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4 HOURS

# Exam 9

## Financial Risk and Rate of Return

### INSTRUCTIONS TO CANDIDATES

1. This 52.5 point examination consists of 25 problem and essay questions.
2. For the problem and essay questions, the number of points for each full question and part of a question is indicated at the beginning of the question or part. Answer these questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors.
  - Write your Candidate ID number and the examination number, 9, at the top of each answer sheet. Your name, or any other identifying mark, must not appear.
  - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – DO NOT WRITE ON THE BACK OF THE PAPER. Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as “Page 1 of 2” on the first sheet of paper and then “Page 2 of 2” on the second sheet of paper.
  - The answer should be concise and confined to the question as posed. When a specified number of items are requested, do not offer more items than requested. For example, if you are requested to provide three items, only the first three responses will be graded.
  - In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.
3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

4. Prior to the start of the exam, you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.
  - Verify that the table of the Normal Distribution is attached to the examination after the last question.
5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.
6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.
7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Nothing written in the examination booklet will be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.
8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

Candidates may obtain a copy of the examination from the CAS Web Site.

All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.
9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.
10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 16, 2013.

**END OF INSTRUCTIONS**

Exam 9, Spring 2013

1. (1.5 points)

Given the following information about investment options:

- $U = E(r_c) - 0.5A\sigma_c^2$
- The risk-free rate of return is 3%.
- The risk premium on the risky portfolio is 5%.
- The reward-to-volatility ratio of the risky portfolio is 0.25.
- The risk aversion parameter is 2.

a. (0.75 point)

Calculate the certainty equivalent rate of the risky portfolio.

b. (0.5 point)

Given that 40% of the available assets are invested in the risky portfolio and the rest in the risk-free asset, calculate the expected return and standard deviation of the complete portfolio.

c. (0.25 point)

Calculate the percentage of available assets to invest in the risky portfolio to maximize utility.

Exam 9, Spring 2013

2. (1 point)

Contrast the topics of risk pooling and risk sharing, including their application to insurance companies.

Exam 9, Spring 2013

3. (3.25 points)

Given the following information:

- The expected risk premium of the market index is 8%.
- The standard deviation of the market index is 20%.
- A single-factor model is used to construct an optimal risky portfolio with two components:
  - i. A passive portfolio containing the market index
  - ii. An active portfolio containing some combination of three stocks, with the following estimated parameters:

Stock <sub>i</sub>	$\alpha_i$	$\sigma^2(e_i)$	$\beta_i$
1	0.06	0.16	0.8
2	0.10	0.20	1.0
3	0.15	0.24	1.2

Calculate the portion of the optimal risky portfolio that is invested in the active portfolio.

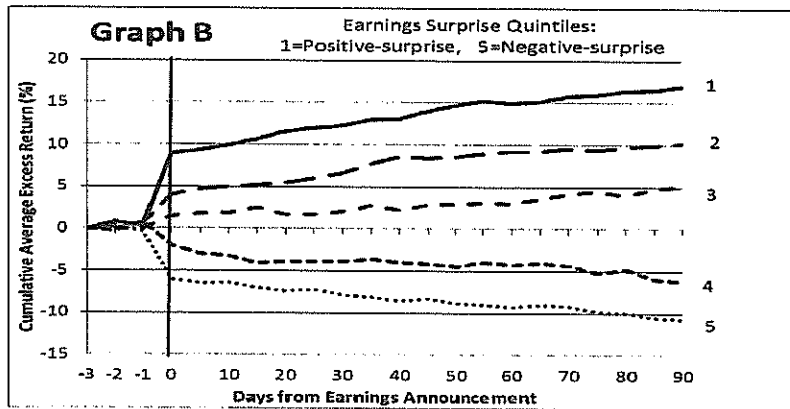
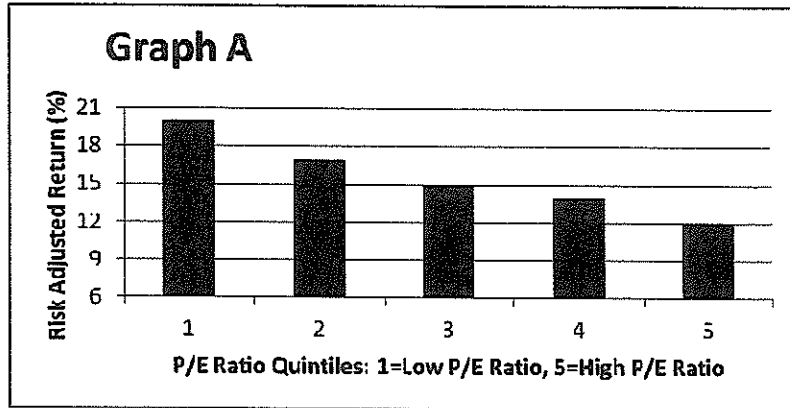
4. (1 point)

An investment manager currently invests all assets in a market index fund with  $\beta$  equal to 1.0. To reduce the risk, the manager plans to sell the market index fund and invest entirely in a single stock with  $\beta$  equal to 0.75.

Using the Capital Asset Pricing Model (CAPM), fully discuss the impact of the manager's strategy on the overall risk of the investment.

5. (2.5 points)

Consider the following graphs in the context of the semi-strong form of the efficient market hypothesis:



a. (0.5 point)

Describe the market anomaly associated with Graph A.

b. (0.5 point)

Describe the market anomaly associated with Graph B.

c. (1 point)

For each anomaly described in part a. and part b. above, describe an information-processing error that would cause the anomaly.

d. (0.5 point)

For each graph describe, either in words or graphically, what it would look like without the anomaly identified in part a. and in part b. above.

Exam 9, Spring 2013

6. (1.5 points)

Given the following information:

- There are no defaults, late payments or prepayments.
- A bank currently offers the following annual rates for one-year and five-year deposits and one-year and five-year loans and is considering modifications to the rates.

Time to Maturity (Years)	Deposit Rate	Loan Rate
1	3.5%	7%
5	3.5%	7%

Using a theory on the term structure of interest rates, describe a risk of financial distress faced by the bank and propose an action the company could take to resolve this risk.



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7. (2.5 points)

Given the following information on a currency swap:

- The swap has a remaining life of 15 months.
- The swap involves exchanging interest at 9% on £20 million for interest at 5% on \$30 million at the end of each year.
- Principal amounts will be exchanged when the swap term expires.
- The term structure of interest rates in both the United Kingdom and the United States is currently flat and if the swap were negotiated today, the interest rates exchanged would be 6% in pounds sterling and 4% in dollars.
- The current exchange rate (dollars per pound sterling) is 1.65.
- All interest rates are quoted with annual compounding.

Viewing the swap as a portfolio of forward contracts, calculate the current value of the swap to the party paying pounds sterling.

Exam 9, Spring 2013

8. (3.5 points)

Given the following information:

- The projected calendar year loss reserve payout as of December 31, 2012 is:

Accident Year	Calendar Year Payments (\$000,000)	
	2013	2014
2012	\$10	\$10
2011	\$10	

- Losses are paid at the end of each year.
  - Liabilities are discounted using the bond yield curve rates.
- The asset portfolio is composed of:

Asset	Market Value (\$000,000)
Cash	\$25
Bonds	\$75

- The duration of cash is zero.
- The bond portfolio is composed of an equal investment in the following bonds:

Time to Maturity (Years)	Coupon Rate	Face Value	Bond Price
1	5%	\$1,000	\$1,020
2	6%	\$1,000	\$1,040

- Bond coupons are paid annually.

Calculate the duration gap of the market value surplus for the insurer as of December 31, 2012.

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9. (1.5 points)

You are given the following information regarding an insurance company.

Written Premium	\$200 million
Loss	\$150 million
Expense	\$50 million
Surplus	\$75 million
Target return on surplus	15%
Client Retention	90%
Risk-free interest rate	5%

Assume premium is collected and expenses are paid at the beginning of the year, and losses are paid at the end of the year.

Calculate the total economic value of the firm.

Exam 9, Spring 2013

10. (2.75 points)

Given the following information regarding a firm:

- The current volatility of the firm's assets is 21.8%.
- The notional value of the firm's debt due in three years is \$1 billion.
- The risk-free interest rate is 5% per annum.
- The risk-neutral probability of default is 10%.

Using Merton's Model, calculate the current value of the firm's debt.

11. (1.5 points)

Recent global market developments and financial innovations have transformed the nature of liquidity risk. Identify two of these developments or innovations and explain how each adds to the challenges of managing liquidity risk.

12. (2 points)

Suppose that three identical bonds are pooled and tranced to create a collateralized debt obligation (CDO). Each bond has the following characteristics:

- The probability of default is 5%.
- Each bond will pay \$0 in the event of a default and \$1 million otherwise.

The CDO is structured as follows:

- The first (or junior) tranche bears the first \$1 million of loss.
- The second (or mezzanine) tranche bears the second \$1 million of loss.
- The third (or senior) tranche bears loss only after the other two tranches have been exhausted.

a. (1 point)

Calculate the difference between default probabilities for the mezzanine tranche assuming bond defaults are perfectly correlated versus completely uncorrelated.

b. (1 point)

Explain how underestimating default probabilities for senior tranches of collateralized debt obligations led to the rise and fall of structured finance.

13. (2.25 points)

A company is considering two risk management strategies. It will either issue a CAT bond or enter into a catastrophe risk swap.

The company's decision will be based on the following three concerns:

- Counterparty default risk
- Basis risk
- Risk diversification

Evaluate the company's two risk management strategies based on each concern and choose the better strategy for the company.

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14. (2.5 points)

A company currently writes coverage for Line A, and is considering expanding into Line B. For both lines, all losses will be paid at time 1. The table below shows the probability distribution of nominal loss amounts.

Probability	Loss for Line A	Total Loss for Line A and B
0.4	\$0	\$0
0.3	\$6,000	\$11,000
0.1	\$8,000	\$12,000
0.1	\$0	\$12,000
0.1	\$14,000	\$23,000

You are also given the following information about the company:

- All assets are invested in risk-free securities.
- The risk-free interest rate is 2.04%.
- Both lines are priced at an expected loss ratio of 100%.
- The company pays no expenses.
- To support Line A on a stand-alone basis, the company requires \$5,800 of risk capital at time 0.
- The company uses a fixed expected policyholder deficit (EPD) ratio to determine its required risk capital.

Calculate the amount of additional capital the company would require at time 0 to support its expansion into Line B.



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15. (2 points)

Management is evaluating two potential new lines of business to pursue next year and has developed the following metrics:

	Line A	Line B
Expected premium	\$9.7 million	\$8.3 million
Required capital	\$9.5 million	\$4.5 million
Expense ratio	8.00%	6.30%
Discounted loss ratio	78.30%	87.00%

For both lines:

- Investment return is 6.5%.
- Expenses are paid immediately.
- Capital is released at the end of the year.
- Losses are paid at the end of the year.
- Target risk-adjusted rate of return on capital is 18%.

Propose which line(s), if any, to pursue and discuss how the recommendation will be impacted by available capital.

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16. (1.5 points)

a. (1 point)

Discuss the importance of economic capital to:

- Policyholders
- Shareholders

b. (0.5 point)

Contrast risk-based capital measures with market value capital.

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17. (1.5 points)

Consider the following assumptions for a new policy:

Expected losses	\$100,000
Expenses	\$20,000
Policyholder supplied funds as a ratio to premium	43.6%
Tax rate	35%
Before-tax return on invested assets	6%
Target return on equity	8%
Surplus-to-premium ratio	2.0
Surplus-to-equity ratio	1.0

a. (1 point)

Using the calendar year return on equity method, calculate the indicated price for the policy.

b. (0.5 point)

Briefly describe two ways that risk is reflected in the calendar year return on equity method.

18. (2.5 points)

An actuary has made the following assumptions for the purpose of pricing an auto policy for ABC Insurance Company:

- Premium is collected in full on the effective date of the policy.
- Fixed expenses equal to \$50 are incurred and paid on the effective date of the policy.
- Variable expenses equal to 15% of premium are incurred and paid on the effective date of the policy.
- Expected future loss payments to be paid at the end of each year:
  - \$400 in year 1
  - \$200 in year 2
  - \$150 in year 3
- The surplus requirement is set to 90% of loss reserves at the beginning of each year.
- Taxes on investment income and underwriting profit are paid at the end of each year at a rate of 35%.
- The risk-free interest rate is 3%, and the pre-tax return on the investment portfolio is 5.5%.
- The market risk premium is 5%.
- ABC is a publicly traded mono-line auto insurance carrier. The expected rate of return on ABC's stock is 7%.
- ABC invests its assets in portfolio XYZ, which yields an expected rate of return of 5.5%.

Using the risk adjusted discounted cash flow model, determine the underwriting profit provision for the policy.

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19. (3 points)

Actuaries analyzing an annual workers' compensation policy have estimated the following:

Expected losses	\$1,200
Expense ratio	35%
Risk-free interest rate	5%
Investment return of the company	11%
Overall market return	9%
Cost of capital	8%

In addition, the actuaries assume the following:

- Losses will be paid according to the following pattern:
  - \$420 at the end of year 1
  - \$780 at the end of year 3
- A loss reserve is set up on the effective date of the policy and decreases as losses are paid.
- Expenses are paid on the effective date of the policy.
- A tax rate of 35% applies to investment income.
- There is no underwriting income tax.
- The company carries surplus equal to 75% of undiscounted loss reserves.
- Equity flows occur annually.

Calculate the minimum premium necessary for this policy to be profitable.

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20. (2 points)

Given the following for an insurance policy:

Premium	\$500
Expected loss ratio	70%
Expected combined ratio	105%
Investment return of the company	11%
Overall market return	9%
Risk-free interest rate	6%

- Taxes are not considered.
- Losses will be paid at the end of the year according to the following pattern:

Year 1	40%
Year 2	35%
Year 3	25%

- Premium and expenses are paid on the effective date of the policy.

a. (1 point)

Using the risk-free interest rate, calculate the net present value of the underwriting cash flows for this policy.

b. (0.5 point)

Describe what the net present value calculated in part a. above represents to the insured.

c. (0.5 point)

The CEO of the company uses the net present value calculated in part a. above to represent the expected profit for this policy. Explain whether the CEO is correct or incorrect.

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21. (3 points)

A property insurer writes two accounts with the following modeled events:

Event	Probability	Loss for Account X	Loss for Account Y
1	0.5%	75,000	35,000
2	1.0%	50,000	25,000
3	2.0%	40,000	10,000
4	5.0%	25,000	5,000

- Target return on equity = 15%.
- Probability of ruin = 2.5%.
- Account X is written first.

a. (2 points)

Under the build up scenario, use the marginal surplus method to calculate the risk load for account X and account Y.

b. (0.5 point)

Calculate the marginal surplus required to support Account Y.

c. (0.5 point)

Briefly describe why the marginal surplus method fails to be renewal additive.

22. (2.25 points)

The following information is available for an insurance policy:

- The expected loss amount for the policy is \$500.
- The standard deviation of the loss is \$3,000.
- The loss safety level is set at \$10,000.
- The target investment return is 12%, with a standard deviation of 30%.
- The risk-free interest rate is 3%.
- No taxes or expenses are associated with the policy.

Using the swap technique, calculate the policy premium.



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23. (2 points)

a. (1 point)

Two techniques for setting reinsurance risk loads are the swap technique and the option technique. Describe the investment strategies that underlie each.

b. (0.5 point)

Briefly describe the two constraints that must be satisfied in both the swap and option techniques.

c. (0.5 point)

A reinsurer has calculated indications for allocated asset and risk load amounts for all four combinations of financial techniques and constraints. Identify the steps the reinsurer would go through to select the risk load for this treaty.

24. (1.5 points)

You are given the following information for an insurance company:

Return on assets	5%
Underwriting profit	3%
Insurance leverage ratio	0.5
Premium-to-surplus ratio	1.5

a. (0.5 point)

Calculate the total return on equity.

b. (0.5 point)

Explain why investment return on assets might increase in a period of increased profitability

c. (0.5 point)

Explain why investment return on assets might decrease in a period of increased profitability

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25. (2 points)

Given the following for an insurance company:

Invested assets	\$30,000,000
Owners' equity	\$15,000,000
Reserve-to-equity ratio	1.0
Investment return	6%
Insurance operating results as a percent of reserves	5%

a. (1 point)

Calculate the return on owners' equity.

b. (1 point)

Discuss the impact of non-equity financing on the quality of an insurer's earnings.

