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FOREWORD

Actuarial science originated in England in 1792 in the early days of life insurance. Because of the technical nature of the business, the first actuaries were mathematicians. Eventually, their numerical growth resulted in the formation of the Institute of Actuaries in England in 1848. Eight years later, in Scotland, the Faculty of Actuaries was formed. In the United States, the Actuarial Society of America was formed in 1889 and the American Institute of Actuaries in 1909. These two American organizations merged in 1949 to become the Society of Actuaries.

In the early years of the 20th Century in the United States, problems requiring actuarial treatment were emerging in sickness, disability, and casualty insurance—particularly in workers' compensation, which was introduced in 1911. The differences between the new problems and those of traditional life insurance led to the organization of the the Casualty Actuarial and Statistical Society of America in 1914. Dr. I. M. Rubinow, who was responsible for the Society's formation, became its first president. At the time of its formation, the Casualty Actuarial and Statistical Society of America had 97 charter members of the grade of Fellow. The Society adopted its present name, the Casualty Actuarial Society, on May 14, 1921.

The purpose of the Society is to advance the body of knowledge of actuarial science in applications other than life insurance, to establish and maintain standards of qualification for membership, to promote and maintain high standards of conduct and competence for the members, and to increase the awareness of actuarial science. The Society's activities in support of this purpose include communication with those affected by insurance, presentation and discussion of papers, attendance at seminars and workshops, collection of a library, research, and other means.

Since the problems of workers' compensation were the most urgent at the time of the Society's formation, many of the Society's original members played a leading part in developing the scientific basis for that line of insurance. From the beginning, however, the Society has grown constantly, not only in membership, but also in range of interest and in scientific and related contributions to all lines of insurance other than life, including automobile, liability other than automobile, fire, homeowners, commercial multiple peril, and others. These contributions are found principally in original papers prepared by members of the Society and published annually in the *Proceedings of the Casualty Actuarial Society*. The presidential addresses, also published in the *Proceedings*, have called attention to the most pressing actuarial problems, some of them still unsolved, that have faced the industry over the years.

The membership of the Society includes actuaries employed by insurance companies, industry advisory organizations, national brokers, accounting firms, educational institutions, state insurance departments, and the federal government. It also includes independent consultants. The Society has two classes of members, Fellows and Associates. Both classes require successful completion of examinations, held in February, May, and November in various cities of the United States, Canada, Bermuda, and selected overseas sites. In addition, Associateship requires completion of the CAS Course on Professionalism.

The publications of the Society and their respective prices are listed in the Society's *Yearbook*. The *Syllabus of Examinations* outlines the course of study recommended for the examinations. Both the *Yearbook*, at a charge of $40, and the *Syllabus of Examinations*, without charge, may be obtained from the Casualty Actuarial Society, 1100 North Glebe Road, Suite 600, Arlington, Virginia 22201.

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NOTICE

Papers submitted to the *Proceedings of the Casualty Actuarial Society* are subject to review by the members of the Committee on Review of Papers and, where appropriate, additional individuals with expertise in the relevant topics. In order to qualify for publication, a paper must be relevant to casualty actuarial science, include original research ideas and/or techniques, or have special educational value, and must not have been previously copyrighted or published or be concurrently considered for publication elsewhere. Specific instructions for preparation and submission of papers are included in the *Yearbook* of the Casualty Actuarial Society.

The Society is not responsible for statements of opinion expressed in the articles, criticisms, and discussions published in these *Proceedings*.

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AN ACTUARIAL APPROACH TO PROPERTY CATASTROPHE COVER RATING

DANIEL F. GOGOL

Abstract

Forty-one years of catastrophe loss data by state are used in this study to produce a model for rating catastrophe covers for insurers in any region of the continental United States. Smooth surfaces are fitted to the data by region, and experience rating is applied in an attempt to give appropriate weight to regional departures from the smoothed results. Severity distributions and frequencies are estimated for each region, and a method for applying them in pricing catastrophe covers is discussed. A method for using the experience of an insurer to produce an experience modification is also presented.

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1. INTRODUCTION

United States catastrophe cover rating is an interesting problem from both practical and theoretical points of view.

On the practical side, it is an important untreated problem. No systematic attempt at using insurance loss data to produce catastrophe cover rates can be found in insurance literature. Discussions of methods involving weather data are in Clark [7] and Friedman [9]. Catastrophe rates fluctuate greatly in the various regions of the country depending on the supply of capacity and whether there has been a large catastrophe in the area recently. Pricing practices were not much different two decades ago when Ingrey [12] stated:

> The general yardstick is the "payback period," or, in how many years will a total loss be amortized in advance. Payback periods depend upon location, type of business written, and past experience in addition to the basic ingredients of amount of capacity required, subject premium, and rate. The adequacy of the initial retention is largely overlooked as are the incremental functions of exposure types; to wit, a company writing mobile homes has a much greater incremental exposure function than another insurer writing private dwellings.

Catastrophe rating is also a challenging theoretical problem. The number of large catastrophes in any region is small, so it is important to use the experience of surrounding areas as well. It is useful to examine the relationship between catastrophe experience and a region's longitude, latitude, and distance from the coast. Also, the size of a region affects the probability of a catastrophe destroying more than a given percentage of property value.

By fitting a smooth surface that is a function of these variables to catastrophe loss data, it is possible to base estimates of expected losses for each region on more than just its own experience. Expected losses by region should generally have a smoother pattern than the sparse data.

An attempt is made in this paper to estimate the appropriate credibility to be given to the actual experience of a region, as opposed to the weight given to the expected losses indicated by a fitted smooth surface. After the indications of smoothed surfaces and the actual experience of a region are credibility-weighted to estimate the expected number of catastrophes for the region in various loss size intervals, a loss distribution is fitted to the estimates in order to smooth them in a reasonable way and to estimate tail probabilities.

2. THE MODEL

A. Data

To compare the relative destructive power of two natural catastrophes hitting different states, it is useful to consider the amount of property insurance premium in each state, as well as the amount of insured property damage in each state. The insured loss in each state will depend not only on the intensity and size of the catastrophe but also on the insured property in the area.

"Catastrophe premium," defined below, will be used as the exposure base to which loss data are related. The definition is based on Ingrey [12]. It is intended that the catastrophe premiums derived from each line of business be in roughly the same proportion as expected catastrophe losses for each line. Ingrey does not present data to support the percentages used in the formula but indicates that they were developed with the cooperation of Allen Hinkelman, Excess and Casualty Reinsurance Association; Daniel Holland, Inland Marine Insurance Bureau; Donald Kifer, New York Fire Insurance Rating Organization; and Allen Royer, Multi-Line Insurance Rating Board. Data on catastrophe losses by line will be discussed in Section 3.

The definition of catastrophe premium used in this paper is a formula often used by underwriters in evaluating a company's catastrophe exposure:

$$\begin{aligned}\text{Catastrophe premium} = {} & (10\% \text{ of inland marine premium})\\ & + (10\% \text{ of commercial multiple peril})\\ & + (80\% \text{ of allied lines})\\ & + (10\% \text{ of auto physical damage})\\ & + (20\% \text{ of farmowners})\\ & + (100\% \text{ of earthquake})\\ & + (20\% \text{ of homeowners})\\ & + (15\% \text{ of ocean marine}).\end{aligned} \tag{2.1}$$

An assumption, for example, that the proportion of homeowners losses caused by catastrophes is twice as high as the proportion for auto physical damage losses is implicit in the formula, since the corresponding percentages of premium are 20% and 10%.

Actually, Ingrey's formula also includes 60% of mobile home premium and 80% of difference in conditions premium, but these premiums are small and are omitted.

Additional insight is given by expressing the loss layer to be reinsured in terms of percentages of the catastrophe premium—for example, 200% excess of 20%. In this paper, layers expressed as percentages of state or regional catastrophe premium are studied. Methods of applying the study to individual company catastrophe cover rating are also discussed.

Catastrophe covers are generally for a high enough layer so that an event must cause losses to several of a company's risks in order to produce a loss to the cover. Windstorms are the most frequent causes of losses to these covers. Other frequent causes are winter freezes, hail, and flooding. Fire is a less frequent cause.

The loss data used in this study were produced by Property Claim Services (PCS) [15]. These data include each United States catastrophe having an estimated insured loss of $1 million or more from 1949 through 1981, and $5 million or more from 1982 through 1989. In order to be included, a loss must affect many insureds, although the exact number of insureds that must be affected has not been defined. (It is generally at least 1,000.) For each catastrophe, the estimated insured loss in each state is given. The PCS estimates are based on an extrapolation of estimates made by a set of insurers writing most of the property premium in the catastrophe area.

Although PCS insured loss estimates are used in the study, a loss development factor is applied in Section 3, where the method of rating catastrophe covers is described.

For each of 28 overlapping regions of the continental United States, catastrophe premium was estimated for 1949 to 1989. Gross written premium data by state from Best's [4], and for older years from *The Spectator* [6], which is no longer published, were used to compute catastrophe premiums by state for approximately every fifth year. Exponential interpolation was used for other years, based on the computed catastrophe premiums.

For each of the 28 regions, the estimated insured loss from each catastrophe from 1949 to 1989 was divided by the region's catastrophe premium for the year of the loss. The ratios, R, of individual losses to corresponding catastrophe premiums were then grouped into the somewhat arbitrarily chosen intervals:

$$8\% < R \leq 16\%,$$

$$16\% < R \leq 32\%,$$

$$32\% < R \leq 64\%, \text{ and}$$

$$R > 64\%.$$

The number of ratios falling in each interval for each region is shown in Exhibit 1. Exhibit 2, a map of the United States, may be helpful in connection with Exhibit 1, as well as later exhibits.

No evidence of a trend in the frequency of any type of catastrophe was found in the data, so no trend factor was applied. The loss trend and the premium trend are assumed to cancel each other out.

