CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

May 8, 2012

Exam 7

Estimation of Policy Liabilities, Insurance Company Valuation, and ERM

4 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 65.25 point examination consists of 24 problem and essay questions.

2. For the problem and essay questions the number of points for each full question or part of a question is indicated at the beginning of the question or part. Answer the questions on the lined sheets provided in your Examination Envelope. Use dark pencil or ink. Do not use multiple colors.

   - Write your Candidate ID number and the examination number, 7, at the top of each answer sheet. Your name, or any other identifying mark, must not appear.
   - Do not answer more than one question on a single sheet of paper. Write only on the front lined side of the paper – **DO NOT WRITE ON THE BACK OF THE PAPER.** Be careful to give the number of the question you are answering on each sheet. If your response cannot be confined to one page, please use additional sheets of paper as necessary. Clearly mark the question number on each page of the response in addition to using a label such as "Page 1 of 2" on the first sheet of paper and then "Page 2 of 2" on the second sheet of paper.
   - The answer should be concise and confined to the question as posed. When a specific number of items is requested, do not offer more items than the number requested. For example, if three items are requested, only the first three responses will be graded.
   - **In order to receive full credit or to maximize partial credit on mathematical and computational questions, you must clearly outline your approach in either verbal or mathematical form, showing calculations where necessary. Also, you must clearly specify any additional assumptions you have made to answer the question.**

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **fifteen-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. A chart indicating the point value for each question is attached to the back of the examination. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until two hours after the start of the examination. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

7. At the end of the examination, place all answer sheets in the Examination Envelope. Please insert your answer sheets in your envelope in question number order. Insert a numbered page for each question, even if you have not attempted to answer that question. Anything written in the examination booklet will not be graded. Only the answer sheets will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc., must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by May 28, 2012.

END OF INSTRUCTIONS

1. (2.75 points)

Given the following information for accident year 2011 as of December 31, 2011:

• Accident year 2011 paid loss:	$700,000
• 2011 earned premium:	$3,000,000
• Initial expected loss ratio:	62.5%
• 12-24 month paid link ratio:	1.50
• 12-ultimate cumulative paid LDF:	2.50

a. (1.25 points)

Calculate accident year 2011 ultimate loss estimates as of December 31, 2011 using each of the following three methods:

i. Chain ladder
ii. Bornhuetter-Ferguson
iii. Benktander

b. (1.5 points)

Determine the accident year 2011 incremental paid loss in 2012 that would result in the Benktander ultimate loss estimate being $50,000 greater than the Bornhuetter-Ferguson ultimate loss estimate for accident year 2011, as of December 31, 2012. Assume all selected development factors remain the same.

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2. (3.5 points)

Given the following information as of December 31, 2011:

Accident Year	On-level Premiums	Cumulative Paid Loss	Fitted Paid Emergence Pattern
2008	$1,300,000	$600,000	70%
2009	1,200,000	350,000	45%
2010	1,200,000	200,000	25%
2011	1,300,000	75,000	10%

- Parameter standard deviation: 300,000
- Process variance/mean scale parameter (σ^2): 10,000

a. (2 points)

Estimate the total loss reserve using the Cape Cod method.

b. (0.5 point)

Calculate the process standard deviation of the reserve estimate in part a. above.

c. (1 point)

Calculate the total standard deviation and the coefficient of variation of the reserve estimate.

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3. (2.5 points)

Given the following information as of December 31, 2011:

Cumulative Incurred Losses ($000)

Accident Year	As of 24 Months	As of 36 Months
2004	2,000	2,600
2005	4,000	6,000
2006	6,500	11,700
2007	6,000	10,000
2008	3,600	6,600
2009	7,600	11,000
2010	5,000	

a. (2 points)

Using a volume-weighted average to calculate the overall age-to-age factor, create a plot of weighted residuals following Mack's methodology.

b. (0.5 point)

Based on the residual plot, assess whether the variance assumption underlying the chain ladder method has been met.

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4. (3 points)

Given the following information:

Incurred Loss Ratio

Accident Year	As of 36 Months	As of 48 Months
2006	0.222	0.375
2007	0.451	0.675
2008	0.446	0.605
2009	0.228	

a. (1.5 points)

Estimate the loss ratio for accident year 2009 as of 48 months using the least squares method.

b. (1.5 points)

An alternate approach to estimating the accident year 2009 loss ratio as of 48 months is to use the arithmetic average of the link ratio method and the budgeted loss ratio method. Using the answer from part a. above, demonstrate whether this averaging approach is optimal.

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5. (2.25 points)

A loss reserve actuary has reviewed three cumulative paid loss triangles to test whether the assumptions underlying the chain ladder method are met.

a. (0.75 point)

State the three chain ladder assumptions as described by Mack.

b. (1.5 points)

For each of the following situations, discuss whether any of Mack's assumptions are violated.

i. The first triangle shows a faster claims settlement pattern in the most recent calendar year resulting from the use of new claims management software.

ii. For the second triangle, the actuary found that the most appropriate selection method for loss development factors was to use an all year volume-weighted average approach.

iii. In the third triangle, the 36-48 month loss development factors are inversely proportional to the 24-36 month loss development factors. These relationships do not appear to be random.

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6. (1.5 points)

Given the following information for an insurance company's book of policies with a $250,000 deductible:

Age-to-Age Loss and ALAE Development Factors

	12-24	24-36	36-48	48-60	60-72
Limited	1.720	1.151	1.072	1.025	1.011
Excess	2.577	1.412	1.350	1.281	1.316

Limited Severity Relativities (R^L)

12	24	36	48	60	72
0.975	0.963	0.955	0.944	0.931	0.912

Calculate the unlimited 36-72 month development factor.

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7. (3 points)

For a set of retrospectively rated workers compensation policies, the following factors apply:

- Basic premium factor: 0.20
- Loss conversion factor: 1.25
- Tax multiplier: 1.04

There will be three retrospective adjustments, with the following estimated factors:

Retro Adjustment Period	% Loss Emerged	Loss Capping Ratio
First	75.1%	0.93
Second	19.5%	0.72
Third	5.4%	0.55

Also assume:

- Total expected unlimited loss for these policies is $205,000,000.
- The expected loss ratio is 75%.
- Total current booked premium is $220,000,000.
- None of the policies has yet had its first retro adjustment.

Estimate the retrospective premium asset for this set of policies.

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8. (3.75 points)

Given the following loss development triangle and selections:

Incremental Incurred Losses ($000)

Accident Year	0-12 Months	12-24 Months	24-36 Months	36-48 Months	48-60 Months
2007	5,800	1,754	875	310	54
2008	6,000	1,675	780	285	
2009	5,700	1,850	888		
2010	5,850	1,500			
2011	6,300				

Selected Age-to-Age Link Ratios

12-24 Months	24-36 Months	36-48 Months	48-60 Months	60 Months to Ultimate
1.290	1.112	1.035	1.006	1.000

- Incremental losses, C_{ij}, follow an over-dispersed Poisson distribution with mean $x_i y_j$ and variance $\phi x_i y_j$.
- The variable x_i represents the expected ultimate losses for accident year i.
- The variable y_j represents the proportion of ultimate losses that emerge in development year j.
- The prior distribution for x_i is gamma with mean α_i/β_i and variance $\alpha_i/(\beta_i)^2$.
- The dispersion parameter, ϕ, for the over-dispersed Poisson distribution is 8.429.
- The accident year 2010 estimates for α and β are 100 and 0.01143 respectively.

The mean of C_{ij} for this Bayesian model is:

$$Z_{ij} \times (\lambda_j - 1) \times D_{i,j-1} + (1 - Z_{ij}) \times (\lambda_j - 1) \times x_i \times \left(\frac{1}{\lambda_j} \right)$$

where:

$$Z_{ij} = \frac{\sum_{k=1}^{i-1} y_k}{\beta_i \times \varphi + \sum_{k=1}^{i-1} y_k}$$

- λ_j is the incremental chain ladder loss development factor for development year j.
- D_{ij} is the cumulative losses for accident year i as of development year j.

QUESTION 8 CONTINUED ON NEXT PAGE

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(8 continued)

a. (3 points)

Use this model to calculate the incremental losses for accident year 2010 expected to emerge between 24 and 36 months of development.

b. (0.75 point)

Explain how this model can be interpreted as a trade-off between the standard chain ladder method and the Bornhuetter-Ferguson method.

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9. (2 points)

a. (0.5 point)

Briefly describe two advantages that non-parametric smoothing models for claims run-off pattern estimates have over models based on parametric curves.

b. (0.5 point)

Describe the role of the θ parameter in the incremental claim predictor:

$$\eta_{ij} = s_{\theta_i}(i) + s_{\theta_j}(\log(j)) + s_{\theta_j}(j)$$

c. (0.5 point)

Briefly describe the fit of the model from part b. above when θ is at each extreme.

d. (0.5 point)

Identify the resulting models when θ is at each of its extremes.

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10. (2.75 points)

Given the following information for two proportional casualty treaties as of December 31, 2011:

Treaty ID	Inception Date	Treaty Coverage Basis	Written Premium	Aggregate Reported Loss
A	Jan 1, 2009	Loss Occurring	$ 80,000	$ 50,000
B	Jul 1, 2009	Policies Written	100,000	20,000

Age of Accident Year (in months)	Age to Ultimate LDF
36	1.25
24	2.00
12	3.00

- Both treaties have a one-year term.
- All underlying policies are one-year, occurrence-based policies with effective dates uniformly distributed over the year.
- Treaty A commission, brokerage and internal expense is 5% of premium.
- Treaty B commission, brokerage and internal expense is 20% of premium.
- The loss development factors apply to all exposures in the applicable accident year.

a. (2 points)

Estimate the ultimate losses for the two treaties combined using the Stanard-Bühlmann method.

b. (0.25 point)

Identify a situation in which the chain ladder method is preferred over the Stanard-Bühlmann method when estimating IBNR for a reinsurer with long-tail exposures.

c. (0.25 point)

Identify a situation in which the Stanard-Bühlmann method is preferred over the chain ladder method when estimating IBNR for a reinsurer with long-tail exposures.

d. (0.25 point)

Identify the key innovation of the Stanard-Bühlmann method over the Bornhuetter-Ferguson method.

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11. (3.75 points)

Identify and explain five technical problems that make loss reserving for a reinsurer more difficult than loss reserving for a primary company.

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12. (3.5 points)

Given the following financial information for an insurance company (in $000,000):

	Actual	Projections				
	2011	2012	2013	2014	2015	2016
Net Income	180	190	200	211	222	234
Beginning Equity	2,000	2,108	2,222	2,342	2,468	2,602
Ending Equity	2,108	2,222	2,342	2,468	2,602	2,742

- The expected market return on equity for peer insurance companies is 10%.
- The risk-free rate based on US Treasury bill rates is 2%.
- The equity beta, based on peer companies, is 0.85.
- The company's business plan contemplates a 60% plowback ratio.
- CAPM is an appropriate model for determining the company's risk-adjusted discount rate.

Determine the value of this company as of December 31, 2011 based on the dividend discount model and the projections for 2012 through 2016.

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13. (4 points)

Given the following data and estimates for a personal lines insurer:

Current Business

	Expected claim count	Expected claim severity
Auto	6,800	$700
Homeowners	9,000	750

Loss Reserves

	Claim count	Claim severity
Auto	2,600	$2,100
Homeowners	400	1,250

The insurer's total losses are modeled by a lognormal distribution with the following Value at Risk (VaR) amounts for certain percentiles:

$$\begin{aligned} VaR_{95} &= 20{,}346{,}511 \\ VaR_{96} &= 20{,}540{,}313 \\ VaR_{97} &= 20{,}795{,}018 \\ VaR_{98} &= 21{,}132{,}866 \\ VaR_{99} &= 21{,}680{,}765 \end{aligned}$$

The risk based capital required for underwriting risk is given by the following factor based formula:

$TVaR_{95}$ – Expected Loss on Current Business – Loss Reserve

a. (1.5 points)

Calculate the estimated risk based capital required to support this insurer's underwriting risk.

b. (1.5 points)

Briefly discuss three limitations with the above model, and suggest changes that would address each.

QUESTION 13 CONTINUED ON NEXT PAGE

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(13 continued)

c. (1 point)

The CEO of this insurer wants to reduce the risk based capital calculated in part a. above by half going forward. The CEO plans to achieve this goal by buying 50% quota share reinsurance on the entire book of business. Fully explain why that will not achieve this goal.

