

Exam LC



CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES



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Exam LC

Models for Life Contingencies

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1.5 HOURS

October 29, 2015

INSTRUCTIONS TO CANDIDATES

1. This 30 point examination consists of 15 multiple choice questions worth 2 points each.
2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.
 - Fill in that it is Fall 2015 and that the exam name is LC.
 - Darken the spaces corresponding to your Candidate ID number. Five rows are available. If your Candidate ID number is fewer than 5 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 00987, enter a zero on the first row, a zero on the second row, 9 on the third row, 8 on the fourth row, and 7 on the fifth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
 - Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Make your marks dark and fill in the spaces completely.
 - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.
3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.
4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.
 - Verify that you have a copy of "Tables for CAS Exam LC" included in your exam packet.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

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5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.
6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.
7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.
8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope. Interoffice mail is not acceptable.

If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

Candidates may obtain a copy of the examination from the CAS Web Site.

All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.
10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by November 14, 2015.

END OF INSTRUCTIONS

1.

You are given the following hazard rate function:

$$\bullet \lambda_x = \frac{2}{110-x}, \text{ for } 0 \leq x < 110$$

Calculate ${}_{15|2}q_{75}$.

- A. Less than 0.050
- B. At least 0.050, but less than 0.055
- C. At least 0.055, but less than 0.060
- D. At least 0.060, but less than 0.065
- E. At least 0.065

2.

A new technology will extend remaining life expectancy at all ages by 10% when it is implemented. Prior to implementation, mortality follows the uniform distribution with $\omega = 100$ years.

Let L equal the life expectancy for an individual that is currently age 30, if the technology is implemented today.

Let L' equal the life expectancy for this individual if the technology cannot be implemented until 20 years from today.

Calculate $(L - L')$.

- A. Less than 0.5
- B. At least 0.5, but less than 1.0
- C. At least 1.0, but less than 1.5
- D. At least 1.5, but less than 2.0
- E. At least 2.0

3.

You are given the following information:

- Deaths are uniformly distributed over each year of age.
- $\mu_{85.5} = 0.50$
- $\mu_{86.5} = 1.00$

Calculate ${}^0e_{85:\overline{2}|}$.

- A. Less than 1.13
- B. At least 1.13, but less than 1.15
- C. At least 1.15, but less than 1.17
- D. At least 1.17, but less than 1.19
- E. At least 1.19

4.

You are given the following information:

- (x) and (y) are two independent lives.
- Deaths are uniformly distributed over each year of age.
- $q_x = 0.05$
- ${}_{0.5}q_{xy} = 0.064$

Calculate q_y .

- A. Less than 0.100
- B. At least 0.100, but less than 0.125
- C. At least 0.125, but less than 0.150
- D. At least 0.150, but less than 0.175
- E. At least 0.175

5.

You are given the following information:

- (x) and (y) are two independent lives.
- (x) and (y) have a constant force of mortality.
- $\mu_x = \mu_y = 0.05$.

Calculate the median future lifetime of the last survivor status of (x) and (y) .

- A. Less than 22.5
- B. At least 22.5, but less than 25.0
- C. At least 25.0, but less than 27.5
- D. At least 27.5, but less than 30.0
- E. At least 30.0

6.

You are given the following information:

- (50) and (60) are independent lives.
- Mortality follows the hazard rate function:

$$\lambda_0(t) = \frac{1}{100-t}, \text{ for } 0 < t < 100.$$

Calculate $E(T_{50:60} \cdot T_{\overline{50:60}})$.

- A. Less than 550
- B. At least 550, but less than 650
- C. At least 650, but less than 750
- D. At least 750, but less than 850
- E. At least 850

7.

You are given a multiple decrement model with ten causes of decrement (1), ..., (10) having constant forces of mortality $\mu^{(1)}, \dots, \mu^{(10)}$ for $t \geq 0$.

You revise the model so that the force of mortality of each individual cause of decrement is halved. You find that the probability of failure by time $t = 2$ due to cause (1) has been reduced to $2/3$ of the original value.

Calculate the original (i.e., prior to the revision) total probability of survival to time $t = 2$.

- A. Less than 0.1
- B. At least 0.1, but less than 0.2
- C. At least 0.2, but less than 0.3
- D. At least 0.3, but less than 0.4
- E. At least 0.4

8.

A company currently offers a 3-year warranty on its flagship product. The warranty replaces the product when the cause of failure is mechanical.

The product has three causes of failure, described in the multiple decrement table below:

Year	Cause of Failure		
	Mechanical	Electrical	All Other
	$q_x^{(m)}$	$q_x^{(e)}$	$q_x^{(o)}$
1	0.005	0.010	0.050
2	0.010	0.005	0.050
3	0.015	0.005	0.050

The company is considering expanding the 3-year warranty to cover not only mechanical failure, but electrical failure as well.

The company expects to sell 3 million independent units next year.

Estimate the *additional* number of replacements the company would be expected to issue on these units as a result of the warranty expansion.

- A. Less than 54,000
- B. At least 54,000, but less than 55,000
- C. At least 55,000, but less than 56,000
- D. At least 56,000, but less than 57,000
- E. At least 57,000

9.