B. Smoothing the Data

The expected values of frequencies in each interval vary more smoothly as a function of regions than the data in Exhibit 1, since the data include random variation.

Most catastrophes are windstorms, and their frequency and severity are related to a region's latitude, longitude, and distance from the coast (Clark [7] and Friedman [9]). The probability distribution of the ratios of catastrophe losses to catastrophe premium is also related to the size of a region. The above facts motivate the attempt to use multiple regression for each interval of R values to fit the frequencies in Exhibit 1 to functions of the latitude, longitude, distance from the coast, and area of the 28 regions.

Multiple regression was used to relate the above variables to frequency of catastrophes in each of the intervals: $8\% < R \leq 16\%$, $16\% < R \leq 32\%$, $32\% < R \leq 64\%$, $R > 64\%$, $R > 32\%$, $R > 16\%$, and $R > 8\%$. The intervals are purposely chosen in an overlapping manner for a reason explained in Subsection 2D.

The details of the regressions are in Appendix A. Exhibit 3 shows a comparison of actual to fitted frequencies for four of the intervals.

C. Experience Rating the Regions

Weights will be selected for the actual and fitted frequencies in Exhibit 3 to produce estimates of expected frequencies by interval and region. The sum of the weights will be one. An explanation of the method of selecting them follows.

For each interval i of R values, and each region j, let the random variable $X_{i,j}$ be the frequency of catastrophes in a randomly selected 41-year period. The fitted values for interval i and region j in Exhibit 3 are estimates of the expected value of $X_{i,j}$. If each fitted value is assumed to be the mean of a probability distribution of possible expected values of $X_{i,j}$, then it can be seen that a more accurate estimate of the expected value can be produced by giving weight (credibility) to the actual frequency as well as to the fitted frequency.

The partly judgmental basis for selecting the following experience rating formula is explained in Appendix B. The number of actual catastrophes in interval i and region j is given credibility $a_{i,j}/(a_{i,j}+k_i)$ where $a_{i,j}$ is the fitted frequency for interval i and region j, $k_i = 9$ for $i = 1, 2, 5, 6$, or 7, and $k_i = 6$ for $i = 3$ or 4; where, for each interval, i is as in Table 4 of Appendix A.

D. Nested Application of Experience Rating System

For each region, experience rating is applied to estimate expected values for the frequencies in each interval of R values.

A nested process is used so that the estimates of expected frequencies for $8\% < R \leq 16\%$ and $R > 16\%$ are based not only on the separate experience for $8\% < R \leq 16\%$ and $R > 16\%$, respectively, but also on the total experience for $R > 8\%$.

By applying the experience rating formula for the interval $R > 8\%$, estimates A_j of the frequency in this interval are produced for each region j. The estimates B_j and C_j produced by applying the experience rating system to the intervals $8\% < R \leq 16\%$ and $R > 16\%$ are then multiplied by a constant D_j such that $A_j = D_j\,(B_j + C_j)$. The estimates $D_j\,B_j$ and $D_j\,C_j$ for the frequencies in region j for intervals $8\% < R \leq 16\%$ and $R > 16\%$, respectively, thus sum to the estimate for region j for the interval $R > 8\%$ and are each in proportion to the estimates B_j and C_j, respectively. It is intended that $D_j\,B_j$ and $D_j\,C_j$ approximate the expected values of the frequencies in region j for intervals $8\% < R \leq 16\%$ and $R > 16\%$, respectively, given that the total of the two expected values is A_j, and that B_j and C_j are the estimates of the two expected values based on their separate data.

The weighted frequencies by region produced by directly applying the experience rating formulas for the intervals $16\% < R \leq 32\%$ and $R > 32\%$ are then adjusted so that their sum equals the estimate for $R > 16\%$. The method is entirely similar to the method used above to adjust the estimates for $8\% < R \leq 16\%$ and $R > 16\%$ so that their sum equaled the estimate for $R > 8\%$.

This nested process is continued until estimates are produced for each of the seven intervals. The estimates for four of the intervals are in Exhibit 4.

E. Loss Distributions by Region

The estimates of expected frequency for each region produced by the above nested application of experience rating for $8\% < R \leq 16\%$, $16\% < R \leq 32\%$, $32\% < R \leq 64\%$, and $R > 64\%$ were divided by the estimate produced for $R > 8\%$; the resulting fractions f_1, f_2, f_3, and f_4 were then fitted to a probability distribution. This probability distribution was used to allocate the estimate of expected frequency for $R > 8\%$ to the above four intervals. The selected yearly frequencies are the above frequencies divided by 41, since 41 years of data were used. The yearly frequencies for $R > 8\%$ are in Table 1.

The single parameter Pareto distribution was used for all 28 regions. It generally was a good fit. A comparison of the estimates produced by the experience rating method in the previous section and by the single parameter Pareto is shown in Exhibit 4. No other tested distribution performed as well. (A study of loss distributions is in Hogg and Klugman [11].)

The single parameter Pareto was used even in regions for which another distribution fit better. This was because the generally good fit of the single parameter Pareto led to the conclusion that it was a good model for the data, and small amounts of data in particular regions were not considered credible enough to counteract this conclusion.

(See Appendix C for a discussion of the method used to fit the single parameter Pareto. The parameters of the Pareto curves used are in Table 1.)

A Pareto parameter of 1 or less implies infinite expected losses for unlimited layers. For $0 < P < 1$, the expected losses in the layer between a and b are $(b^{1-P} - a^{1-P})/(1 - P)$, which approaches infinity as b approaches infinity. In reality, catastrophe losses are limited by the total insured value, so the frequency distribution falls below a

TABLE 1
FREQUENCIES (F^1) AND PARAMETERS (P)

Region	F^1	P	Region	F^1	P
1	0.213	0.96	15	0.292	0.94
2	0.335	1.21	16	0.190	1.07
3	0.727	1.26	17	0.312	1.00
4	0.682	0.95	18	0.212	1.08
5	0.419	0.60	19	0.244	1.44
6	0.431	0.86	20	0.590	0.92
7	0.749	1.61	21	0.507	1.13
8	0.184	1.24	22	0.450	1.78
9	0.235	1.27	23	0.265	1.25
10	0.566	1.49	24	0.196	0.93
11	0.788	1.54	25	0.183	1.17
12	0.453	1.59	26	0.487	1.33
13	0.254	1.16	27	0.265	1.00
14	0.282	0.98	28	0.393	1.54

Pareto at some point. Although Pareto parameters of 1 or less were selected for some regions, they are only intended to be used in estimating expected losses for limited layers of sizes that are actually reinsured. The Pareto's overestimate of frequency far out in the tail does not have a great effect in estimating expected losses for these layers. The frequency of losses above x times the truncation point is x^{-P} times the frequency above the truncation point. Since $P > 0$, this fraction x^{-P} approaches zero as x approaches infinity.

3. RATING CATASTROPHE COVERS

A. *Using the Model*

Rates for catastrophe covers include a risk charge, but this discussion is of expected losses rather than risk.

A reinsurer evaluating a catastrophe cover often receives a breakdown of the ceding company's subject property premium by state and line. The commercial multiple peril, homeowners, farmowners, and auto physical damage premiums that are considered to be subject to a catastrophe treaty are sometimes only a percentage (usually approximately 65%, 90%, 90%, and 35%, respectively) of the total premiums for those lines. It is necessary to adjust for this reduction to apply the catastrophe premium formula in this paper to the cedent.

If the cedent does not provide this information, estimates of catastrophe premium by state for a primary company can be made by using the company's major direct premium writings by state and its net written premiums by line from *Best's Insurance Reports* [3]. Based on this information and on Table 2, one of the 28 regions may be selected judgmentally as being approximately representative of the region in which the company writes.

TABLE 2

1988 CATASTROPHE PREMIUMS BY REGION (IN 000S)

Region	Premium	Region	Premium
1	$1,757,793	15	$890,083
2	473,889	16	973,760
3	881,629	17	789,209
4	521,551	18	2,231,681
5	668,967	19	546,455
6	700,932	20	1,403,180
7	478,800	21	1,848,699
8	365,904	22	1,484,958
9	180,551	23	1,793,682
10	238,494	24	2,653,051
11	273,418	25	2,778,136
12	973,046	26	3,366,938
13	1,110,098	27	5,816,632
14	683,584	28	11,961,706

For any region selected as representative of the company, the selected yearly frequency for catastrophe losses greater than 8% of catastrophe premium and the selected Pareto distribution may be found in Table 1. They may be used to compute an estimate of expected losses for any layer of a catastrophe cover by expressing the layer in terms of percentages of the company's total catastrophe premium. An example of the rating method will be given at the end of this section, but several related points are discussed first.

The method used in the example is based on historical data. However, due to the potential for an enormously damaging earthquake in California and the small number of earthquakes in the historical data used, expected losses from catastrophes in California are widely believed to be greater than the estimate that would be based on historical data. The very severe 1906 earthquake is not included in the available data.

An adjustment will be made in the rating method for catastrophe covers to reflect that the model in this paper is based on data for regions rather than for individual reinsurers. By the use of certain definitions and reasonable assumptions, the following statement could be made more precise and proven mathematically. On average, for catastrophe losses as defined by PCS, the probability distribution of ratios of catastrophe losses to catastrophe premiums has the same mean for an insurer within a region as for the region—but it has a greater variance.

The rating method, which will be applied to individual insurers, uses 0.85 times the Pareto parameter in Table 1 for the region selected as representative of the insurer. This adjustment reflects that the distributions for individual insurers have greater variance, on the average, than the distribution for the region.