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14. (2 points)

The Chief Risk Officer for an insurance company is considering U.S. Treasury interest rate models for the company's Enterprise Risk Management process.

a. (0.75 point)

Provide the formula for the Vasicek model and briefly describe the process it models.

b. (0.5 point)

Describe the key difference between the Cox, Ingersoll, Ross (CIR) model and the Vasicek model.

c. (0.75 point)

Provide and support a recommendation as to which of the two models, Vasicek or CIR, is more appropriate to use in the current environment of historically low interest rates.

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15. (2.75 points)

Given the following auto liability claims information:

Claim #	Case Incurred Indemnity ($)	Case Incurred ALAE ($)
1	800	100
2	5,000	250
3	2,100	0
4	12,000	1,800
5	77,000	4,000

a. (1 point)

Calculate the Pearson's correlation coefficient.

b. (1.25 points)

Using a dependency measure other than Pearson's correlation coefficient, draw a conclusion with respect to the appropriateness of using Pearson's correlation coefficient for this set of claims information.

c. (0.5 point)

Identify one similarity and one difference between Pearson's correlation coefficient and the dependency measure in b. above.

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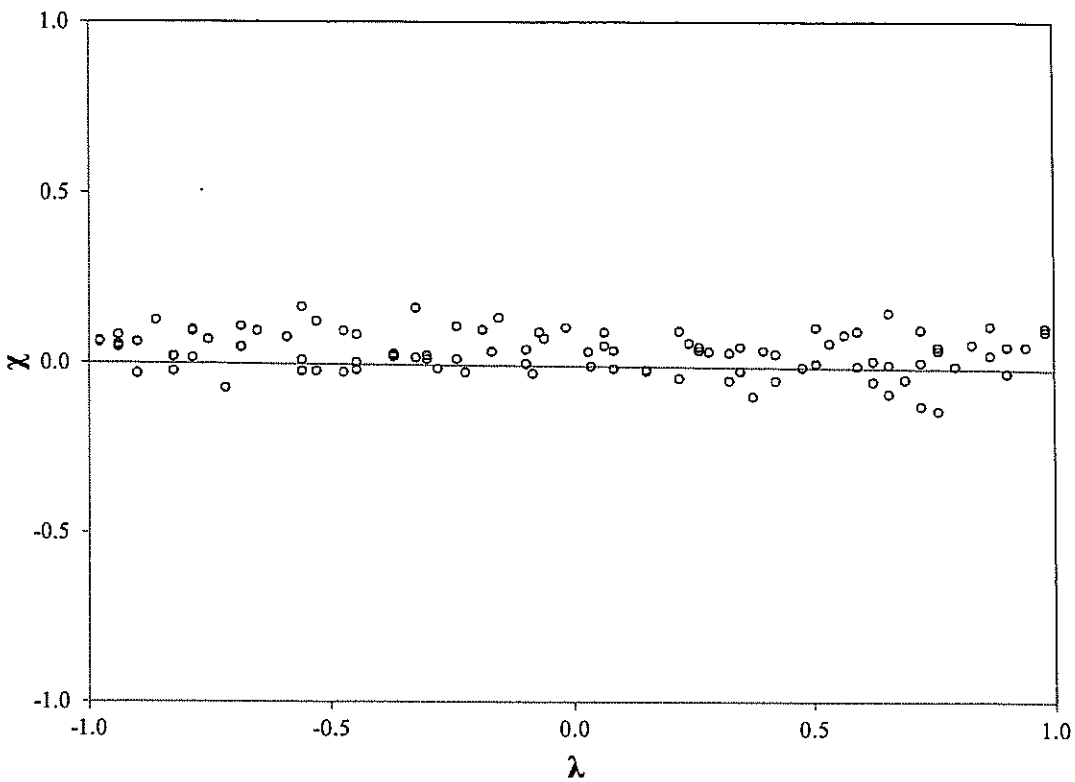
16. (1.5 points)

A χ-plot of indemnity amounts and ALAE amounts for auto physical damage claims is shown below.



a. (1 point)

Explain what the chi (χ) and lambda (λ) variables measure.

b. (0.5 point)

Based on the χ-plot, draw a conclusion regarding the auto physical damage book of business.

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17. (4 points)

An insurer writes property and workers compensation insurance in California. To model the total annual losses for this insurer, the following two copulas are being considered:

i. $C(u,v) = uv$

ii. $C(u,v) = \min(u,v)^{0.25}(uv)^{0.75}$

a. (1 point)

Define and briefly explain the right-tail concentration function that may be used to characterize a copula.

b. (2 points)

Derive and graph the right-tail concentration functions for the two copulas.

c. (1 point)

Explain which copula is more appropriate to model total losses for this insurer. Briefly describe how the graph in part b. above supports your answer.

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18. (3 points)

Two insurance companies want to select appropriate risk measures, based on their aggregate loss distributions, for capital allocation purposes. The four risk measures under consideration are:

Standard deviation
Semi-standard deviation
VaR
TVaR

a. (1.5 points)

Company A writes homeowners and commercial property insurance in several coastal states. Its main concern is hurricane risk. Assess the appropriateness of each risk measure for Company A and determine the best one.

b. (1.5 points)

Company B writes personal auto insurance in several mid-western states. Its main concern is the price adequacy of their book. Assess the appropriateness of each risk measure for Company B and determine the best one.

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19. (2.5 points)

ABC Company is a small personal lines insurance company. ABC is lightly capitalized relative to its written premium. The company's CFO worries that the rating agencies might downgrade the company if it experiences extremely adverse underwriting results in the upcoming accident year. ABC Company has received quotes for stop-loss reinsurance programs from two reinsurance companies. The CFO is considering the following options.

Option	Reinsurer's rating	ABC's expected return on GAAP equity
Reinsurer X	AAA	10%
Reinsurer Y	A– with negative outlook	15%
No reinsurance		25%

a. (1.5 points)

Discuss the CFO's reinsurance options with regard to the costs of financial distress and taxation.

b. (1 point)

Discuss, in terms of agency theory, a CFO bonus structure for ABC Company in which the CFO's bonus is maximized by achieving a 30% return on GAAP equity and minimized by achieving less than a 15% return on GAAP equity.

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20. (3 points)

a. (1.5 points)

Describe three reasons for an insurer to purchase reinsurance.

b. (1.5 points)

For each reason given in part a. above, explain whether proportional or non-proportional reinsurance would best satisfy that reason for purchasing reinsurance.

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21. (2.25 points)

    a. (1 point)

       Describe two costs of financial distress that apply specifically to insurance companies.

    b. (0.75 point)

       Briefly describe three other potential costs of financial distress that might apply to a firm in any industry.

    c. (0.5 point)

       Explain why firms with higher market-to-book ratios or higher research and development expenditures may gain more value from risk management than other firms.

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22. (2 points)

An insurance company is planning to upgrade its current claims system. The project is expected to take several months, during which time the claims system will be periodically offline. In addition, much of the design and installation of the new system will be outsourced to a third party.

a. (0.5 point)

Describe what is meant by "operational risk."

b. (1.5 points)

Identify two potential sources of operational risk this project entails and suggest a way to mitigate each one.

CONTINUED ON NEXT PAGE

23. (2 points)

    a. (1.5 points)

       Describe three main sources of underwriting risk that should be modeled separately in an insurance company's internal solvency model.

    b. (0.5 point)

       Explain the difficulty in determining the cross-class correlation component of the internal model.

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24. (2 points)

a. (0.5 point)

Contrast a capital model calibrated to rating agency capital with one calibrated to an economic capital level.

b. (0.5 point)

Explain how an internal model calibrated to an economic impairment earnings level is different than the models described in part a. above.

c. (1 point)

Describe two examples of why it may not be practical to use a single internal model for multiple ERM, operational, and capital applications.

END OF EXAMINATION

Exam 7
May 2012

Examiners' Report
with Sample Solutions

Question 1 Sample Answer

Solution 1

a) AY ultimate chain ladder = paid to date × $LDF_{12\text{-ult}}$
= 700,000×2.5
= 1,750,000

AY ultimate B-F = Paid to date + expected ultimate × % unpaid
= 700,000 + (3,000,000 × .625) × (1 – (1/2.5))
= 1,825,000

AY ultimate G-Benktander = Paid to date + BF ultimate × % unpaid
= 700,000 + 1,825,000 × (1 – (1/2.5))
= 1,795,000

24-ultimate factor = (2.5/1.5) = 1.667

b) G-Benktander ultimate = (700,000 + x) + U_{BF} × (1 – (1/1.667))

BF ultimate = (700,000 + x) + (3,000,000 × .625) × (1 – (1/1.667))
= 700,000 + x + 750,000
= 1,450,000 + x

∴ G-Benktander ultimate = (700,000 + x) + (1,450,000 + x) (1 – (1/1.667))
= 700,000 + x + (1,450,000 + x) (0.4)
= 700,000 + x + 580,000 + 0.4x
= 1,280,000 + 1.4x

Benktander ultimate = BF ultimate + 50,000

1,280,000 + 1.4x = 1,450,000 + x + 50,000
= 1,500,000 + x

0.4x = 220,000
x = 550,000

Sample 2

a) Chain ladder 700k × 2.5 = 1.75M
BF 700k + (3000k) (0.625) (1 – (1/2.5)) = 1.825M
Benktander:

p_k = 1/2.5 = 0.4
q_k = 1 – 0.4 = 0.6

(0.6^2) (3000k) (0.625) + $(1 – 0.6^2)$ (1.75M)
= 675 + 1120 = 1795k

b) 24-ult LDF = 2.5/1.5 = 5/3

p_{24} = 0.6 q_{24} = 0.4

$Ult_{Benk} – Ult_{BF} = Resv_{Benk} – Resv_{BF} = q_K U_{BF} – q_K U_0$ = 50,000

= (0.4) [x + 0.4 (3000k) (0.625)] – (0.4) (3000k) (0.625) = 50k

x – (0.6) (3000k) (0.625) = 125k

x = 1250k

2012 incremental paid loss 1250k – 700k = 550k

Examiner Comment

The a. part of this question was fairly straightforward. Common errors were arithmetic errors and errors in the formulas for each method.

The b. part involved setup of both the Bornheutter-Ferguson and the Benktander methods and solving a system of equations to determine the incremental paid loss. Common errors were in calculating q_k, errors in recognizing that part b is a different time (12/31/2012) period than part a (12/31/2011), errors in the setup of the problem, and other calculation errors.

Question 2 Sample Answer

Solution 1

a) OLEP × Paid Emergence

AY	Used up EP	Paid Loss
2008	910,000	600,000
2009	540,000	350,000
2010	300,000	200,000
2011	130,000	75,000
Total	1,880,000	1,225,000

SB ELR = 1.225M/1.880M = 65.159% (Call this "x")
SB IBNR = ∑ (ELR) × (1 – % paid) × OLEP

= .30 (1.3M) (x) + .55 (1.2M) (x) + .75 (1.2M) (x) + .90 (1.3M) (x)
= 2,032,978.72

b) Process variance = $\sigma^2 \sum_i R_i$ = 10000(2032979)
Process SD = √(P Variance) = 142,582.56

c) Total Variance = Process Variance + Parameter Variance
= 10000(2032979) + 300000^2

Total SD = √(Total Variance) = 332,159.28

CV reserve estimate = (total SD)/($\sum_i R_i$) = 332159.28/2032978.72 = .1634

Solution 2

a)

	(1) On lvl prem	(2) paid loss	(3) % paid	(4)=(1)×(3) used up prem	(5)=(1)ELR(1-(3)) (5) loss reserve
08	1,300,000	600,000	.70	910,000	254,122
09	1,200,000	350,000	.45	540,000	430,053
10	1,200,000	200,000	.25	300,000	586,436
11	1,300,000	75,000	.10	130,000	762,367
		1,225,000		1,880,000	2,032,978

ELR = ∑(2)/ ∑(4) = 1,225,000/1,880,000 = .6516

b) Process variance = $\sigma^2 R$
$= 10{,}000(2{,}032{,}978)$
$= 2.033 \times 10^{10}$

Process st. dev = $\sqrt{(2.033 \times 10^{10})} = 142{,}583$

c) Total st. dev = $\sqrt{((2.033 \times 10^{10}) + 300{,}000^2)} = 332{,}159$
Coefficient of variation = st. dev/mean = 332,159/2,032,978 = .163

Examiner Comment

The a. part of this question was a basic question on the Cape Cod methodology. Common errors included errors in calculation, and failure to recognize that the fitted emergence pattern was the % used (not the % unused).

The b. part requires the understanding that process variance = $\sigma^2 \times$ mean and that standard deviation is the square root of the variance. Common errors included misunderstanding of these concepts, as well as arithmetic errors.

The c. part requires the understanding that total variance is the sum of parameter and process variance, and that standard deviation is the square root of the variance. Common errors included adding the parameter standard deviation to the process deviation to find the total standard deviation, general misunderstanding of these concepts, as well as arithmetic errors.