# Exam 9

## Financial Risk and Rate of Return

May 2, 2013

### POINT VALUE OF QUESTIONS

QUESTION	TOTAL POINT VALUE OF QUESTON	SUB-PART OF QUESTION					
		(a)	(b)	(c)	(d)	(e)	(f) (g)
1	1.50	0.75	0.50	0.25			
2	1.00	1.00					
3	3.25	3.25					
4	1.00	1.00					
5	2.50	0.50	0.50	1.00	0.50		
6	1.50	1.50					
7	2.50	2.50					
8	3.50	3.50					
9	1.50	1.50					
10	2.75	2.75					
11	1.50	1.50					
12	2.00	1.00	1.00				
13	2.25	2.25					
14	2.50	2.50					
15	2.00	2.00					
16	1.50	1.00	0.50				
17	1.50	1.00	0.50				
18	2.50	2.50					
19	3.00	3.00					
20	2.00	1.00	0.50	0.50			
21	3.00	2.00	0.50	0.50			
22	2.25	2.25					
23	2.00	1.00	0.50	0.50			
24	1.50	0.50	0.50	0.50			
25	2.00	1.00	1.00				
TOTAL	<u>52.50</u>						

# Tables of the Normal Distribution



## Probability Content from $-\infty$ to $Z$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

### Values of $z$ for selected values of $\Pr(Z < z)$

z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr (Z < z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995

Exam 9

**Question 1a)**

Model Solution 1

a)  $r_f = .03$   
 $E(r_p) = .05 + .03 = .08$

$$S = \frac{E(r_p) - r_f}{\sigma_p} \rightarrow .25 = \frac{.08 - .03}{\sigma_p}$$

- $\sigma_p = .20$
- $U_p = .08 - .5(2)(.2)^2 = .04$
- $U_{ce} = .04 = E(r_{ce}) - 0$
- Certainty equivalent rate = .04

Model Solution 2

a)

- $A = 2$        $E(r_p) - r_f = 0.05$
- $r_f = 0.03$        $\frac{E(r_p) - r_f}{\sigma_p} = 0.25$

$$E(r_p) = 0.08 \qquad \frac{0.05}{\sigma_p} = 0.25 \qquad \sigma_p = \frac{0.05}{0.25} = 0.2 \qquad \sigma_p^2 = 0.04$$

$$U_p = 0.08 - 0.5(2)(0.04) = 0.04$$

Certainty equivalent Rate = 0.04

Model Solution 3

a) Certainty equivalent rate = the rate of the risk free rate that produces the same U as the .... portfolio

- $U_p = E(r_p) - 0.5Ar^2f$       where  $E(r_p) = 0.05 + 0.03 = 0.08$

$$= 0.08 - 0.5 \cdot 2 \cdot 0.2^2$$

$$= 0.04$$

$$\text{sharpe ratio} = \frac{E(r_p) - r_f}{\sigma_p}$$

$$= \frac{0.05}{0.25} = 0.2$$

- $U_f = E(r_f) - 0.5Ar^2f$   
 $= E(r_f)$   
 Thus,  $E(r_f) = 0.04$  SO the certainty equivalent rate is 4%

**Question 1b)**

Model Solution 1

$$G(r_c) = .4G(r_p) + .6r_f = .4(.08) + .6(.03) = .05$$

$$S = \frac{r_c - r_f}{G_c} = \frac{.08 - .03}{G_c} = .25 \rightarrow G_c = .08$$

Model Solution 2

$$w_p^2 = 0.4 \quad w_{r_f} = 1 - w_p = 0.6$$

$$E(r_c) = 0.4(0.08) + 0.6(0.03) = 0.05$$

$$E(r_c) = 0.05$$

$$\sigma^2 = w^2p \sigma^2 + w^2r_f \sigma^2r_f + 2 w_p w_{r_f} Cov(p, r_f)$$

$$G_c = 0.08$$

$$= 0.4^2 0.2^2 + 0.6^2(0)^2 + 2(0.4)(0.6)(0)$$

$$= 0.0604 \quad G_c = 0.08$$

Model Solution 3

$$E(r_c) = 0.4(0.08) + 0.06(0.03) = 0.05$$

$$G_c = 0.4(0.2) = 0.8$$

**Question 1c)**

Model Solution 1

$$V = E(r_c) - 0.5(A)\sigma^2$$

$$V = YE(r_p) + (1 - y)r_t - 0.5(A)y^2\sigma_p^2$$

$$dv/dy = E(r_p) - r_t - 0.5(A)(2)\sigma_p^2y = 0$$

$$E(r_p) - r_f = (A)\sigma_p^2y$$

$$y = \frac{E(r_p) - r_f}{A\sigma_p^2} = \frac{0.08 - 0.03}{2(0.2)^2} = 0.625$$

Invest 62.5% of available assets in the risk portfolio.

Model Solution 2

$$Y^* = \frac{E(r_p) - r_t}{A\sigma_p^2} = \frac{0.05}{2(0.2)^2} = 0.625$$

## Question 2)

### Model Solution 1

Risk pooling refers to adding a number of uncorrelated risks together. In the context of insurance it would be adding to a portfolio, many independent insurance exposure together. In doing so, the Sharpe ratio of the portfolio increases. Assuming each risk share an expected risk premium of  $R$  and standard deviation of  $S$ . Charge Ratio is  $R$ . Adding  $n$  independent risk increase sharpe ration to  $\sqrt{n} \times \frac{r}{s}$  for the total portfolio. ( $R$  increase to  $nR$  and  $S$  to  $\sqrt{n} S$ ). Risk sharing assume that we keep the same size of portfolio buy diversity between risky anet. With risk sharing total risk premium of portfolio will still be  $R$  but the standard deviation reduces to  $\frac{s}{\sqrt{n}} \Rightarrow$  the sharpe  $\frac{r}{\sqrt{n}} = \sqrt{n} \frac{r}{s}$ . Sharpe ratio increases and standard deviation decrease therefore Risk pooling is not enough to reduce risk there has to be risk sharking.

### Model Solution 2

Risk pooling refers to the practice of adding uncorrelated risks to reduce risk. However, what actually reduces is the risk per unit, not total risk (so sharpe ratio increases, but total risk also increases). This is what insurance companies are in the business of doing

Risk sharing refers to selling shares of an attractive risky portfolio. So that total investment remains constant. It is spreading an investment of a fixed size over multiple uncorrelated risks. Unlike risk pooling, this will actually reduce total risk. It will continue to increase the sharpe ratio. Insurance compares can produce risk sharing by ceding business to reinsurers share for example Write more uncorrelated risks, than cede a portion of total portfolio so that the total amount of insurance written (net of reinsurance) is the same as before.

### Model Solution 3

Risk pooling is piling on correlated risks into a portfolio, so that reward-risk-increases. Through diversification, it allows for a smaller avg risk; however, because of increase in size of portfolio, total risk increases as well.

Risk sharing is taking a fixed amount of money and instead of investing it into 1 asset I, investing inot different correlated assets. This way, reward-to-risk ratio  $\uparrow$  Because of diversification, Avg Risk  $\downarrow$  as before, but since now the exposure to risk is not increased Total risk  $\downarrow$  as well. Thus it is the right away to  $\uparrow$  Reward-to-Risk ration and  $\downarrow$  Risk.

It is a similar idea when applied to insurance. If an insurer writes more & more polices, via diversification, its  $\frac{\text{reward}}{\text{risk}} \uparrow$ , but because of increased total exposure to loss, its total risk  $\uparrow$  too. (Risk Pooling) If however the insurer takes its portfolio above + spreads its risk by among many investors, its reward-to risk ratio  $\uparrow$ , while total risk  $\downarrow$ . (Risk Sharing)



### Question 3

#### Model Solution 1

Stock	$W^0_i = \frac{\sigma_i}{\sigma^2 e_i}$	$W_i = \frac{W_i^0}{\sum W_i^0}$	$W_i^* = W_i \times W_i^*$
1	.375 = .06/.16	0.25 = .375/1.5	=.1922 = (.25 × .7687)
2	.5	0.333	.2562
3	.625	0.4167	.3203
Sum	1.5	1	

1. We calculate initial weights  $W^0_i = \frac{\sigma_i}{\sigma^2 e_i}$
2. We then normalize those weights so that they add up to 1  $W_i = \frac{W_i^0}{\sum W_i^0}$
3. Use those weights to calculate  $\sigma_a = \sum w_i \sigma_i = (.25) \times .06 + (.333) \times .1 + (.4167) \times .15 = .1108$   
 $\sigma^2 a = \sum w_i^2 \sigma_i^2 = (.25)^2 (.16) + (.333)^2 (.1) + (.4167)^2 (.225) = .0739$
4. Calculated the initial weight in active portfolio:  $W A^0 = \frac{\sigma_a / \sigma^2 a}{E(R_m) \sigma^2 m} = \frac{.1108 / .0739}{.08 / (.2)^2} = .7497$
5. Calculate:  $W A^* = \frac{W A^0}{1 + (1 - \beta) W A^0} = \frac{.7497}{1 + (1 - 1.033) .7497} = .7687$
6. The portion of the optimal risky portfolio invested in Active is  $W a^* = .7687$

### Question 4)

#### Model Solution 1

Investment Manager (From here on: "I M") wants to reduce risk.

Risk of an investment is measured by its volatility  $\sigma$

IM wants to reduce  $\sigma$  of the investment. The current investment (market index)  $\beta=1$  will only have systematic risk (non-syst. Risk diversified away). Here  $\sigma(e_i) \approx \emptyset$

The proposed investment will have the following volatility @  $\beta = .75$ :  $\sigma^2 = \beta^2 \sigma^2 m + \sigma^2 (e_i)$

The new investment will have a smaller systematic risk component  $\beta^2 \sigma^2$  but will have an additional firm specific risk  $\sigma^2(e_1)$  which will increase total  $\sigma$

IM believes the additional risk  $\sigma^2(e_1)$  Will be smaller than the decrease in systematic risk  $\beta^2\sigma_m^2$

However, return only reward systematic risk. Therefore, IM will be losing returns disproportionate to the change in risk.

Would not advise- losing diversification benefit of risk sharing!

### Question 5a)

#### Model Solution 1

a.) P/E effect: .../low P/E ratio Earn Higher Risk Adjusted Returns on Average.

#### Model Solution 2

a.) P/E ratio effect: the lower P/E ratio companies tends to generate a superior return. Since P/E ration is public information, by efficient market hypothesis, this should not happen.

### Question 5b)

#### Model Solution 1

b.) Post Earnings Announcement Drift: prices take time to react to New information given in earnings announcements

#### Model Solution 2

B.) Post-earning announcement drifting effect the compact of earning announcements seems to continue to impact the stock price for an extended period after the announcement. By efficient market hypothesis, prices should be reflected instantaneously when the announcement is made.

### Question 5c)

#### Model Solution 1

c. P/E forecasting Errors → Giving too much weight to recent Data such as earnings Data → Giving too High Price.

Drift= Conservatism → not adjusting expectations, given new data quickly, takes time for expectations to change.

#### Model Solution 2

c. Graph A- Forecast error- investors put too much weight into recent data (in this case) and derive an incorrect mobility Distribution- investors put too much weight into the recent earning, and forecast that the stock is going to under perform.

Graph B- Conservatism- Investors are often too slow/conservative in updating this portfolio to reflect updated information. Investors tend to be too slow to buy more shares ( which drives up the share price) after positive surprises,... up the stock price for an extend period of time.

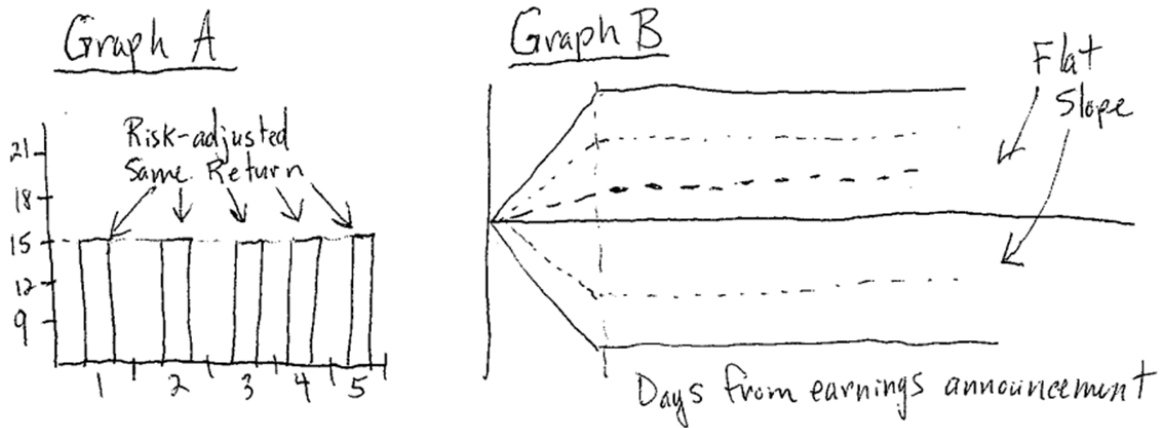
**Question 5d)**

Model Solution 1

Graph A: Each quintile should have same risk-adjusted return

Graph B: Excess return would immediately reflect new price data. No slope, vertical change, then flat

Model Solution 2



**Question 6)**

Model Solution 1

Under the liquidity preference theory, investors will tend to choose shorter term investments if the yield is the same as the larger term yield. For investor liabilities, the opposite is true; borrowers will be done at a long term if yields are the same.

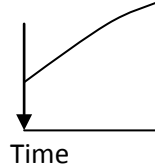
This is the cause here. If the liq. Pref. theory is correct, the bank is at risk: depositors will always choose to deposit at the 1-year rate, and loan buyers will always lock in the 5-year rate since this is a safer choice for them for guaranteeing available funds for no cost + for longer

The risk if interest rates rise significantly, depositors will demand a higher rate, but the loans will still largely be locked in at 7%. The bank needs to duration match assets + liabilities. To achieve this, the short term deposit and loan rate could be reduced until investors find them attractive in proportions that achieved the duration match.

Model Solution 2

Current dates suggest that yield curve is flat so expectations of future short rates are equal to forward rates and there is no liquidity premium reflected in rates to compensate people for hold onto investments longer (due to pricing risk or investment risk).

Typically investments w/ longer times maturity reflect a liquidity premium so you int. rate term struct might look like this →



If the bank kept its rates the same it will likely attract too many short term investors because their deposit rates for long for long term investors is too low. Similarly, they mat attract too many long term borrows b/c their loan rat is low relative to short term loan rate. If they have too many people only depositing money for a short time and too many people borrowing for long time, often they will face liquidity issues when different obligations are due

### Model Solution 3

→Solution would be to increase deposit rate for 5 yrs period and decrease loan rate for 1 year period to attract appropriate amount of customers in each time to maturity market (segmentation theory) and be able to manage liquidity risk as a whole much better.

-Yield Curve is positive sloping due to a higher percentage of short term investors who need to be compensated to give up that liquidity and invest in longer term investments.

- the structure in the example shows the same role for both 1 and 5 year investments. Most likely, more people are investing short term (1 year) and borrowing long term (5-year) due to the structure.

→This causes the bank to have very mismatched Asset-liability matching in durations.

→If interest rates increase, everybody is investing short term and can then take advantage of higher rates and the bank will still have longer loans out at old rate.

Solution: Structure the rates to be increasing with longer maturities to get more balance between short and long term borrowers and investors.

**Question 7)**

Model Solution 1

<u>Time</u>	<u>Dollars</u>	<u>Pounds</u>	<u>Fx rate</u>	<u>Pounds as\$</u>	<u>CF</u>	<u>PV CF</u>
0			1.65			
3 mths	1.5M	21.8M	$1.65 \left(\frac{1.04}{1.06}\right)^{.125}$	2.96 M	-1.46	-1.44M
15 mths	31.5M	21.8M	$1.65 \left(\frac{1.04}{1.06}\right)^{1.25}$	35.12 M	-3.62	-3.45M
						<hr/>
						-4.89M
						↑
						Use 4% discount rate

### Question 8)

#### Model Solution 1

$$DG_S = \frac{MVA \times D_A - MVL \times D_L}{MVS}$$

Finding Duration of bonds portfolio

$$P = \frac{\sum PV(Cf_t) \times t}{\sum PV(Cf_t)} = \frac{304.754}{2060} = 1.4766$$

$$\sum PV(Cf_t) = 1020 + 1040 + 2060$$

$$\sum PV(Cf_t) \times 6$$

$$1020 = \frac{1050}{1+r_1} = r_1 = .0294$$

$$= \frac{1050+60}{1.0294} + \frac{1060 \times 2}{1.0391^2} = 3041.754$$

$$1040 = \frac{60}{1+r_1} + \frac{1060}{(1+r_2)^2}$$

$$1040 = \frac{60}{1+r_1} + \frac{1060}{(1+r_2)^2}$$

$$(1+r_2)^2 = 1.07974 \quad r_2 = .0391$$

$$MVA = 100M \quad D_{mva} = (.75)(1.4766) + (.25)(0) = 1.10745$$

Loss Payments: 2013/20 2014/10

$$Duration_L = \frac{\sum PV(Cf_t) \times G}{\sum PV(Cf_t)} = \frac{37.952}{28.69} = 1.323$$

$$\sum(Cf_t) = \frac{20}{1.0294} + \frac{10}{1.03912} = 28.69$$

$$\sum(Cf_t) \times t = \frac{20}{1.0294} + \frac{10 \times 2}{1.0391^2} = 37.952$$

$$D_L = 1.323$$

$$MVL = 28.69M$$

$$DG_S = \frac{100(1.17045) - 28.69(1.323)}{100 - 28.69}$$

$$= 1.0207$$

#### Model Solution 2

$$\frac{1050}{1+r_1} = 1020 \quad r_1 = 2.94\%$$

$$\frac{60}{1.0294} + \frac{1060}{(1+r_2)^2} = 1040 \quad \frac{1060}{(1+r_2)^2} = 981.71 \quad r_2 = 3.91\%$$

### Bonds

T	PMT	PV	PV·T
1	1110	1078.30	1078.30
2	1060	<u>981.73</u>	<u>1963.46</u>
		2060.03	3041.76

$$\text{Duration} = \frac{3041.76}{2060.03} = 1.4766$$

### Losses

T	PMT	PV	PV·T
1	20M	19,428,793	19,428,793
2	10M	<u>9,261,585</u>	<u>18,523,170</u>
		28,690,378	37,951,963

$$\text{Duration} = \frac{37,951,963}{28,690,378} = 1.32228$$

$$\text{Duration of assets} = \frac{25(0) + 75(1.4776)}{100} = 1.1075$$

Value of liabilities = 28,690,378

Mv surplus = 100M – 28,690,378 = 71,309,622

$$\text{Dur MVs} = \frac{1.1705(100M) - 1.3228(28,690,378)}{71,309,622}$$

Dur = 1.0209

Dur Gap = 1.0209 – 0 = 1.0209

### Model Solution 3

Spot Rates

$$1020 = \frac{1050}{1+s_1} \Rightarrow S_1 = 2.94\%$$

$$1040 = \frac{60}{1+s_1} \times \frac{1020}{1050} + \frac{1060}{(1+s_2)^2} \Rightarrow S_2 = 3.91\%$$

$$\text{PV}(L_{2012}) = \frac{10}{1.0294} + \frac{10}{(1.0351)^2} = 18.98$$

$$PV(L_{2011}) = \frac{10}{1.0294} = 9.71$$

$$D_{2012} = \frac{1}{PV(L_{2012})} \times \left( 1 \times \frac{10}{1.0294} + 2 \times \frac{10}{(1.0391)^2} \right) = \frac{28.24}{18.98} = 1.49$$

$$D_{2011} = \frac{1}{PV(L_{2011})} \times \frac{10}{1.0294} = 1$$

$$D_{MVL} = \frac{18.88 \times 1.49 \times 9.71 \times 1}{18.98 + 9.71} = 1.32$$

$$MVL = 18.98 + 9.71 = 28.69$$

Duration of Bonds

$$D_{B1} = 1$$

$$D_{B2} = \frac{1}{1040} \left( \frac{60}{1.0294} + \frac{2 \times 1060}{(1.0391)^2} \right) = 1.94$$

$$D_B = \frac{1020 \times 1 + 1040 \times 1.94}{1020 + 1040} = 1.47$$

$$D_{MVA} = \frac{0 \times 25 + 1.47 \times 75}{100} = 1.10$$

Duration Surplus

$$D_{MVS} = \frac{MVA \times D_{MVA} - MVL \times D_{MVL}}{MVA - MVL} = \frac{100 \times 1.1 - 28.69 \times 1.32}{100 - 28.69} = \frac{72.13}{71.31} = 1.01$$