The expected frequencies from Table 1 will be used, unadjusted, for individual insurers. The expected frequency of catastrophe losses, as defined by PCS, is less for an individual insurer than for the surrounding region. However, the assumption of a smaller Pareto parameter for individual insurers implies that for some percentage W, the expected frequency for $R > W\%$ is the same for the individual insurer as for the region. The estimate that $W = 8\%$ is implicit in the use of the expected frequencies from Table 1 for individual insurers.

The estimate that ultimate insured losses for catastrophes, on the average, are 1.33 times as great as the PCS estimates will be used in estimating expected losses for catastrophe covers. Since the PCS estimate is made within a few days of the catastrophe, it is natural to expect development. Also, the PCS estimate excludes allocated loss adjustment expense, all ocean marine and crop losses, and some inland marine and business interruption losses. Lastly, the model in this paper used gross losses and premiums while catastrophe reinsurance covers losses net of excess reinsurance. Studies (e.g., Ludwig [13]) have shown that net catastrophe losses are a higher percentage of net premiums than gross catastrophe losses are of gross premiums. An adjustment for this is included in the 1.33 factor.

The 0.85 factor for Pareto parameters and the 1.33 factor for losses have the combined effect of significantly raising estimated expected losses for catastrophe covers. The resulting expected losses, as a percentage of actual premiums charged, have been found to be a reasonable match to actual loss ratios for the catastrophe cover premium of two reinsurers over a 20-year and a 12-year period, respectively. (In addition, an adjustment was made to include the catastrophic year 1992.) This premium totaled almost $300 million and consisted of shares of a much greater amount of premium.

Example

Suppose that a primary insurer, in the latest year for which data are available, had writings for which region 28 is considered the best match.

Suppose that, using cp to represent the insurer's catastrophe premium, the layer to be reinsured can be expressed as $(2.00cp)$ excess of $(0.20cp)$.

The selections in Table 1 for region 28 were 0.393 catastrophe losses per year greater than 8% of catastrophe premium and a Pareto parameter of 1.54. The loss development factor of 1.33 and the adjustment factor to the Pareto parameter of 0.85, which were discussed above, are used. Therefore, 0.393 is the frequency for $R > 10.64\%$, and the Pareto parameter becomes 1.31. The expected losses to the layer in one year therefore are:

$$0.393\,(0.1064cp)\left[\frac{(0.20/0.1064)^{-0.31} - (2.20/0.1064)^{-0.31}}{0.31}\right] \tag{3.1}$$

(See Philbrick [14].) This amount equals 5.82% of catastrophe premium.

If it is not clear which region is the best match for the primary insurer, the above method may be used for more than one region and a final estimate may be judgmentally selected.

B. Underwriting Judgment

Since the above estimate is based on data from the entire region, it may be useful to judgmentally modify it if the ceding company is not believed to be typical of the region. For example, the ceding company may have a very high or low percentage of its insured property near the coast, where exposure to hurricanes is greatest. An estimate of how a ceding company compares to a region could also be made by using Clark's model [7], since that software can be applied to both regions and individual companies.

C. The Catastrophe Premium Formula

The estimated expected catastrophe losses for individual insurers were affected by the choice of percentages by line in the catastrophe premium formula defined in Section 2.

If the percentages by line that were used in the formula are multiplied by the corresponding premiums in Table 3, an approximation of the relative amounts of expected catastrophe losses by line can be derived. (Although fire premium is a portion of the property premium in Table 3, it was not included in the catastrophe premium formula; it was considered to account for only a negligible portion of catastrophe losses.)

Some data suggest that, for hurricanes, a much lower percentage of losses comes from auto physical damage than would be estimated based on the catastrophe premium formula. In [1], the All-Industry Research Advisory Council (AIRAC) estimated the following percentages of losses by line for seven hurricanes from 1983 to 1985:

Homeowners Multiple Peril	46.8%
Commercial Multiple Peril	22.2%
Auto Physical Damage	3.7%
All Others	27.3%.

TABLE 3

INDUSTRY PREMIUMS FOR SELECTED LINES —1990

	Premiums Earned (Millions)
Fire	$ 4,494
Allied Lines	2,097
Farmowners Multiple Peril	968
Homeowners Multiple Peril	18,116
Commercial Multiple Peril	17,626
Ocean Marine	1,169
Inland Marine	4,441
Earthquake	459
Auto Physical Damage	35,185

Another source of data on catastrophe losses by line was produced by the Insurance Services Office (ISO) for homeowners losses by individual catastrophe for the period 1970 to 1978 [2]. Those data indicate that homeowners and dwelling extended coverage losses are 19.6% and 2.7%, respectively, of total catastrophe losses as estimated by PCS for the same catastrophes. (The ISO estimates, like the PCS estimates, are an extrapolation of total insured losses based on data from a set of insurers in the region.) The percentage of total catastrophe losses covered under homeowners is much less in the ISO data for all catastrophes combined than in the AIRAC hurricane data. Therefore, the percentage of auto physical damage losses may well be much greater for all catastrophes combined than for hurricanes.

Hurricanes produced $6.35 billion in catastrophe losses from 1981 to 1990, compared to $9.7 billion in losses from hail and tornadoes and $3.7 billion in losses from winter storms, according to PCS.

If so desired, the catastrophe cover rating method used in this paper can be applied with a catastrophe premium formula having different percentages by line from those used. Any alternative percentages used should be chosen so that, when multiplied by the premiums in Table 3, they produce the same catastrophe premium as the percentages in this paper's formula. If this is done, then Table 1 approximates the corresponding table that would have been created if the alternative catastrophe premium formula had been used in the study. Therefore, the rating method used in this paper still gives an estimate of expected losses from catastrophes if the alternative catastrophe premium formula is used.

D. Experience Rating a Catastrophe Risk

Suppose the amount of each catastrophe loss of the ceding company for a certain time period is known. The frequency of these losses in intervals expressed in terms of ratios to the company's catastrophe premium can be compared to the experience of the region selected as being representative of the company. Exhibit 5, which shows experience from 1949 to 1969 and from 1970 to 1989 separately, may be useful for this comparison. An example of a judgmental experience rating is given below.

Example

Suppose that Insurance Company A had eight catastrophes greater than 10.64% (i.e., 8% times our selected development factor) of catastrophe premium for the period of 1970 to 1989 and that the region selected as corresponding to it had five catastrophes greater than 10.64% of catastrophe premium in the same period.

Suppose that the formula $n/(n+9)$, where n is the number of catastrophes in the region from 1970 to 1989, is the credibility assigned to the experience of Company A. (This formula is similar to one used in this paper to assign credibility to the actual frequency of catastrophes in a region.)

The credibility weighted frequency is then

$$(5/(5+9))\,(8) + (9/(5+9))\,(5),$$

which equals 6.07. The modifier produced by the experience rating is thus 6.07/5.00; that is, 1.21. This modifier is then applied to the expected losses for the reinsured layer that are estimated as in Equation 3.1.

4. CONCLUSION

A model that can be used to estimate expected losses to catastrophe covers based on insured loss data has been presented. An example of the application of the model to a specific cover was given. The obstacles to using actuarial methods in catastrophe rating are not as great as has sometimes been suggested.

The application of actuarial science gives a very useful and much needed perspective in this area.

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EXHIBIT 1

FREQUENCIES BY REGION

Region	Interval of Ratio R			
	$8\% < R \leq 16\%$	$16\% < R \leq 32\%$	$32\% < R \leq 64\%$	$R > 64\%$
1. CA	3	1	2	0
2. AZ, NM, NV, UT, CO	10	4	1	1
3. TX	22	1	4	3
4. AL, MS, LA	14	3	5	5
5. FL	4	5	2	5
6. GA, SC, NC	8	6	4	2
7. TN, AR, OK	23	8	1	0
8. OR, WA, ID	4	1	0	1
9. ND, SD, WY, MT	4	5	1	1
10. MN, WI	13	6	5	1
11. NE, KS	22	9	4	1
12. IA, MO, IL	11	6	0	0
13. MI, IN, OH	6	2	1	1
14. KY, WV, PA	6	1	4	0
15. VA, NJ, DE, MD, DC	6	2	1	2
16. NY, VT	2	2	1	0
17. ME, NH, MA, RI, CT	7	5	0	2
18. 1, 2 (above)	3	3	1	0
19. 8, 9	8	3	0	1
20. 3, 4	8	7	2	6
21. 5, 6, 7	18	4	3	1
22. 10, 11, 12	14	4	0	0
23. 13, 14	7	3	1	0
24. 15, 16, 17	1	2	1	2
25. 1, 2, 8, 9	3	1	2	0
26. 3, 4, 7, 10, 11, 12	11	4	3	1
27. 5, 6, 13, 14, 15, 16, 17	5	2	3	1
28. Continental U.S.	9	4	2	0
	252	104	54	37