Question 3 Sample Answer

Solution 1

a)

Mack residuals for volume weighted average $= \dfrac{C_{i,k+1} - C_{ik} f_k}{\sqrt{C_{ik}}}$

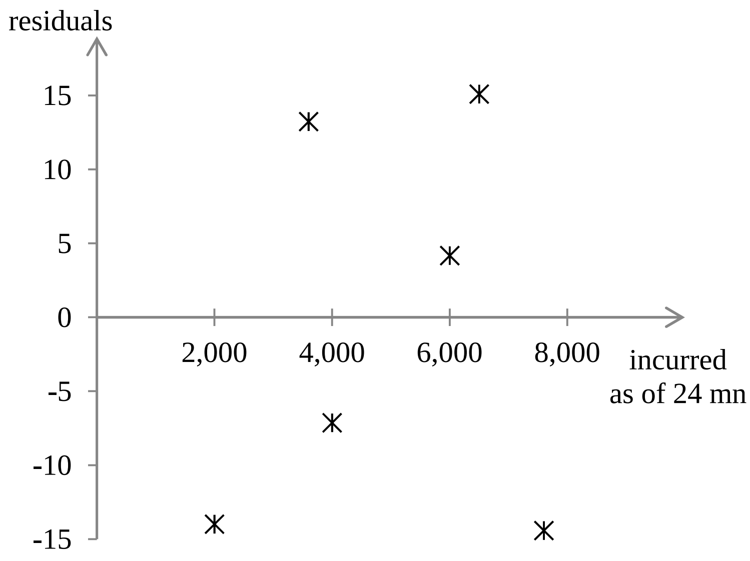
$$f = \frac{2600 + 6000 + \ldots + 11000}{2000 + 4000 + \ldots + 7600} = \frac{\sum \text{cuml losses as of 36 months}}{\sum \text{cuml losses as of 24 months}}$$

$$= \frac{47{,}900}{29{,}700} = 1.613$$

AY	residuals
04	-14
05	-7
06	15
07	4
08	13
09	-14

$$\leftarrow \frac{2600 - 2000 \times 1.613}{\sqrt{2000}}$$

Incurred as of 24 mths
2000
4000
6500
6000
3600
7600



An underlying assumption of the chain ladder method is linearity of the development. We expect to see residuals scattered evenly about the y-axis. These residuals do appear randomly scattered about the x-axis ∴ the variance assumption is met.

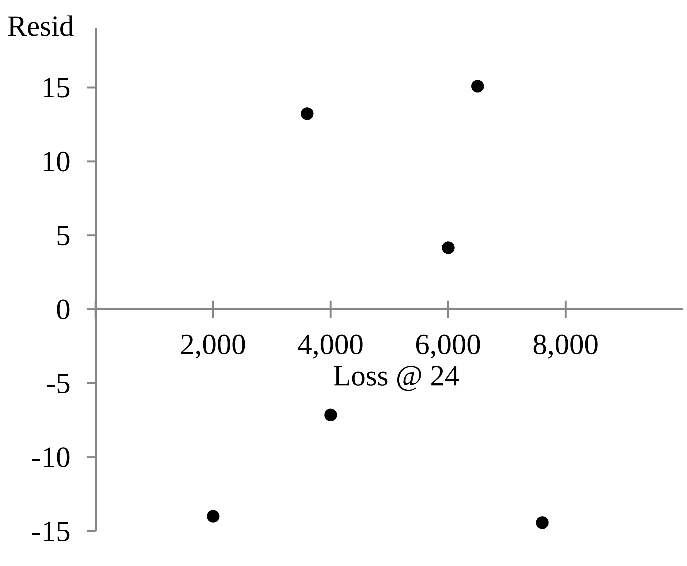
Solution 2

a)

$$\text{Age-Age} = \frac{(2600 + \ldots + 11,000)}{(2000 + \ldots + 7600)}$$

$$= \frac{47900}{29700} = 1.613$$

Yr	(a) 24 loss × 1.613	$\frac{\text{Loss@36} - (\text{a})}{\sqrt{\text{Loss@24}}}$	
4	3,226	–14	= (2000–3226)/√2000
5	6,452	–7.15	
6	10,485	15.07	
7	9,678	4.16	
8	5,807	13.22	
9	12,259	–14.44	



b) The points are fairly randomly scattered around 0. They don't seem to be increasing or decreasing. So it appears the variance assumption has been met.

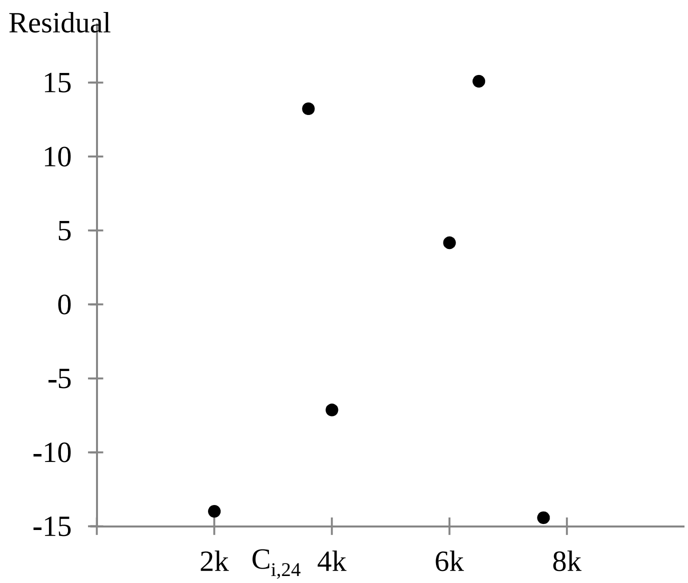
Solution 3

a)

$$\text{24 to 36 LDF} = \frac{\sum \text{losses @ 36 months years '04 to '09}}{\sum \text{losses @ 24 for years '04 to '09}}$$
$$= 1.613$$

$$\hat{C}_{ij} = 1.613\left(C_{ij-1}\right) \qquad \text{residual} = \frac{C_{i36} - \hat{C}_{i36}}{\sqrt{C_{i24}}}$$

$C_{i,36}$	$\hat{C}_{i36}$	Residual
2600	3226	–14.00
6000	6452	–7.15
11700	10484.5	15.08
10000	9678	4.16
6600	5806.8	13.22
11000	12258.8	–14.94



b) The residuals should be random around the zero, with no clear patterns as $C_{i,24}$ increases. This appears to be the case. So we can assume the variance assumption is met.

Examiner Comment

Most candidates knew how to calculate the weighted LDF. There were a number of variations in the form of the residual calculation. Partial credit was given if the residual numerator had Actual – Expected. Many candidates did not use the square root of the 24 month actuals in the denominator. The graph was accepted if the plot was of residuals vs. loss @ 24 months, even if residuals were calculated incorrectly. For the assessment of the variance assumption, candidates frequently lost at least partial credit for not fully assessing and explaining the reasoning behind their answer. Candidates are encouraged to state what the assumption is and how the graph did or did not meet the criteria.

Question 4 Sample Answer

Solution 1

a)

x	y
0.222	0.375
0.451	0.675
0.446	0.605

$$L(x) = a + bx + \varepsilon$$

$$\hat{b} = \frac{\overline{xy} - \overline{x} \cdot \overline{y}}{\overline{x^2} - \overline{x}^2} = \frac{(0.219) - (.373)(0.552)}{(0.151) - (0.373)^2}$$

$\overline{x} = 0.373$ $\quad \overline{xy} = 0.219$ $\quad \hat{b} = 1.104$

$\overline{y} = 0.552$ $\quad \overline{x^2} = 0.151$ $\quad \hat{a} = \overline{y} - \hat{b}\overline{x} = 0.140$

2009: L(0.228) = 0.140 + 1.104(0.228) = 0.392

b)

Link ratio method: $L(x) = x \cdot \dfrac{\overline{y}}{\overline{x}} = \dfrac{x}{d}$ where $d = \dfrac{\overline{x}}{\overline{y}}$

Budget Loss: $L(x) = \overline{y} = EY$

cred-wtd: $L(x) = Z\dfrac{x}{d} + (1 - z)EY$

Arithmetic average ⇔ Z = ½

Least squares is equivalent to a credibility weighing between the link ratio method and the budget loss method with weight Z given to link ratio.

where Z = b × d = (1.104) (0.373/0.552) = 0.746

Since arithmetic approach is equivalent to Z = ½ ≠ 0.746 it is not optimal

Solution 2

a)

$$b = \frac{\overline{xy} - \overline{x} \cdot \overline{y}}{\overline{x^2} - \overline{x}^2} = 1.175 \qquad a = .1135 \qquad y'(.228) = \boxed{.381}$$

b) $L(x) = ZU_{LR} + (1-Z)U_0$ LSE: $Z = \frac{b}{c}$ b from above, $c = \overline{y}/\overline{x} = 1.979$

$Z = .7944 > .5$

A straight average approach would not give the chain ladder enough weight

Solution 3

a)

AY	As of 36 X	As of 48 Y
06	0.222	0.375
07	0.451	0.675
08	0.446	0.605

$$b = \frac{\overline{xy} - \overline{x} \cdot \overline{y}}{\overline{x^2} - (\overline{x})^2} = 1.175$$

$$a = \overline{y} - \overline{x}b = 0.1135$$

$$\% \text{ loss ratio} = 0.1135 + (1.175)(0.228) = \boxed{38.14\%}$$

b)

$$\text{LDF} = \frac{\overline{y}}{\overline{x}} = \frac{1.655/3}{1.119/3} = 1.479$$

According to least square, other optimal Z in $b/(\overline{y}/\overline{x}) = 1.175/1.479 = 0.794$

By taking the arithmetic average of CL and budget loss ratio method, you are assuming that Z is 0.5. Thus, the alternate approach is <u>NOT</u> optimal.

Examiner Comment

The vast majority, over 95%, of candidates were able to apply the Lease Squares credibility formulas to calculate a, b, and y_{48}. Errors, if any, were generally found in arithmetic errors

It was expected that the candidates would conclude that the arithmetic average of the two methods would not produce the optimal result. Full credit was given when the candidate recognized that the least squares method in (a) is the best linear approximation to the Bayesian approach with a credibility weighting of the two methods. The weight given to the chain ladder approach, would be Z = b(hat) / c, where c is the weighted average LDF. The candidate should also recognize that the alternate approach is an approach where Z = 0.50.

Common errors which did not receive full credit for (b) involved the candidate calculating Z and proving that by using Z, you should arrive at the same answer as in part (a). These papers failed to actually answer the question asked about the optimality of the alternative approach.

Note that the actual calculation of the arithmetic average loss ratio need not be computed to receive credit for part (b).

Question 5 Sample Answer

Solution 1

a)

1. $E(C_{i,K}| C_{i,K-1}, \ldots, C_{i,0}) = f_K \times (C_{i,K-1} + C_{i,K-2} + \ldots)$
   Incremental losses equal a loss developmental factor times the cumulative losses for that AY from the prior point.

2. Losses for a given accident year are independent of losses for any other AY.

3. Variance of incremental losses equals a constant times the incremental losses where the constant only depends on the development age.

b)

1. Assumption #2 above is violated. The incremental losses along the diagonal will be influenced and will affect all AY's violating the independence assumption.

2. This does not violate any assumptions and is needed for assumption #3 above to hold

3. This violates assumption #1 above. The incremental losses in 36-48 are dependent upon the losses from 24-36. For #1 to hold, they are only dependent on the loss at time 36, not what happened prior to 36 months.

Solution 2

a)

1) Expected increments in next period equals loss paid to date (in AY) times a factor based on age.

2) Variance is proportioned to losses paid to date (times a factor α_k^2 based on age)

3) Losses not in the same accident year are independent on each other.

b)

1) Yes, (c) would be violated. Calendar year effects mean that losses in various accident years have dependencies. Also, (a) may be violated; same factor based on age may not apply with calendar year effects.

2) None violated. Mack uses this too. Variance assumption depends on it.

3) Yes, if higher dev factor follows a lower one and vice versa, this implies that (a) is violated. The factors based on age are not randomly varying around some true factor, so we don't want to use a model that assumes one true underlying factor based on age to apply to cumulative losses.

<u>Solution 3</u>

a)

1) The cumulative claims in a period only depend on the previous level of cumulative claims and a factor based on age. $\sum(C_{i,K+1}) = C_{iK}f_K$

2) Accident years are independent of each other.

3) The variance of the next period estimates is proportional to the previous cumulative. $Var(C_{i,K+1}) = C_{i,K} \cdot \alpha_k^2 \leftarrow$ factor based on age.

b)

1) The second assumption <u>is violated</u> as all of the accident years are being affected by this new claims management software and cannot assume that the AYs are independent.