Duration Gap MVS =  $D_{MVS} = 1.01$

MVS discounted lab

MKV of assets

$$r_1 \ 2) \ 1020 = 1050 (1 + r_1)^{-1} \qquad i_1 = 2.94\%$$

$$r_2, \ 2) \ 1040 = 60 (1 + r_1)^{-1} + 1060 (1 + r_2)^{-2} \qquad i_2 = 3.91\%$$



### Question 9)

#### Model Solution 1

- $TEV=C+F$
- $C= P-E+S$
- $= 200-50+75- 150/1.05$
- $= 82.14 \text{ Mil}$

testing P

$$p = \frac{5(15\% - 5\%) + 1}{1.05} + E = 200 \text{ Mil}$$

- $F = (P-E-L) \frac{d}{1+y} \frac{1}{1-d}$

$$d = \frac{cr}{1+y} \quad 1-d$$

- $= (200-50-150) \frac{.9}{1.05} \frac{1}{.15}$

$$= .9 / 1.05 = 0.15 / 1.05$$

- $= 42.86 \text{ Mil}$

$$TEV = 42.86 \text{ mil} + 82.14 \text{ Mil} = 124.997 \text{ mil} \approx 125 \text{ mil}$$

#### Model Solution 2

$$TEV = CEV + FV$$

- Using formulae from Panning Paper:

- $CEV = S + \frac{(k-y)S}{1+y} = 82.14$

$$FV = \frac{(k-y)S}{1+y} \times \frac{cr}{1+y-cr} = 42.86$$

$$TEV = 82.14 + 42.86 = 125.0$$

$$\text{(assuming) } P-E-C/1 + 4 = \frac{(k-y)S}{1+y}$$

Panning formula can be used to calculate the franchise value.

$$F = \frac{c \cdot s (a + by)}{(1+y)(1+y-cr)}$$

Cr= Reflection rate= 90X. Here the pricing does not depend on the.....rate so b=0 a= 15%

$$= \frac{(.9)(75)(.15-.05)}{(1.05)(1.05-.9)} = \$42.857 \text{ M}$$

$$\text{Current value} = S + P - E - \frac{L}{1+y}$$

$$= 75 + 200 - 50 - \frac{150}{1.05}$$

$$= \$82.14 = (75 + 7.14)$$

Total value = \$42.857 + 82.14

Sum of Franchise and current economic value = 125 Mil

### Question 10)

#### Model Solution 1

$$E = VN(d_1) - D e^{-rt} N(d_2)$$

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_v^2/2)T}{\sigma_v \sqrt{T}} \quad d_2 = d_1 - r\sqrt{t}$$

$$N(-d_2) = .1 \rightarrow d_2 = 1.282 \quad d_1 = 1.282 + .218\sqrt{3} = 1.66$$

$$\ln(v) = 1.66(r\sqrt{t}) - t\left(r + \frac{1}{2}r^2\right) = 4.055 \Rightarrow v = 1.5B$$

$$E = 1.5(.9515) - e^{-.15}(.9) = .65B$$

$$Debt = V - E = 1.5 - .65 = .85B$$

#### Model Solution 2

$$\text{Prob of default} = N(-d_2) = 0.1$$

$$d_2 = \frac{(\ln \frac{v}{d}) + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{t}}$$

$$1.282 = \frac{\ln(\frac{v}{1}) + (0.05 - \frac{1}{2}0.218^2) \cdot 3}{0.218\sqrt{3}}$$

$$\ln(v) = 0.4054$$

$$V = 1.4998$$

$$\text{Equity} = 1.4998 N(d_1) - 1 e^{-0.05 \cdot 3} N(d_2)$$

$$= 1.4998 \cdot 0.9515 - e^{-0.05 \cdot 3} \cdot 0.1$$

$$= 0.05245$$

$$\text{Debt} = 1.4998 - 0.05245$$

$$= 0.84738 \text{ billion}$$

### Model Solution 3

$$N(-d_2) = P(\text{default}) = .1$$

$$r = .05$$

$$d_2 = 1.282$$

$$=.04879$$

$$d_2 = d_1 - \sigma_v \sqrt{t}$$

$$1.282 = d_1 - .215 \cdot 3^{.5}$$

$$d_1 = 1.66$$

$$N(d_1) = .9515$$

$$N(d_2) = .9$$

$$d_1 = \frac{\ln\left(\frac{v_0}{p}\right) + \left(r + \frac{\sigma^2 v}{2}\right) \cdot 3}{\sigma v \sqrt{t}}$$

$$1.66 = \frac{\ln\left(\frac{v_0}{p}\right) + (0.4879 + .218 \frac{2}{2}) \cdot 3}{.218 \cdot 3}$$

$$v_0 = 1,505,520$$

$$e_0 = v_0 \cdot d_1 - d e^{-rt} \cdot v(d_2)$$

$$= 1,505,520 \cdot .9515 - 1,000,000 e^{-.048 \cdot 879 \cdot 3} \cdot 9$$

$$= 655,048$$

$$\text{CV debt} = MV_A - MV_e = 1,505,520 - 655,048 = 850,472$$

## Question 11)

### Model Solution 1

- 1) Intra-day payments are now possible instead of everything being settled at the end of the day. This makes liquidity risk more challenging since payments have to be made sooner and funds need to be made available quicker.
- 2) Banks now securitize many of their mortgage loans to make them more liquid. But this process takes a long time and banks can be caught warehousing these risks until securitization process is complete. If there is a market disruption during this process the bank could get caught without the liquid funds it needs.

### Model Solution 2

1. Cross border flows: increased globalization has allowed easy transfers b/w currencies. However, due to this, liquidity shocks are also able to easily ride through into other markets. This might freeze some funds in certain currencies, which cannot easily be converted to meet obligations in other currencies.
2. Complex financial instruments- a lot of people do not fully understand them and may not grasp their liquidity. Since they are fairly new, its tough to tell what kind of liquidity they might have in stress scenarios. Many are also embedded w/ call options; so another aspect of uncertainty with the cash flows.

### Model Solution 3

1. Increase use of complex financial instrument for example CDO
  - i. Due to short history of the instrument, hard to get data to predict cash flow and earning
  - ii. These instruments are not actively trades so hard; so hard to know the market value
  - iii. They have call feature or collateral, hard to predict is value  
Due to above reason it increases the risk of liquidity
2. Increase use of collateral  
Parties use collateral to reduce credit risks, but banks will involve more liquidity risk due to the short notice of collateral calls.

### Model Solution 4

1. The securitization of assets in the marketplace can take time. Due to this time element, assets can lose their market value and as a result actually increase the liquidity risk of the company.
2. Companies receive more funding from the capital markets these days. Because this type of funding can be volatile, this can actually increase the liquidity risk of the company.

### Question 12a)

#### Model Solution 1

Completely correlated  $\rightarrow$   $p_r = 5\%$  (if one defaults, they all default)

$$P=0 \rightarrow 0.5^3 + 3(0.5)^2 \times .95 = 0.725\%$$

$$\text{Diff} = 5\% - 0.725\% = 4.275\% \text{ diff}$$

#### Model Solution 2

Perfect correlation = 5% chance of 3M loss

= mezzanine tranches defaults

No correlation: Looking for P (loss > 2m) at least 2 bonds default

$$= 1 - P(\text{only one bond defaults}) - P(\text{no defaults})$$

$$= 1 - 3 \times 0.05 \times 0.95 \times 0.95 - 0.95^3$$

$$= 0.725\%$$

$$= \text{difference} = 5\% - 0.725\% = 4.275\%$$

### Question 12b)

#### Model Solution 1

Because  $Pr(\text{def})$  was underestimated, the senior tranches were thought to be very safe b/c the junior and mezzanine tranches would absorb defaults 1<sup>st</sup>. As a result, the Sr. tranches received AAA credit ratings. Since bank capital requirements are tied to credit ratings. There was huge demand for AAA securities, so structured finance grew rapidly.

When  $Pr(\text{def}) \uparrow$ , the Sr. tranches became much less safe, and their market value dropped substantially. The structure of CDOs amplified this effect. Those who held Sr. tranches lost a lot of \$ and now demand for CDOs/ structured securities is very low b/c people know how sensitive they are to estimation errors.

#### Model Solution 2

Rise: since correlation of losses not considered gave the impression that senior tranches were very safe. Furthermore, rating agencies gave AAA ratings to these tranches leading to their popularity.

Fall: Since losses were indeed correlated when junior/mezz tranches defaulted, so did many senior tranches. This led the market to be suspicious of security of structure finance products and reliability of agency ratings.

### Question 13)

#### Model Solution 1

Counterparty default risk

→CAT bond: risk limit is fully collateralized (from investor → SPR→ trust) NO credit risk

→Risk swap: Since its only an agreement to pay in times of credit event, it is not prefunded and therefore is exposed to credit default risk.

Basis risk

→Both depends on the trigger selected. Most CAT bonds are issued on an index trigger basis which exposes insurers to substantial risk of actual loss not perfectly correlated with actual payoff. Swap, on the other hand, trigger is predefined by the contract. If index trigger →no benefit over CAT bond If identity trigger → beneficial. Most contracts however are on index trigger basis to prevent ..... hazard NO Difference

Risk diversification: CAT bond is essentially buying XOL reinsurance no diversification benefit swap allows insurers to diversify risk.

In the event of CAT company should be more concerned if they can receive payment or not as it is a matter of solvency vs. insolvency. Compared to this achieving risk diversification (swap) is an immaterial issue. Therefore propose company to use CAT Bond.

#### Model Solution 2

CAT Bonds are fully collateralized and thus there is no counterparty default risk. Basis risk depends on the trigger for Cat Bond; indemnity trigger carries the least basis risk but investors do not like these triggers. A parametric trigger carries more basis risk but is reasonable if the parameters are chosen carefully. The same applies to index triggers. Other triggers are available as well. A CAT bond does not really diversify risk, it just acts as CAT reinsurance.

Catastrophe risk swaps are an arrangement between two entities which are exposed to different catastrophe risk. Risk diversification can be achieved by pairing uncorrelated risks (say on opposite sides of the world). However there remains counterparty default risk with this arrangement. Basis risk depends on how the arrangement is structured-if loss settlement is based on an index there may be considerable basis risk.

For this company, a CAT bond does not achieve any risk diversification. A risk swap could be arranged, in theory, with a low amount of counterparty default risk by making sure the other participant is financially strong and the exposures are uncorrelated. Similarly, basis risk can be managed by selecting appropriate triggers. Risk diversification is achieved select the risk swap.

**Question 14)**

Model Solution 1

Capital for Line A alone = 5,800 →  $A - E[L] = 5.8 \rightarrow A = 9.8k$

Prob	$Loss_A$	$Assets_A@T1$	Def.	EPD	Scen#
.4	0	$A \times 1.204$	0	0	1
.3	6	"	$6 - A \times 1.204$	$1.8 - .30612A$	2
.1	8	"	$8 - A \times 1.0204$	$.8 - .10204A$	3
.1	0	"	0	0	4
.1	14	"	$14 - A \times 1.0204$	$1.4 - .10204A$	5

$$4k = E[L]$$

Now def. in 5 only →  $1.4 - .10204 \times 9.8 = 0.4k = 400 = EPD$

$$\rightarrow EPD \text{ ratio} = 400/4000 = 10\%$$

Save for lines A+B

Prob	$Loss_A$	$Assets_A@T1$	Def.	EPD	Scen#
.4	0	$A \times 1.204$	0	0	1
.3	11	"	$- A \times 1.0204 + 11$	$3.3 - .36012A$	2
.1	12	"	$- A \times 1.0204 + 12$	$1.2 - .10204A$	3
.1	12	"	$- A \times 1.0204 + 12$	$1.2 - .10204A$	4
.1	23	"	$- A \times 1.0204 + 23$	$2.3 - .1204A$	5

$$8 = E[L]$$

EPD ratio = 10% →  $EPD/8 = .1 \rightarrow EPD = 800$

→ Assume def. in seen .5 only:  $2.3 - 1.0204A = 800 \rightarrow A = 14,700$

$$14.7 \times 1.0204 = 15 > 12k \rightarrow \text{assumption holds}$$

→Capital 14.7k-8k= 6.7k

→ Incr. 0.9k or 900

### Model Solution 2

$$\text{Line A: } E(L) = 6k(.1) + 8k(.1) + 14k(.1) = 4,000$$

$$C = 5,800 = A - E(L) \Leftrightarrow A = 4,000 + 6,800 = 9800 \text{ @ } T = 0$$

$$A = 9,800(1.0204) = 9999.92 \text{ @ } T = 1$$

$$\text{EPD Ratio} = \frac{14,000 - 9999.92(1)}{4,000} = 10\%$$

Line A+B

$$E(L) = 11k(.3) + 12k(.1) + 23k(.1) = 8,000$$

$$\text{EPD Ratio} = .10 \frac{[(23,000 - A(1.0204))(.1)]}{8,000} \Leftrightarrow A = 14,700.12 \text{ @ } t = 0$$

$$C = A - E(L) = 14,700.12 - 8,000 = 6,700.12$$

$$\text{Additional Capital} = 6,700.12 - 5800$$

$$= 900.12$$



### Question 15)

#### Model Solution 1

Assume the disc. Loss ratio given discounts to the end of year 1.

Only considering investment income on the policy hold supplied funds

Line A's EVA:

$$\begin{aligned} &= 9.7\text{m} (1-0.08)(1.065) - 9.7\text{m}(0.7830) - 9.5\text{m}(0.18) \\ &= 0.19896\text{M} \end{aligned}$$

Line B EVA=

$$\begin{aligned} &= (8.3\text{m})(1.0063)(1.065) - (8.3\text{m})(0.87) - (4.5\text{m})(0.18) \\ &= 8.2826 \quad - \quad 7.221 \quad - \quad 0.81 \\ &= 0.2516\text{m} \end{aligned}$$

Both are positive EVA if enough capital is available to pursue both, do that.

If Capital is only available to pursue one, we would choose the higher expected valued added (EVA) which is line B.

#### Model Solution 2

$$\text{RAROC} = \frac{(P-E) \times (1+y) - PV(L)}{\text{Capital}}$$

$$\text{Line A Raroc} = \frac{9.7 \times (0.92) \times 1.065 - 78.3\% \times 9.7}{9.5} = 0.2$$

$$\text{Line B Raroc} = \frac{8.3(0.937) \times 1.065 - 87\% \times 8.3}{4.5} = 0.236$$

I would invest in both lines since Raroc > target Raroc if, however, there is not enough capital say, there is enough capital to invest in only one line I would go with line B since it has higher RAROC.

### Question 16a)

#### Model Solution 1

Shareholders-they view econ capital is that is needed to achieve capital adequacy objective (such as earning adequate returns, continue to grow business etc

Policyholders views economic capital from solvency objective viewpoint. They want to make more that there is enough capital for insurer to meet its obligation to PM

#### Model Solution 2

Two objectives for economic capital.

Maintain:

1. Sufficient capital to ensure all of an insurance company's obligations to the policy holder is able to be bought  
→Solvency objectives primary to policyholders.
2. Shareholders: company needs different objectives for them. It is to maintain sufficient capital to ensure:
  - A. Company continues to pay dividends.
  - B. Company continues to grow as outlined in company's business plans
  - C. Company maintains its financial structure in the market place.

Then objectives are to maximize franchise value of a company to the benefit of the shareholders.

**Question 16b)**

Model Solution 1

Market value includes the measurable/tangible value of the firm ( approximate book value) as well as the intangible value of the firm usually termed franchise value which reflects such as things as broad value/ reputation. Risk based capital does not reflect the 'intangible' value of a firm so risk based return measures could arguably overstate the company's return.

Model Solution 2

Market value of capital is the market value of equity and includes franchise value but It is not risk-adjusted.

Risk based capital measures have and unassociated probability to meet an objective eg pay claims. Risk based capital usually lower than market value capital.