EXHIBIT 2

THE CONTINENTAL UNITED STATES

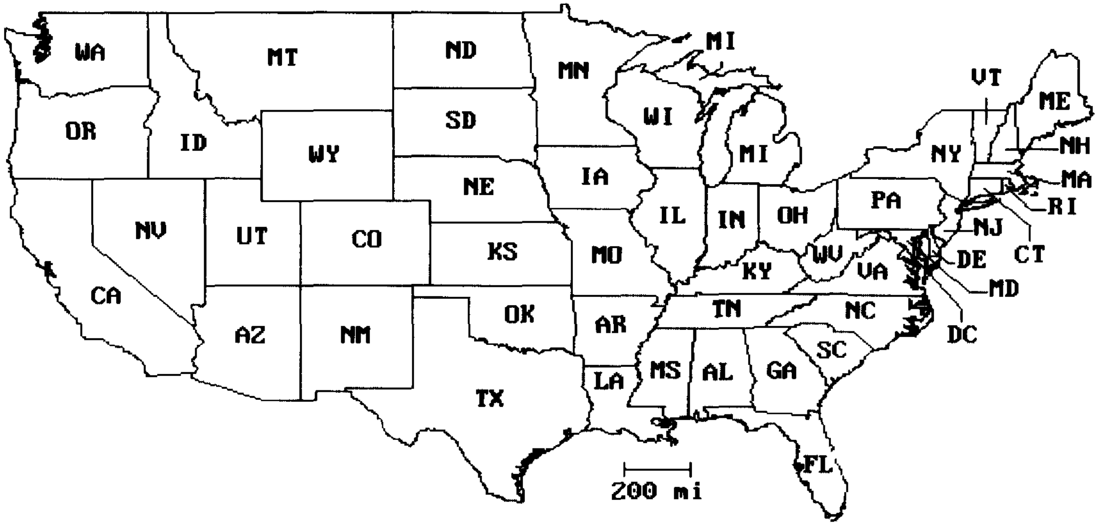

EXHIBIT 3

Comparison of Actual (A) to Fitted (F) Frequencies

	Interval of Ratio R							
	$8\% < R \leq 16\%$		$16\% < R \leq 32\%$		$32\% < R \leq 64\%$		$R > 64\%$	
Region	A	F	A	F	A	F	A	F
1	3	5.71	1	2.91	2	1.84	0	1.71
2	10	5.61	4	2.62	1	1.84	1	0.74
3	22	17.60	1	5.31	4	3.82	3	1.86
4	14	17.77	3	5.61	5	3.82	5	3.25
5	4	7.23	5	3.24	2	4.32	5	4.45
6	8	6.15	6	2.93	4	2.44	2	1.65
7	23	16.11	8	5.53	1	2.61	0	1.35
8	4	4.73	1	2.79	0	0.90	1	0.73
9	4	4.59	5	2.69	1	0.82	1	0.44
10	13	12.72	6	5.65	5	1.01	1	0.68
11	22	14.46	9	5.53	4	1.70	1	0.87
12	11	14.43	6	5.46	0	1.70	0	0.85
13	6	5.14	2	2.95	1	1.20	1	0.70
14	6	5.54	1	3.00	4	1.59	0	0.88
15	6	5.60	2	3.20	1	1.59	2	1.51
16	2	4.97	2	3.20	1	0.99	0	1.07
17	7	4.92	5	3.24	0	0.94	2	2.75
18	3	5.59	3	2.58	1	1.84	0	0.82
19	8	4.62	3	2.60	0	0.86	1	0.51
20	8	17.48	7	5.12	2	3.82	6	1.95
31	18	6.21	4	2.71	3	2.69	1	2.02
22	14	13.73	4	5.07	0	1.48	0	0.79
23	7	5.27	3	2.79	1	1.38	0	0.71
24	1	5.05	2	2.88	1	1.14	2	1.13
25	3	5.10	1	2.47	2	1.32	0	0.57
26	11	15.70	4	4.80	3	2.61	1	1.22
27	5	5.53	2	2.61	3	1.75	1	0.91
28	9	14.44	4	4.49	2	1.97	0	0.89
	252	252.00	104	103.98*	54	53.99*	37	37.01*

*Totals do not always match exactly, due to rounding.

EXHIBIT 4
Part 1

COMPARISON OF EXPERIENCE RATED FREQUENCIES WITH FITTED PARETO FREQUENCIES

	Experience Rated Frequencies			
Region	$8\% < R \leq 16\%$	$16\% < R \leq 32\%$	$32\% < R \leq 64\%$	$R > 64\%$
1	4.28	1.97	1.45	1.03
2	7.83	3.07	1.92	0.90
3	20.14	3.61	3.92	2.14
4	15.00	4.81	4.29	3.87
5	5.73	3.43	3.34	4.67
6	7.54	4.19	3.73	2.22
7	20.72	6.72	2.14	1.11
8	4.30	2.03	0.63	0.61
9	4.72	3.46	0.94	0.53
10	13.52	7.04	1.83	0.82
11	20.22	8.29	2.70	1.08
12	11.62	5.15	1.15	0.64
13	5.61	2.65	1.33	0.83
14	5.86	2.43	2.41	0.88
15	5.91	2.79	1.57	1.72
16	3.67	2.48	0.86	1.79
17	6.28	4.18	0.57	1.75
18	4.31	2.31	1.44	0.63
19	6.12	2.65	0.70	0.52
20	10.95	6.32	3.56	3.36
21	12.58	3.32	2.99	1.89
22	13.03	4.06	0.86	0.51
23	6.09	2.74	1.37	0.66
24	3.43	2.50	0.98	1.12
25	4.04	1.84	1.17	0.42
26	12.02	4.52	2.37	1.03
27	5.41	2.44	2.07	0.94
28	10.17	4.02	1.39	0.54
	251.10	105.04	53.68	37.23

EXHIBIT 4
Part 2

COMPARISON OF EXPERIENCE RATED FREQUENCIES WITH FITTED PARETO FREQUENCIES

	Fitted Pareto Frequencies			
Region	$8\% < R \leq 16\%$	$16\% < R \leq 32\%$	$32\% < R \leq 64\%$	$R > 64\%$
1	4.26	2.18	1.12	1.18
2	7.77	3.37	1.46	1.12
3	17.37	7.25	3.03	2.17
4	13.53	6.99	3.61	3.85
5	5.85	3.86	2.54	4.92
6	7.91	4.37	2.42	2.99
7	20.63	6.76	2.21	1.08
8	4.36	1.85	0.78	0.57
9	5.64	2.34	0.97	0.69
10	14.96	5.32	1.89	1.04
11	21.21	7.28	2.50	1.31
12	12.38	4.13	1.38	0.69
13	5.76	2.57	1.15	0.93
14	5.70	2.89	1.47	1.51
15	5.74	2.99	1.56	1.69
16	4.09	1.94	0.92	0.84
17	6.39	3.20	1.60	1.60
18	4.58	2.17	1.03	0.93
19	6.31	2.33	0.86	0.50
20	11.39	6.03	3.19	3.58
21	11.31	5.16	2.35	1.97
22	13.07	3.81	1.11	0.46
23	6.29	2.65	1.11	0.81
24	3.82	2.00	1.05	1.16
25	4.15	1.85	0.82	0.66
26	12.02	4.77	1.90	1.25
27	5.41	2.71	1.36	1.36
28	10.58	3.64	1.25	0.66
	252.49	106.41	46.64	41.51

EXHIBIT 5

Regional Frequencies by Time Period

	Interval of Ratio R							
	$8\% < R \leq 16\%$		$16\% < R \leq 32\%$		$32\% < R \leq 64\%$		$R > 64\%$	
Region	1949-69	1970-89	1949-69	1970-89	1949-69	1970-89	1949-69	1970-89
1	1	2	1	0	1	1	0	0
2	2	8	3	1	0	1	0	1
3	10	12	0	1	1	3	1	2
4	4	10	0	3	4	1	3	2
5	2	2	1	4	1	1	5	0
6	4	4	3	3	2	2	1	1
7	9	14	4	4	0	1	0	0
8	0	4	0	1	0	0	1	0
9	2	2	3	2	1	0	1	0
10	3	10	2	4	3	2	1	0
11	9	13	3	6	3	1	1	0
12	7	4	4	2	0	0	0	0
13	4	2	2	0	1	0	0	1
14	2	4	0	1	2	2	0	0
15	3	3	2	0	0	1	2	0
16	1	1	1	1	1	0	0	0
17	1	6	4	1	0	0	2	0
18	0	3	2	1	0	1	0	0
19	3	5	2	1	0	0	1	0
20	3	5	3	4	1	1	3	3
21	7	11	3	1	3	0	0	1
22	7	7	4	0	0	0	0	0
23	4	3	3	0	0	1	0	0
24	1	0	0	2	1	0	2	0
25	1	2	1	0	1	1	0	0
26	7	4	1	3	1	2	1	0
27	2	3	1	1	2	1	1	0
28	1	8	3	1	0	2	0	0

APPENDIX A

Details of Regressions

The "center" of a region is defined as the point such that half the area is to the north, half to the east, half to the west, and half to the south. For each of the 28 regions, the latitude and longitude of the centers of the regions were estimated and considered to be the latitude and longitude of the region. The "distance to the coast" of a region is defined as the length of the shortest line from the center to any ocean.

The independent variables used in the regression were x_1, x_2, x_3, and x_4, such that, for each region,

$$x_1 = \text{latitude of region;}$$

$$x_2 = \begin{cases} 0, & \text{if } 92 \le \text{longitude of region} \le 99, \\ |\,\text{longitude} - 99|, & \text{if } 99 < \text{longitude} < 105, \\ 6, & \text{if longitude} \ge 105, \\ |\,\text{longitude} - 92|, & \text{if } 86 < \text{longitude} < 92, \\ 6, & \text{if longitude} \le 86; \end{cases}$$

$$x_3 = \ln(\ln(\text{area of region, in thousands of miles}));$$

$$x_4 = \ln(\ln(\text{distance from coast of region, in miles})).$$

The values of x_1, x_2, x_3, and x_4, for the 28 regions are given in Exhibit 6.

For each of the seven intervals for R, the dependent variable used in the regression for the interval was ln(frequency of catastrophes). (In cases where the frequency was zero, ln (⅓) was judgmentally used instead of the undefined ln (0).) This dependent variable was chosen so that, for each independent variable, a given amount of change would produce a fixed multiplicative effect on the fitted frequencies defined below.

This approach produced a better fit than any other dependent variable and avoided the problem of negative or unreasonably small fitted values. An attempt was made to use (frequency of catastrophes)$^{0.5}$ as the dependent variable, since its variance is relatively close to being independent of the expected frequency of catastrophes, and this is desirable when using regression. However, it did not produce the most acceptable fitted values.

The use of ln (ln (x)) for x_3 and x_4 resulted from the observation that it produced values of x_3 and x_4 that came reasonably close to having the desired linear relationship with the values of the dependent variable.

