>
<td>beta</td>
<td>83.32</td>
</tr>
<tr>
<td>E[Impact]</td>
<td>1.000</td>
</tr>
<tr>
<td>SD[Impact]</td>
<td>0.252</td>
</tr>
</tbody>
</table>

The solver iteration that produces the values in Table 4 column 6 uses the values from Tables 4 and 5, column 8-14 as follows:

- Column (8) shows random values from a uniform distribution.
- Column (9) shows the observations $y_{err,i}^{Rev}$, generated from a beta distribution with the parameters in (6) and the random variable values in (8).
- Column (10) = column (9) * $y_{err,i}^{Rev}$, for i=1 to 1000, from Table 2, column 4. Column 10 is the new modeled $y_{err}^{Rev}$ distribution.
We continue the calculation as follows:

- Column 11 shows the selected cumulative distribution function percentiles at which we compare $y_{err}^{Rev}$ and the newly calculated $y_{err}^{Orig}$ distribution.

- Column 12 is the value of $y_{err}^{Orig}$ at the cumulative distribution probability level in column 11. In this example, the original model is lognormal, so these are the inverse cumulative lognormal values for the CDF levels in column 11. The method, however, does not require a parametric distribution for column 12.

- Column 13 shows the new $y_{err}^{Rev} = y_{err}^{Orig} \times y_{err}^{Rev}$.

- Column 14 is the difference between column 12 and column 13, squared. The objective function is the sum of column 14. We determine the beta parameters in column 6 using Excel Solver to minimize the sum of column 14. You should be aware that there are an infinite set of beta parameters that will result in $y_{err}^{Rev}$ fitting our needs and rerunning the solver multiple times will return a different set of parameters. That is, the beta parameters are unstable, but that does not affect the utility of the outcome as, in our tests, the distribution of $y_{err}^{Rev}$ is stable.
The quality of the fit between $y_{err}^{\text{Orig}}$ and the newly constructed $y_{err}^{\text{Rev}}$ is good, as evidenced by a comparison of columns 12 and 13.

**Conclusions**

We believe this is an interesting application of common risk drivers. Moreover, it is a demonstration of a process that can be used to incorporate informative external variables or other sources into an existing economic capital model with minimum disruption to the company’s existing ECM.
Upgrading an Existing Capital Model – A Common Risk Driver Application

Supplementary Material

We provide a linked illustrative Excel workbook that shows our calculations and the Tables and Figures we present in this paper.

Authors

Principal Authors: Mario E. DiCaro and Allan M. Kaufman.

Allan M. Kaufman is a Managing Director at FTI Consulting. He also serves as Chairman of the Board of a Lloyd’s managing agency. He is a Fellow of the CAS and a Member of the American Academy of Actuaries. He currently participates on the American Academy of Actuaries Property& Casualty Risk Based Capital Committee and recently led the CAS Risk Based Capital Dependency and Calibration Research Working Party. Allan has served in a wide variety of other CAS and American Academy of Actuaries roles.

Mario E. DiCaro is a Vice President at Tokio Marine HCC and is responsible for the capital modeling work done at the group level. He is a Fellow of the CAS and a Chartered Enterprise Risk Actuary (CERA). He currently serves on the committee of the Joint Risk Management Section, a collaboration of the CAS, CIA, and SOA.

Assistance from Shira Jacobson, FCAS

Further assistance provided by Predictive Modeling Working Group

References


Upgrading an Existing Capital Model – A Common Risk Driver Application


http://www.variancejournal.org/issues/01-01/variance01-01.pdf
Appendix 1

In this Appendix we discuss three further issues:

1. Correlations to other LOBs
2. Alternative Models
3. Sensitivity Testing Extreme Values

Correlation to other LOBs

In the existing capital model, after separating the risk into components, $y_{err}^{Rev}$, the component not related to the economic variable, is still part of the existing correlation matrix, but $y_{err}^{Rev}$, the economic component, is not. Absent other adjustments, the separation reduces the correlation to other LOBs of business in ways that are not desirable.

There are two approaches to retaining the desired correlation between the D&O LOB and other LOBs. The complex approach is to update all LOBs with risk drivers. That may not be practical.

Instead, in practice, we increase the correlation factors in the copula between the D&O LOB, $y_{err}^{Rev}$, and the other LOBs. That is, we increase the row/column in the matrix until the measured output correlations were consistent with what they were prior to the model changes.

In that way, we retain the pre-existing LOB correlation but we add explicit correlation between D&O loss ratios and investment risk.

Alternative Models

In this paper we used a quadratic regression model to relate LRs to the S&P Index.

We considered alternative structures and alternative variables that we outline here.

For structure, we chose quadratic rather than linear for the reasons described in the paper, even though, as noted, the data to support that decision is limited. We also considered a kernel function that uses nearest neighbors to estimate values at various simulated points. The results were interesting but given the limited historical data for calibration we chose the quadratic model.

For variables we considered the following:

- Unemployment
- Interest rates
Upgrading an Existing Capital Model – A Common Risk Driver Application

- Changes in corporate bond spreads and yields,
- Change in average CEO salaries
- The number of securities class action lawsuits

We chose to consider only one variable because the historical data for calibration is limited.

For practical reasons, we also choose to consider only variables for which we have simulation forecast values from the ESG. The last two variables are not available in the ESG.

Unemployment, interest rates, corporate spreads and yields, and the S&P 500 index are variables simulated by the ESG. Plotting the relationships between the historical loss ratios and the index movements quickly revealed that many different relationships could have been used. None were clearly better than the S&P Index, particularly for use in an illustration such as the one presented in this paper. With more historical experience, it might have been clear that a different model was superior.

**Sensitivity Testing Extreme Values**

In this paper we did not demonstrate any tests for model validity at extreme S&P error points. That is, what would the predicted loss ratio be if the ESG produces an S&P error of 2.5, or 0.2? In practice these sorts of outcomes need to be tested for or your model may produce embarrassingly impossible outcomes in those extremes.