2) <u>none are violated.</u> The all year volume wtd approach follows from the variance assumption. This is the basic chain ladder.

3) If columns of development factors are not independent, this violates Mack's first assumption. The development in a period should only be dependent on the prior cumulative and a factor for that age, not a prior development factor. Since it seems like the development is dependent on the prior development factor, it appears to violate Mack's assumptions.

<u>Solution 4</u>

a)

i. Accident year losses are independent of each other.

ii. Expected incremental losses are based on cumulative losses for the accident period to date.

iii. Variance of the accident period is proportional to losses (paid/reported) to date and age.

b)

i. Violated. This shows that accident year losses are not independent as there are calendar year effects.

ii. Appropriate. The volume weighted link ratios produce minimum variance in the Mack model.

iii. Violated. This shows correlations between the columns.

Examiner Comment

We also accepted the formulas that underlie Mack's assumptions

Many candidates lost credit for not explaining why the assumptions were valid or violated.

<u>Question 6 Sample Answer</u>

<u>Solution 1</u>

36-72 dev. factor = 36-48 factor × 48-60 factor × 60-72 factor
Limited = 1.072 × 1.025 × 1.011 = 1.111
Excess = 1.350 × 1.281 × 1.316 = 2.726

$LDF_{36\text{-}48} = LDF^{L}_{36\text{-}48} \cdot R_{36} + LDF^{XS}_{36\text{-}48}(1 - R_{36})$
$= 1.072(0.955) + 1.350(1–0.955)$
$= 1.085$

$LDF_{48\text{-}60} = 1.035(0.944) + 1.281(1–0.944)$
$= 1.039$

$LDF_{60\text{-}72} = 1.011(0.931) + 1.316(1–0.931)$
$= 1.032$

total $LDF_{36\text{-}72} = 1.088 \times 1.039 \times 1.032 = 1.163$

<u>Solution 2</u>

Limited $LDF_{36\text{-}72} = (1.072)(1.025)(1.011) = 1.111$
XS $LDF_{36\text{-}72} = (1.35)(1.281)(1.316) = 2.276$

$R^L = 9.55$ – the 36 month rel

Unlimited $LDF_{36\text{-}72} = (LimLDF_{36\text{-}72}(R^L) + (XSLDF_{36\text{-}72})(1–R^L)$
$= (1.111)(.955) + (2.276)(1-.955)$
$= 1.163$

<u>Solution 3</u>

From $\frac{LDF^{L}_{36\text{-}72}}{LDF_{36\text{-}72}} = \frac{R^{L}_{72}}{R^{L}_{36}}$, and $LDF^{L}_{36\text{-}72} = 1.072 \times 1.025 \times 1.011 = 1.1109$

$R^{L}_{72} = 0.912 \qquad R^{L}_{36} = 0.955$

$\therefore LDF_{36\text{-}72} = \frac{1.1109}{0.912} \times 0.955 = 1.163$

Solution 4

$LDF_t = LDF_t^L \times R^L + XSLDF_t^L(1 - R^L)$
$LDF_t^L = LDF_t \, \Delta R^L$
$XSLDF_t^L = LDF_t \, \Delta(1 - R^L)$

	36-72
Limited:	$1.072 \times 1.025 \times 1.011 = 1.1108868$
Excess:	$1.35 \times 1.281 \times 1.316 = 2.2758246$

Using $LDF_t^L = LDF_t + \Delta R^L$

$\Rightarrow 1.1108868 = LDF_t + (.912/.955)$
$\Rightarrow LDF_t = (1.1108/.95497) = \underline{\underline{1.163}}$

If using $XSLDF_t^L = LDF_t \Delta \, (1 - R^L)$

$$2.2758 = LDF_t + ((1 - .912) / (1 - .955)) = LDF_t \times 1.955$$

$LDF_t = \underline{\underline{1.163}}$

Examiner Comment

For this question, most candidates received full credit.

The following was a common error that yielded a correct final factor but used incorrect methodology. We gave partial credits for this response.

$LDF_t^L = 1.072 \times 1.025 \times 1.011 = 1.1109$

$XSLDF_t^L = 1.316 \times 1.281 \times 1.35 = 2.2758$

$LDF_t = 1.1109 \times (.912/.955) + 2.2758 \times (1 - .912/.955)$

$LDF_{t\,72} = 1.1633$

Question 7 Sample Answer

Solution 1

Since no adjustments have been made, find the cumulative capping ratio.

$$\frac{CL_{UH}}{L_{UH}} = .751(.93) + .195(.72) + .054(0.55) = .869$$

L_{UH} = 205M (given) $\Rightarrow$ CL_{UH} = 205M(.869) = 178.05M

Standard premium = 205M / .75 = 273.33M

Premium Asset = [0.2(273.33M) + (178.05M)(1.25)](1.04) – 220M
= 68,316,578

Solution 2

Retro Period	%Emerged × 205M Total loss in period	(Total loss in period)×(LCF) Capped loss	
1	153,955,000	143,178,150	←153,955,000×0.93
2	39,975,000	28,782,000	
3+	11,070,000	6,088,500	
		178,048,650	

Initial Prem = 205,000,000 / 0.75 =
BP = basic prem 205M/0.75 = 54,666,667
Prem = (BP + Capped Loss × LCF) × TM

Premium= (54,666,667 + 178,048,650 × 1.25) × 1.04
including adjustments

288,316,579

Premium asset = (estimated premium including adjustments) – booked premium

68,316,579

Solution 3

$$PDLD_1 = \left[\frac{BP \times TM}{SP \times \%\text{Loss} \times ELR}\right] + \left[\frac{CL_1}{L_1} \times LCF \times TM\right]$$

$$= \left[\frac{(0.2)(1.04)}{(.751)(.75)}\right] + \left[(.93) \times 1.25 \times 1.04\right] = 1.578$$

$$PDLD_2 = \frac{CL_1 - CL_2}{L_2 - L_1} \times LCF \times TM = 0.72 \times 1.25 \times 1.04 = 0.936$$

$$PDLD_3 = 0.55 \times 1.25 \times 1.04 = 0.715$$

$$CPDLD = \frac{(1.578)(.751) + (.936)(.195) + (.715)(.054)}{(.751) + (.195) + (.054)} = 1.406$$

$$\text{Asset} = (205\text{M} \times 1.406) - 220\text{M} = \$68\text{M}$$

Examiner Comment

Many candidates interpreted the loss capping ratios to be cumulative rather than incremental, producing nonsensical results. Candidates should have recognized that additional loss dollars will not reduce the premium as implied in this approach.

A number of candidates calculated estimated premium incorrectly by applying the CPDLD to loss dollars at each period and summing.

Many candidates incorrectly calculated the basic premium.

Question 8 Sample Solution

Solution 1

a)

$$\text{expected } c_{ij} = \frac{\alpha}{\beta} = \frac{100}{.01143} = 8748.91 \qquad \text{24-36 for AY10}$$

$$\begin{aligned} \text{LDF}_{\text{12-ult}} &= 1.29 \cdot 1.112 \cdot 1.035 \cdot 1.006 \\ &= 1.4936 \\ \text{LDF}_{\text{24-ult}} &= 1.1578 \end{aligned}$$

	0-12	12 -24
1/LDF = D_k =	.66952	.1941814

Z = (.66952 + .1941814) / (.01143 · 8.429 + .66952 + .1941814) = .899647

C_{ij} = .899647 × (1.112-1) · (5850+1500) + (1-.899647)(1.112-1) · (8748.91/1.112)

= 829.02

Solution 2

a)

	12-Ult	24-Ult	36-Ult	48-ult	60-Ult
	1.4936	1.1578	1.04121	1.006	1.000
% Rptd.	66.95%	86.37%	96.04%	99.4%	100%

$$Z_{24\text{-}36} = \frac{86.37\%}{(8.429)(0.01143) + 86.37\%} = 0.8996$$

$$\lambda_{24\text{-}36} = 1.112$$

$$D_{2010,24} = 5850 + 1500 = 7350$$

$$x_{2010} = \text{mean of Gamma} = \frac{\alpha}{\beta} = \frac{100}{0.01143} = 8749$$

Incremental losses AY 2010 btw 24 and 36 months =
7350(1.112 – 1) × 0.8996 + 8749 (96.04% – 86.37%) × (1-.8996)
= 740.55 + 84.941

= 825.49

Solution 3

a) CL Estimate:

Expected incremental 2012 IBNR losses @ 24-36m
= (1,500 + 5,850)(1.112 – 1)
= 823,200

BF Method Estimate:

Expected losses = α/β = 100/.01143 = 8,749

IBNR Increment Estimate = Exp losses × ($D_{36\text{-}48}$ – $D_{24\text{-}36}$) → Need $D_{24\text{-}36}$
$D_{24\text{-}36}$ = (1/1.041) – (1/1.158) = 9.71%

	24:36	36:48
Cum LDF:	1.158 = (1.041)(1.112)	1.041 = (1.035)(1.006)

⇒ IBNR BF estimate = 8,749(.0971) = 849,151

$$Z = \frac{\frac{1}{1.158}}{.01143(8.429) + \frac{1}{1.158}} = \frac{D_{k-1}}{\beta\phi + D_{k-1}} = .8996$$

2010 Inc Losses Estimate = Z × CL estimate + (1 – Z) BF Estimate
= .8996(823,200) + (1-.8996) (849,151)

= $825,805

Solution 4

a) Cum losses AY 2010 @ 24 months = 5850+1500 =

Cum LDF	12-ult	24ult	36ult	48ult
		1.1578		

$$Z_{2010,24} = \frac{1/1.1578}{0.01143 \times 8.429 + \frac{1}{1.1578}} = 0.8996$$

$$C_{2010,36} = \underbrace{Z \times (1.1578 - 1) \times 7350}_{1043.38} + \underbrace{(1-Z)(1.1578-1) \times \frac{100}{0.01143} \times \frac{1}{1.1578}}_{119.7184}$$

$$\boxed{= 1163}$$

Solution1

b) Notice that the equation is $Z \cdot (\lambda - 1) \cdot D + (1 - Z)(\lambda - 1) \cdot x \cdot (1/\lambda)$

- $(\lambda - 1) \times D$ is a chain ladder estimate of incremental loss.
- $(\lambda - 1) \cdot x/\lambda$ is an expected emergence of our prior distribution.

Then both are credibility weight by Z, a factor that considers losses reported to date giving more weight to CL as more loss are reported. BF is similar in that it weights CL estimate with an expected ultimate based on Z of % loss reported.

Solution 2

b) The credibility weighting uses the chain ladder estimate ($Z(\lambda_j - 1) D_{ij-1}$) and the Bornhuetter Ferguson estimate ($(1 - Z)(\lambda_j - 1) x_i \times 1/\lambda_j$) weighted by Z and (1 – Z) respectively.

Examiner Comment

The model solution is based on the original paper interpretation of calculating E[C] rather than the formula in the exam. There were a couple differences between the formula on the exam and the formula in the paper. Because of the potential confusion, a number of model solutions were given full credit.

The formula for the mean of Cij for the Bayesian model is slightly different from the exam problem. Instead of $1/\lambda j$, the correct formula is $1/(\lambda_j \lambda_{j+1} \ldots \lambda_n)$. Candidates who used this formula instead will receive full credit as well. Additionally, candidates who used alternative formulas to calculate the BF estimate were given full credit as long as the method was accurate.

Additionally, the top of the summation term when calculating Z should have been j-1(j-1=2) rather than i-1(i-1=3). Most candidates ignored this difference but both answers were accepted.

Some of the common errors where candidates lost points include:

- Using the incremental LDF, or percent reported between 24 and 36 months, to calculate y
- Calculating x using the chain-ladder method rather than using α and β.
- Incorrect calculation of λ (e.g. using 36 to ultimate rather than 24 to ultimate)

The majority of candidates had little problem with the b. part. To receive full credit, candidates must discuss how the formula is a credibility weighted average of the chain ladder or BF method and must identify which part of the formula was C-L and which part of the formulas was BF.

Due to confusion in part a., some candidates discussed how the second half of the formula was "like a BF" but not exactly. These responses received full credit as well.

The most common errors were not mentioning "credibility" or "Z," and confusing the chain-ladder and B-F terms.

Question 9 Sample Answer

Solution 1

a) Two advantages =
   1. Not as rigid of a fit, so they can fit the data better, using a blending of 2 well known models.
   2. Parametric curves may end up showing trends that aren't in the data when extrapolating, or showing non-intuitive increasing development. Non-parametric models can control this.

Solution 2

a) There are fewer parameters required. Parametric models have been critiqued for being over parameterized. Smoothing models allow you to extrapolate beyond the end of your data.