You are given the following information:

- Actuarial exam takers transition through the examination process according to a homogenous Markov chain with the following four states:
 - State 1 = Actuarial student
 - State 2 = ACAS
 - State 3 = FCAS
 - State 4 = Change careers

$$Q = \begin{bmatrix} 0.7 & 0.2 & 0.0 & 0.1 \\ 0.0 & 0.8 & 0.2 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

- Bob is an actuarial student and Judy is an ACAS.
- Bob and Judy transition independently of one another.

Calculate the probability that either Bob or Judy will be an FCAS after three transitions.

- A. Less than 0.40
- B. At least 0.40, but less than 0.45
- C. At least 0.45, but less than 0.50
- D. At least 0.50 but less than 0.55
- E. At least 0.55

10.

Consider a professional sports league in which players transition at the end of each year among three states: Major League, Minor League, and Retirement.

The transition probabilities between states are as follows:

- 80% of Major League players remain in the Major League, 15% are demoted to the Minor League, and 5% retire.
- 60% of Minor League players remain in the Minor League, 10% are promoted to the Major League, and 30% retire.
- Retired players never return to Major or Minor Leagues (i.e. retirement is permanent).

Calculate the probability that a player currently in the Minor League will be in the Major League exactly 3 years from now (i.e., after 3 transitions).

- A. Less than 0.115
- B. At least 0.115, but less than 0.125
- C. At least 0.125, but less than 0.135
- D. At least 0.135, but less than 0.145
- E. At least 0.145

11.

A retirement fund gives a retiree, (x) , two options for collecting benefits:

1. Collect as a continuous temporary life annuity, paying at a rate of \$10,000 per year until death or time $t = 20$, whichever occurs first.
2. Collect as a continuous deferred whole life annuity, paying at a rate of \$10,000 per year starting at time k and continuing until death.

You are given:

- (x) has constant force of mortality, $\mu = 0.04$
- $\delta = 0.06$

Calculate the time k at which the two options have the same actuarial present value.

- A. Less than 2
- B. At least 2, but less than 4
- C. At least 4, but less than 6
- D. At least 6, but less than 8
- E. At least 8

12.

You are given the following information:

- $\ddot{a}_x = 10$
- ${}^2\ddot{a}_x = 6$
- $i = \frac{1}{24}$

Calculate $Var(\ddot{Y}_x)$.

- A. Less than 90
- B. At least 90, but less than 100
- C. At least 100, but less than 110
- D. At least 110, but less than 120
- E. At least 120

13.

A special fund is established by collecting an amount of M from each of 1,000 independent lives age 75:

- The fund will pay \$10,000, payable at the end of the year of death, to each person who dies before age 77.
- The fund will pay M , payable at age 77, to each person who survives to age 77.
- Mortality follows the Illustrative Life Table.
- Interest rate $i = 6\%$.
- M is calculated according to the equivalence principle.

Calculate M .

- A. Less than 4,500
- B. At least 4,500, but less than 5,000
- C. At least 5,000, but less than 5,500
- D. At least 5,500, but less than 6,000
- E. At least 6,000

14.

A company is determining the amount it should charge for its annual subscriptions.

You are given the following information:

- The subscription fee is paid at the beginning of the year.
- Using a homogenous Markov chain model with states S = Subscribed and C = Cancelled, the company has determined that:
 - A price of \$80 results in transition matrix Q^A :

$$Q^A = \begin{matrix} & \begin{matrix} S & C \end{matrix} \\ \begin{matrix} S \\ C \end{matrix} & \begin{bmatrix} 0.80 & 0.20 \\ 0.02 & 0.98 \end{bmatrix} \end{matrix}$$

- A price of $\$X$ results in transition matrix Q^B :

$$Q^B = \begin{matrix} & \begin{matrix} S & C \end{matrix} \\ \begin{matrix} S \\ C \end{matrix} & \begin{bmatrix} 0.90 & 0.10 \\ 0.05 & 0.95 \end{bmatrix} \end{matrix}$$

- Transitions occur at the end of each year.
- $i = 10\%$
- A price of $\$X$ produces the same actuarial present value of revenue as a price of \$80 over 3 years for a currently subscribed customer whose subscription is due to expire tomorrow.

Calculate X .

- A. Less than \$62.50
- B. At least \$62.50, but less than \$65.00
- C. At least \$65.00, but less than \$67.50
- D. At least \$67.50, but less than \$70.00
- E. At least \$70.00

15.

You are given the following information:

- Policyholders are in one of three states based on the number of claims they had in the previous year: 0, 1, or 2.
- It is impossible for a policyholder to have more than two claims in a given year.
- Their experience in the next year is described by the following transition matrix Q :

$$Q = \begin{pmatrix} 0.90 & 0.08 & 0.02 \\ 0.80 & 0.15 & 0.05 \\ 0.70 & 0.20 & 0.10 \end{pmatrix}$$

- The current average cost for each claim is 5,000.
- Interest rate is 6%.
- Claim inflation rate is 6%.
- There are 1,000 policyholders, none of whom have had claims in the previous year.

Calculate the expected present value of the total cost of claims over the next two years.

- A. Less than 1,000,000
- B. At least 1,000,000, but less than 1,250,000
- C. At least 1,250,000, but less than 1,500,000
- D. At least 1,500,000, but less than 1,750,000
- E. At least 1,750,000

Fall 2015 Exam LC Solution Key

1. D

2. D

3. E

4. A

5. B

6. A

7. C

8. E

9. D or C

10. E

11. A

12. C

13. B

14. B or E

15. C or B