### Question 17a)

#### Model Solution 1

$$U = \frac{1}{1-t_u} \left[ r \cdot \frac{QSR}{PSR} - i_{AFIT} \left( PHSF + \frac{1}{PSR} \right) \right]$$

$$U = \frac{1}{1-.35} \left[ .08 \left( \frac{1}{\frac{1}{2}} \right) - .06(1 - .35) \left( .436 + \frac{1}{\frac{1}{2}} \right) \right]$$

$$U = 0.1$$

$$P = \frac{L + FX}{1-V-U} = \frac{100,000 + 20,000}{1-.1}$$

$$= \$133,333$$

#### Model Solution 2

$$U = \frac{1}{1-Tax} \left[ ROE \times \frac{\frac{E}{S}}{\frac{P}{S}} - 1Aft \times \left( PHSF + \frac{1}{\frac{P}{S}} \right) \right]$$

$$= \frac{1}{-.35} \left[ .08 \times \frac{\frac{1}{\frac{1}{2}}}{\frac{1}{\frac{1}{2}}} - .06(1 - .35) \times (.436 + 2) \right]$$

$$=.09999$$

$$\text{Price} = \frac{\text{Losses} + \text{Expenses}}{1 - U}$$

$$= \frac{100k + 20k}{1 - .0999}$$

$$= 133,332$$

#### Model Solution 3

$$U = \frac{1}{1-t} \left[ r \left[ \frac{Eg}{Prem} \right] - Iaf \left[ PHSF + \frac{stur}{prem} \right] \right]$$

$$I \text{ after tax} = .06(1-.35) = 39\%$$

$$U = \frac{1}{1-.35} \left[ .16 - .039(2.436) \right] = 10\%$$

$$P = \frac{100,000 + 20,000}{1-.10} = 133,333$$

#### Model Solution 4

$$ROE = \frac{UW \text{ profit} + II - \text{taxes}}{\text{Equity}}$$

$$8\% = \frac{([P - 100k - 20k] + 6\%(.436P + S)) \times (1-t)}{\text{Equity}}$$

$$S:P = 2 \rightarrow S = 2P$$

$$S: Eq = 1 \rightarrow S = \text{Equity}$$

$$8\% = \frac{(1-.35)[P-120K+.06(2.436P)]}{2P} \quad .08 = \frac{-7800}{2P} + \frac{.745P}{2P} = .3725 - \frac{3900}{P} \quad P = \$133,332$$

### Model Solution 5

$$\text{lafit} = 0.06(1-0.35) = 0.039$$

$$E/p = S/p \cdot E/s = 2 \cdot 1 = 2$$

$$U = \frac{r\left(\frac{e}{p}\right) - \text{lafit}\left(\text{PHSF} + \frac{s}{p}\right)}{1 - tu}$$

$$U = \frac{0.08(2) - 0.039(0.436 + 2)}{1 - 0.35} = 0.100$$

$$P = \frac{100,000 + 20,000}{1 - 0.1000} \quad \rightarrow \text{I assume all expenses are fixed}$$

$$P = 133,333.33$$

### **Question 17b)**

#### Model Solution 1

The target return on equity,  $r$ , reflects the riskiness of the investment for the shareholders. The premium to surplus ratio, PSR reflects the amount of surplus required to support the riskiness of the written business.

#### Model Solution 2

It is reflected in the – selection of the target return on equity; this will be based on the risk of policy or line. – Selection of the leverage ratio (premium to surplus) or allocated surplus- this is based on the risk of the line.

#### Model Solution 3

By selecting target return on equity; By selecting surplus to premium ratio.

#### Model Solution 4

Uses benchmark leverage ratios to ensure an amount of equity that reflects risk inherent to the policy. Selected target ROE is set high enough to ensure an adequate return for the risk.

#### Model Solution 5

Based on the selected target return on equity, (ROE) higher ROE= more conservative= can absorb more risk

Lower ROE= less conservative- can not absorb as much risk

Based on the allocated surplus, Higher surplus- more cushion to absorb risk. Lower surplus= Less Cushion to absorb risk

**Question 18)**

Model Solution 1

Draw a table with all cash flows:

	0	1	2	3
Premium	P			
Expenses	50+ .15p			
Losses Pd		400	200	150
Reserves	750	350	150	
Reg. Surplus	675	415	135	0

Risk Adjusted Discounted Cash flows Model @T= 1  $PV(P_1 i_f) = PV(L, i_a) + PV(E_1 i_f) + PV(FiT \cdot i_f)$

$$PV(P_1 i_f) = P(1.03)$$

$$PV(\sigma, i_f) = (50 + .15p)(1.03)$$

$PV(\text{Investment income}) = 675(.055) + 315(.055)(1.03)^{-1} + 135(0.55)(1.03)^{-2}$  (For this method is only on supplies.)

$$PV(\text{VW Income}) = 1.03P - (50 + .15P)(1.03) - 400 - \frac{200}{1+.015} - 150/(1 + .015)^{-2}$$

$$-.7 = .03 + \beta(.05) \rightarrow \beta = .8$$

Adjustment for risk = .07 - .055 = .015

$$i_a = .03 - .015 = .015$$

$$= .8755 - 794.14$$

$$\rightarrow 1.03P = 51.5 + 1.545P + 742.64 + (.35) [60.94 + .8755P - 794.14]$$

$$.569 P = 537.52 \Rightarrow P = 944.67$$

$$CR = \frac{944.67 (.15) + 50 + 750}{944.67} = .9969 \rightarrow v = 1 - cr = .003$$

$$944.67$$

Model Solution 2

Time	Prem	PV Factor	PV(prem)	ion	Pv Factor	Pv Loss	Expense	PV EXP
0	P	1.03	1.03P	-	1.0515	-	50+.15p	51.5+.1545P
1	-	-	-	400	1.00	400	-	-
2	-	-	-	200	1.015 <sup>-1</sup>	197.04	-	-
3	-	-	-	150	1.015 <sup>-2</sup>	<u>145.60</u>	-	-
						742.64		<u>51.5+ 1545P</u>

\*Premium and Expense discounted using risk free rate

\*Ion discounted at risk adjusted rate.

$$i_r = i_f + B_l(i_m - i_f)$$

- Need to determine the liability  $\beta$  from the equity beta and asset  $\beta$

$$\text{ABC's equity: } r_e = r_f + \beta_e(R_m)$$

$$.07 = .03 + \beta_e(.05) \Rightarrow \beta_e = .8$$

$$\text{Assets: } r_a = r_f + \beta_a(R_m)$$

$$.055 = .03 + \beta_a(.05) \Rightarrow \beta_c = .5$$

- Do not have relative weights so I will just assume that  $\beta_c = \beta_A - \beta_e = .5 - .8 = -.3$   
 $i_r = .03 - .3(.05) = .015$

Time	PV VW income	PV( Tax on VW income)
0	.8755P-51.5	.3064P – 18.025
1	- 400	- 140
2	-145. 60	-50.96

Time	Less Reserve	Surplus Inv. Income	Tax on INV. Inc.	Pv. Factor	PV tax on INV. INC
0	750	675 -	-	1.03	-
1	350	315 37.125	12.99	1.00	12.99

2	150	135	17.325	6.06	$1.03^{-1}$	5.88
3	0	-	7.425	2.60	$1.03^{-2}$	2.45
						<hr/> 21.32

$$PV(P) = PV(L) + PV(EXP) + PV(\text{Tax})$$

$$1.03P = 742.64 + 515 + .1545P + (.3064P - 2777.95 + 21.32)$$

$$1.03P - .1545P - .3064P = 742.64 + 51.5 - 277.94 + 21.32$$

$$.5691P = 537.52$$

$$P = 944.51$$

$$U = 1 - \frac{\text{Loan} + \text{EXP}}{\text{Premium}} = 1 - \left( \frac{750 + 50 + .15(944.51)}{944.51} \right) = 0.3\%$$



**Question 19)**

Model Solution 1

T	0	1	2	3
Premium	P			
Losses pd	0	420	0	780
Expenses	35%			
Loss reserve	1200	780	780	0
Required Surplus	900	585	585	0
Total Assets	2100	1365	1365	0
u/w income	p- 35% - 1200 = 65%P - 1200			
I.I income taxes	0	231= 2100x 11%		150.15 150.15
Equity Cf	65%p – 2100	150.15+ 215	97.6	97.6+585= 682.6 = 465.15

U/W income= EP- IL- incurred Exp.

Assume premium is earned at t= 0, expenses are incurred at 0. Equity CF= U/W income + After tax I.I.- Change in surplus.

$$65\%p - 2100 + \frac{465.15}{1+8\%} + \frac{97.6}{(1+8\%)^2} + \frac{682.6}{(1+8\%)^3} = 0$$

$$P= 1605.784$$

Model Solution 2

IRR- Calculate Premium- Let premium be P

Assume premium is all collected on effective date.

	0	1	2	3
Premium	P			
Expenses	.35P			

Losses paid	0	420	-	780
Loss reserve	1200	780	780	-
Required Surplus	900	585	585	-
Contribution equity holders	1 (2100-.65P)			
Required Assets	2100	3) 1830.15	6) 1462.6	10) 682.6
Distribution to equity holds	-	5) 465.15	8) 97.6	682.6
Required Assets (post Distribution)	2100	4) 1365	1365	-
Taxes	-	2) 80.85	7) 52.55	9) 52.55

$$- 2100 - 65P + \frac{465.18}{1+r} + \frac{97.6}{1+r^2} + \frac{682.6}{1+r^3} = 0$$

$$.65P - 2100 + 1056.24 = 0$$

$$P = 1605.78$$

**Question 20a)**

Model Solution 1

$$EL = 500 \times .1 = 350$$

$$exp = .35 / sov = 175$$

T	Premium	Loss	PV CF
0	500	-175	325
1		$(350 \times .4) = -140$	$-140 / 1.06 = 132.08$
2		-122.5	$-122.5 / 1.06^2 = -109.02$
3		-87.5	-73.47
			10.43

Model Solution 2

Assumieng Expenses = 35%

$$1.05 - .7 = .35$$

$$NPV = 500(.65) - \frac{500(.7)(.4)}{1.06} - \frac{122.5}{(1.06)^2} - \frac{87.5}{(1.06)^3} = 10.43$$

Prem = 500    L = 500 x .7 = 350    CR = 1.05%    i = 11%     $r_f = 6\%$

$$CR = \frac{350 + 6}{500} = 1.05 \quad EX = 175$$

	Prem	Loss	Expense	Cash	PV Cash
0	500		-175	325	325
1		$-350 \times .4 = -140$			$-140 / 1.06 = -132.08$
2		$-350 \times .35 = -122.5$		-122.5	$-122.5 / 1.06 = 109.02$
3		$-350 \times .25 = -87.5$		-87.5	$-87.5 / 1.06^3 = 73.47$
					10.43

Model Solution 3

Time	P	Expense	Loss	Risk free Discount	PV Income
0	500	175	-	1.00	325
1	-	-	140	1.06	-132.08

2	-	-	122.5	$1.06^2$	-109.02
3	-	-	87.5	$1.06^3$	<u>-73.47</u>

NPV= 10.43

Model Solution 4

Since combined ratio = 105%

$$105\% = \text{Ion} + \text{Expense} / \text{Premium}$$

$$= .70 + E/P$$

$$E = (105\% - 70\%) P$$

$$= 35\% (500) = 175$$

Losses: Year	Payment	\$
1	40%	140
2	35%	122.5
3	25%	<u>87.5</u>
		350

$$\text{Total Ion} = 70\%(500) = 350$$

$$\text{PV(Prem)} = 500 \quad \text{PV (Express)} = .7500( .4/1.06 + .35/1.06^2 + .25/1.06^3) = 314.57$$

$$\text{PV (Express)} = 175$$

$$\text{Prem} = \text{Loss} + \text{Express} / 1 = U = \frac{7500 \times \text{Express}}{1 - (-.05)} = 500 \quad \text{Exp} = 175$$

$$\text{NPV} = 500 - 314.57 - 175 = 10.43$$

Time	Paid Prem.	Exp	Paid Loss
0	500	175	
1			140
2			122.5
3			87.5
4			

Assume uw cash flows are in line w/ cash flows as losses are paid not incurred.

PV PV P- PVE- PVL

$$PV \text{ at time } = 0 \quad 500 - 175 - \frac{140}{1.06} - \frac{122.5}{1.06^2} - \frac{87.5}{1.06^3} = 10,433 \text{ } Pv \text{ of Profit}$$

Model Solution 5

	Prem	Expense	Paid loss	VwCf
0	500	175	0	325
1	0	0	140	-140
2	0	0	122.5	-122.5
3	0	0	87.5	-87.5

Expenses= 1.05 - .7= .35

Expected Loss= .7(500)= 35-

$$NPV = 325 - \frac{140}{1.06} - \frac{122.5}{1.06^2} - \frac{87.5}{1.06^3} = 10,433$$

Model Solution 6

Expense Ratio = 105% - 70% = 35%

	P	EXP	Exp paid loss	Net Cash Flow
0	500	175	0	+325
1			140	-140
2			122.5	-122.5
3			<u>87.5</u>	-87.5
			350	

$$NPV \text{ of VW cash flows} = 325 - \frac{140}{1.06} - \frac{122.5}{1.06^2} - \frac{87.5}{1.06^3} = 10,433$$

Model Solution 7

	Prem	Loss	Expenses	CF	PV	PVCF
0	500		175	325	1	+325
1		140		-140	.9434	-132.08

2	122.5	-122.5	.8890	-108.90
3	87.5	-87.5	.8396	-73.47
				<hr/>
				+10.55

Expenses 105% - 70% = 35% off prem                      NPV = +10.55

Model Solution 8

$$500 - (500)(.4)(.7)/1.06 - (500)(3.5)(.7)/1.06^2 - (500) \cdot \frac{(25)(.7)}{1.06^3} - .35(500)$$

$$CR = 1.05 = \frac{L+T}{p} = \frac{.7(500) + x(500)}{500} \quad x = .35 = \text{expense ratio}$$

NPV = 10.43

Model Solution 9

	P	EXP	Exp paid loss	Net Cash Flow
0	500	175	0	+325
1			-140	-140/1.06
2			-122.5	-122.5/1.06 <sup>2</sup>
3			<hr/> -87.50	<hr/> -87.5/1.06 <sup>3</sup>
				\$10.43

Model Solution 10

NPV = PV of premium – PV of expenses – PV of losses (\*no need to account for taxes)

EXP come ratio = Exp. Loss Ratio + exp/prem

EXP/Prem = 105% - 70% = 35%

$$NPV = 500(1 - 0.35) - 0.7 \left( \frac{0.4}{1.06} + \frac{0.35}{1.06^2} + \frac{0.25}{1.06^3} \right) \times 500 = 325 - 0.7 \times 500(0.8988) = 10.4333$$

Model Solution 11

	0	1	2	3
Prem	500			
Exp	500x (105 - .7)			
Loss		350 x .4	350 x .35	350 X .25



This NPV represents the opportunity cost to the insured. This is the amount of invest income they lose out on because they bought the policy and paid up front.

#### Model Solution 5

NPV represents the present value of the expected cash flows associated with underwriting a given insured. This only considers cash flows UW, and represents the opportunity cost to the policy holder for pay premium to the insured now, and being identified for losses in the future.

#### Model Solution 6

This represents the opportunity cost to the insured of what they could have earned at risk free rates if they didn't buy ins. I.e. Pay the premium earlier than when the losses needed to be paid. This is the amount of insured should be given credit for via less premium b/c the insurer is now able to earn inv income off of this( policy holder only given credit for inv income could earn e risk free).

#### Model Solution 7

NPV at the risk-free rate is the opportunity cost of the insured. Insured's pay premiums at the start of the policy before losses are paid. This is what they are losing by buying a policy instead of investing the money on their own.

#### Model Solution 8

It is the opportunity cost of the insured, the risk- free present value of the cost of insurance (paying up front for later claims), including investment income on PHSF @ $r_f$

#### Model Solution 9

The NPV represents the (opportunity cost) of not investing the funds elsewhere opting to purchase insurance instead.

#### Model Solution 10

This represents the opportunity cost for the insured. The arises due to the deferred nature of the insurance, and represents the "cost" of putting their money somewhere else in a risk free asset. Premiums must be paid before expenses and claims are paid, thus giving rise to an opportunity cost.

#### Model Solution 11

It represents the opportunity cost expected to be suffered by the average policy holder for the risk free income lost due to prepayment of funds not yet required for losses and infrastructure (buildings software/computer systems/desks).

#### Model Solution 12



It's the opportunity cost of forgone interest on premium. Insured pays premium before losses are paid and thus loses out on investment income from investing this premium. The dates of premium payment and loss payments.

#### Model Solution 13

This represents the opportunity lost of upfront payment of premiums to the insurer (as opposed to the losses which are paid out at a later date.)

#### Model Solution 14

The NPV represents the opportunity cost to insured (i.e. cost that insured pays premium before losses are paid.)

#### Model Solution 15

It represents the opportunity cost for the policyholders, who must forego this amount to purchase insurance protection. The lost amount of investment income s/he would have earned due to the fact that she must pay prem. Up front and be reimbursed for loss payment later.

#### **Question 20c)**

#### Model Solution 1

This is not correct, the expected profits will be greater because they expect to get a greater investment return than the risk free rate of return- policy holders should not be given credit for this higher return because they don't bear any investment risk, so we use the risk free rate to discount the cash flows when determining their opportunity.

#### Model Solution 2

The CEO is incorrect, the actual profit should be higher because the insured should expect to earn greater than the risk free rate on investments. They should also expect to earn investment income on surplus

#### Model Solution 3

Incorrect: insurer should earn more than risk free rate for investment portfolio besides there is investment income from surplus.

#### Model Solution 4

This is not expected profit for the policy, This value ignores investment income on the policyholder supplied funds above the risk free rate. Also ignores all investment income on the supplies.

#### Model Solution 5

This is not correct because it ignores important cash flows to the company, such as investment income on surplus and actual investment yield instead of risk free yield.

### Model Solution 6

From CEO perspective this is not his expected profit. It is only profit on policyholder supplied funds at risk free rate. This ignores INV income on surplus. Also ignores inv income on these dollars above risk free rate which company should expect to earn because they are taking a risk that losses will be greater than prem. So they should earn more than risk free.

### Model Solution 7

The CEO is incorrect. This is only the profit earned from the risk-free rate. The company actually earns 11% on its investments instead of 6%. The investment income from surplus is also not included in this amount as it does not belong to the policy holders.

### Model Solution 8

Incorrect. This excludes investment income on surplus or inv income on PHSF above  $r_f$ . The policy gives insurer leverage to increase ROE, so part B is not the full profit.

### Model Solution 9

The CEO is incorrect for a few reasons: 1. Does not allocate surplus to this line, which would generate II 2. Company can earn 11% investment not 6% so more profit should be expected. 3. Some of expense should go to infrastructure so these will not be invisible ( ↓II)

### Model Solution 10

The CEO should actually expect to earn considerably higher profit that this due to two items. 1. Investment income should be earned on surplus that is supporting the business, which the policy holder should not be given credit for when calculating opp cost. 2. Investment returns should be greater than the risk free rate, which should also not be considered when calculating opp cost.

