Exam ST

CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

April 30, 2015

Exam ST

Models for Stochastic Processes and Statistics

2.5 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Spring 2015 and that the exam name is ST.
   - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. <u>No additional time will be allowed for this after the exam has ended.</u> Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. <u>Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils.</u> <u>You will also not be allowed to use calculators.</u> The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam ST" included in your exam packet.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 15, 2015.

END OF INSTRUCTIONS

1.

You are given:

- The number of messages sent follows a Poisson process with rate $\lambda(t) = \frac{t}{4}$, $t \geq 0$.
- Each message sent is received with probability 0.80.

Calculate the probability that at least 2 messages will be received by time t = 4.

A. Less than 0.35

B. At least 0.35, but less than 0.45

C. At least 0.45, but less than 0.55

D. At least 0.55, but less than 0.65

E. At least 0.65

EXAMINATION CONTINUED ON NEXT PAGE

2.

You arrive at a bus station at exactly 8:30 am and you have the option of either taking Line 1 or Line 2 to bring you to your destination.

Buses along Line 1 and Line 2 arrive independently, both according to a Poisson process. On the average, one Line 1 bus arrives every 15 minutes and one Line 2 bus arrives every 10 minutes.

If you board Line 1, it will take you 8 minutes to reach your destination. If you board Line 2, it will take you 20 minutes to reach your destination.

You decide to take the first bus that arrives.

Calculate the expected length of time, to the nearest minute, that will take you to reach your destination.

A. Less than 22 minutes
B. At least 22 minutes, but less than 24 minutes
C. At least 24 minutes, but less than 26 minutes
D. At least 26 minutes, but less than 28 minutes
E. At least 28 minutes

EXAMINATION CONTINUED ON NEXT PAGE

3.

You are given:

- Claim frequency follows the Poisson process with a rate $\lambda(t) = 3t,\ t > 0$.
- Frequency and severity of claims are independent.
- Claim severity follows the distribution given in the following table:

Amount	Probability
5	0.6
10	0.3
75	0.1

- The aggregate claim amount by $t = 5$ was 505.

Calculate the variance of the aggregate claim amount at $t = 25$.

A. Less than 500,000

B. At least 500,000, but less than 520,000

C. At least 520,000, but less than 540,000

D. At least 540,000, but less than 560,000

E. At least 560,000

EXAMINATION CONTINUED ON NEXT PAGE

4.

Let $X_1, X_2, \ldots, X_n$ be a random sample from a Bernoulli distribution with success probability q and let $\bar{X} = \frac{1}{n}\sum_{k=1}^{n} X_k$ denote the sample mean.

The sample mean is used as an estimator for q.

Determine the correct expression for the mean squared error of this estimator.

A. 0

B. $q(1-q)$

C. $nq(1-q)$

D. $\frac{1}{n}q(1-q)$

E. $\frac{1}{n^2}q(1-q)$

EXAMINATION CONTINUED ON NEXT PAGE

5.

Let $X_1, X_2, \dots, X_n$ be a random sample from a population X with probability mass function:

$$p(x) = \theta(1-\theta)^x \text{ for } x = 0,1,2,\dots$$

θ is an unknown parameter between 0 and 1 and the expected value of X is:

$$E[X] = \frac{1-\theta}{\theta}$$

Determine the Cramer-Rao lower bound for the variance of all unbiased estimators of θ.

A. $\frac{1}{n}\theta(1-\theta)$

B. $n\theta(1-\theta)$

C. $\frac{1}{n^2}\theta(1-\theta)$

D. $\frac{1}{n}\theta^2(1-\theta)$

E. $\frac{1}{n^2}\theta^2(1-\theta)$

EXAMINATION CONTINUED ON NEXT PAGE

6.

For a general liability policy the paid claim amounts, X, follow the Weibull distribution which is shown below.

$$F(x) = 1 - e^{-\left(\frac{x}{\theta}\right)^2} \qquad 0 < x$$

You are given 5 paid claim amounts of 1, 5, 6, 8, and 10. Also, it is known that the paid amounts for each of three additional claims exceed 1.

Calculate the maximum likelihood estimate of θ.

A. Less than 5
B. At least 5, but less than 6
C. At least 6, but less than 7
D. At least 7, but less than 8
E. At least 8

7.