Solution 3

a)
   1. Allows for bimodal distributions.
   2. Uses empirical data, which may give a closer approximation of the true dist. than the smoothed model especially @ early maturities. Allows for some volatility.

Solution 4

a) Advantage – you can reduce the number of parameters in your model which can decrease your estimation (parameter) variance.

   Disadvantage – The curve may not be able to match the actual shape of the development pattern and may overstate or understate factors.

Solution 5

a)
   - smoothing models respond more to data than parametric curves (at extreme = CL method)
   - Parametric curves may have too few parameters, improved if early dev periods get own wt.

Solution 6

a)

1) For non-parametric smoothing models no further assumptions on loss distribution parameters need to be made.

2) Moments beyond the first two moments can be estimated for the loss distribution.

Solution 7

a)

1. Non-parametric models are easy to incorporate judgment into selection of reserves. Parametric models will sometimes pick up false signal in the data.

2. Non-parametric models can handle negative development for salvage and subrogation or beneficial case reserve development. Many parametric models do not support negative development (like over dispersed Poisson and Gamma)

Solution 1

b) θ is the level of smoothing that takes place, trading off between a rigid fit and the CL method.

Solution 2

b) The θ parameter works as a smoothing parameter between the analytic result (say an over dispersed Poisson resulting in chain ladder estimates) and the Hoerl curve. The model works to fit and smooth simultaneously. It keeps your output from being too jumpy.

Solution 1

c) When $\theta = 0$, there is no smoothing and the fit is to the data itself. When $\theta = \infty$, the model is completely smoothed and linear.

Solution 2

c) When θ is small, more dependently is on the emerging data and when θ is large more dependency is on the parametric model.

Solution 1

d) When $\theta = 0$, this is the chain ladder method. When $\theta = \infty$, this is the Hoerl Curve.

Solution 2

d) When $\theta = 0 \Rightarrow$ chain ladder method
When $\theta \to \infty \Rightarrow$ Hoerl curve

Examiner Comment

For the a. part, a common answer that did not receive credit was the ability to extrapolate the data as extrapolation is not an unique characteristic/advantage of non-parametric smoothing models.

A common incomplete response for the b. part stated that theta was the smoothing parameter, but failed to identify a tradeoff aspect between goodness of fit and smoothing. Stating that theta "credibility weighted" two curves was also not a valid response.

Credit was given in the c. part if the response explained that the two extremes were unsmoothed and fully smoothed. The two extremes resulted in reliance on the empirical data and a smoothed parametric curve was also a common valid response.

Credit for the d. part was only given for stating the Chain Ladder or Hoerl/Gamma curve. Responses that did not mention the extreme values or note which curve went to each extreme received credit, but responses that matched the extreme with the wrong curve did not.

Question 10 Sample Answer

Solution 1

a) Remove commission, brokerage, & internal expenses

A 80,000 × (1–.05) = 76,000
B 100,000 × (1–.2) = 80,000

At 12/31/11, treaty A is 36 months mature
treaty B is 24 months mature (avg written date = 1/1/10)

So used-up premium = 76,000(1/1.25) + 80,000 (1/2)
= 100,800

Total losses = 70,000 so ELR = 70/100.8 = .694

IBNR = .694 (76(1– (1/1.25) + 80(1– 1/2)) = 38,333.33

Ult loss = IBNR + rep to date = $108,333.33

b) CL is preferred when it is difficult to bring premium from all yrs to same rate level.

c) SB is preferred when there are fluctuations in rep losses but we expect ELR (to on-level earned premium) has remained constant.

d) ELR is based on loss data, not just judgmentally selected.

Solution 2

a)

		@12/31/11 % Earned	Adj. Prem	
A	loss occurring (1/1/09–12/31/09)	100%	80(.95)=76K	@36 mo
B	pol written (7/1/09–6/30/09)	100%	80K	1/8@36 3/4@24 1/8@12

	Prem		% Rpt	Used Prem
2009	76 + (1/8)(180k)	= 86,000	1/1.25 = .8	68,800
2010	(3/4)(180k)	= 60,000	.5	30,000
2011	(1/8)(180k)	= 10,000	.333	3,000
		= 156,000		102,133

ELR = (50k+20k) / 102,133 = .685

(.685)(156,000) = 106,910

b) When the premium is difficult to adjust to current rate level or net the expenses, it is more appropriate to use chain ladder.

c) When the Stanard-Bühlmann method is preferred to the chain ladder.

d) Stanard-Bühlmann uses actual experience to estimate the expected loss ratio, where the Bornhuetter Ferguson method uses an apriori loss ratio.

Examiner Comment

Many candidates lost points for not removing expenses from the premium. Almost all candidates lost points for not correctly allocated the treaty premiums and losses over the 3 year subject period (AY's 09 – 11). If the development of the SB ELR and Ultimate Losses followed appropriate form, even without expense removal or proper AY allocation, full credit was available for that step. If there was no distinction in lag factors by AY, partial credit was subtracted. No credit was given for candidates that did not know to add the 2 contracts together for determining IBNR.

For b., c., and d., candidates answered fairly accurately. However, a number of candidates didn't relate the need for premiums at current rate levels as the main disadvantage of the SB method.

<u>Question 11 Sample Answer</u>

<u>Solution 1</u>

1) Reporting lags are greater for reinsurers. This can be due to many factors including:
   - Long pipeline through which claims data must travel to get from being reported to the cedent to being recorded by the reinsurer.
   - Claims may be undervalued and therefore remain below the claims reporting threshold for some time.

2) Claims reporting patterns differ by reinsurance line, type of contract, attachment point, etc.
   - Data is very heterogeneous making it difficult to use traditional reserving methods.

3) Industry statistics may not be useful
   - Schedule P does not have line broken down into categories that are homogeneous enough.
   - ISO statistics would need to be significantly revised for the specific reinsurance specifications in order to be used.

4) Reports received by reinsurers may be lacking important information (i.e. may have CY or UW data instead of AY).

5) Data coding and IT systems may not be able to keep up with the complex needs of reinsurers.

<u>Solution 2</u>

1) Report lags longer for the reinsurance than primary. This is because first has be reported to primary and decide if reportable to reinsurer. Also, lag because of time enter primary system then report to reinsurer and enter in their system.

2) Increasing emergence in reinsurance-combination of claim being reserved at modal value until more information or at minimum value. Also have economic/social inflation impact.

3) Reporting pattern varies for different types of reinsurance treaties and lines of insurance. Not consistency in reporting patterns so difficult reserve analysis in larger groups.

4) IT system deficiencies – because of delay in reporting and constant change in market the IT data systems for reinsurance always behind and need to be updated to capture necessary info.

5) Reinsurance requires more capital/surplus, more volatility and uncertainty in reinsurance plus regulatory requirements for capital.

Solution 3

1) Longer development pattern then primary insurer due to
    - Extended by cedant's reporting pipeline.
    - Cedant tend to under evaluate large claim.
    - Extreme delays in searching and reporting latent claims.

2) Consistent upward development, due to:
    - Inflation impact.
    - Cedant's tend to under evaluate ALAE.
    - Cedant reserve large claim at modal value when claims are initially reported.

3) Industry data is not helpful due to:
    - No breakdown of reinsurer's exposure in homogeneous group.
    - Severity of development increase with attachment point.

4) Missing claim information
    - Reinsurer's exposure is not completely measured in most recent year.
    - Miss detail claim information on excess loss level.

5) Claim development is extreme different due to:
    - Reinsurance contract is unique
    - Significant fluctuation during development because single large claims.

Solution 4

1) The claim reporting lag for reinsures is larger that for a primary reinsurer. Generally, the claim will not be reported to the reinsurer until the dollar amount reaches half the attachment point. The claim also has to go through a filter before entering the reinsurer's claim system.

    - Cedant's claim system
    - Cedant's reinsurance accounting dept.
    - Broker
    - Reinsurer books claim
    - Claim enters reporting system

2) There is persistent development trend upward for reinsurer loss data. This is due to inflation, tendency for cedant to book at modal value and tendency to underestimate ALAE.

3) Exposures are heterogeneous in the reinsurer loss data. Lines that have longer tails will be mixed with lines that have shorter tails. It is difficult to determine the expected loss and reporting pattern when they are combined.

4) There is a lack of important information available to the reinsurer on claims reported to them. Usually, the cedant only provides a summary of claims information. This lack of detail hampers the reinsurer from setting an accurate estimate of reserves for a claim.

5) Due to the heterogeneous exposures with different reporting patterns, reinsurers face data coding and IT reporting issues. The claims take a longer time to enter the reinsurer's claim reporting system as a result.

<u>Examiner Comment</u>

Partial credit was awarded for each *problem* as follows:

- for identifying the problem
- for explaining

No partial credit was given for any problem and/or explanations that were repeated. In order to receive full credit per problem, the candidate must list the problem and a corresponding reason. If problems and reasons were mismatched, partial credit was awarded where the mapping of reasons to problems could be identified.

Question 12 Sample Answer

Solution 1

$k = r_f + B \times (r_m - r_f)$
$= 0.02 + 0.85 \times (0.1 - 0.02)$
$= 0.088$

p = 0.6, pay 40% dividend = 1 – p

Yr	2012	13	14	15	16
Dividends = NI × 0.4	180(0.4) = 76	80	84.4	88.8	93.6
ROE = NI / Beg Equity	190/2108 = 0.090	0.090	0.090	0.090	0.090
Select ROE = 0.09					

g = p × ROE = 0.6 × 0.090 = 0.054

$$\text{Value} = \left.\frac{76}{1.088} + \frac{80}{1.088^2} + \frac{84.4}{1.088^3} + \frac{88.8}{1.088^4} + \frac{93.6}{1.088^5}\right\}\text{PV Div}$$

$$\left. + \frac{93.6(1.054)}{(1.088)^5\underbrace{(.088-.054)}_{k-g}}\right\}\text{TV @ Dec 2011}$$

$$= 327.73 + 1903.24$$

$$= 2230.97$$

Solution 2

Dividend % of NI = 1 - p = 40%

YR	NI	Dividends	ROE =NI/Beg Eq	PV Dividends =div/(1+k)^i
2012	190	76	9%	69.85
2013	200	80	9%	67.58
2014	211	84.4	9%	65.53
2015	222	88.8	9%	63.37
2016	234	93.6	9%	61.39
Total				327.72

$k = r_f + \beta(E(r_m) - r_f) = 2\% + 0.85(.1 - .02) = 8.8\%$

$g = p \times ROE = (60\%)(9\%) = 5.4\%$

$$\text{PV(Forecast Horizon)} = \frac{93.6 \times 1.054}{(0.088 - 0.054)(1.088)^5} = 1903.24$$

Value of Company = 327.72 + 1903.24 = 2230.96

Examiner Comment

Many candidates used the free cash flow to equity method to calculate dividends when the question calls for the dividend discount model. While this produces similar results, the approach is incorrect.

Many candidates assumed that the initial or final ROE’s were appropriate for the firm over the forecast horizon rather than evaluating trends in ROE over the six year period.

Question 13 Sample Answer

Solution 1

a)

	Exp Loss	Loss Reserve
Auto	4.76M	5.46M
HO	6.75M	.5M
	11.51M	5.96M

Var_{95} = 20,346,511

$Tvar_{95}$ = 20,346,511 + mean excess @ 95%

Lognormal has increasing mean excess function. So we would expect $Tvar_{95} > Var_{97.5}$. Take the average of the mean excess at 98% and 99% as an estimate.

$$\begin{aligned} \text{Tvar}_{95} &\approx 20,346,511+\left(\frac{21,132,866+21,680,765}{2}-20,346,511\right) \\ &= 21,406,816 \\ \text{RBC} &= \text{Tvar}_{95}-11.51\text{M}-5.96\text{M} \\ &= 21,406,806-11.51\text{M}-5.96\text{M} = 3,936,816 \end{aligned}$$

Solution 2

a) Loss reserve = 2600(2100) + 400(1250) = 5,960,000

=Auto count × severity + HO count × severity

Expected Ultimate = 6800(700) + 9000(750) + 11,510,000

$$\begin{aligned} \text{TVaR}_{95} &= \text{Average loss above the } 95^{\text{th}} \text{percentile} \\ &= \frac{\text{VaR}_{96}+\text{VaR}_{97}+\text{VaR}_{98}+\text{VaR}_{99}}{4} = 21,037,240.5 \end{aligned}$$

$$\text{TVaR}_{95}-\text{Expected}-\text{Reserve} = 3,567,240.5$$

Solution 3

a)

	Current Biz			Reserves
LOB	Expected Ultimate		LOB	Expected Ultimate
Auto	6,800(700)=4,760,000		Auto	2,600(2,100)=5,460,000
HO	9,000(750)=6,750,000		HO	400(1,250)=500,000
Total	11,510,000		Total	5,960,000

$TVaR_{95} = (VaR_{95} + VaR_{96} + VaR_{97} + VaR_{98} + VaR_{99}) / 5 = 20{,}899{,}095$

RBC = 20,899,095 – 11,510,000 – 5,960,000 = 3,429,095

Solution 4

a) Loss = 6800×700 + 9000×750 = 11,510,000

Reserve = 2600×2100 + 400×1250 = 5,960,000

→ $TVaR_{95} = E[X \mid X > 20{,}346{,}511]$
→ Must be a formula for TVaR for lognormal, but I don't know it
→ I will assume $TVaR_{95}$ = 21,000,000 to continue

Required capital = $TVaR_{95}$ – Loss – Reserve
= 21,000,000 – 11,510,000 – 5,960,000
= 3,530,000

Solution 1

b)

- These lines could be prone to catastrophes. We may want to add an additional piece to the RBC required for a provision for catastrophes
- We are using a relatively heavily total lognormal, were using TVaR to help calculate the requirement. TVaR is linear in the tail. Thus it may not be picking up on enough the extreme tail losses, which potentially you could get with auto liability or horrendous catastrophe. May want to use something like a distortion measure instead.
- Losses between these two lines may be more highly correlated in the extreme tails (due to weather, etc.) If we aren't taking this in account with something like a copula, we may be exposed to much higher losses in extreme scenarios than the capital from RBC that we are holding.