### Model Solution 11

The CEO is incorrect since he is not accounting for profit resulting from returns on investment of policy holder provided funds excess of the risk-free rate. He is also not accounting for total investment income earned on surplus. The excess of risk free investment returns on policy holder funds is left out of the above because the insured does not have a liability on or claim on actual earnings/losses of the insurer and the investment income or surplus is excluded because the surplus is meant to protect the insured but it does not belong to them.

### Model Solution 12

Incorrect. This NPV assumes that interest is earned only at risk- free rated and only on policy holder- provided funds. In reality, the insurer invest, both policy holder- provided funds as well as supplies (funds owned by shareholders) and earns the investment return of the company on them (in this case 11%) Thus the profit of the firm should be larger.

### Model Solution 13

The CEO is incorrect. There are various reasons why this amount does not represent the profit such as discounting may be done at a higher rate than risk free since actual investment income is likely higher, riskiness is not incorporated, stock insurers have to pay dividends on  $s/\mu$  which needs to be incorporated. Surplus has a cost which should be incorporated.

### Model Solution 14

CEO is not correct. Note that we use the risk free rate to discount the cash flow. The investment return in excess of risk free rate is excluded from the calculation. In addition, the investment return on surplus is also excluded from calculation. Hence, the expected profit is different than the above amount.

### Model Solution 15

CEO is incorrect because: .1. The NPV in a0 is calculated in a discount rate of risk free rate. In reality, the insurer should expect to earn a higher rate than risk free rate such as the 11% investment return cited. This will increase the expected profit. 2. The NPV in a) also doesn't consider investment income on the surplus supporting this policy on the policy holders' supplied funds. This will also increase the expected profit. Hence the expected profit should be higher than a).

### Question 21a)

#### Model Solution 1

$$\text{Prob}(\text{rain}) = 0.25 = N(-z)$$

$$z = 1.96$$

- $\sigma_x^2 =$   
 $(.005)(.995)(75,000)^2 + (.01)(.99)(50,000)^2 + (.02)(.98)(40,000)^2 + (0.5)(.95)(25,000)^2$
- $\sigma_x = 10666.86$
- $\sigma_y^2 =$   
 $(.005)(.995)(35,000)^2 + (.01)(.99)(25,000)^2 + (.02)(.98)(10,000)^2 + (0.5)(.95)(50,000)^2$
- $\sigma_y = 3928.02$
- $\sigma_{x+y} = (.005)(.995)(11,000)^2 + (.01)(.99)(75,000)^2 + (.02)(.98)(50,000)^2 + (0.5)(.95)(30,000)^2$
- $\sigma_{x+y} = 14409.55$

$$X = \frac{yz}{1+y} = \frac{(.15)(1.96)}{1.15} = .25565$$

$$r_x = X\sigma_x = (.25565)(10666.86) = 2726.98$$

$$r_y = X(\sigma_{x+y} - \sigma_x) = (.25565)(14409.55 - 10666.86) = 956.82$$

### Question 21b)

#### Model Solution 1

- $(V^1 - V) = z(S^1 - S) - R$
- $(V^1 - V) = 1.96 (14409.55 - 1066686) - .15((V^1 - V))$
- $(V^1 - V) = 6378.85$

#### Model Solution 2

$$V = Zs - R \rightarrow V = 1.96 \times 10667 - 2727 = 18180$$

- $V^1 = ZS^1 - R^1 \rightarrow V^1 = 1.96 \times 14410 - (2727 + 957) = 24560$
- $V^1 - V = 24560 - 18180 = 6380 \leftarrow$  Marginal surplus for Y

#### Model Solution 3

$$\text{Marginal surplus} = Z(S^1 - S) - R = 1.96 (14,410 - 10667) - 957 = 6,379.28$$

- $V_y = \frac{956.91}{.15} = 6379.32$

### Question 21c)

#### Model Solution 1

Marginal surplus method uses the change in standard deviation, which is the square root of variance. The square root function is sub-additive, resulting in renewal risk loads for pieces of the portfolio that add up to be less than the risk load for the aggregate portfolio.

#### Model Solution 2

Marginal surplus is not renewal additive because the square root operator is sub-additive. i.e.

$$\sqrt{x + y} < \sqrt{x} + \sqrt{y}$$

$$\text{At renewal, } r_x = \lambda (\sigma_{x+y} - \sigma_y)$$

$$r_y = \lambda (\sigma_{x+y} - \sigma_x)$$

Because of sub-additivity  $r_x + r_y < r_x + r_y$  and all accounts will be undercharged at renewal

## Question 22)

### Model Solution 1

$$A = \frac{S - \mu_L}{1 + y} = \frac{10,000 - 500}{1 + .12} = 8,482.14 \qquad A = \frac{\sigma_L}{\sigma_y} = \frac{3000}{.3} = 10,000$$

Select (the) method with highest assets which is Variance

$$R = A \left( \frac{y - r_f}{1 + r_f} \right) = 10,000 \left( \frac{.12 - .03}{1 + .03} \right) = 873.86$$

$$P = R + \frac{\mu_L}{1 + r_f} = 873.86 + \frac{500}{1.03} = 1,359.2 = \text{Prem}$$

### Model Solution 2

$$\mu_L = 500 \qquad s = 10,000$$

$$\sigma_L = 3,000 \qquad y = .12 \qquad \sigma_y = .3$$

$$r_f = .03$$

Assume all losses pd at time t=1

-2 constraints, choose the one with the largest assets, A

$$(1 + y)A = (1 + r_f)F - \mu_L$$

$$\text{Where } F = P + A = \frac{\mu_L}{1 + r_f} + R + A$$

$$\text{W/ Safety constraint: } A_s = \frac{S - \mu_L}{1 + y} = \frac{10,000 - 500}{1.12}$$

$$A_s = 8482.14$$

$$\text{W/ Variance constraint } A_v = \frac{\sigma_L}{\sigma_y} = \frac{3000}{.3} = 10,000$$

Use  $A_v$  to calculate risk load and premium.

$$(1 + y)A = (1 + r_f)(P + A) - \mu_L$$

$$1.12(10,000) = 1.03(P + 10,000) - 500$$

$$P = 1,359.22$$

## Question 23a)

### Model Solution 1

Swap= reinsurer invests funds in risk free investment instead of target investment. The advantage is reduction in risk and the disadvantage is loss of investment income. Put option: reinsurer invests funds in target investment and also purchase a put options which gives it the right to sell the investment at prices which provides it a rate of returns equals to the risk free rate of return. If the actual rate of return is below the risk free rate the reinsurer will exercise the option. The advantage is reduction in risk and disadvantage is the cost of the option.

### Model Solution 2

Swap: invest all funds at the risk free rate from time 0 until loss payment occurs. The funds to invest include premium and additional assets required by the various cost rights. We remove all uncertainty regarding investment risk by using risk-free rate.

Put option- Collect premium and additional assets at the time 0, and then purchase a Put option that will reimburse you the difference between the actual investment rate you earn and the risk free rate in the event that the actual instant yield is below the risk free rate. This costs money up front but allows for a riskier investment strategy with higher potential yields. This yield is earned between when premium is collected and when the Cost payment is required.

### Model Solution 3

Swap tech. Write in and invest funds in risky portfolio. But in swap technique you enter into a swap to earn risk free rate instead of risky. So funds grow at risk free rate.

Option you write ins and invest funds in risky portfolio. You also buy an option ( reducing funds to invest). The option allows you to at a minimum earn what you would @ risk free. So if risky inv't portfolio does well keep returns if does power you have right to sell your invested funds for  $Fe^{rt}$ . The option eliminates downside risk of inv't strategy.

### Model Solution 4

Swap: Move assets A that would have been invested in the target investment into the risk free investment and use the assets A to support the reinsurance contract.

Option: Purchase an options so that for assets A return or the target investment will be at least of ( the risk free return) the assets A will support the reinsurance contract.

### Model Solution 5

Swap Technique: Take allocated assets out of target investments and put them into the risk free investment.

IRR equation  $A(1+IRR) = (P+A)(1+r_f) - w_l$

Option technique: leave allocated assets in target investment and also purchase a put option to guarantee that return wont fall risk free.

IRR equation  $(A(1+IRR) = \frac{P+A}{1+r})(1+i) - w_l$   $r =$  per unit cost of option  $i =$  put protected return

### Model Solution 6

Swap technique under this the reinsurers invests the funds in the risk-free asset instead of the target investment.

The cost of the technique is the loss of investment income in excess of the risk-free rate. Benefit is the reduced volatility in relations

Option technique: under this the reinsurers invests the funds in the target investment but also purchases a put option. The option gives the right to sell the target inv. At maturity at a strike price equal to the risk-free rate of returns. Cost is the cost of the option. Benefit in higher returns and lower volatility of returns.

### **Question 23b)**

#### Model Solution 1

Safety constraint- the funds available to pay claims at the end of the year is at least equal to the safety level

Variance constraint: the return of combined reinsurance and investment strategy is no more volatile than the direct investment in risky investment.

#### Model Solution 2

Safety constraint: the amount of money available after investing your initial funds must be high enough to exceed some given \$ amount threshold based on the funds required in a selected adverse loss outcome.

Variance constraint: the Volatility of the return to investors for investing in the insurance company (which is in part determined by the volatility of losses) has to be no worse than the volatility of a similar investment they could make in the market (with similar expected returns).

#### Model Solution 3

Safety constraints says you must have enough money to pay losses a certain amount given a selected prob of min. safety level is an amount in loss dist that augs with a specified min prob and inv't strategy must ensure you can pay losses at that level.

Variance Constraint says the variance or volatility of the ins+ investment strategy cant be more than it would be in the pure inv't in risk portfolio. IE in swap tech and option

#### Model Solution 4

Safety constraint: Ensures that enough assets will be available to react obligation of the reinsurance contract up to the deserved loss safety level (higher level more likely the loss will be paid in full)

Investment constraint: Standard deviation if unite the reinsurance contract will be  $\leq 1$  standard deviation if don't write the reinsurance contract (ie if assets A were invested in the target investment)

#### Model Solution 5

Safely constraint: The insurer needs to have funds available to pay claims up to the safely level, which is some high percentile of loss distribution.

Variance Constraint- the combination of the reinsurance policy and the investment strategy should have total risk less than the risk of the target investment.

#### Model Solution 6

Loss safely constraint- The initial funds invested by the reinsurer should sufficient at the end of the year to pay for losses alone above a safety level.  $(1 + y_t) F > S$

Variance constraint: the contract should not have more risk than ..... of the target invest meat.  
 $S \cdot D \cdot \text{Of IRR} \leq \sigma_y$

#### **Question 23c)**

#### Model Solution 1

Steps: 1. For each of the techniques select the more restrictive constraint. The constraint that is higher. 2. Select the lower risk load from two techniques this will produce more competitive premium.

#### Model Solution 2

- 1) Within a given technique choose the constraint that required higher assets as the dominant constraint. This Asset level ensures both constraints are met for a given investment techniques.
- 2) Compare the resulting risk load for the dominant constraints of the two. Techniques and select the smaller risk load as the preferred technique. This ensures that we choose the most economically efficient method for meeting all required constraints
- 3) Implement the selected techniques.

#### Model Solution 3

For each technique select the risk load that is greater. This ensures both constraints are met. Then compare R.L coming out of each technique. i.e. swap option and select the less amounts as to offer most competitive premium.

#### Model Solution 4

1. For each technique select the constraint (risk load) that results in higher asset amount A so higher risk load.



2. Now compare the 2 risk loads for the 2 techniques and select the technique (+ risk load) that results in a lower risk load.
3. This is the risk load

#### Model Solution 5

Step 1. Choose higher A for each financial technique, as this will satisfy both constraints.

Step 2. Calculated R for each financial technique, using Chose A values.

Step 3. Choose lower R for competitive purposes

#### Model Solution 6

1. Identify the dominant constraint under eacer technique. The constraint with higher assets is dominated
2. Compare their risk loads to the dominant constraint for eacer technique.
3. Reinsurer selects the lower risk load if the wants his ratio to be more competitive. Reinsurer select the higher risk load if he wants more prem. And the weakest can support the higher risk load.

#### **Question 24a)**

$$T/S = \frac{I}{A} \left( 1 + \frac{R}{S} \right) + \frac{U}{p} \left( \frac{P}{S} \right)$$

$$T/S = 5\% (1 + .5) + (3\%)(1.5)$$

$$= 12\%$$

#### **Question 24b)**

When an insurer is profitable they can afford to be more aggressive in their investment strategy. Which could lead to higher investment returns on assets.

#### **Question 24c)**

##### Model Solution 1

If more profitable they may decide to write more business however this additional business may result in more uninvested assets due to outstanding agent's balances, etc.

##### Model Solution 2

U/P ↑ → P/S ↑ → I/A ↓

If the insurer has increased profitability it is expected the insurer will want to write more business to receive more profit. However when writing more business, the amount of insurance risk is increased. To help offset this additional risk, the insurer invests in safer investments. This leads to a lower investment return.

### Question 25a)

#### Model Solution 1

$$T/S = \frac{T}{A} + \frac{R}{S} \left( \frac{I}{R} + \frac{U}{R} \right)$$

Assume 6% is return on assets I/A

$$= 6\% + 1(6\% + 5\%)$$

$$= 17\%$$

#### Model Solution 2

Reserve = 15m

Op Results =  $ios \times 15m = .75m$

Investment results =  $.06 \times 30M = 1.8M$

$$ROE = \frac{\text{invest} + \text{op results}}{\text{Equity}} = \frac{1.8M + .75M}{15M} = 17\%$$

### Question 25b)

#### Model Solution 1

This is not financing in the sense of collecting debt. The non-equity financing arises from the delayed nature of paying losses. Can think of reserves as “borrowed” money from p/h’s ..... is kept until it makes payments. Underwriting losses, if there are any, can be thought of as the “interest.” Will allow more even written: ↑ earnings stream → ↑ value ↑ variability → ↓ value.

#### Model Solution 2

Non equity financing makes and insurers earnings more volatile. This will result in a larger discount rate when valuing the insurer. This may also result in increased earnings and the two effects will offset to some degree depending on the amount of leverage. An insurer's reserves are a form of non-equity financing and provide a form of leverage for the insurer. Greater reserves will increase volatility when accompanied by increased premium in different lines that diversifies the insurer which could decrease volatility.

### Model Solution 3

For an insurer, non-equity financing comes from the amount of premium the insurer writes and the corresponding reserves. This is reflected in the leverage ratio, which is calculated as  $R/S$ . The greater the leverage ratio, the greater the expected return as well as volatility, for that insurer. Thus increasing non-equity financing will increase the insurer's expected earnings, but also increase the earnings volatility.

Comments about the exam in general from the Chair:

Overall, the committee was very pleased with 2013 sitting of exam 9. This exam had a pass score at roughly 70% of the available points which produced an effective pass ratio of 41%. The feedback from the survey responses was very positive as well with many responders indicating this exam was a good blend of syllabus coverage, difficulty and length. Many survey responders commented they finished the exam and had time left over to review some answers.

The exam committee did find students struggled to answer the contrast type questions. For these types of questions, the correct answer requires more than just providing the definition for the items being contrasted, they also require thoughts on why and what makes them different.

QUESTION 1:

- A) THE MAJORITY OF CANDIDATES DID WELL ON THIS PART, EARNING FULL POINTS. COMMON ERRORS INCLUDED THE FOLLOWING:
- I) SOLVING FOR THE RISKY RATE OF RETURN THAT WOULD GIVE THE SAME UTILITY AS THE RISK-FREE ASSET
  - II) SIMPLY SELECTING THE RISK-FREE RATE OF RETURN AS THE CERTAINTY EQUIVALENT RATE
  - III) SIMPLY SELECTING THE EXPECTED RETURN OF THE RISKY PORTFOLIO AS THE CERTAINTY EQUIVALENT RATE.
  - IV) SOLVING FOR THE CERTAINTY EQUIVALENT RATE OF THE WRONG PORTFOLIO. IN SOME CASES, THE CANDIDATE CALCULATED THE CERTAINTY EQUIVALENT FOR THE "COMPLETE" PORTFOLIO GIVEN IN PART (B); IN OTHER CASES THE CANDIDATE SOLVED FOR THE OPTIMAL "COMPLETE" PORTFOLIO (PART (C)) AND USED THE EXPECTED RETURN AND VOLATILITY OF THAT PORTFOLIO TO CALCULATE THE CERTAINTY EQUIVALENT RATE IN PART (A).
  - V) MISTAKING THE RISK PREMIUM ON THE RISKY PORTFOLIO FOR THE EXPECTED RETURN ON THE RISKY PORTFOLIO
  - VI) MISTAKES IN APPLYING THE REWARD-TO-VOLATILITY RATIO
- B) THE VAST MAJORITY OF CANDIDATES SCORED FULL POINTS ON THIS PART. THOSE WHO DIDN'T GET FULL CREDIT TYPICALLY MADE ONE OF THE FOLLOWING COMMON ERRORS:
- I) REVERSING THE WEIGHTS APPLIED TO THE RISKY PORTFOLIO AND RISK-FREE ASSET.
  - II) CALCULATING ONLY ONE OF THE VALUES REQUIRED BY THE QUESTION – EITHER THE EXPECTED RETURN WITHOUT THE

STANDARD DEVIATION, OR VICE VERSA.

III) MISTAKES IN APPLYING THE INFORMATION GIVEN IN THE PROBLEM  
(RETURN-TO-VOLATILITY RATIO, RISK PREMIUM, ETC.)

C) AGAIN, ALMOST ALL CANDIDATES RECEIVED FULL CREDIT FOR THIS PART. THOSE WHO DIDN'T GENERALLY MADE AN ERROR IN THE CALCULATION OR IN SUBSTITUTING A VALUE FOR A VARIABLE IN THE FORMULA TO CALCULATE THE OPTIMAL WEIGHT TO GIVE THE RISKY PORTFOLIO.