Let $X_1, X_2, \ldots, X_5$ be independent and identically distributed observations for a random variable from a population with probability density function:

$$f(x;\theta) = \frac{x^{(1-\theta)/\theta}}{\theta}, \qquad 0 < x < 1, \qquad 0 < \theta$$

Given the following observations:

0.250	0.200	0.800	0.750	0.050

Calculate the maximum likelihood estimate of θ.

A. Less than 0.5
B. At least 0.5, but less than 1.0
C. At least 1.0, but less than 1.5
D. At least 1.5, but less than 2.0
E. At least 2.0

8.

You are given the probability distribution for observing a given Genotype:

Genotype	Probability
AA	θ^2
Aa	$2\,\theta(1-\theta)$
aa	$(1-\theta)^2$

The results from a random sample of 122 people are:

Genotype	Number of People
AA	15
Aa	31
aa	76

Calculate the maximum likelihood estimate of θ.

A. Less than 0.15

B. At least 0.15, but less than 0.21

C. At least 0.21, but less than 0.27

D. At least 0.27, but less than 0.33

E. At least 0.33

9.

Determine the formula for the Fisher Information of n independent samples from a geometric distribution with mean β where each of the n samples gives you the number of trials required to obtain a successful outcome in that particular sample.

A. $1/\beta$

B. n/β

C. $n/\beta - n/(\beta + 1)$

D. $n\beta/(\beta + 1)$

E. $\beta(\beta + 1)/n$

EXAMINATION CONTINUED ON NEXT PAGE

10.

An insurance company is examining its offer of reduced rates for car insurance premiums to owners of small vehicles.

Some analysts suggest that when small vehicles are involved in accidents, the chances of serious injury are higher than that for larger sized vehicles.

A random sample of 1000 accidents is classified according to severity of the injuries and the size of the car. The results are:

	Size of Car			
	Small	Medium	Large	**Total**
Fatal/Critical	235	120	60	**415**
Non-critical	300	165	120	**585**
Total	**535**	**285**	**180**	**1000**

The following null and alternate hypotheses are created:

H_0: There is no association between size of car and severity of injury.

H_A: There is association between size of car and severity of injury.

Calculate the chi-squared test statistic to evaluate the null hypothesis.

A. Less than 6.6

B. At least 6.6, but less than 6.8

C. At least 6.8, but less than 7.0

D. At least 7.0, but less than 7.2

E. At least 7.2

EXAMINATION CONTINUED ON NEXT PAGE

11.

You are given:

- A random sample, $X_1, X_2, \ldots, X_{10}$ from the Bernoulli distribution with $q = P(X = 1)$
- $H_0: q = 0.5$ and $H_a: q > 0.5$.
- The critical region, $C = \{\sum_{k=1}^{10} x_k > 6\}$.

Calculate the probability of Type I error.

A. Less than 0.1
B. At least 0.1, but less than 0.2
C. At least 0.2, but less than 0.3
D. At least 0.3, but less than 0.4
E. At least 0.4

EXAMINATION CONTINUED ON NEXT PAGE

12.

You are given two samples of paid claim amounts for two Hospitals, A and B, which, after the natural logarithm transformation, independently follow the normal distribution with means μ_A and μ_B. The results for the paid claim amounts below are after the natural logarithm transformation has been applied.

Hospital A

6.21	7.34	5.67	7.88	3.89

Hospital B

7.89	10.12	9.71	5.55	4.33	12.48

The unbiased standard deviations of the above two samples are $s_A = 1.56$ and $s_B = 3.04$. Assume that variances of paid claim amounts on the natural logarithm scale for these hospitals are equal.

Calculate the upper bound of the 95 percent symmetric confidence interval for the difference $\mu_B - \mu_A$.

A. Less than 4.9
B. At least 4.9, but less than 5.1
C. At least 5.1, but less than 5.3
D. At least 5.3, but less than 5.5
E. At least 5.5

13.

For a general liability policy you are given:

- The natural logarithm of paid claim amounts follows the normal distribution with mean μ and standard deviation 3.
- The null hypothesis H_0: $\mu = 8$
- The alternate hypothesis H_A: $\mu = 7$
- The sample size is 160 and the probability of error of type I, α, is equal to 0.05.

Based on Neyman-Pearson Lemma, calculate the probability of error of type II, β.