For each interval I_i of R values, there is a corresponding set of frequencies by region $\{f_{i,j}\}$, where j is an integer from 1 to 28.

Fitted values $\hat{y}_{i,j}$ were produced by regression. Then the function

$$g_i(\hat{y}_{i,j}) = \exp(\hat{y}_{i,j}) \left(\left(\sum_{j=1}^{28} f_{i,j} \right) \bigg/ \sum_{j=1}^{28} \exp\left(\hat{y}_{i,j}\right) \right) \tag{A.1}$$

was used to produce values $g_i(\hat{y}_{i,j})$ such that

$$\sum_{j=1}^{28} g_i(\hat{y}_{i,j}) = \sum_{j=1}^{28} f_{i,j} .$$

The values $g_i(\hat{y}_{i,j})$, rather than $\hat{y}_{i,j}$, were used as final fitted values for the frequencies $f_{i,j}$.

Tornadoes are more prevalent in the region between longitudes 92 and 99, which helps explain the motivation for the definition of the variable x_2.

The interval $R > 64\%$ was the only one for which x_4 was used. It appears that distance from the coast is a useful variable for large hurricanes, but not for smaller catastrophes such as tornadoes. The variable x_4 didn't work well for intervals for which $R \leq 64\%$, possibly due to collinearity with the longitude variable. The coefficient came out only negligibly negative or even positive.

Positive coefficients for any of the variables x_1, x_2, x_3, and x_4 were considered counter to the overall indications of the data and not appropriate for use in the study. For all intervals, all the variables x_1, x_2, and x_3 were used unless one of them had a positive coefficient. In these cases, a regression was done without using that variable.

To find confidence intervals for the regression coefficients or for the expected values of the dependent variables, it would have to be true that:

1. A linear relationship exists between the independent variables used and the dependent variable used.
2. The conditional distributions of the dependent variables, given values of the independent variables, are uncorrellated and have a common variance.

Neither condition is satisfied. Nothing can be done to satisfy the first condition unless a way is known to transform the variables so that they satisfy a linear relationship. Therefore, it was considered better to avoid the complications involved in transforming variables to come closer to satisfying the second condition. The results of the regression are considered to be simply a useful method of smoothing the data.

The functions resulting from the regressions are shown in Table 4.

TABLE 4
REGRESSION FUNCTIONS

i	Interval	Function
1	$8\% < R \le 16\%$	$-0.024x_1 - 0.167x_2 - 0.083x_3 + 3.694$
2	$16\% < R \le 32\%$	$-0.00005x_1 - 0.108x_2 - 0.461x_3 + 2.312$
3	$32\% < R \le 64\%$	$-0.095x_1 - 0.035x_2 + 4.169$
4	$R > 64\%$	$-0.030x_1 - 0.069x_2 - 0.241x_3 - 2.719x_4 + 6.457$
5	$R > 32\%$	$-0.102x_1 - 0.002x_2 - 0.808x_3 + 6.150$
6	$R > 16\%$	$-0.047x_1 - 0.087x_2 - 0.720x_3 + 5.172$
7	$R > 8\%$	$-0.035x_1 - 0.119x_2 - 0.596x_3 + 5.393$

EXHIBIT 6

VALUES OF INDEPENDENT VARIABLES

Region	x_1	x_2	x_3	x_4
1	37	6	1.6.12	1.535
2	37	6	1.838	1.824
3	31.5	0	1.715	1.708
4	31.5	0	1.596	1.513
5	28	6	1.381	1.303
6	34	6	1.596	1.582
7	35.5	0	1.626	1.790
8	44.5	6	1.703	1.758
9	45.5	6	1.787	1.924
10	45.5	0	1.581	1.936
11	40	0	1.626	1.903
12	40	0	1.654	1.909
13	41.5	6	1.581	1.818
14	38.5	6	1.548	1.767
15	38.5	6	1.405	1.582
16	43.5	6	1.405	1.652
17	44	6	1.381	1.303
18	37	6	1.876	1.780
19	45	6	1.862	1.868
20	31.5	0	1.796	1.684
21	33	6	1.767	1.504
22	41.5	0	1.813	1.902
23	40	6	1.703	1.817
24	42	6	1.640	1.629
25	40.5	6	1.970	1.868
26	35.5	0	1.935	1.798
27	37.5	6	1.854	1.740
28	38.5	0	2.078	1.870

APPENDIX B

DERIVATION OF CREDIBILITY FORMULA

To approximate an experience rating formula, we assume:

1. Given that $g_i(\hat{y}_{i,j})$ is the fitted value for interval i and region j in the smoothing method of this paper, the probability distribution of the random variable $E_{i,j}$, which represents the expected value of the frequency of catastrophes in interval i and region j, has mean $g_i(\hat{y}_{i,j})$.
2. For each i, the probability distribution of $E_{i,j}$ has the same coefficient of variation C_i for each j.

It follows that, for each interval i and each region j, the Z such that

$$Z\,(\text{actual frequency in interval } i \text{ and region } j) + (1-Z)\, g_i(\hat{y}_{i,j}) \tag{B.1}$$

is the best least squares estimate of the expected value of the frequency in interval i and region j is

$$Z = g_i(\hat{y}_{i,j})/(g_i(\hat{y}_{i,j}) + 1/C_i^2). \tag{B.2}$$

The proof is as follows. By Bühlmann's theorem (Bühlmann [5], Herzog [10]), $Z = H_{i,j}/(H_{i,j} + V_{i,j})$ where $H_{i,j}$ equals the variance of the probability distribution of the expected value of the frequency for interval i and region j, and $V_{i,j}$ equals the expected value of the variance of the frequency, given the above probability distribution for the expected value of the frequency.

For each possible value $e_{i,j}$ for the expected value of the frequency, the probability distribution of actual values is assumed to be Poisson and thus has variance $e_{i,j}$. Therefore, by Assumption 1 above, $V_{i,j} = g_i(\hat{y}_{i,j})$. By Assumption 2 above, $H_{i,j} = (C_i\, g_i(\hat{y}_{i,j}))^2$. Therefore,

$$
\begin{aligned}
Z &= C_i^2\, g_i\, (\hat{y}_{i,j})^2 / (C_i^2\, g_i\, (\hat{y}_{i,j})^2 + g_i\, (\hat{y}_{i,j})) \\
&= g_i\, (\hat{y}_{i,j}) / (g_i\, (\hat{y}_{i,j}) + 1/C_i^2).
\end{aligned}
\tag{B.3}
$$

This completes the proof.

The estimates of the numbers C_i^2 will now be discussed.

Consider the frequency in interval i and region j during the 41-year period used for the data to be the outcome of an experiment. Let the random variable $X_{i,j}$ represent the outcome. The expected value of $(g_i\, (\hat{y}_{i,j}) - X_{i,j})^2$, given that $e_{i,j}$ is the expected value of the frequency, equals $(g_i\, (\hat{y}_{i,j}) - e_{i,j})^2$ plus the expected value, given that $e_{i,j}$ is the expected value of the frequency, of $(e_{i,j} - X_{i,j})^2$. (This is left for the reader to verify.) Therefore, the mean of $(g_i\, (\hat{y}_{i,j}) - X_{i,j})^2$ equals the mean of $(g_i\, (\hat{y}_{i,j}) - E_{i,j})^2$ plus the mean of $(E_{i,j} - X_{i,j})^2$.

By Assumption 2 above, the mean of $(g_i\, (\hat{y}_{i,j}) - E_{i,j})^2$ equals $C_i^2\, (g_i\, (\hat{y}_{i,j}))^2$.

Given that $e_{i,j}$ is the expected value of the frequency, the mean of $(e_{i,j} - X_{i,j})^2$ is $e_{i,j}$. Therefore, the mean of $(E_{i,j} - X_{i,j})^2$ equals the mean of $E_{i,j}$, which is $g_i\, (\hat{y}_{i,j})$.

Therefore, the mean of $(g_i\, (\hat{y}_{i,j}) - X_{i,j})^2$ equals $C_i^2\, (g_i\, (\hat{y}_{i,j}))^2 + g_i\, (\hat{y}_{i,j})$. So C_i^2 equals the expected value of

$$
\left(\sum_{j=1}^{28} (g_i\, (\hat{y}_{i,j}) - X_{i,j})^2 - \sum_{j=1}^{28} g_i\, (\hat{y}_{i,j}) \right) \Big/ \sum_{j=1}^{28} g_i\, (\hat{y}_{i,j})^2.
\tag{B.4}
$$

The estimate of the expected value of

$$
\sum_{j=1}^{28} (g_i\, (\hat{y}_{i,j}) - X_{i,j})^2
$$

will depend partly on judgment and intuition, due to problems in estimating it purely mathematically.

Assume for the sake of approximation that the following two conditions are satisfied.

1. The values $g_i(\hat{y}_{i,j})$ are the function values produced directly by a regression, and a linear relationship with coefficients $a_{i,j}$ actually exists between the independent variables used and the expected values of the dependent variables.
2. The differences between the dependent variables and their expected values have independent probability distributions with a common variance σ^2.

Under these conditions,

$$\left(\sum_{j=1}^{28} (g_i(\hat{y}_{i,j}) - f_{i,j})^2\right) / (\text{degrees of freedom}), \qquad \text{(B.5)}$$

where $f_{i,j}$ is the actual frequency in interval i and region j, is an unbiased estimate of σ^2 (Draper and Smith [8]). If the values $g_i(\hat{y}_{i,j})$ are not the true expected values of the frequencies in interval i and region j, then the expected value of

$$\left(\sum_{j=1}^{28} (g_i(\hat{y}_{i,j}) - X_{i,j})^2\right) / 28$$

is greater than σ^2.