Solution 2

b)

1) Lognormal distribution may not be heavily enough in the tails to make estimates of risk capital based on the tails a good measure. Consider measuring the losses and reserves stochastically to capture the heaviness in the tail.

2) TVaR is linear in large losses which does not reflect the desirable property that losses 2x as large are more then 2x as bad. Using RTVaR would add in a factor loading of the standard deviation which would weight the tail more.

3) Just using a tail measure fails to capture risk not in the tail but which could be significant. Perhaps measure the distribution at a lower value than at .95.

Solution 3

b)

1) Capital levels using TVaR are linear; i.e. a loss 2x as bad does not hurt 2x as much. → use RTVaR or WTVaR

2) It doesn't mention any correlations being considered. We should model the combined lines with a copula to capture tail dependencies.

3) We have not considered any premium risk. We could model out a premium distribution to capture that risk.

Solution 4

b)

i) TVaR increases linearly with loss since a loss twice the size is more than as bad. It is preferable to use a measure increasing more than linearly with loss. Suggestion: use RTVaR.

ii) The $TVaR_{95}$ does not meet the strict Basil II requirements of capital. A 1/20 loss threshold is not sufficient. Suggestion: it would be preferable to use a 1/200 threshold.

iii) The risk based capital measure only captures underwriting risk, this may leave the firm undercapitalized against many other risk types. Suggestion: it would be preferable to include operating risk in the risk based capital measure.

Solution 5

b)

1) Does not consider correlation between auto and homeowners losses

- Use a copula to model tail dependency

2) TVaR assumes losses are linear in XS region. (A loss that is twice as big is twice as bad).

- Use risk adjusted TVaR

3) Large losses are included in total loss, it may distort analysis

- Model attritional and large losses separately

Solution 1

c) Reinsurance helps reduce risk some. However, there are still credit risk associated with reinsurance; the reinsurer may default, especially in extreme scenarios. The regulator is not likely to give credit for reinsurance ceded.

Also, quota share treaties decrease losses uniformly, but if you have losses in the extreme tails, your losses will still be very high regardless. One of the main functions of RBC is to have sufficient capital for these extreme scenarios. So RBC would not be reduced by half. Also, as noted above, extreme scenarios are also the points where reinsurers may likely to default in which case you would have to pay near the full loss anyway.

Solution 2

c) This will not work completely. It will greatly reduce the capital by ceding such a large %. However, there is some non-zero chance of the reinsurer defaulting. This does not allow for a full offset to capital. Also, there is a timing risk between when the reinsurer is billed and when they pay. Capital must be held for this also.

Solution 3

c) While a 50% quota share will reduce the expected losses on future business it will not affect business previously written. Thus, the volatility will still exist for the loss reserves, which comprise the most severe losses and current business, which cannot be retroactively ceded.

Solution 4

c) Because there is more risk than just the policies liabilities. It must keep capital for other risks like market risk, credit risk, and operational risk. Also, other risk cannot be calculated with a formula. Need to account them also.

Examiner Comment

For the a. part we accepted anything reasonable as long as it was well explained. Some examples include TVaR based on VaR > 95%, average of excess, and assumed TVaR based on formula.

Common mistakes included using VaR_{95}, arithmetic errors, and just writing down the numeric result without any explanation.

Common mistakes for the b. part included:

- Gave problem but no solution
- Creating a hypothetical situation that didn't make sense based on information provided

For the c. part, also accepted:

The RBC calculated in part a is for underwriting risk only so a 50/50 quota share reinsurance should cut it in half.

Common mistakes:

- TVaR will not be cut in half
- Quota share won't take care of large losses and use excess of loss instead
- Bringing in premium as the issue

Question 14 Sample Answer

Solution 1

a) $dr_t = (b - ar_t)dt + sdw_t \rightarrow$ form of Vasicek model

- Models a mean reverting Brownian notion where b is the mean term and a corresponds to the autocorrelation

b) CIR model is $dr_t = (b\text{-}ar_t)dt + s\sqrt{r}\, dw_t$
- It introduces the rate as a factor of the variation which allows the variation to vary with the rate.

c) I would recommend the CIR model because it has the added feature where the square root of the note is used in the variability. This means that the volatility is linked to the rate, so in the current environment in low interest rates there is limited or little volatility. Also because of this feature, rates can never be negative, which in the current environment of low interest rates is a good thing because we could potentially model negative rates otherwise.

Solution 2

a) $dr_t = (b\text{-}ar_t)dt + sdW_t$
- Vasicek is mean reverting Brownian motion model for the short rate r, with mean b/a

b) CIR model: $dr_t = (b - ar_t)dt + s\sqrt{r}\, dW_t$
- The key difference is the inclusion of the $\sqrt{r}$ term in the standard deviation. This term makes it so the CIR model can't produce negative rates and so that volatility increases with the rate, which is seen empirically.

c) CIR is more appropriate as it is a more robust model. Because interest rates are so low, there is an increased risk of Vasicek producing a negative rate, which is impossible. CIR doesn't have this issue. Also, it is seen empirically that volatility does not increase with higher rates, so this feature of the CIR model is preferable to include.

Examiner Comment

For the a. part, in addition to the formula, a brief description of the Vasicek model using at least 2 key words such as “mean reverting”, “Brownian motion”, “short rate”, “continuous” was required to receive full credit. It was not sufficient to only say that the Vasicek Model models US treasury interest rates since this is defined in the problem.

Some candidates used alternative letters for r, a, b, etc. when defining the formula. Full credit was provided as long as the formula was correct.

Common errors included:

- Calling the Vasicek model a discrete distribution rather than a continuous distribution
- Not including a dt in the first term (the dt shows that this is a continuous distribution, which is a key point in the model)
- Using r in the second term
- Using $a(b-r_t)dt$ as the first term, as the b term is not multiplied by a
- Saying that the model measured interest rates, as this was defined in the problem
- Using just 1 key word in the description

For the b. part, the key difference is the addition of the square root of r term in the formula which does not allow the CIR model to produce negative rates. Partial credit was given for a number of alternative differences between the CIR model and the Vasicek model, as long as the difference was accurate.

One common error was that candidates took the square root of s rather than r.

For the c. part, candidates must include discussion about the low interest rate environment when selecting the appropriate model. Only partial credit was awarded if the low interest rate environment was not discussed.

Question 15 Sample Answer

Solution 1

a) X – Case Incurred Indemnity
Y – ALAE

$E(X) = 1/5(800+5000+2100+12000+77000) = 19380$

$E(X^2) = 1/5(800^2+5000^2+2100^2+12000^2+77000^2) = 1,220,610,000$

$E(Y) = 1/5(100+250+0+1800+4000) = 1230$

$E(Y^2) = 3,802,500$

$E(XY) = 1/5[(800(100) + 5000(250) + 2100(0) + 12000(1800) + 77000(4000)]$

$= 66,186,000$

$\sigma_X = [E(X^2) - E(X)^2]^{½} = 29069.32404$

$\sigma_Y = [E(Y^2) - E(Y)^2]^{½} = 1532.840501$

$$\text{Pearson's } \rho = \frac{E(XY) - E(X)E(Y)}{\sigma_X \sigma_Y}$$

$$= \frac{66186000 - (19380)(1230)}{(29069.32404)(1532.840501)}$$

$$= 0.950402$$

b) Use Kendall's Tau (τ)

- Rank the Indemnity (I)

Claim #	Indemnity (I)	ALAE
1	800	100
2	2100	0
3	5000	250
4	12000	1800
5	77000	4000

# of swaps Q = 1 (swap 0 & 100)

$$\tau = 1 - \frac{Q}{\text{\# of pairs}/2}$$
$$= 1 - \frac{1}{10/2}$$
$$= 0.8$$

$$\text{\# of pairs} = n(n-1)/2$$
$$= 5(4)/2$$
$$= 10$$

It can be concluded that the case incurred indemnity is positively correlated with the ALAE due to the high correlation measure from Pearson's ρ and Kendall's τ (both close to 1)

c) Similarity: Both summarize the overall dependency in one single number.
Difference: Pearson's correlation uses value in determining correlation whereas Kendall's τ uses ranks in determining correlation.

Solution 2

a) Pearsons $\rho = \text{Cor}(x,y) / (\sigma_X \sigma_Y) = .95$

Cor(x,y) = E(XY) – E(Y)E(X)

$\sigma^2_X = E(X^2) - E(X)$

$\sigma^2_Y = E(Y^2) - E(Y)$

E(X) = (800+5000+2100+12000+77000)/5 = 19380

$E(X^2) = (800^2+5000^2+2100^2+12000^2+77000^2)/5$

E(Y) = 1230

$E(Y^2) = (100^2 + 250^2 + 0^2 + 1800^2 + 4000^2) /5$

E(XY) = [(800(100) + 5000(250) + 2100(0) + 12000(1800) + 77000(4000)]/5

b) Spearman's $\rho = 1 - (6s/(n(n^2 - 1))$ where s = sum of squares of difference in ranks

Claim	Indem	ALAE	Indem Rank	ALAE Rank	(I-rank – A-rank)^2
1	800	100	1	2	$(2-1)^2 = 1$
2	2100	0	3	3	$(3-3)^2 = 0$
3	5000	250	2	1	1
4	12000	1800	4	4	0
5	77000	4000	5	5	0
					2

$$\text{Spearman's } \rho = 1 - \frac{6(2)}{5(25-1)} = .9$$

Pearson's appears to appropriate since it is similar to Spearman's. One of the Pearson's weaknesses is that it is severely affected by outliers but the entire distribution appears correlated so it's not a problem here.

c) Similarity: both weight by the squares of the differences
Different: Spearman is an ordinal measure of correlation where as Pearson is a cardinal measure

Solution 3

a)

$$\text{Pearson's } \rho = \frac{\sum \tilde{x}_i \tilde{y}_i}{\sqrt{\sum \tilde{x}_i^2 \sum \tilde{y}_i^2}}$$

$$\tilde{x}_i = x_i - \overline{x}$$
$$\tilde{y}_i = y_i - \overline{y}$$

Claim	Incurred (x_i)	ALAE(y_i)	$\tilde{x}_i$	$\tilde{y}_i$
1	800	100	-18580	-1130
2	2100	0	-14380	-980
3	5000	250	-17280	-1230
4	12000	1800	-7380	570
5	77000	4000	57620	2770

$$\overline{x} = 19,380 \qquad \overline{y} = 1230$$
$$\sum \tilde{x}_i \tilde{y}_i = 2.117 \times 10^8$$
$$\sum \tilde{x}_i^2 = 4.225 \times 10^9 \qquad \sum \tilde{y}_i^2 = 11,748,000$$
$$\therefore \rho = 0.95$$

b) I will use Spearman's ρ

$$\text{Spearman's } \rho = 1 - \frac{s}{\left(\frac{n\left(n^2-1\right)}{6}\right)} = 1 - \frac{2}{\left(\frac{5\left(25-1\right)}{6}\right)} = 0.9$$

$$s = \sum \left(\text{difference of ranks}\right)^2 = 1^2 + 0^2 + \left(-1\right)^2 + 0^2 + 0^2$$
$$= 2$$

Claim	rank of incurred	rank of ALAE	Difference of rank
1	5	4	1
2	3	3	0
3	4	5	-1
4	2	2	0
5	1	1	0

Conclusion: Pearson is heavily affected by outliers. In this case the effect was not so bad because the huge case incurred was in the "right" direction i.e. larger ALAE as well as larger Incurred. If it had been in the opposite direction, i.e. much smaller incurred; it would have dragged the correlation down and possibly made it negative.