A. Less than 0.0049

B. At least 0.0049, but less than 0.0051

C. At least 0.0051, but less than 0.0053

D. At least 0.0053, but less than 0.0055

E. At least 0.0055

14.

You are given:

- Ten independent observations which follow the normal distribution with unknown mean μ and variance $\sigma^2 = 20$.
- Based on your observations, the $(1-\alpha)$ confidence interval for μ is $(-6.84, -1.30)$.
- H_0: $\mu = -5$ and H_1: $\mu \neq -5$.

Calculate the critical region at significance level α.

A. $(-\infty, -6.84) \cup (6.84, \infty)$

B. $(-6.84, 6.84)$

C. $(-7.77, -2.23)$

D. $(-\infty, -7.77) \cup (-2.23, \infty)$

E. $(-\infty, -10.54) \cup (0.54, \infty)$

CONTINUED ON NEXT PAGE

15.

You are given:

- The following pairs of observed values:

Observation	X	Y	Y - X
1	100.9	101.4	0.5
2	100.4	100.4	0
3	136.9	138.2	1.3
4	149.4	149.6	0.2
5	81.9	81.4	-0.5
6	140.7	141.9	1.2
7	104.4	106.2	1.8

- H_0: $E[X] = E[Y]$
- H_1: $E[X] \neq E[Y]$

Calculate the p-value for the null hypothesis.

A. Less than 0.01

B. At least 0.01, but less than 0.02

C. At least 0.02, but less than 0.05

D. At least 0.05, but less than 0.10

E. At least 0.10

EXAMINATION CONTINUED ON NEXT PAGE

16.

You are given:

- X_1, X_2, and X_3 are independent observations from an exponential distribution.
- $E[X] = 5$
- $Y_{(1)}$, $Y_{(2)}$, and $Y_{(3)}$ are the order statistics of those observations.
- The probability density function of the order statistic $Y_{(k)}$ from a sample of size n is:

$$g_k(y_{(k)}) = \frac{n!}{(k-1)!\,(n-k)!}[F(y_{(k)})]^{k-1}[1 - F(y_{(k)})]^{n-k} f(y_{(k)})$$

Calculate $E(Y_{(2)})$.

A. Less than 3.0
B. At least 3.0, but less than 4.0
C. At least 4.0, but less than 5.0
D. At least 5.0, but less than 6.0
E. At least 6.0

EXAMINATION CONTINUED ON NEXT PAGE

17.

You are given:

- (X_1, Y_1), (X_2, Y_2), (X_3, Y_3), and (X_4, Y_4) is a random sample from a bivariate continuous distribution function $F(x, y)$.
- $R(X_i)$ denotes the rank of X_i among $\{X_1, X_2, X_3, X_4\}$.
- $R(Y_i)$ denotes the rank of Y_i among $\{Y_1, Y_2, Y_3, Y_4\}$.
- The ranks are:

Observation	$R(X_i)$	$R(Y_i)$
1	3	1
2	1	?
3	2	?
4	4	?

- There are no tied ranks.
- X and Y are independent according to Spearman's rho.

Determine $R(Y_2)$, $R(Y_3)$, and $R(Y_4)$.

A. 2, 4, 3

B. 3, 2, 4

C. 3, 4, 2

D. 4, 2, 3

E. 4, 3, 2

18.

You are given:

- The following paired samples:

Pair	X	Y
1	29	23
2	24	26
3	26	26
4	12	11
5	14	14
6	15	11
7	25	23

- H_0 : Median of X = Median of Y
- H_1 : Median of X > Median of Y

Calculate the exact p-value using the sign test.

A. Less than 10%

B. At least 10% but less than 15%

C. At least 15% but less than 20%

D. At least 20% but less than 25%

E. At least 25%

EXAMINATION CONTINUED ON NEXT PAGE

19.

You are given:

- A tutor claims that students will get more than 89.5 out of 100 questions right on a particular exam if the students use the tutor's service.
- A random sample of 160 previous students was taken.
- No student scored exactly 89.5.
- H_0: Students using the service have a median score of 89.5 on the exam.
- H_1: Students using the service have a median score greater than 89.5 on the exam.
- H_0 was rejected at the 0.05 significance level using the sign test.
- The normal approximation with continuity adjustment is used.