Assuming Equation B.5 is equal to or less than the expected value of

$$\left(\sum_{j=1}^{28} (g_i(\hat{y}_{i,j}) - X_{i,j})^2\right) / 28,$$

Equation B.4 gives the following lower bound for C_i^2 :

$$(\text{Equation B.5} - \sum_{j=1}^{28} g_i(\hat{y}_{i,j}))/\sum_{j=1}^{28}(g_i(\hat{y}_{i,j}))^2. \tag{B.6}$$

We now discuss an upper bound for C_i^2.

It clearly appears that the expected value of

$$\sum_{j=1}^{28}(g_i(\hat{y}_{i,j}) - X_{i,j})^2$$

is less than

$$\sum_{j=1}^{28}\left(\left(\sum_{j=1}^{28} g_i(\hat{y}_{i,j})\right)/28 - f_{i,j}\right)^2,$$

where $f_{i,j}$ is the actual frequency in interval i and region j. The value

$$\left(\sum_{j=1}^{28} g_i(\hat{y}_{i,j})\right)/28$$

is a mere average of the values $g_i(\hat{y}_{i,j})$, so the individual estimates $g_i(\hat{y}_{i,j})$ intuitively appear to be better estimators for the expected values of the variables $X_{i,j}$ than is

$$\left(\sum_{j=1}^{28} g_i(\hat{y}_{i,j})\right)/28.$$

Therefore, it follows, based on the above arguments and Equation B.4, that the following is an upper bound for C_i^2 :

$$\sum_{j=1}^{28}\left(\left(\sum_{j=1}^{28}(g_i(\hat{y}_{i,j})/28 - f_{i,j})^2 - \sum_{j=1}^{28} g_i(\hat{y}_{i,j})\right)/\sum_{j=1}^{28}(g_i(\hat{y}_{i,j}))^2\right). \tag{B.7}$$

Thus we have (Equation B.6) $< C_i^2 <$ (Equation B.7). Using the actual values of the expressions in Equations B.6 and B.7 for $i = 1$ through 7, and averaging inequalities, gives

$$0.049 < ((C_1^2 + C_2^2 + C_5^2 + C_6^2 + C_7^2)/5) < 0.146 \qquad \text{(B.8)}$$

and

$$0.065 < ((C_3^2 + C_4^2)/2) < 0.215. \qquad \text{(B.9)}$$

The reason for considering C_3 and C_4 separately from C_1, C_2, C_5, C_6, and C_7 is that the numbers $g_i(\hat{y}_{i,j})$ for $i = 3$ and $i = 4$ were based on less data than for $i = 1, 2, 5, 6$, and 7. Thus, the expectation is that they are less accurate. Therefore, it can be seen from Equation B.1 that C_i^2 would be expected to be greater for those intervals.

By Equation B.2, the choices of $k_i = 9$ for $i = 1, 2, 5, 6$, or 7 and $k_i = 6$ for $i = 3$ or 4 in Subsection 2.C imply choices of $1/9$ for each of C_1^2, C_2^2, C_5^2, C_6^2 and C_7^2 and $1/6$ for C_3^2 and C_4^2.

Thus, the selected values for k_i are toward the low end of the range of inequalities B.8 and B.9. Still, the numbers $g_i(\hat{y}_{i,j})$ have a much greater effect than the numbers $f_{i,j}$ on the tails of the loss distributions selected by region in Subsection 2E.

APPENDIX C

METHOD OF FITTING PARETO

Iteration was used to find the single parameter Pareto distribution that minimizes

$$\sum_{i=1}^{4} ((f_i - P_i)^2 / P_i^{1.5}),$$

where f_i is as defined in Subsection 2E, and P_i is the corresponding fraction for the Pareto distribution.

The above method of fitting a Pareto to the numbers f_i is different, for theoretical reasons, from methods that would be used to fit a Pareto to actual frequencies. An explanation of the method is as follows.

Let the random variable X_i equal the f_i produced by performing the experiment of using the method of this paper on the data for the 41-year period. Assume that there is some Pareto distribution A such that each A_i, defined similarly to P_i, is the mean of X_i.

The Pareto that minimizes

$$\sum_{i=1}^{4} ((f_i - P_i) / \sigma_i)^2 ,$$

where σ_i is the standard deviation of X_i, is an estimate of A. If the probability distribution of X_i is Normal, then it is the maximum likelihood estimate of A.

If $P_i = A_i$, then based on the process used in computing the number f_i, it is judgmentally estimated that, for some constant c, each σ_i^2 equals approximately $cP_i^{1.5}$. Each f_i results from a weighting of actual data and a smoothed estimate. If only actual data were used, each σ_i^2

would be approximately in the same proportion to P_i. On the other hand, if only smoothed estimates were used to produce each f_i, and if the coefficient of variation were the same for each X_i, each σ_i^2 would be in the same proportion to P_i^2. The value $P_i^{1.5}$ was selected above because it is approximately midway between P_i and P_i^2. Thus the Pareto that minimizes

$$\sum_{i=1}^{4} ((f_i - P_i)^2 / P_i^{1.5})$$

is an estimate of the Pareto that minimizes

$$\sum_{i=1}^{4} ((f_i - P_i) / \sigma_i)^2.$$

A viable alternative method, which avoids the somewhat arbitrary choice of exponent on P, would be to use iteration to find the Pareto that maximizes the likelihood function $\Pi\, P_i^{f_i}$. This is numerically no more difficult than the approach used.

AGGREGATE RETROSPECTIVE PREMIUM RATIO AS A FUNCTION OF THE AGGREGATE INCURRED LOSS RATIO

ROBERT K. BENDER

Abstract

The aggregate premium returned to a group of individual risks that are subject to retrospective rating depends upon the retrospective rating formula, the aggregate loss ratio of the risks, and the distribution of the individual risks' loss ratios around the aggregate. As the aggregate incurred loss ratio for a group of risks increases, the aggregate returned premium decreases, but not as rapidly as the loss ratio increases.

In this paper a simple equation is developed for the relationship between the aggregate incurred loss ratio and the aggregate retrospective return premium. The equation relies on the tabular charges and savings of Table M, thereby eliminating the need to perform Monte Carlo style simulations.

Using the relationship expressed in terms of Table M values, the response of several retrospective rating formulas to changes in the aggregate incurred loss ratio is determined.

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A balanced individual risk retrospective rating plan is one in which the aggregate premium retained for all risks is equal to the aggregate premium that would have been collected if all of the risks had been written on a guaranteed cost basis. While charging an amount equal to the guaranteed cost premium in the aggregate, the retrospectively determined individual risk premiums are allowed to vary (within limits) as a function of the individual risk's actual loss experience.

An attempt is made to anticipate and reflect all of the possible individual risk loss outcomes of the guaranteed cost rates. However, only those outcomes which produce retrospective premiums that lie between the specified minimum and maximum premiums enter the formula explicitly. Those loss outcomes that produce retrospective premiums less than the minimum premium have the same effect on the aggregate retrospective premium as those which yield the minimum premium exactly. Likewise, risks that produce retrospective formula premiums greater than the specified maximum contribute no more premium to the aggregate than those with losses that exactly produce the maximum premium.

The loss "capping" effect of the minimum and maximum retrospective premium constraints makes achieving a balance with the corresponding guaranteed cost rates a non-trivial exercise. The rather well known device by which a balance can be achieved is the insurance charge. The insurance charge is used to modify the retrospective premium formula in such a way that the aggregate retrospective and guaranteed cost premiums become equal. The mechanics of how one determines the appropriate insurance charge can be found in John Stafford's monograph [7] as well as the Retrospective Rating Plan Manuals for both the National Council on Compensation Insurance (NCCI) [4] and the Insurance Services Office, Inc. [3]. More theoretical treatments can be found in several monographs and papers (see [2], [5], and [6], for example). Both the guaranteed cost (GC) rates and the individual risk retrospective rating (IRRR) formula (together with the specified minimum and maximum premiums) are established prospectively. Only the individual risk premiums are determined retrospectively. In order to determine the GC rates and the insurance charge component in the IRRR formula, one must forecast the incurred loss ratio (*ILR*) for the aggregation of all policies to be written under these rates. A look at recent rate filings for workers' compensation shows that, in many jurisdictions, the *ILR* anticipated in the original filing was quite different from the *ILR* actually experienced.

Guaranteed cost rates offer no immunity to the insurer from the effects of missing the "target" loss ratio. Retrospective rating, on the other hand, does possess the ability to offset some of these effects by increasing or decreasing the aggregate retrospectively determined premiums in response to the error in the estimated *ILR*. The maximum and minimum premium constraints, however, place limits upon the degree to which a retrospective rating plan can respond to changes in the aggregate *ILR*.

It is desirable to have a quantitative measure to prospectively determine the degree to which a particular retrospective rating plan will respond to differences between the underlying expected aggregate loss ratio and the actual aggregate loss ratio. Section 1 provides a theoretical treatment of the problem. It concludes with the derivation of a formula expressing the ratio (*RP*) of aggregate retrospectively determined return premium to standard premium as a function of the aggregate *ILR* for all retro-rated policies. Rather than being explicitly dependent on a distribution about the mean of individual policy *ILR*s, the functional relationship is expressed in terms of Table M charges and savings. Appendix A displays a tractable, albeit unrealistic, numerical illustration of the theory that is introduced in Section 1.

In Section 2, the results of Section 1 together with Table M insurance charges are used to obtain sets of ordered pairs of aggregate *RP*s and *ILR*s for a set of risks that will be subject to retrospective rating. Using this set of ordered pairs, the sensitivity of a retrospective rating plan's *RP* to changing *ILR*s is examined. In particular, the influence of four factors (the individual risk *ILR* distribution, the plan loss conversion factor (*LCF*), the plan minimum premium ratio, and the plan maximum premium ratio) is discussed. Section 2 continues with some remarks about curve fitting. Appendix B provides the details of one of the simulations that is presented in Section 2.