A NOTE ON NOTATION: MANY CANDIDATES USED NOTATION THAT WAS INCORRECT, INCONSISTENT WITH THE TEXT, OR INCONSISTENT WITHIN THE CANDIDATE'S RESPONSE, IN A FEW PRIMARY CATEGORIES:

1. IDENTIFYING A PORTFOLIO'S RETURN, EXPECTED RETURN, OR RISK PREMIUM,
2. VOLATILITY/STANDARD DEVIATION VERSUS VARIANCE
3. IDENTIFYING WHICH PORTFOLIO A GIVEN STATISTIC REFERS TO.

ALTHOUGH WE DID NOT DEDUCT ANY POINTS DIRECTLY FOR USING INCONSISTENT NOTATION, IT APPEARED THAT IN SOME CASES INCONSISTENCY IN NOTATION TRIPPED UP CANDIDATES, CAUSING MISTAKES IN THE CALCULATIONS.

QUESTION 2

		TOPIC
<b>RISK POOLING</b>	to Contrast	<b>RISK SHARING</b>
<p>WRITING/INVESTING IN ADDITIONAL SIMILAR RISKY ASSETS TO INCREASE THE PREDICTABILITY OF OVERALL PORTFOLIO.</p> <p>CREATING PORTFOLIO BENEFITS FROM LACK OF CORRELATION ACROSS BETS (ASSETS/STOCKS), WITHOUT REDUCING OVERALL RISK.</p>	<p>Generic Definition</p>	<p>Selling shares in an attractive portfolio to limit risk and maintain the profitability. As more bets are pooled, continue to sell percentage shares in the portfolio to maintain your position of total funds invested. Spreading a fixed amount of funds over more and more uncorrelated risk assets. Buying "n" assets and taking 1/n position in each, investing the same total amount as before.</p>
<p>WRITING MORE POLICIES INCREASES THE PREDICTABILITY OF THE AVERAGE PROFIT RETURN (LOSS RATIO).</p> <p>"LAW OF LARGE NUMBERS"</p>	<p>Insurance Definition or Application</p>	<p>Writing more policies/risks, but taking a smaller percentage of the portfolio. Add n-times dollar exposure, reduce share to 1/n of portfolio. (i.e.) Reinsurance or Quota Share Reinsurance Lloyds of London Syndicates Spreading risk across investors/stockholders .</p>
<p>BECOME MORE PREDICTABLE, BUT GROWTH IN SIZE MAKES IT MORE RISKY IN TOTAL DOLLAR LOSS FROM YEAR TO YEAR.</p>	<p>Expected Losses</p>	<p>May remain the same if writing similar risks, outcome becomes more stable.</p>

<p>WRITING MORE POLICIES REQUIRES MORE CAPITAL AND ASSETS TO BACK THE BOOK. INSURERS CANNOT GROW INDEFINITELY WITHOUT ADDITIONAL CAPITAL.</p>	<p>Capital Requirements</p>	<p>Insurer maintains the same capital requirements, while lowering risk. Can grow portfolio indefinitely, while lowering the percentage held in the portfolio, if they can sell shares.</p>
<p>AVERAGE EXPECTED LOSSES MORE PREDICTABLE, BUT OVERALL LOSSES LESS PREDICTABLE. <b>OVERALL RETURN INCREASING IN NOMINAL AMOUNTS.</b></p> $E(R) = N * Y * R$	<p>Expected Return</p> <p>Defining E ( r ) :</p> $E ( r ) = y * R$	<p>May remain the same, if similar risks added.</p> $E ( r ) = y * R$
<p>INSURANCE PRINCIPLE - "RISK INCREASES LESS THAN PROPORTIONALLY TO THE NUMBER OF POLICIES INSURED."</p> <p><b><u>INCREASES VARIANCE</u></b> - BY FACTOR OF N.</p> $VAR = N * ( Y^2 * \Sigma^2 )$	<p><b>"RISK"</b> - Either Variance or Std. Dev. Variance <math>Var = ( y^2 * \sigma^2 )</math></p>	<p><b><u>Lowers Variance</u></b> - by factor of n. <math>Var = ( y^2 * \sigma^2 ) / n</math></p>
<p><b><u>INCREASES</u></b> BY FACTOR OF <math>( N )^{1/2}</math></p> $STD. DEV. = ((N)^{1/2}) * ( Y * \Sigma )$	<p>Standard Deviation "Risk" Std. Dev. = <math>( y * \sigma )</math></p>	<p><b><u>Decreases</u></b> by factor of <math>( n )^{1/2}</math></p> $Std. Dev. = ( y * \sigma ) / ((n)^{1/2})$
<p><b><u>INCREASES</u></b> BY A FACTOR OF <math>( N )^{1/2}</math></p> $SHARPE = ( N )^{1/2} * [ Y * R ] / \Sigma$	<p><b>Sharpe Ratio</b> (Risk/Reward Ratio) <math>[ y * R ] / \sigma</math></p>	<p><b><u>Increases</u></b> by a factor of <math>( n )^{1/2}</math></p> $Sharpe = ( n )^{1/2} * [ y * R ] / \sigma$
<p>OTHER VALID STATEMENTS:</p> <p>PREFERRED POSITION IS POOLING THAN SHARING TO SPREAD THE RISK, AND INCREASE THE SHARP RATIO.</p>		

CANDIDATES MUST PROVIDE THE FOLLOWING ITEMS TO RECEIVE FULL CREDIT:

(1) INSURANCE DEFINITIONS OF RISK SHARING AND RISK POOLING  
(CAN HAVE OR NOT HAVE INSURANCE APPLICATIONS IN ADDITION TO INSURANCE DEFINITIONS)

OR

(2) GENERAL PORTFOLIO DEFINITIONS AND INSURANCE APPLICATIONS OF RISK SHARING AND RISK POOLING

AND (3) COMPARE THE EFFECTS ON RISK (VAR OR STD. DEV., ETC.) FOR THE TWO STRATEGIES  
AND (4) COMPARE THE EFFECTS ON RETURN (TOTAL RETURN OR SHARPE RATIO, ETC) FOR THE TWO STRATEGIES

COMMON ERRORS:

1) ONLY PROVIDE THE INSURANCE DEFINITIONS OF THE TWO STRATEGIES WITHOUT GIVING COMPARISONS.

2) ONLY PROVIDE THE GENERAL DEFINITIONS WITHOUT REFERENCE TO INSURANCE APPLICATIONS.

3) ONLY COMPARE THE EFFECTS ON RISK, BUT NOT THE EFFECTS ON RETURN

4) USE VAGUE, INSTEAD OF SPECIFIC WORDING:

E.G. FOR RISK SHARING EFFECT ON RISK, INSTEAD OF SAYING THAT THE TOTAL RISK/VARIANCE DECREASES, SAYING THAT THE RISK DOESN'T INCREASE.

THIS IS NOT CLEAR ENOUGH AND THE CANDIDATE WON'T RECEIVE FULL CREDIT.

E.G. FOR RISK POOLING EFFECT ON RISK, INSTEAD OF SAYING THAT THE TOTAL RISK/VARIANCE INCREASES, SAYING THAT THE RISK DOESN'T DECREASE.



### Question 3

This question was a straight-forward application of the methodology found on p 266 of the BKM textbook.

The most common mistake candidates made was to be either totally or partially unfamiliar with the method. A couple of other common mistakes are mentioned below.

There are seven steps (formulae) needed to calculate the weight of the active portfolio.

Step 1: Compute the initial position of each security in the active portfolio

$$w_i^{\circ} = \alpha_i / \sigma^2(e_i) \text{ [see table below]}$$

Step 2: Scale the initial positions so that their sum is unity

$$w_i = w_i^{\circ} / \sum w_i^{\circ} \text{ [see table below]}$$

Stock	$\alpha_i$ (given)	$\sigma^2(e_i)$ (given)	$\beta_i$ (given)	$w_i^{\circ}$ (Step 1)	$w_i$ (Step 2)
1	0.06	0.16	0.8	0.3750	0.2500
2	0.10	0.2	1.0	0.5000	0.3333
3	0.15	0.24	1.2	0.6250	0.4167
$\Sigma$				1.5000	1.0000

Risk Premium of the Market Index ( $R_M$ ) 0.08 (given)

Standard Deviation of the Market Index ( $\sigma_M$ ) 0.20 (given)

Step 3: Compute the alpha of the active portfolio

$$\alpha_A = \sum_{i=1}^n w_i \alpha_i = [0.25 * 0.06 + 0.333 * 0.10 + 0.417 * 0.15] = 0.110833$$

Step 4: Compute the residual variance of the active portfolio

$$\sigma^2(e_A) = \sum_{i=1}^n w_i^2 \sigma^2(e_i) = [0.25^2 * 0.16 + 0.333^2 * 0.20 + 0.417^2 * 0.24] = 0.073889$$

A common mistake in Step 4 was failing to square the weights.

Step 5: Compute the initial weight of the active portfolio

$$w_A^{\circ} = [(\alpha_A / \sigma^2(e_A)) / (E(R_M) / \sigma_M^2)] = [(0.110833 / 0.073889) / (0.08 / 0.20^2)] = 75.0\%$$

Step 6: Compute the beta of the active portfolio

$$\beta_A = \sum_{i=1}^n w_i \beta_i = [0.25 * 0.8 + 0.333 * 1.0 + 0.417 * 1.2] = 1.033333$$

Step 7: Adjust the initial weight of the active portfolio

$$w_A = w_A^{\circ} / (1 + (1 - \beta_A)w_A^{\circ}) =$$

$$0.75 / (1 + ((1-1.033333)*.75)) = 0.769231$$

A common mistake in Step 7 was to put  $(1+\beta_A)$  in the denominator rather than  $(1-\beta_A)$ . While the point of this step is to adjust for  $\beta_A$  being different from unity, the graders chose to only take a quarter-point off for this mistake.

**Question 4:**

*Examiner's Comments:*

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*This question showed that many candidates were familiar with the basic concepts of CAPM.*

*The most common mistake was missing what the item asked candidates to comment on: what is the impact of the investment strategy change on the risk profile of the position? Many candidates focused on the change in expected returns of the two positions as well as the Sharpe ratios. What was missing was a discussion of the fact that CAPM assumes, among other things, that investors all hold the efficient portfolio which has firm-specific risk diversified away. Some candidates forgot that, while the systematic risk component is lower than that of the market portfolio, the total risk profile may not be lower because of the introduction of firm-specific risk.*

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Examiner's Report. Exam 9, Question 5

To get full credit for this question an answer has to have the following:

Part A.

Identify the anomaly ("P/E effect" or "P/E Ratio Anomaly") 0.25

Description/explanation of the anomaly as shown on the graph that stocks with low P/E ratios tend to outperform stocks with high P/E ratios. High P/E stocks underperform low P/E stocks was also acceptable. 0.25

Part b.

Identify the anomaly "Post Earning Announcement Price Drift " 0.25

Description/explanation of the anomaly including discussion of the adjustment to the earnings news occurring over an extended period of time (not immediately). 0.25

Part c.

Clearly mentioned term "Forecasting Error" 0.25

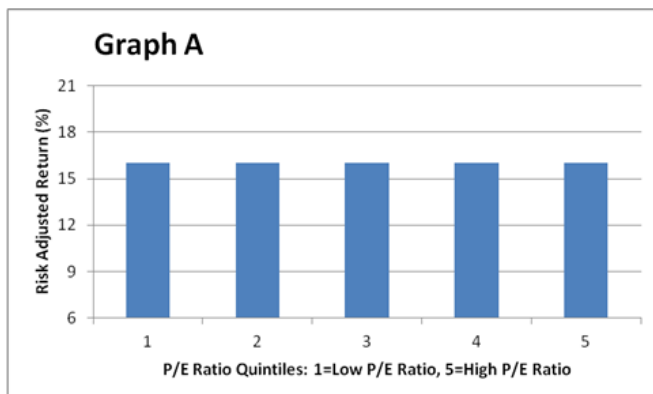
Reference to "placing too much weight to recent experience" 0.25

Clearly mentioned term "conservatism" 0.25

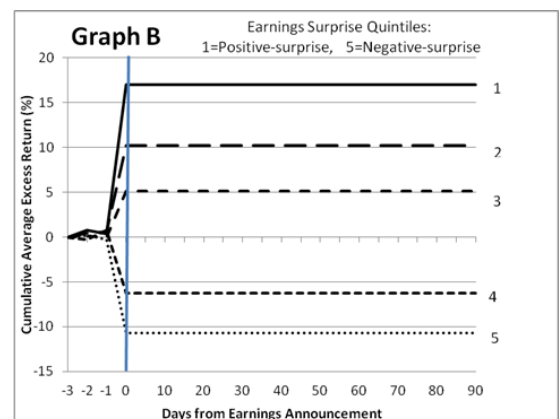
Reference to "slow reaction to new information" 0.25

Part d.

Either graph or text answers acceptable:



0.25



0.25

Alternative:

For Graph A. Bars of the same height. 0.25

For Graph B. Jump at time of announcement (t=0), then a horizontal or flat line. 0.25

## Common Errors

### Part a.

1. Failure to identify the anomaly (common)
2. Reference to “Book to Market” ratio. Graph clearly states “P/E ratio”
3. Stating that high P/E leads to higher returns
4. Describing semi-strong form of the efficient market and how it leads to identical risk adjusted returns, instead of describing the anomaly.

### Part b.

1. Failure to identify the anomaly by name (uncommon)
2. Reference to momentum. The question clearly concerns the announcement of earnings.
3. Citing pre-earning announcement price movement (“leakage”) as the anomaly in question
4. Describing semi-strong form of the efficient market and how it leads to no price change after the announcement, instead of describing the anomaly.

### Part c.

1. Failure to state the term “Forecast Error” (common)
2. Listing all possible information errors, without identify which one applies
3. Siting the following as reasons for “P/E effect”
  - a. Sample size
  - b. Wrong model
  - c. Hot stocks (?)
  - d. Projecting too far into future
  - e. Neglected firm effect
4. Failure to state the term “Conservatism” (very uncommon)
5. Using “Forecast error” for “Post Earning Announcement Price Drift” and “Conservatism” for “P/E effect”

### Part d.

1. Graph A
  - a. Reversing the graph – High P/E ratio should earn higher returns
  - b. Risk adjusted returns should be random and not exhibit a pattern based on P/E ratio
2. Graph B
  - a. Referring to “Straight” line instead of “Flat”, “Horizontal”, “Zero slope” etc.
  - b. Reverting to 0 excess returns.
  - c. Only eliminating the slow increase in Cumulative Average Excess Returns prior to announcement.

## Question 6

### Common mistakes

- Not connecting the risk and the theory used. E.g. Identifying the expectations hypothesis as a theory in a way that does not explain the resulting asset-liability mismatch.
- Not identifying a scenario leading to financial distress. (E.g. asset-liability mismatch doesn't imply automatic distress without interest rates rising dramatically, or deposits being withdrawn).
- Not identifying an action, or identifying an action inconsistent with the theory and risk (e.g. many candidates suggested raising long-term deposit rates and lowering long-term loan rates to fix ALM).
- Only discussing deposits (or loans). Without understanding the relative balance of loans and deposits, one can't determine whether the bank sensitive to interest rate changes.
- Assuming the institution was already in distress, and discussing potential problems, as opposed to issues that could lead to distress.

## Question 7

---

The following are several of the common mistakes made:

1. The swap was calculated for the party paying dollars rather than pounds (answer would be positive rather than negative).
2. One of the most common mistakes was calculating the value of the swap using the bond method rather than a portfolio of forward contracts. The bond method did not involve one of the key testing points of the question which was calculating forward rates. Partial credit was given for those that solved using the bond method as this answered the “value of the swap” aspect of the question.
3. The wrong time was used in discounting (i.e. something other than .25 and 1.25)
4. One or more cash flows were excluded from the swap. Often only the first cash flow was calculated and then principal.
5. The principal was excluded from the swap calculation.
6. The interest rates used in the cash flow calculations were switched between dollars and pounds.
7. The interest rates intended for discounting were used for the cash flow amount calculation and vice versa.
8. Interest rates were not converted to a continuous basis but used as if it was continuous.
9. Interest rates were converted to a continuous basis but then used as if it were on an annual.
10. The wrong currency was converted using the exchange rate. If exchange rate was left on a dollars per pound sterling basis the exchange rate was multiplied by dollars or if the exchange rate was converted to pounds sterling per dollar it was multiplied by pounds sterling.
11. Attempted to calculate using one of the several versions of the Forward Rate Agreement formulas in the Hull textbook rather than calculating the value of the swap.
12. Attempted to calculate as an interest rate swap rather than a currency swap.