I would not recommend Pearson's for this data is much better to use either Spearman's ρ or Kendall's τ which focus on order not values.

c) Similarity: They both summarize the correlation in a single value.
Difference: Pearson's correlation focuses on the values of the data and their distance from the mean. Spearman's correlation focuses on the order of the data points and their ranks.

<u>Examiner Comment</u>

Candidate can calculate either Kendall's Tau (0.80) or Spearman's (0.90) rank correlation. Both give results similar to Pearson, that is, strong positive correlation. Candidates should conclude that Pearson's correlation is appropriate in this case. We also give credit if they say that Pearson's may be appropriate, but the 77K outlier is driving up the correlation since it's not truly linear. Basically any reasonable conclusion based on assumptions is valid for full credit. If candidate's comment was a correct conclusion based on incorrect calculation, we will give full credit for the comment.

Question 16 Sample Answer

Solution 1

a)

$$\chi_i = \frac{H_i - F_i G_i}{\sqrt{F_i(1-F_i)G_i(1-G_i)}}$$

measures how far the joint distribution is from the independent distribution

$$\lambda = 4 \times \text{sign}\left(\tilde{F}_i \tilde{G}_i\right) \times \max\left(\tilde{F}_i^2 \tilde{G}_i^2\right)$$

λi =

where $\tilde{F}_i = F_i - \frac{1}{2}$ and $\tilde{G}_i = G_i - \frac{1}{2}$

F_i and G_i use the marginal distribution and H_i is the joint distribution

λ measures where in the distribution with *H* indicating the top left or bottom right and 1 indicating the top right or bottom left, and 0 indicates near the mid point.

Solution 2

a) χ measures independence, since the numerator of the χ calculation takes joint distribution minus the product of the marginals. This would be 0 for independence.

λ measures distance to the center of the distribution.

Solution 3

a) χ measures the difference between the value of the empirical copula at a specific point (H) and the null hypothesis of independence of the two distributions (F·G).

$$\chi = \frac{H - FG}{\sqrt{F(1-F)G(1-G)}}$$

→ difference described above

→ scaling factor to keep between -1 and 1

λ is the measure of the signed distance between the point and the point in the bivariate distribution with the median coordinates, where F and G = .5

Solution 1

b) If the bulk of the observations are near the χ=0 line then the 2 series are independent. Based on this chart the auto PD book is independent for the losses and ALAE.

Solution 2

b) There appear to be more positive X than negative ones. If they were randomly scattered around 0 you could say that indemnity an ALAE were independent, but with more positives than negatives it looks like the empirical relationship shows a little more dependence than none. The dependence does not appear to vary though, along the distribution. It appears to be steady.

Solution 3

b) The data points are uniformly random around the x=0 line, this implies they are independent. If there was a pattern (like an arc), then we would draw a different conclusion.

Examiner Comment

A few candidates defined lambda as the distance from the mean of the distribution rather than the median. Also, some candidates failed to illustrate the signed nature of lambda.

A few candidates defined chi as a residual and while mathematically correct, this definition does not provide any insights into how chi measures dependency.

A few candidates did not adequately explain their conclusions from the graph.

Question 17 Sample Answer

Solution 1

a)

$$\text{Right tail concentration factor} = R(z) = \frac{\Pr\left(X > z \text{ and } Y > z\right)}{1-z}$$
$$= \frac{1+2z+C\left(z,z\right)}{1-z}$$

The right tail concentration factor is used to look for right tail dependencies is loss distributions. Copulas such as the Gumbel and heavy-right-tail (HRT) have high values of $R\left(R = \lim_{z\to 1} R\left(z\right)\right)$ while symmetric distributions like the normal and Frank's have R = 0.

Solution 2

a) Right-tail concentration function

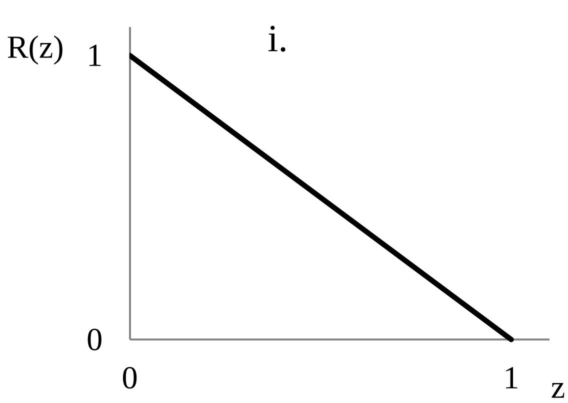
$$\text{R(z) = Prob(U>z and V>z) = } \frac{1-2z+C\left(z,z\right)}{1-z}$$

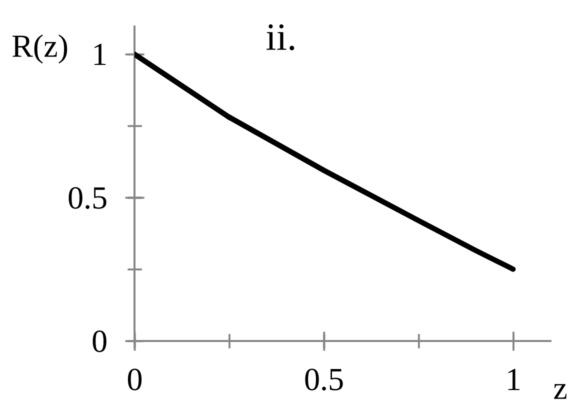
The right tail concentration function calculation how much density a copula has in its right tail. If it is large, the copula can be used to model distributions with strong correlation in the right tail.

Solution 1

b)

i. $R\left(z\right) = \frac{1-2z+z^2}{1-z} = 1-z$





ii. $$R(z)=\frac{1-2z+\min(z,z)^{.25}(zz)^{.75}}{1-z}=\frac{1-2z+z^{.25}\left(z^2\right)^{.75}}{1-z}$$
$$=\frac{1-2z+z^{1.75}}{1-z}$$

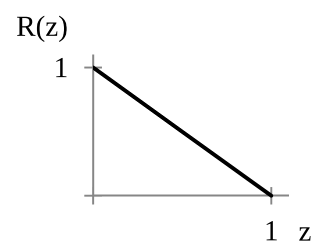
z	*R(z)*	*z*	*R(z)*
0	1	.9	.316
.25	.78	.95	.283
.5	.595	.99	.257
.75	.42	.999	.251

Solution 2

b)

i) c(u,v) = uv

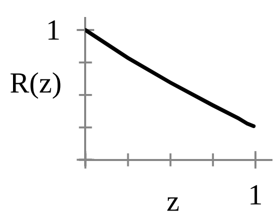
$$R(z)=\frac{1-2z+z^2}{1-z}=\frac{(1-z)^2}{1-z}=\boxed{1-z}$$



ii) c(u,v) = min (u,v)$^{0.25}$(uv)$^{.75}$

$$R(z)=\frac{1-2z+\left(z^{.25}z^{1.5}\right)}{1-z}=\boxed{\frac{1-2z+z^{1.75}}{1-z}}$$

z	R(z)
0	1
.25	.785
.5	.595
.75	.42
.9	.32
.95	.28
.99	.26



Solution 1

c) Property and worker's compensation in California might become highly correlated in the event of a large earthquake. Therefore we need a copula with a heavy right tail. The second copula has a heavier right-tail. The second copula has a heavier right-tail. Therefore it would be more appropriate than the first (the product copula)

Solution 2

c) WC & property are generally independent. However, there may be correlations in extreme events. For example, an earthquake during a work day could cause property damage and injuries to employees. Thus $R(z) > 0$ as $z \rightarrow 1$ could be appropriate. For copula ii, $R(z) \rightarrow .25$ as $z \rightarrow 1$ but for copula i, $R(z) \rightarrow 0$. Thus copula ii is more appropriate (the two are similar for smaller z, which also what we'd expect: normally independent)

Examiner Comment

For the a. part, many candidates offered a definition for right-tail concentration function or the right-tail concentration formula itself. It was common for candidates not to have both the written explanation and the formula.

Most candidates had difficulty with the b. part. Candidates often failed to both derive the function and graph the function as required by the question. When graphing the functions, Candidates commonly failed to label their graphs or show any work as to how they plotted the points on the graph. Additionally, it was common for candidates on the second copula to intercept R(1) with the x-axis, this was incorrect.

For the c. part, many candidates correctly selected the proper copula but failed to explain why that copula was the best choice, and in some cases there were no graphs or data present in part b. but the candidate was still able to guess and select correctly. We required a visual graph to be present in b. in order to get full credit in c. We also gave limited partial credit to candidates who selected the first copula as the answer and provided an explanation that property and workers compensation losses are independent.

Question 18 Sample Answer

Solution 1

a) Standard deviation: Not appropriate since it doesn't work well with skewed distributions like lines subject to hurricane losses

   Semi standard deviation: Not appropriate since it will end up ignoring part of the distribution, and won't highlight hurricane risk enough (since it's total loss)

   VaR – Not appropriate since its only one point, VaR(95) may not be high enough to capture hurricane risks.

   T-VaR – While not ideal (I would use distortion of RTVaR) it's the best one in the list because it will average all loss values above VaR, better capturing hurricane risk.

b) Standard deviation: Not appropriate here because it builds in positive or beneficial deviations

   Semi-standard: Best one since it uses the whole distribution, and focuses only on negative outcomes, even the small ones.

   VaR an T-VaR: Not appropriate for two reasons
   - Estimating at high levels ignores too much risk
   - Estimating at low levels puts too little weight on high risk (tail events)

   Also, you would really want to look at the whole distribution here

Solution 2

a) The standard deviation and semi-standard deviation are mean based risk measures they would not be appropriate in this case because for hurricane risk you are more concerned with the tail. As hurricane losses might be large but infrequent.

   VaR and TVaR would be more appropriate as they are tail risk measures. TVaR might be even more applicable because it measures the average loss in the tail not just the probability of experiencing a loss above a certain threshold like the VaR.

b) To asses the risk of price inadequacy a mean-based risk measure would be more appropriate because it takes into account the entire loss distribution. Therefore I would use either the standard deviation or the semi-standard deviation. Semi-standard might be slightly more appropriate as it focuses on only adverse outcomes which is what we are concerned about.

Examiner Comment

Most candidates did fairly well at assessing the risk measures, although oftentimes they defaulted to defining the measure as opposed to truly assessing its value for the given insurer. Semi-standard deviation was the most misunderstood of the four risk measures. A selection of the most appropriate risk measure for each situation was also required. A number of candidates did not do this. Of those candidates who did correctly identify the best measure, some failed to explain why that was indeed the best measure for the given situation.

Question 19 Sample Answer

Solution 1

a) If the insurance company is downgraded it could incur significant costs trying to raise new capital, which is already thin. High yield bond issues or issuance of discounted stock are not preferred. It should go with reinsurer x to prevent volatility in its U/W results. While its expected ROE will be less, it will be more stable. This in turn will reduce potential tax effects, as excessive income in one period will be taxed at a higher bracket than if it were stable overall years. With no reinsurance or the reinsurance that produces a downgrade, the ROE may be higher but the volatility could result in risk of insolvency or financial distress costs.

b) If the CFO's bonus is maximized with a 30% ROE, the company is basically rewarding the CFO to take risks with the shareholders money. Under agency theory you want to align the interests of principals (shareholders, and the agents, the CFO) who you put in the charge of your money for your natural gain. The CFO will not want to mitigate the risk because it will lower his bonus as the ROE declines. This may result in financial distress costs for the shareholders if the insurer is downgraded and needs to raise capital.

Solution 2

a) Financial distress is expensive: if debts (bonds) are issued, they will be at a high interest rate. If stock is issued, it will be at a discount. May need to quickly sell assets, but the sale will be at depressed prices. Having reinsurance reduces the chance of being placed in this situation.

Having stable & predictable results will minimize taxes.

– Reins x has a great rating (low credit risk) X seems to be more stable, therefore choose this reinsurance.	y has moderate rating	w/ no reins, a large loss may cause distress if not enough capital available.

b) CFO acts on behalf of the principal for mutual benefit. A compensation setup like this will incentivize the CFO to be riskier and purchase no reinsurance (since that option has a 25% expected return). Stakeholders (especially policyholders) would not like this. They prefer stable, predictable outcomes. P/h's are willing to pay more from a more stable insurer, & demand a discount on premium if it's not stable. The uncertainty of future indemnification may also scare customers away.