Section 3 concludes by summarizing the results of Section 2 and suggesting practical applications of the theory to the evaluation of residual market retro plans. The establishment of a retrospective unearned premium liability is also briefly discussed.

1. THE FUNCTIONAL RELATIONSHIP BETWEEN *ILR* AND *RP*

In this section a functional relationship is derived for the aggregate *ILR* for a group of individual risks defined by a particular loss ratio density function, $f(s)$, and the resulting aggregate retrospective premium returned ratio *RP*. The distribution, f, will be defined by two moments, the familiar charge, $X(r)$, and savings, $Y(r)$, of Table M or Table L. (See [5] and [6].)

For simplicity, assume that the retrospective rating plan does not involve any per claim (or occurrence) loss limit, nor does it incorporate retrospective development factors (either of these could be handled within the theoretical framework that follows, but neither would add to the exposition). A retrospective rating plan consists of a retrospective rating *formula*

$$R = [e*S + c*I*S + c*L]*TM\,, \tag{1.1}$$

subject to the limiting *constraint* that

$$H*S \le R \le G*S, \tag{1.2}$$

where:

R	is the retrospectively determined premium;
S	is the standard premium for the risk;
e	is the ratio of non-loss-based expenses to S;
c	is the loss conversion factor (*LCF*) which consists of unity plus a provision for any loss-based expenses;
I	is the net insurance charge as a ratio to S;
TM	is the tax multiplier, $TM = 1/(1-\text{tax rate})$;

L is the actual incurred loss for the policy;

$H*S$ is the agreed upon minimum retrospective premium; and

$G*S$ is the agreed upon maximum retrospective premium.

Dividing both sides of Equation 1.1 by S gives

$$R/S = [e + c*I + c*ILR]*TM, \tag{1.3}$$

where

$$ILR = L/S \tag{1.4}$$

is the incurred loss ratio for the policy. Even if we limit our discussion to a priori identical policies that have the same individual risk expected loss amount $<L>$ and expected loss ratio $<ILR>$ (where the brackets, $<\ldots>$, denote the expected value of the variable that they enclose), the individual risk loss ratios, ILR, can be expected to differ from the expected one.

Following the notational conventions that are used with the Table M of insurance charges, we define an individual policy entry ratio, s, as follows

$$s = L/<L> = ILR/<ILR>. \tag{1.5}$$

Assume that the probability density, f, is such that $f(s)ds$ gives the probability of finding an individual risk with an entry ratio between s and $s + ds$. The function f can, and in Table M does, vary with $<L>$. We note that $f(s)$ need not correspond to any published Table M. "Table M" is used in a generic sense to describe a set of ILR distributions and the charges and savings that are implied by them.

Given $f(s)$, we define two functions,

$$X(r) = \int_r^\infty (s - r) f(s) ds, \tag{1.6}$$

and

$$Y(r) = \int_0^r (r - s)\, f(s) ds, \tag{1.7}$$

which are the familiar charge and savings, respectively, of Table M or Table L (depending upon the particular density f and the definition of ILR).

Returning to Equations 1.2 and 1.3 we find that the minimum entry ratio, $r_{\min}$, is given by

$$r_{\min} = [H/TM - e - c*I\,]/(c*\langle ILR\rangle), \tag{1.8}$$

and the maximum entry ratio, $r_{\max}$, is given by

$$r_{\max} = [G/TM - e - c*I\,]/(c*\langle ILR\rangle)\;. \tag{1.9}$$

If we define a *capped* incurred loss ratio, ilr, as follows:

$$ilr = \langle ILR\rangle * \begin{cases} r_{\min} & \text{for } s < r_{\min} \\ s & \text{for } r_{\min} \leq s \leq r_{\max} \\ r_{\max} & \text{for } r_{\max} < s \end{cases} \tag{1.10}$$

then Equation 1.3 can be recast into a form that does not require the explicit constraint condition, namely:

$$R/S = [e + c*I + c*ilr]*TM. \tag{1.11}$$

The average ratio of the retrospective premium to the standard premium, over all policies described by f, is given by

$$\langle R/S\rangle = [e + c*I + c*\langle ilr\rangle]*TM, \tag{1.12}$$

where

$$\langle ilr \rangle = r_{\min} \int_{0}^{r_{\min}} f(s)ds + \int_{r_{\min}}^{r_{\max}} sf(s)ds + r_{\max} \int_{r_{\max}}^{\infty} f(s)ds. \tag{1.13}$$

Equation 1.13 can be recast into the following form:

$$\langle ilr \rangle = \langle ILR \rangle * [1 + Y(r_{\min}) - X(r_{\max})], \tag{1.14}$$

in which X and Y appear, implicitly representing all of the necessary details contained in $f(s)$.

The interpretation of Equation 1.14 is that the expected capped loss ratio, the one "seen by" the retrospective rating plan, differs from the uncapped expected value by the addition of some losses from risks with formula premiums below the minimum, $\langle ILR \rangle * Y(r_{\min})$, and by the removal of some losses for risks that produce formula premiums above the maximum premium, $\langle ILR \rangle * X(r_{\max})$.

If we require that

$$I + \langle ilr \rangle = \langle ILR \rangle, \tag{1.15}$$

then $\langle R/S \rangle$ will, indeed, balance to the guaranteed cost premium. The determination of the insurance charge, I, is not as trivial as it appears. Solving Equation 1.15 for I gives

$$I = \langle ILR \rangle - \langle ilr \rangle, \tag{1.16}$$

but $\langle ilr \rangle$, itself, depends upon I because the $r_{\min}$ and $r_{\max}$ depend on I. For the purpose of this paper, we can assume that a solution has been obtained, although nothing in what follows depends on a balance being achieved. The interested reader can refer to any of references [2] - [7] to see how the trial and error procedure to determine I, given a table of $X(r)$ and $Y(r)$, is usually performed. Even if we do not impose the requirement that R be balanced to the guaranteed cost premium, Equation 1.14 can be substituted into Equation 1.12 to determine the ordered pair ($\langle ILR \rangle$, RP).

Regardless of whether or not $\langle R/S \rangle$ balances to the guaranteed cost premium, it can be assumed that increasing values of $\langle ILR \rangle$ for *a fixed insurance charge* will produce increasing values of $\langle ilr \rangle$ and hence $\langle R/S \rangle$. The question is: By how much will $\langle ilr \rangle$ increase when $\langle ILR \rangle$ increases? That depends, of course, on the percentage of risks that are either no longer subject to the minimum constraint or are now subject to the maximum constraint. A change in the aggregate *ILR* after the retrospective plan has been established will have no effect on the minimum and maximum loss ratios to be seen by the plan. Only the corresponding entry ratios, $r_{\max}$ and $r_{\min}$, will differ from those originally anticipated.

If $\langle ILR \rangle_1$ is the actual aggregate loss ratio for a portfolio of risks, and

$$\langle ILR \rangle_1 = g * \langle ILR \rangle_0 \, , \tag{1.17}$$

where the constant, g, is defined by Equation 1.17, then, in terms of the actual distribution f_1,

$$r_{\min 1} = r_{\min 0} / g \text{ , and} \tag{1.18}$$

$$r_{\max 1} = r_{\max 0} / g. \tag{1.19}$$

We have adopted the indicator 0 for the originally assumed distribution parameters and density and 1 for the actual distribution parameters and density. The parameters indicated with zeroes may alternatively be thought of as being based upon a priori estimates.

As in Equation 1.13, the actual average loss ratio seen by the retrospective rating plan, given the actual density f_1 and the actual mean $\langle ILR \rangle_1$, will be:

$$\langle ilr \rangle = g * \langle ILR \rangle_0 * \left[(r_{\min} / g) * \int_0^{r_{\min}/g} s f_1(s) ds \right.$$

$$\left. + \int_{r_{\min}/g}^{r_{\max}/g} sf_1(s)ds + (r_{\max}/g) * \int_{r_{\max}/g}^{\infty} f_1(s)ds \right]$$

$$= g{*}{<}ILR{>}_0{*}[1 + Y_1(r_{\min}/g) - X_1(r_{\max}\ / g)]. \qquad (1.20)$$

Upon subtracting Equation 1.20 from 1.14, we find that the difference between the actual and estimated aggregate loss ratio as seen by the retrospective rating plan is given by

$$\Delta{<}ilr{>} = {<}ILR{>}_0 - {<}ILR{>}_0{*}[g{*}X_1(r_{\max} / g) - X_0(r_{\max})]$$

$$+ {<}ILR{>}_0{*}[g{*}Y_1(r_{\min} / g) - Y_0(r_{\min})]. \qquad (1.21)$$

The corresponding change in the retrospective premium ratio is found by substituting Equation 1.21 into 1.12 as follows:

$$\Delta{<}R/S{>} = c{*}\Delta{<}ILR{>}_0{*}TM$$

$$- c{*}{<}ILR{>}_0{*}[g{*}X_1(r_{\max}/g) - X_0(r_{\max})]{*}TM$$

$$+ c{*}{<}ILR{>}_0{*}g{*}Y_1(r_{\min} / g) - Y_0(r_{\min})]{*}TM. \qquad (1.22)$$

Because most retrospective rating plans are designed to return premium to policyholders in the aggregate, we shall refer to return premium ratios (RP), given by

$$RP = 1 - R/S \qquad (1.23)$$

and

$$\Delta RP = -c\Delta{<}ILR{>}_0{*}TM$$

$$+ c{*}{<}ILR{>}_0{*}[g{*}X_1(r_{\max}/g) - X_0(r_{\max})]{*}TM$$

$$-c{*}{<}ILR{>}_0{*}[g{*}Y_1(r_{\min}/g) - Y_0(r_{\min})]{*}TM. \qquad (1.24)$$

Appendix A provides a detailed analysis of the significance of the three terms that appear in Equation 1.24.