**Question 7:**

**MODEL SOLUTIONS – 2013 Part 9**

<u>Using Continuous Rates</u>							
Convert to Continuous Rate							
LN(1.06) = .05827							
LN(1.04) = .03922							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Cash	Cash		Cash	Net		Discounted
	Flow In (\$)	Flow Out (£)		Flow Out (\$)	Cash Flow		Net Cash Flow
I	(000s)	(000s)	Forward Exchange Rate	(000s)	(000s)	Discount	(000s)
				= (2) x (3)	= (1) - (4)		= (5) x (6)
0.25	.05 x 30M = 1,500	.09 x 30M = 1,800	$(1.65)e^{(0.03922-0.05827)(0.25)} = 1.6422$	\$ 2,956	\$ (1,456)	$e^{(-0.03922)(0.25)} = .9902$	\$ (1,442)
1.25	1.05 x 30M =31,500	1.09 x 20M =21,800	$(1.65)e^{(0.03922-0.05827)(1.25)} = 1.6112$	\$ 35,124	\$ (3,624)	$e^{(-0.03922)(1.25)} = .9522$	\$ (3,451)
						Total:	\$ (4,893)
<u>Alternative 1</u>							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Cash	Cash		Cash	Net		Discounted
	Flow In (\$)	Flow Out (£)		Flow In (£)	Cash Flow		Net Cash Flow
I	(000s)	(000s)	Forward Exchange Rate	(000s)	(000s)	Discount	(000s)
				= (1) x (3)	= (4) - (2)		= (5) x (6)
0.25	\$ 1,500	£1,800	$(1/1.65)e^{(0.05827-0.03922)(0.25)} = 0.6090$	£914	-£887	$e^{(-0.05827)(0.25)} = .9855$	-£874
1.25	\$ 31,500	£21,800	$(1/1.65)e^{(0.05827-0.03922)(1.25)} = 0.6207$	£19,552	-£2,248	$e^{(-0.05827)(1.25)} = .9298$	-£2,090
						Total:	-£2,964
<u>Using Annual Rates</u>							
<u>Alternative 2</u>							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Cash	Cash		Cash	Net		Discounted
	Flow In (\$)	Flow Out (£)		Flow Out (\$)	Cash Flow		Net Cash Flow
I	(000s)	(000s)	Forward Exchange Rate	(000s)	(000s)	Discount	(000s)
				= (2) x (3)	= (1) - (4)		= (5) x (6)
0.25	\$ 1,500	£1,800	$(1.65)(1.04/1.06)^{(0.25)} = 1.6422$	\$ 2,956	\$ (1,456)	$1.04^{(-0.25)} = .9902$	\$ (1,442)
1.25	\$ 31,500	£21,800	$(1.65)(1.04/1.06)^{(1.25)} = 1.6112$	\$ 35,124	\$ (3,624)	$1.04^{(-1.25)} = .9522$	\$ (3,451)
						Total:	\$ (4,893)
<u>Alternative 3</u>							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Cash	Cash		Cash	Net		Discounted
	Flow In (\$)	Flow Out (£)		Flow In (£)	Cash Flow		Net Cash Flow
I	(000s)	(000s)	Forward Exchange Rate	(000s)	(000s)	Discount	(000s)
				= (1) x (3)	= (4) - (2)		= (5) x (6)
0.25	\$ 1,500	£1,800	$(1/1.65)(1.06/1.04)^{(0.25)} = 0.6090$	£914	-£887	$1.06^{(-0.25)} = .9855$	-£874
1.25	\$ 31,500	£21,800	$(1/1.65)(1.06/1.04)^{(1.25)} = 0.6207$	£19,552	-£2,248	$1.06^{(-1.25)} = .9298$	-£2,090
						Total:	-£2,964

## Question # 8

### Grading Key

#### Points

<b>Awarded</b>	<b>Action demonstrated to receive points</b>
0.25	Calculating interest rate 1 correctly
0.25	Calculating interest rate 2 correctly
	<u>For duration of liabilities:</u>
0.25	Present Value of CY 2013 payments/AY 2011 payments
0.25	Present Value of CY 2014 payments/AY 2012 payments
0.25	DMV(L) having numerator correct
0.25	DMV(L) having denominator correct
	<u>For duration of assets:</u>
0.5	Duration of bond 2
	Duartion of bond portfolio (Alternatively, 1.00 point for calculating the entire bond
0.5	portfolio correctly)
0.5	Total duration of assets, including the cash at duration of 0
	<u>Duration Gap of the Market Value of Surplus:</u>
0.25	Numerator = $DMV(A) * MV(A) - DMV(L) * MV(L)$
0.25	Denominator = $MV(A) - MV(L)$

#### Common

#### Errors

- Used equal weights, 50/50, for combining bond portfolio instead of the bond price, resulted in -.25 points
- Reversed the liability cash flows (10 for CY 2013 and 20 for CY 2014, instead of 20, then 10, respectively) resulted in -.25 points
- Using 2 year spot rate (3.88%) instead of short rate (3.91%), resulted in -.25 points
- Not contemplating cash at 0 duration in asset portfolio, resulted in -.25 points
- Using full value of liabilities (30) insated of MV(L) (28.69 after taking the present value) for duration gap, resulted in -.25 points
- Instead of just using DMV(S) as the amount that varies from 0, some subtracted an additional amount from the answers, resulted in -.25 points



### Question 9

Of the responses that lost credit, there were two that were by far the most common:

1. The first was incorrectly using the firm's target rate of return in place of the risk free rate. Most often, candidates just made this error when calculating the 'd', however, others did make the incorrect substitution in the Current Economic Value section as well.
2. The second common error was an incorrect formula for the Current Economic Value, either by omitting the surplus, omitting the first year of profits, or by adding in an additional investment income term that should not appear.

A few candidates used algebraic manipulations to make the formulas easier to memorize, and there were errors in the algebra causing them to lose points. Several candidates recalculated the premium, in some cases incorrectly, despite the value being given in the problem. The remaining errors were primarily calculation errors or substitution errors (using the value of surplus for losses, for example).

### Question 10

There are 6 values that must be calculated to get full credit for this problem:  $d_2$ ,  $d_1$ ,  $N(d_1)$ ,  $V_0$ ,  $E_0$  and  $D_0$ . Calculating  $d_2$  is worth 0.25 point, so no partial credit was possible if the correct value of 1.28 was not calculated. To calculate  $d_1$ , the candidate must know the formula  $d_1 = d_2 + \sigma_v \times \sqrt{T}$ , and perform arithmetic. 0.25 point is awarded for writing the formula, and 0.25 point is awarded for the correct arithmetic. If the candidate incorrectly writes the formula, no credit is awarded for the formula, nor the arithmetic. The candidate is awarded 0.25 point for  $N(d_1)$  if the correct value is provided, relative to the value calculated for  $d_1$  (the candidate need not calculate  $d_1$  correctly to get credit for  $N(d_1)$ ). The candidate must calculate  $V_0$  - the current value of the assets. This requires knowing the following formula:

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_v^2/2)T}{\sigma_v \sqrt{T}}$$

If the candidate correctly calculates  $V_0$ , he/she was awarded 1 point. 0.25 point is awarded for writing down the above formula correctly. If an error was made in writing down the formula, the candidate loses the full 1 point. If a calculation error is made in the arithmetic needed to calculate  $V_0$ , 0.25 point is deducted, assuming the work is sufficiently shown. If an incorrect value was substituted into the formula (for instance if  $T=1$ ), 0.25 point is deducted. If the same mis-substitution was performed elsewhere on the paper, no additional points were deducted.

To calculate  $E_0$ , the candidate must write down the formula:  $E_0 = V_0 N(d_1) - D_0 e^{-rT} N(d_2)$ , substitute the appropriate values, and calculate. 0.25 point was awarded for the correct formula, and 0.25 point was awarded for the correct calculation (the calculation is correct if the candidate

substitutes and correctly performs the arithmetic with the values they already calculated in the problem) If the formula is wrong, 0.5 point was deducted.

Finally, to calculate  $D_0$ , worth 0.25 point, the candidate must recognize  $D_0 = V_0 - E_0$ , and correctly calculate a  $D_0$  value based on the values previously calculated by the candidate in the problem.

Most candidates were able to approach this problem using the framework of the Merton Model, as stated in the item. It was common for candidates to perform some, but not all, of the required calculations for full credit. For instance, candidates often were able to calculate  $V_0$  and  $E_0$ , but did not calculate  $D_0$  - this would result in the loss of .25 point. Other candidates calculated  $V_0$ , but not  $E_0$  or  $D_0$ , which would result in the loss of .75 point assuming all other calculations were performed correctly.

Many candidates approached the problem by taking the present value of  $D$  ( $De^{(-rT)}$ ). No credit was awarded for this approach, as the problem clearly stated the Merton Model should be used, and it did NOT ask for the risk-free present value. If, however, a candidate included additional calculations that were assigned points in the grading rubric, those points were awarded. For instance, if the candidate calculated  $d_2 = 1.28$  then  $De^{(-rT)} = \$0.774B$ , 0.25 point would be awarded for calculating  $d_2$ .

Several candidates interpreted the interest rate as annually compounded, and therefore calculated the equivalent continuously compounded rate for use in calculations. No points were deducted for this. However, if a candidate applied interest as an annually compounded rate, 0.25 point was deducted as the Merton framework always uses continuous compounding.

Several candidates made the assumption that volatility = variance (not standard deviation). 0.25 point was deducted in this case, as the graders believed that is an incorrect assumption.

### Rubric

According to the Merton-Model:

$$N(-d_2) = .10 \Rightarrow d_2 = 1.28 \quad (0.25)$$

$$d_2 = d_1 - \sigma_v \times \sqrt{T} \rightarrow d_1 = 1.28 + .218 \times \sqrt{3} = 1.66 \quad (0.5)$$

$$N(d_1) = .951 \quad (0.25)$$

$$d_1 = \frac{\ln(V_0/D) + (r + \sigma_v^2/2)T}{\sigma_v \sqrt{T}}$$

$$\sigma_v \sqrt{T} \quad (0.25)$$

$$d_1 = \frac{\ln(V_0/1) + (.05 + .218^2/2) \times 3}{0.218 \times \sqrt{3}} = 1.66 \quad (0.25)$$

$$\Rightarrow \ln(V_0) + 0.2213 = 0.6268 \quad (0.25)$$

$$\Rightarrow V_0 = \exp(0.6268 - 0.2213) = \$1.5B \quad (0.25)$$

$$E_0 = V_0N(d_1) - De^{-rT}N(d_2) \quad (0.25)$$

$$= 1.5B(0.951) - 1Be^{-0.05*3}(0.9) = \$0.652613B \quad (0.25)$$

$$D_0 = V_0 - E_0 = \$1.5B - \$0.652613B = \mathbf{\$0.847387B} \quad \underline{(0.25)}$$

**2.75**

Question 11:

The question was meant to test the students' understanding of liquidity risk and various mechanisms to liquidity risk.

In general, the question was well-answered. More than half of the candidates received full credit while the majority of candidates received at least two-thirds of the total points. Candidates had no problems naming two developments/innovations that have transformed liquidity risk. Explanations of the two developments/innovations that were listed were generally thorough.

# GRADING KEY FOR EXAM QUESTION 12

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## PART A)

Default probability of mezzanine tranche (perfectly correlated) = **0.05** (or 5%)

- **Worth 0.25 points; all or nothing**

Default probability of mezzanine tranche (uncorrelated) =  $[ 0.05 \times 0.05 \times 0.05 ] + [ 3 \times 0.05 \times 0.05 \times 0.95 ]$   
= **0.00725** (or 0.725%)

- **0.25 points for the first half:  $[ 0.05 \times 0.05 \times 0.05 ]$**
- **0.25 points for the second half:  $[ 3 \times 0.05 \times 0.05 \times 0.95 ]$**

### Alternative Solution to Uncorrelated Default Probability

Default probability of mezzanine tranche (uncorrelated) =  $1.0 - [ 0.95 \times 0.95 \times 0.95 ] - [ 3 \times 0.05 \times 0.95 \times 0.95 ]$

= **0.00725** (or 0.725%)

- **0.25 points for the first half:  $1.0 - [ 0.95 \times 0.95 \times 0.95 ]$**
- **0.25 points for the second half:  $[ 3 \times 0.05 \times 0.95 \times 0.95 ]$**

Difference between default probabilities =  $0.05 - 0.00725 = \mathbf{0.04275}$  (or 4.275%)

- **Worth 0.25 points; all or nothing**

Note: If the test taker made an error in one of the calculations above, but correctly carried that result forward to this last step, then no additional points should be deducted.

Note: Deduct 0.25 points for any of the following:

- Calculator error

- Not showing complete work in calculating the uncorrelated default probability
- Having sloppy work that is hard to follow (ex. not properly labeling a probability of default as uncorrelated)

*Examiner's Comments:*

~~~~~  
 Almost half the candidates received full credit. The most common reasons that candidates lost points was either 1) not calculating the difference once the correlated and uncorrelated default probabilities were calculated, or 2) forgetting the  $0.05^3$  term in the uncorrelated default probability calculation.  
 ~~~~~

## **PART B)**

### **Point A**

- The underestimation of default probabilities led to credit rating agencies giving the senior tranches AAA ratings.
  - **Worth 0.25 points**

### **PointB**

- This made them appear safer than they really were and thus were very desirable to investors
- This provided them with a yield advantage over single bonds with AAA ratings and thus were very desirable to investors
- Because investors are required to hold less capital for AAA-rated securities they were very desirable to investors
  - **Any worth 0.25 points**

### **PointC**

- Thus when the underlying securities began to default, the senior tranches defaulted at a higher rate than was expected.
- Thus when the underlying securities began to default, the credit rating agencies downgraded the senior tranches
- Thus when the underlying securities began to default, the structure of the CDOs magnified the effect of underestimating default probabilities
- Thus when the underlying securities began to default, the senior tranches began to suffer significant losses

- Thus when the underlying securities began to default, senior tranches began defaulting despite their AAA-rating.

- Any worth 0.25 points

**PointD**

- This caused investors to turn away from these securities, either not purchasing them or selling them off.
- Therefore, investors became reluctant to invest in securities they did not fully understand.
- This caused investors to no longer trust structured finance.

- Any worth 0.25 points

*Examiner's Comments:*

~~~~~  
*We found that many test takers described the fall of structured finance in terms of systemic risk rather than underestimated default probabilities (the Coval, Jurek and Stafford article clearly lists these as separate reasons for the rise and fall of structured finance) and thus received no points for explaining the fall of structured finance.*

*In addition, we found that many candidates' explanations were not specific enough. For example, many candidates lost a quarter-point for Point A for only stating that senior tranches received high credit ratings rather than AAA credit ratings.*

~~~~~

### Question 13:

#### **Counterparty Default Risk**

Most candidates did very well comparing the counterparty default risk associated with the CAT bond and the CAT risk swap.

#### **Basis Risk**

Most candidates correctly discussed how the selection of the trigger in a CAT bond could enable the issuer to mitigate basis risk. However, many candidates failed to understand the importance of the trigger selection in CAT risk swaps as well. Instead, many incorrectly assumed that because risks were “swapped” in this structure, it meant that losses were automatically paid on an indemnity basis.

Candidates were given substantial partial credit for an adequate discussion of trigger selection with regards to either CAT bonds or CAT risk swaps, but a full credit response required the candidate to recognize that both instruments provided an opportunity to limit basis risk if structured appropriately.

Candidates also incorrectly associated the cat risk assumed to be related to basis risk. While another company’s cat risk can result in a loss, the risk of this loss is not ‘basis’ risk.

#### **Risk Diversification**

Most candidates also did very well discussing the risk diversification provided by CAT risk swaps, but many incorrectly stated that a CAT bond would provide the company with risk diversification because catastrophe risks are uncorrelated with investment returns. While it is true that catastrophe risks are indeed uncorrelated with investment returns, this risk diversification benefit is provided to the investors of the CAT bond, not the issuing company. As the question asked the candidate to evaluate the strategies from the company’s perspective, the candidate was required to correctly state (in some form) that the company received no risk diversification from issuing a CAT bond in order to receive full credit.

Many candidates also noted that the risk diversification provided to the investors of a CAT bond could provide a lower spread than high layer reinsurance to the issuing company, which is indeed a correct statement, but full credit still required the candidate to state the distinction between this benefit and a true risk diversification benefit, of which the latter is not present for a company issuing a CAT bond.

#### **Choose the Better Strategy**

As a last step, candidates were also required to state their final selection based on correct and detailed rationale. Some candidates simply commented on the three concerns without appropriately making a final selection, or instead stated their final selection without giving any rationale or statements of preference in the preceding comments about each concern. These candidates received substantial partial credit but not full credit, given the lack of a robust final argument. Also, some candidates provided concerns other than the concerns mentioned in the question. The question clearly states that “the company’s decision will be based on the ... three concerns.” Credit was not deducted for other legitimate concerns, nor was credit given for other concerns in isolation.

## Question 14

### Examiner's Comments:

This question was generally well answered. The majority of candidates correctly applied the concepts required. Many candidates received full credit.

There were several common mistakes. The most common mistakes were not recognizing that premium is an investable asset when losses are paid at time 1 and also not properly accounting for the time value of money. Candidate responses that correctly accounted for the time value of money – regardless of whether interim calculations were shown at time 0 or 1 – received full credit. As such, the model solution could be adapted in this vein.

Other common mistakes included:

- Failure to recognize that losses were given for line A and B combined. Many candidates double-counted the losses for line A.
- Assuming all loss scenarios resulted in a deficit position when evaluating lines A and B combined, which is essentially not understanding that the level of assets impacts the expected deficit calculation.
- Using an interest rate of 2.4%, whereas the question states the interest rate as 2.04%.
- Calculating the total required capital whereas the question addresses the additional amount of required capital.



### Question 15

Model solution 1:

$$\text{RAROC} = [(P - E) * (1 + y) - PV(L)] / \text{Capital}$$

$$\begin{aligned} \text{Line A RAROC} &= (9.7 * 0.92 * 1.065 - 9.7 * 78.3\%) / 9.5 \\ &= 20\% \end{aligned}$$

$$\begin{aligned} \text{Line B RAROC} &= (8.3 * 0.937 * 1.065 - 8.3 * 87\%) / 4.5 \\ &= 23.6\% \end{aligned}$$

I would invest in both lines, since RAROC > target RAROC. If, however, there is only enough capital to invest in one line I would go with line B since it has higher RAROC.

Model solution 2:

EVA = (Premium less expenses \* (1 + inv returns)) – (Losses discounted to time 1) – (Cost of capital held for one year)

Line A's EVA:

$$\begin{aligned} &= 9.7\text{M} * (1 - 0.08) * (1.065) - 9.7\text{M} * (78.3\%) - 9.5 * (18\%) \\ &= \$198.9\text{k} \end{aligned}$$

Line B's EVA:

$$\begin{aligned} &= 8.3\text{M} * (1 - 0.063) * (1.065) - 8.3\text{M} * (87\%) - 4.5 * (18\%) \\ &= \$251.6\text{k} \end{aligned}$$

Both lines are EVA positive. If enough capital is available to pursue both, do that. If capital is only available to pursue one, we would choose the higher expected value add, which is line B

#### Examiners Comments

1 point of credit was given for the mathematical calculations of RAROC / EVA and 1 point was given for candidate's decision of which lines the company should pursue. The point for the written explanation of which lines to pursue were split into half a point for explaining that both lines should be pursued if enough capital is available, and half a point for explaining which line to pursue if capital is constrained. Only slightly more than half of candidates were able to recall the correct RAROC or EVA formula.