Calculate the minimum number of students that scored more than 89.5 in the sample.

A. 89 or fewer

B. 90

C. 91

D. 92

E. 93 or more

20.

You are given the following loss ratios for two lines of business, A and B, over a four-year period:

A	80	75	75	90
B	70	75	60	95

You are asked to set up a linear regression model for estimating loss ratios of line B using loss ratios of line A.

Based on the above sample, the estimated slope is $\frac{11}{6}$, and the unbiased estimate of the error variance is 72.92.

Calculate the upper bound of the symmetric 95 percent confidence interval for the intercept of the regression line.

A. Less than -100
B. At least -100, but less than 0
C. At least 0, but less than 100
D. At least 100, but less than 200
E. At least 200

21.

The following two linear regression models were fit to 20 observations:

- Model 1: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \varepsilon$
- Model 2: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \varepsilon$

The results of the regression are as follows:

Model Number	Error Sum of Squares	Regression Sum of Squares
1	13.47	22.75
2	10.53	25.70

The null hypothesis is H$_0$: $\beta_3 = \beta_4 = 0$ with the alternative hypothesis that the two betas are not equal to zero.

Calculate the statistic used to test H$_0$.

A. Less than 1.70

B. At least 1.70, but less than 1.80

C. At least 1.80, but less than 1.90

D. At least 1.90, but less than 2.00

E. At least 2.00

22.

You are given the following linear regression model fitted to 12 observations:

$$Y = \beta_0 + \beta_1 X + \varepsilon$$

The results of the regression are as follows:

Parameter	Estimate	Standard Error
β_0	15.52	3.242
β_1	0.40	0.181

Determine the results of the hypothesis test H_0: $\beta_1 = 0$ against the alternative H_1: $\beta_1 \neq 0$.

A. Reject at $\alpha = 0.01$

B. Reject at $\alpha = 0.02$, Do Not Reject at $\alpha = 0.01$

C. Reject at $\alpha = 0.05$, Do Not Reject at $\alpha = 0.02$

D. Reject at $\alpha = 0.10$, Do Not Reject at $\alpha = 0.05$

E. Do Not Reject at $\alpha = 0.10$

EXAMINATION CONTINUED ON NEXT PAGE

23.

You are given:

- The number of claims per year for a policyholder follows a Poisson distribution with mean λ.
- The prior distribution for the mean, λ, follows a gamma distribution with mean 0.2 and variance 0.005.
- Last year, 60 annual policies produced a total of 12 claims.

Calculate the variance of the posterior distribution of λ.

A. Less than 0.0015
B. At least 0.0015, but less than 0.0025
C. At least 0.0025, but less than 0.0035
D. At least 0.0035, but less than 0.0045
E. At least 0.0045

24.

You are given:

- The score in a computer-based test follows a binomial distribution with parameters m and q.
- The prior distribution for q follows a beta distribution with parameters $a = 5$ and $b = 5$.
- After 15 questions, the posterior mean of the test score is 0.64.

Calculate the number of questions answered correctly.

A. Fewer than 9

B. 9

C. 10

D. 11

E. More than 11

25.

You are given:

- The size of a claim follows a normal distribution with mean μ and variance 169.
- The prior distribution of μ follows a normal distribution with mean 50 and variance 25.
- Two claims were observed: 35 and 45.

Calculate the lower bound of the symmetric 95% Bayesian confidence interval for μ.

A. Less than 42

B. At least 42, but less than 45

C. At least 45, but less than 48

D. At least 48, but less than 51

E. At least 51

END OF EXAMINATION

Spring 2015 Exam ST Answer Key

Question#	Answer
1	C
2	A
3	D
4	D
5	D
6	C
7	C
8	C
9	C
10	A
11	B
12	E
13	B or C
14	D
15	D or E
16	C
17	A
18	C
19	C
20	D
21	E
22	D
23	B
24	D
25	A

There was a clerical error which resulted in item 22 being scored with the wrong key. This should have been caught when we reviewed the performance stats, and we have changed our internal procedures to prevent it from reoccurring. This error was corrected before the final list of passing candidate names was released and the posted final answer key is correct. Unfortunately it is not feasible to produce a corrected set of candidate failing reports. This means that the analysis provided for item 22 will be inaccurate for some candidates.