If the CFO were aware of this, he may choose some reinsurance as opposed to more and receive a minimal bonus instead of being out of a job.

Examiner Comment

For the a. part, candidates would have to discuss at least 3 costs of financial distress. Credit would also be given to mentioning the following:

- Reduction in earnings and market share
- Policyholders are less willing to may as much for a financially distressed insurer

Common errors included the following:

- Just saying that the firm would experience a greater chance financial distress without reinsurance, or saying that financial distress is costly, without mentioning what these costs are.
- Saying that a firm would be downgraded when in financial distress. This was already mentioned in the question as a concern for the company's CFO.
- Saying that a firm would pay lower taxes because of the costs of reinsurance
- If a candidate said that a firm would pay lower taxes with reinsurance because of a lower expected ROE, .25 points were deducted. While it is technically correct, it does not fully explain why the expected ROE is lower (because of greater earnings stability).

For the b. part candidates also got credit for saying that the CFO is the agent to the policyholders and bondholders (though in this last case it wouldn't be a direct relationship). Candidates also got credit for saying that the interests of the CFO and the principals are misaligned. Common errors included the following:

- Not mentioning the relationship between the CFO (agent) and the shareholders/policyholders/debtholders (principals). This is a key concept of agency theory.
- Not discussing the conflict of interest between the CFO and the principals. This is another key concept of agency theory.
- Saying that the CFO was the agent to the company. The company is not a principal.
- Not specifically discussing the interest of the principals. For example, saying that the CFO was putting the company at risk without specifically saying that the objective of the principal was to minimize risk.

Question 20 Sample Answer

Solution 1

a)

i. Protect against large losses. May be able to hold less capital and decrease risk of insolvency, with adequate reinsurance problem from extreme events.
ii. More stable earnings over time. Investors tend to prefer this. Also, taxes may be lower with more stable earnings.
iii. Underwriting experience from reinsurer. Reinsurer may have more experience in a particular line and may help primary insurer be more profitable.

b)

i. Non-proportional(excess), to protect against extreme tail events. (Proportional would decrease losses across the board, but not extreme losses by enough).
ii. Non-proportional, since it would reduce the very large impact extreme events could have on earnings.
iii. Proportional would probably be best here. Gives the reinsurer a stake in profitability of the line and encourages them to give good recommendations.

Solution 2

a)

1) Reduce catastrophe exposure - limited on largest losses
2) Exit line of business or geographic region
3) Surplus relief – need to adjust surplus by reinsurance commission

b)

1) Non-proportional treaty to reduce catastrophe – excess of loss
2) Proportional treaty that takes 100% of certain line of business or region
3) Proportional – Quota share to move premium and losses on % for surplus relief since also get commission premium back.

Solution 3

a)

i. Risk mitigation: Insurer may want to lower its exposure of insolvency
ii. Spreading risk over time: It may be more profitable for the insurer to have constant profits instead of fluctuating ones because of taxes and regulation
iii. Achieving company goals: The company may have objections of expanding market but cannot support the rapid growth

b)

i. Risk mitigation: non-proportional is better as it caps losses above a certain threshold, protecting the insurer from extreme losses.
ii. Spreading risk: Non-proportional is better as high losses are capped in exchange for a lesser profit due to premium ceding
iii. Goals: Proportional is better because it allows the insurer to recognize the reinsurance commission right away, thus decreasing its premium to capital ratio

Solution 4

a)

i. So that they have sufficient capital to satisfy the policyholders. People want to buy insurance from a company who will be able to pay all claims made against them!
ii. Taxation – reinsurance smoothes the income of the insurer which can assist in planning and optimizing the taxes they must pay.
iii. Avoid cost of financial distress. By transferring risk they can avoid financial distress (where they must desperately seek to raise capital by issuing bonds with a very high yield). People also don't like to buy insurance from a company in distress, which makes the problem even worse!

b)

i. Non-proportional to cover the extreme and unpredictable losses that could cause the insurer to go insolvent w/o reinsurance
ii. Non-proportional to smooth out the spikes in profit and losses, thus leading to a more predictable income.
iii. Non-proportional to cover extreme and unpredictable losses that could cause the insurer to go insolvent w/o reinsurance.

Examiner Comment

Credit for part b. was given when a solid foundational response was given for either proportional or non-proportional. In general, the non-proportional would be used to limit extreme results or the tails of distributions which may aid in stakeholder outlook, tax implications, or ratings. In general, proportional would be preferred to divest a segment of business, gain surplus relief via commissions received, and gaining insight from the reinsurer on the underlying lines of business.

For example:

To provide protection to the insurer against the effect of large losses, non-proportional reinsurance could be purchased to cover high-severity, low frequency losses which may not be fully accounted for in the underlying policy pricing.

For an unfamiliar line of business the insurer is thinking of expanding into, the purchase of proportional reinsurance coverage may provide an opportunity for the reinsurer to bring its expertise to the pricing of the policy and the managing of the claims, since it would be sharing in the fortunes (profit or loss) of the insurer.

A wide variety of possible responses was considered and as such, candidates fared particularly well on the question. For a number of the part b. responses, either proportional or non-proportional may satisfy the reason for purchase in part a., provided an appropriate explanation accompanied the selected type of insurance.

Credit was typically not given when:

1. Proportional was suggested to reduce the overall risk - this statement would be ignoring the top-line loss of premium associated with the ceded premium
2. Proportional was suggested as a means to stabilize earnings - as proportional reinsurance does not change one of the key metrics in earnings….the loss ratio
3. Non-proportional was suggested as a means to cede away the non-profitable excess layers - the excess layers may not necessarily be the unfavorable experience layers, as it depends on the associated premiums charged. Similarly, the retained layer may not be priced appropriately for the actual loss experience

Question 21 Sample Answer

Solution 1

a)

1) Rise in cost and insuring policyholders. If an insurer is deemed risky, the insurers will demand a greater discount for buying a policy, thus reducing premium without reducing assumed risk. Discount demand often greater than the expected loss from increased risk.
2) Sufficient capital is required by the state regulators for the insurer to operate. If the regulator requires more capital to be held when the company is in distress this capital will be more expensive to acquire, making matters even worse.

Solution 2

a)

- Insurers may have to write less business, thus possibly forgoing future profitable business
- Insurer may lose policyholders who expect a discount for assuming the increased risk of being with a distressed company

Solution 1

b)

i. Issue of debt will be done at a higher yield since debt holders are less likely to be paid back.
ii. Stocks are sold at a discount, since growth prospects are diminished.
iii. Assets are sold at below market value since the seller can't wait for the optimal price and buyers know it.

Solution 2

b)

1. Agency problems – agent may not act for the mutual benefit of agent & principal
2. Regulation and taxation – stable outcomes may minimize taxes; could be a regulatory requirement for capital
3. Relationship w/ stakeholders – they prefer stable and predictable outcomes

Solution 1

c) R&D is very capital intensive with long times to payout. R&D doesn't produce stable cash flow, so capital is needed to keep it going. Firms who need capital will benefit the most from risk management since they need to be in solid financial shape when they go to markets to acquire capital. Don't want these capital costs to be too high.

Solution 2

c) These firms anticipate more growth opportunities & thus have more to lose. These firms are also lightly capitalized with few tangible assets, making financial distress even more costly. Riskier, so raising capital more expensive.

Examiner Comments

Candidates who answered with insurance specific costs of distress for part b. were not given credit if they had attempted to describe two costs of insurer-specific distress in part a.

Candidates who only mentioned the high value of "intangible assets" or "intellectual property" for part c. were not given credit.

Question 22 Sample Answer

Solution 1

a) Operational is the risk due to failure of people, processes, systems, or due to external events. Includes legal, but excludes reputational, strategic, and systematic.

b)

- Business Disruption and System Failures – since the claims system may sometimes be offline so the claims dept can't get their job done of serving customers. Could mitigate by having a back-up system in place for when the new system is down.

- External fraud – since this project is being outsourced, a third party may have access to their data and could hack the computer systems. Mitigate by requiring passwords to access certain info, and have heightened security.

Solution 2

a) The risk of loss due to failed process, people or systems, or from external events. This includes legal risk but generally excludes reputational or strategic risk.

b)

1. Business disruption/systems failures, due to the system being offline. You should have 2 settings running in parallel, 1 for development and 1 for regular use, until the new system is ready. Make sure any outages occur late at night.

2. Execution, Delivery & Process Mgmt. There is a risk the vendor won't deliver the system, or that the system won't function properly. To mitigate, you should ensure clarity & communication w/ your own IT personnel & the offshore personnel, so that expectations of the new system are clear. You should also fund for this risk in the event that your customer claims aren't processes accurately.

Examiner Comment

Candidates generally selected two operational risks and provided valid means of mitigation. Some candidates included reputational risk in their solutions where the definition of operational risk excludes this risk. A few candidates included operational risks that were not relevant to the example given in the problem. While grading we were considerate that "operational risk" can have numerous examples depicting it.

<u>Question 23 Sample Answer</u>

<u>Solution 1</u>

a)

1. Attrition losses – this is the risk that there will be more claims than expected, or claims will be more severe than expected.

2. Large losses – the risk that there will be more large losses or larger losses than expected.

3. Catastrophe risk – the risk that a single event will cause many independent risks to become correlated.

<u>Solution 2</u>

a)

1. Attritional claims need to be modeled – this is the risk of having many (high frequency) small (low severity) claims.

2. Large claims need to modeled. These are the larger claims that have the potential to cause large unexpected losses that the company needs to be able to survive.

3. Catastrophe claims must be modeled. These are low frequency, high severity stress losses that occur in the extreme tail of the loss distribution. These can cause insolvency quickly if a company is not prepared.

<u>Solution 3</u>

a)

1. Pricing – will premiums be enough to cover business written/ profit

2. Claims – will you have more claims than expected or perhaps a higher number of large claims or just higher than expected severity on small claims

3. Product design risk – will coverage be interpreted to be more broad than anticipated? Exposing one to unanticipated loss types or latent exposures?

<u>Solution 1</u>

b) This is difficult to determine due to the nature of cross class correlation during extreme scenarios. There is very little, if any, data available illustrating extreme events and scenarios. This causes the correlation to primarily be based on actuarial judgment.

Solution 2

b) Correlations may be stronger in the tails of distributions. The problem is: The lack of data in tails within the company to estimate appropriately & verify. Need to use:

   - Industry/expert advice
   - Copulas or other technical theory to model structural dependencies not observed yet.

Examiner Comment

For the a. part, alternate solutions:

- Latent risk
- Correlation
- Expenses

Common mistakes:

- Not explaining the risk
- Several candidates did not explain the risk but how they would change the model. That is not what the question asked.

For the b. part, credit was lost for not explaining enough. For example, saying it would be difficult in the tail, but not explaining why (data) would result in partial credit.

Question 24 Sample Answer

Solution 1

a) Rating agency capital is the amount of capital needed to maintain a certain rating. It can be calculated based on the rating agency's factor based model or adjusted internal model of the insurer.

   Economic capital is the amount of capital to keep the probability of ruin of an insurer below a target level. Usually based on an internal model of the insurer and at different confidence level.

b) Economic impairment earning usually use a much lower confidence level than the 2 above.

c)
   1) A single firm wide internal model is difficult to incorporate day-to-day management decisions.
   2) A single model may be simpler than needed for product pricing etc.

Solution 2

a) Rating agency capital is one capital required by the rating agency to maintain credit rating. This rating agency model.

   Economic capital level is the capital required for the company to maintain its solvency. It reflects company's own risk measures.

b) Its internal model estimated at a lower return period, i.e. lower confidence level. It's not capital $\Longrightarrow$ it measures losses that will put company in economic impairment.

c)

   1) Models for business units may be more complex than the single internal model, e.g. pricing, bonus structure calculation is complex.
   2) A single internal model may not be able to handle day-to-day business decision making. e.g., a simple decision of whether underwriting a policy may take a day to run, so it's not feasible due to long run time.

Examiner Comment

It was common for the candidates to know the exact definitions of rating agency capital and economic capital. Depending on how complete the definitions were an explicit "contrast" of the two types was not always needed since it was inherent in the full definitions. Some candidates contrasted by assuming one produces a higher capital level than the other. However in definition and in practice this can vary by company.

For the b. part, it was common for the candidates to know the exact definition of economic impairment earnings level and successfully compare to part a. Occasionally, candidates would interchange the definitions of economic impairment with economic capital.

For the c. part the question was answered correctly the majority of the time. A few candidates provided only one example.   We did want to see some commonality in the answer with the examples found in the syllabus article. In responses that were too generic or simply irrelevant deductions were made.