If an error was made in the RAROC or EVA calculations, we still gave credit for correct answers in the written section. A correct answer would be judged relative to the calculations done by the candidate. For instance, if a candidate erroneously calculated that line A had a higher RAROC; we would give credit for them advising to pursue line A over line B in an environment with constrained capital.

Common mistakes included:

- RAROC or EVA formula errors.
- A calculation that was done for a profitability metric which was not a risk-adjusted measure.

- Only considering a single scenario of capital availability: either enough to write both lines or only enough to write a single line.
- Identifying which lines they would pursue, but without an explanation or justification for how they arrived at that decision.
- Advising to pursue a line simply because it requires less capital than the other, without consideration for the profit potential of those lines
- Interpreting the written part of the question as how the decision would be affected if capital was allocated differently between the two lines, rather than how the decision would be affected by the level of capital available.

### Question 16:

- a. Candidates described the policyholder view of economic capital reasonably well. The majority of the candidates alluded to the necessity of the insurer having enough capital to pay claims. The more prepared candidates articulated that the capital was used to pay for claims in excess of expected levels, thus protecting the company from insolvency.

Candidates had various levels of success in articulating the shareholder view of economic capital. Most candidates succeeded in explaining that economic capital is important to help achieve a return on their investment (i.e., dividends), but the more prepared candidates also explained how economic capital is used to achieve this objective; for instance by supporting the growth of the business or supporting a particular credit rating.

The most common reasons that candidates did not get full credit on this question were:

- Simply defining economic capital; the question specifically asked for the viewpoint of policyholders and shareholders
  - Not explaining how economic capital supports the shareholder goal of return on investment
  - Not adequately explaining when economic capital would be needed by policyholders; that is, some reasonable description of solvency. Also candidates who described capital as paying claims directly without implying that capital is only deployed in tail or excess loss scenarios
  - Implying that economic capital was directly provided by the shareholders
- b. Candidates generally struggled with this part of the question. We gave full credit for showing meaningful differences between risk-based capital measures and market value capital. The most common reasons that candidates did not get full credit on this question were:
- Simply defining the two terms without contrasting.
    - i. Often there was little overlap between the definitions given to give any basis for contrasting usually due to market value capital being minimally defined
  - Also many candidates focused only on regulatory RBC requirements rather than considering broader risk-based capital measures used by companies.

## **Question 17**

### **Model Solution 1:**

a)

$$\text{ROE} = [U * P + i * (\text{PHSF}\% * P + S)] * (1 - \text{tax}\%) / E$$

$$8\% = [(1 - \text{CR}) * P + .06 * (43.6\% * P + 2 * P)] * (1 - 35\%) / E$$

$$(8\% * E / P) / .65 = 8\% * 2 / .65 = 1 - \text{CR} + .06 * 2.436$$

$$\text{CR} = (\text{Loss} + \text{Expense}) / \text{Premium} = 120,000 / \text{Premium}$$

$$\text{CR} = 1 - 8\% * 2 / .65 + .06 * 2.436 = 0.900$$

$$\text{Premium} = 120,000 / 0.900 = \$133,333$$

b)

1. Risk can be reflected in the selection of the surplus requirement
2. Alternatively we can adjust for risk by selection a different target return.

### **Model Solution 2:**

a)

$$U = 1 / (1 - t_u) * [r * \text{QSR} / \text{PSR} - i_{\text{AFIT}} * (\text{PHSF}\% + 1 / \text{PSR})]$$

$$U = 1 / (1 - 0.35) * [0.08 * 1 / (1/2) - 0.06 * (1 - 0.35) * (0.436 + 1 / (1/2))] = 0.1$$

$$P = (L + \text{FX}) / (1 - V - U) = (100,000 + 20,000) / (1 - 0.1) = \$133,333$$

b)

The target return on equity,  $r$ , reflects the riskiness of the investment for the shareholders; The Premium to Surplus ratio, PSR, reflects the amount of surplus required to support the riskiness of the written business.

Candidates who listed out the correct formula, plug in the right number, and accurately calculate the final answer for  $P$  will receive full credit for part a. They may use formula in either model solution #1 or #2.

Those who pointed out the two key items – selection of target ROE and selection of allocated surplus or leverage ratio received full credit for part b. Missing either item will result in a 0.25 point deduction.

### **Common Errors:**

1. Part a – many candidates did not lay out the formula; rather they directly plug in the numbers. 0.25 point was deducted if the candidate failed to write out one or more formula.
2. Part a – Some candidates express figures in \$1000s without specifically noting it on the paper with a “K” or “in \$1000s”, as a result they get final answer of  $P = \$133.33$ , rather than \$133,333. 0.25 point was taken out in this situation. This deduction was necessary because the problem asked for a premium number and there is a big difference in the context of the problem between a premium of \$133.33 and \$133,333.
3. Part a – many candidates did not correctly use after-tax return on invested assets, rather they used the before tax interest rate.
4. Part a – many candidates miscalculated the ratio of QSR / PSR, and/or  $1/\text{PSR}$  as used in the underwriting profit provision formula.

5. Part b – some candidates layout the pros and cons of the CY ROE method (i.e. how results can be distorted by using calendar year data) as there were similar questions asked in prior exams, however, this was not exactly what this question is asking for.
6. Part b – note that the question asks one to “describe two ways that risk is reflected in the Calendar Year Return on Equity method”, but not to “identify which component of CY ROE formula contains risk or indicates risk”. The model answer gives a couple examples “via the selection of target ROE and via the selection of allocated surplus”, both of which are items that can be judgmentally selected based on your contemplation of the risk level. In other words, the question is looking for “measures/ways” CY ROE method allows user to adjust for risk, but not simply a list of risks the user faces.

Some candidates identified underwriting risk, investment risk, asset risk, and etc. They also explained what these risks are or why they exist. Although those risks mentioned are relevant to the insurance business, they are not “ways” with which user can use CY ROE method to reflect risks. Therefore, no credit was given.

### Question 18

Candidates were expected to demonstrate working knowledge of CAPM, use Robbin’s discounted cash flow (DCF) model to calculate premium, and convert premium, losses and expenses into an underwriting profit provision.

The most difficult part of the problem was appropriately identifying the risk-adjusted rate using CAPM. In the ideal response, the examiners were looking for a calculation of the company (equity) beta and asset beta, and using the difference between the two betas (to determine the liability beta) in a third CAPM formula to arrive at the risk adjusted rate. Partial credit was provided if a candidate used CAPM to calculate a beta and provided additional information that connected the liability beta to the company and asset betas, such as writing “liability betas are usually negative” or that “the risk-adjusted rate is normally less than the risk-free rate”. Most candidates received no credit on the risk-adjusted rate calculation.

The next section looked for correct application of the key DCF formula  $PV(P; i_t) = PV(L; i_t) + PV(FX+VX; i_t) + PV(FIT; i_t)$ . Note that there is an algebraically-rearranged version of the key formula that many candidates used:  $PV(P; i_t) = PV(L; i_t) + PV(FX+VX; i_t) + \frac{Tax Rate}{1-Tax Rate} PV(Investment Income; i_t)$ . This formula was also valid for full credit.

While Robbin’s DCF model discounts cash flows to time 1, solutions that were equivalent and used any other time adjustment were considered valid. It was important that the discount amounts and timing be kept appropriate for all data.

A common error in the calculation of tax on investment income was earning investment income on surplus + loss reserves instead of on surplus alone.

The calculation of tax on the underwriting income was the most difficult calculation. In most responses the losses were either undiscounted or discounted at a different rate than the risk-adjusted rate (usually at the risk-free rate). The latter error occurred most commonly when the candidate’s response was organized in a tabular format. Prior to discounting, all the cash flows to be discounted would be combined into one column and then discounted at the risk-free rate. Note that it was not necessary to use a tabular format to respond to the item.

The item instructed candidates to calculate an underwriting profit provision. Several candidates omitted this calculation, stopping once they solved for the premium. The other common error was using discounted, rather than undiscounted, losses to determine the underwriting profit provision.

Question 19:

This question had many candidates receiving full credit; however grades also had a large standard deviation. Candidates seemed to either “get it” or not. A correct answer with a chart filled in generally yielded full credit; however incorrect entries in the chart without a clear formula or thought trail shown for each line minimized partial credit. Please note a chart was not necessary for full credit but adequate documentation of calculations was.

Common mistakes included:

- 1) Investment income rate applied to surplus only (not total assets)
- 2) Ignoring tax
- 3) Ignoring cash flows at time 2
- 4) Creating an unearned premium reserve (such that P is shifted to time 1, not time 0)
- 5) Mixed up directions of cash flows (signs wrong on SH contribution/distributions)
- 6) Difficulties with the calculation of SH distributions (e.g., calculating Investment Income, subtracting change in surplus, but then also subtracting paid)
- 7) Wrong cost of capital (use of risk free rate)

## **Question 20:**

### Part a)

The vast majority of candidates scored perfectly on this part. To receive full credit, candidates should have used the correct loss ratio, expense ratio, timing of cash flows, discount factors, and performed all calculations correctly using the correct interest rate.

The most common mistake was an incorrect calculation of the expense ratio. Candidates should have used  $LR + ER = CR$ . The CR and LR were both given, so the ER is the difference (35%). Candidates lost a small amount for this error.

The next most common mistake was including investment income (on premium and/or reserves) in the calculation. The NPV of underwriting cash flows should not have considered this. This was treated as a more serious mistake, as the point of this question is to test the concept of "opportunity cost" as it relates to NPV. Furthermore, the question specified "underwriting cash flows", which do not include investment results.

Other minor mistakes included using the incorrect discount rate (the problem specified using the risk-free rate). Some used incorrect timing of cash flows on the premium, expenses, and/or losses. Others had simple arithmetic errors.

### Part b)

The majority of candidates scored perfectly on this part. To receive full credit, candidates should have noted that the NPV of underwriting cash flows represents the opportunity cost suffered by the insured for the risk-free income lost through advance payment of funds not yet required by the company for infrastructure, loss payments, or expense payments.

This part was worth 0.5 point and asked for a description - therefore candidates should have provided more explanation than just a few words. A simple statement of "opportunity cost" lost a small amount of credit, as they did not describe what this concept actually means.

Full credit was also awarded to candidates who answered it is the cost for the insured to smooth their cash flows (remove uncertainty, or purchase protection).

The most common mistake was interpreting the NPV from the wrong perspective, e.g. from the insurer's point of view. Any explanation that described a meaning relative to the insurer, without any mention of the insured, lost credit.

The next most common mistake was stating that the amount represented the markup or profit that insured could expect was going to the insurer. This is not correct, as the amount does not exactly equal the markup/profit the insurer will receive (as Part c of this question will ask). Candidates that answered "markup" or "profit" lost credit unless they explained in more detail how this is related to opportunity cost to the insured.

Part c)

This part was worth 0.5 point - so candidates needed to explain two major reasons why the CEO was incorrect. Simply stating the CEO was incorrect was not enough to receive credit.

Candidates' scores varied on this part, with the vast majority receiving half-credit or full-credit. Candidates should have not only stated that the CEO is incorrect, but explained that the amount does not represent profit to the insurer but rather opportunity cost to the insured. The insurer should expect greater profit than this due to the fact that investment income should be considered. Additionally, the insurer should expect to earn a higher rate of return than the risk-free rate, notably the 11% mentioned in the problem. It will also earn a return on investment of reserves/surplus. Finally, the amount still does not consider taxes, other income, or capital needed which would affect profitability.

The most common mistake was providing too brief an answer, typically mentioning that the insurer should expect to earn a higher rate of return than the risk-free rate. While correct, there are a variety of additional reasons that could have been provided.

The problem was also not concerned with a comparison of the NPV/IRR methods; candidates who answered this way lost credit.



Question 21:

- a. Most people responded correctly. Common mistakes included:
  - i. Calculating SD as  $\text{SQRT} ( E[X^2] - E[X]^2 )$
  - ii. Not translating  $p\text{-ruin}=2.5\%$  to  $z=1.96$ , and calculating risk load  $Y$  on a stand-alone basis, or not at all (sometimes included as answer to part b).
  
- b. Most people did not respond correctly. Common mistakes included answering with marginal risk load of  $Y$ , rather than the marginal surplus requirement for  $Y$ . Four methods accepted for calculation of surplus, see model responses.
  
- c. Most people responded correctly that the standard deviation or square root is sub-additive for partial credit. Most people did not provide sufficient explanation of what renewal additivity was. Common mistakes for the latter included the candidate not specifying the component risk loads ( $X$  and  $Y$ ) were calculated under a renewal scenario when compared to total portfolio risk load (rather than build-up or stand-alone scenario).

Question 22:

Overall, the candidates did well on this question, but there were a few common mistakes. The most common mistakes involved using the wrong interest rates in the denominator of the Safety Constraint calculation (using the risk free rate instead of the target investment return) or the Risk Load calculation (using the target investment return instead of the risk free rate). Another common mistake involved the discounting of the expected loss in the Premium calculation – candidates either did not discount at all or discounted using the target investment return instead of the risk free rate.

### Question 23:

#### Part a:

- 1) Swap: For full credit, candidates must (at least) state exchange of risky asset for risk-free asset (or some notion of exchanging a higher risky return for lower risk free return).
- 2) Option: For full credit, candidate must (at least) state that (a) the insurer invested in target investment and (b) bought a put option to (c) guarantee at least a risk free return

#### Part b:

- 1) In general, the candidate must state both sides of the constraint and be specific about what is being constrained.
- 2) For example:
  1. Funds available must be greater than some safety level at the end of the period (acceptable).
  2. Must be greater than safety level (unacceptable).
- 3) A lot of candidates did not get credit for the variance constraint portion of the question. Some common mistakes are listed below:
  1. Variance of losses less than variance of returns on target investment - incorrect since this is only true for the swap technique but not the option technique, but the constraint should apply to both techniques
  2. Variance of investments less than variance of target investment – it is unclear whether the candidate included the investment in the reinsurance policy (which would be correct), or just the investment in the financial instruments (which would be incorrect).
  3. Variance of IRR less than variance of return on target investment – this was a tough call, because this would have been taken verbatim from the paper. However, the paper would have set the context for this wording. At the end of the day, we decided to not award credit for this formulation of the answer because the candidate has not made it clear that the IRR is the IRR of the combined investment + reinsurance policy, and not just the IRR of the investment alone (to be consistent with the grades awarded in pt. 2 above).
- 4) Candidates who only wrote down the relevant formulas with no explanations whatsoever of the formulas also did not receive credit for their answers as the question specifically requested candidates to “Briefly **describe**”.

#### Part c:

- 1) There are two common mistakes made in this part:
  1. doing the steps in the incorrect order (i.e. selecting technique first and then comparing the results of the constraints), or
  2. selecting the min assets (between techniques) after selecting the constraint that results in the max assets for each technique.

Question 24:

- a) Common mistake was to assume  $1+R/S=.5$ , with answer 7%. This was given 0.25.  
Also underwriting as -3% instead of 3% and insurance leverage as  $S/R$  given 0.25.  
Also some wrote correct formula and inputs but ended up with another amount, given 0.25.  
No points for writing down the formula.
- b) Full points for aggressive/risky investments.  
There were a few cases given 0.25 – e.g. one who wrote progressive investments when they may have meant aggressive investments.  
Nothing if indicated more cash or paying off debt (i.e. no ratio).  
Nothing if first answer wrong.
- c) Most common correct answer was higher premium leads to more uninvestible assets (whether mentioned agents balances or not).  
Full points for higher  $P/S$  is riskier so have more conservative investments.  
Nothing for riskier investments may not turn out well (about 20 candidates).  
No points for the following if did not mention growth:
- More owners' equity is at risk.
  - May want to invest in safer assets.
- Nothing for profits may be invested with low-return securities.  
Nothing for higher commissions and expenses.  
Nothing for larger asset base.  
Nothing for may have shifted to short tail lines with less opportunity to invest funds – not clear re relationship to  $I/A$ .  
Nothing if first answer wrong.

### Question 25:

#### **Part a:**

This part of the question was generally responded to well, with the majority of the candidates receiving full credit. Some candidates were confused about the sign of the U/R term in the Ferrari formula, writing  $T/S = I/A + R/S*(I/A - U/R)$  rather than  $T/S = I/A + R/S*(I/A + U/R)$ . Several other candidates incorrectly included investment return in the insurance operating results.

#### **Part b:**

This part of the question was significantly more challenging for candidates, as it required the candidate to recall a specific conceptual lesson from within Ferrari's paper rather than a formula. Ferrari discusses at length the need to consider the effect of insurance operations on both the magnitude and quality of an insurer's earnings, and summarizes that "the central issue of the optimal capital structure is the effect of non-equity financing such as reserves on the quality (variance) of the insurer's earnings, and, hence, on the rate at which the earnings are capitalized by the market for valuation purposes." Full credit was awarded only to candidates who demonstrated (i) understanding that "non-equity financing" for insurers referred to the use of reserves for investment and (ii) that the impact on the quality of earnings was to increase the volatility or variance.

Many candidates focused instead on the effect of U/R on the magnitude of an insurer's earnings (or on the relationship between I/A and U/R in determining whether or not to continue writing business). While this misses the key point that insurance underwriting increases earnings volatility, the graders gave significant partial credit to such responses in recognition of potential confusion over the term "quality of an insurer's earnings." Several candidates discussed the use of (true) debt financing rather than reserve financing; since the question does not define non-equity financing (though it is clear in context from the reference to insurers), partial credit was awarded to such responses. Overall, due to potential ambiguity in the question wording and the diversity of exam responses, the graders scored this part of the problem leniently, awarding credit where the response was deemed to demonstrate some understanding of the underlying concept.