2. SENSITIVITY TESTING BY MEANS OF SIMULATION

By means of the simulation model, Equation 1.24, we may test the sensitivity of a particular retro plan to changes in any of its parameters. We shall begin by considering a set of risks with an expected loss ratio of 60 percent of standard premium. Furthermore, we shall assume that the individual risk loss ratios have the same distribution as expected loss group 60 of the NCCI Table M. Using the July 1, 1991 expected loss groupings, this expected loss group corresponds to a set of risks with approximately $53,000 of standard premium. If the expense provision in the rates (excluding taxes but including profit) is 26 percent of standard premium (about midway between the NCCI Table XIV stock and non-stock expense provision for this policy size), and taxes are 4.4 percent of collected premium (i.e., the taxes which lead to a 1.046 tax multiplier), then 10 percent of the standard premium will be available for retrospective premium returns. Using the standard Table M algorithms, and the information given above, the insurance charge for any c, G, and H can be determined. Once those items have been specified, the retrospective rating formula and constraints will be known. Equation 1.24 can be used to generate a set of simulation points for the particular plan. (See Appendix B for the details of one such simulation.)

To quantify the sensitivity of a retro plan to changes in any parameter, we must have a measure of the retro plan's response to changing $\langle ILR \rangle$s. As described in Appendix B, three curves were fit to each simulation: linear, geometric, and exponential. Even when the geometric or exponential model produced a better fit to the data, the linear model for the RP as a function of the $\langle ILR \rangle$ was a close runner-up (as measured by the mean squared error), and the linear model frequently performed best. Because of its simplicity, the slope of the linear curve has been selected as the best measure of a retro plan's response to changes in the $\langle ILR \rangle$. A slope of -.25, for example, means that an eight point increase in the aggregate *ILR* over and above the expected *ILR* results in a two point decrease in the *RP*.

The remarkable feature of our simulations using Table M loss group 60 is how small the slopes are. In other words, large changes in the <*ILR*> do not have a large impact on the *RP* for any of the retrospective rating formulas that were tested.

Table 1 provides us with a summary of the sensitivity of the generic formula,

$$R/S = [.260 + 1.000 * I(G) + 1.000 * ILR] * 1.046, \tag{2.1a}$$

subject to

$$.70 \leq R/S \leq G, \tag{2.1b}$$

where H, the ratio of minimum premium to standard premium, has been set equal to .70.

TABLE 1

RESPONSE (SLOPE) AS A FUNCTION OF THE MAXIMUM PREMIUM RATIO, G
[For Table M Expected Loss Group 60 and 60 Percent Expected Loss Ratio]

			MSE (x 1,000,000)			
LCF	*G*	*H*	Linear	Geometric	Exponential	Slope
1.000	1.15	0.70	6	4	1	-0.10
1.000	1.20	0.70	6	14	2	-0.12
1.000	1.25	0.70	9	24	4	-0.13
1.000	1.30	0.70	10	36	8	-0.14
1.000	1.35	0.70	5	94	36	-0.16
1.000	1.40	0.70	8	111	43	-0.17
1.000	1.45	0.70	9	177	80	-0.18

By varying the maximum premium, we are able to test the formula's sensitivity to changes in *G*, the maximum premium. Here, *I(G)* is the net insurance charge that places *R/S* into balance with the corresponding guaranteed cost rates for an <*ILR*> of 60 percent. Intuitively, we expect that as *G* increases, fewer risks will "max" out, and Equation 2.1a should reflect a greater portion of the actual losses. As expected, the slope does become larger (in absolute value) as the maximum premium increases from 1.15 to 1.45 times the standard premium.

Our intuitive notion concerning the shape of the best model is also confirmed. While both the linear and the exponential model fit the first four *G*s well, the exponential model with its negative first *and* second derivatives fits better. The negative second derivatives are indicative of the law of diminishing returns, which is consistent with the upper bound on *R/S*. For maximum premiums above 130 percent of standard premium, the capping effect is less noticeable, and the linear model produces a better fit. It is interesting to note that, while the slope as a function of *G* has the expected monotonic behavior, even with *G* equal to 145 percent of standard premium, the slope is a modest 18 percent of the change in <*ILR*>. In the limit as *G* goes to infinity, one would expect the slope to approach -1.00 (i.e., equal to the *LCF*). Obviously 1.45 is not anywhere near being effectively infinite for this group of risks with its rather widely spread out *f(s)*.

In Table 2 we freeze *G* at 135 percent of standard premium and attempt to achieve greater response by varying the *LCF*. At the individual risk level, a greater *LCF* will make the formula more responsive to changes in the *ILR*, unless the risk is pinned to the minimum or maximum premium. Even with a 1.35 maximum, the aggregate response for *LCF*s between .700 and 1.200 is essentially flat at 16 percent. While the choice of the *LCF* is a significant factor as far as individual rate equity is concerned, it has almost no effect in providing a cushion for the carrier against missing the aggregate <*ILR*> target!

TABLE 2

RESPONSE (SLOPE) AS A FUNCTION OF THE LOSS CONVERSION FACTOR, *LCF* [For Table M Expected Loss Group 60 and 60 Percent Expected Loss Ratio]

			MSE (x 1,000,000)			
LCF	*G*	*H*	Linear	Geometric	Exponential	Slope
0.700	1.35	0.70	6	46	12	-0.15
0.800	1.35	0.70	8	52	14	-0.15
0.900	1.35	0.70	8	56	16	-0.15
1.000	1.35	0.70	5	94	36	-0.16
1.100	1.35	0.70	7	75	25	-0.16
1.150	1.35	0.70	10	71	23	-0.16
1.200	1.35	0.70	10	79	26	-0.16

Table 3 is identical to Table 2 except that there is no stated minimum for the retro plan. No stated minimum implies a minimum that is equal to the basic premium, $B = e + c*I$, times the tax multiplier, *TM*. While a $B \times TM$ plan is slightly more responsive for large *LCF*s than the corresponding $H = .70$ plan, the slope as a function of *LCF* is still very flat. The additional responsiveness for the $B \times TM$ plans can be attributed to changes in *R* for risks which have an *R* that falls below 70 percent of the standard premium (and would have been pinned to the minimum for those loss ratios under the $H = .70$ plan).

Table 4 is a larger version of Table 1, but for $B \times TM$ plans. The response has been determined for various maximum premium factors, *G*. Plans with $G < 1.15$ are possible for $H = B \times TM$ but because their $B \times TM$ is greater than .70, they could not be considered in Table 1. A quick comparison of the slopes for the plans that are common to Tables 1 and 4 shows that there is no significant difference in response between a "no stated minimum" and a ".70 minimum" plan for NCCI loss group 60.

TABLE 3

RESPONSE (SLOPE) AS A FUNCTION OF THE LOSS CONVERSION FACTOR, *LCF*, WITH NO SPECIFIED MINIMUM PREMIUM RATIO, *H* [For Table M Expected Loss Group 60 and 60 Percent Expected Loss Ratio]

			MSE (x 1,000,000)			
LCF	*G*	*H*	Linear	Geometric	Exponential	Slope
0.500	1.35	$B \times TM$	5	24	4	-0.13
0.600	1.35	$B \times TM$	6	32	7	-0.14
0.700	1.35	$B \times TM$	8	38	8	-0.15
0.800	1.35	$B \times TM$	6	58	17	-0.15
0.900	1.35	$B \times TM$	9	67	20	-0.16
1.000	1.35	$B \times TM$	10	79	24	-0.16
1.100	1.35	$B \times TM$	8	125	49	-0.17
1.150	1.35	$B \times TM$	10	119	46	-0.17
1.200	1.35	$B \times TM$	11	115	42	-0.17

TABLE 4

RESPONSE (SLOPE) AS A FUNCTION OF THE MAXIMUM PREMIUM RATIO, *G* WITH NO SPECIFIED MINIMUM PREMIUM RATIO, *H* [For Table M Expected Loss Group 60 and 60 Percent Expected Loss Ratio]

			MSE (x 1,000,000)			
LCF	*G*	*H*	Linear	Geometric	Exponential	Slope
1.000	0.95	$B \times TM$	2	2	2	-0.02
1.000	1.00	$B \times TM$	1	1	1	-0.05
1.000	1.05	$B \times TM$	4	1	2	-0.06
1.000	1.10	$B \times TM$	6	2	2	-0.08
1.000	1.15	$B \times TM$	4	7	1	-0.11
1.000	1.20	$B \times TM$	9	9	1	-0.12
1.000	1.25	$B \times TM$	7	22	3	-0.13
1.000	1.30	$B \times TM$	8	45	11	-0.15
1.000	1.35	$B \times TM$	10	79	24	-0.16

The more compact the incurred loss ratio distribution is, that is, the smaller its variance is, the more responsive a retrospective rating formula should be for a given value of G. That is because a smaller percentage of its risks should have loss ratios that pin R to G. As the Table M expected loss group numbers decrease, the underlying distributions become more compact.

Table 5 tests the responsiveness of a $H = B \times TM$, $LCF = 1.000$, $G = 1.35$ retro plan for different Table M expected loss groups (i.e., different premium sizes). Our intuitive notion is supported by the resulting slopes. Again, the striking feature of the slopes as a function of expected loss group is how small (in absolute value) they are. Even for group 40, only 37 percent of the change in *ILR* translates into a change in *RP*.

TABLE 5

RESPONSE (SLOPE) AS A FUNCTION OF EXPECTED LOSS GROUP, WITH ALL OTHER PARAMETERS HELD CONSTANT
[Expected Loss Ratio Equals 60 percent]

			Loss	Standard		MSE (x 1,000,000)		
LCF	*G*	*H*	Group	Premium	Slope	Linear	Geometric	Exponential
1.000	1.35	$B \times TM$	70	24,000	-0.12	4	9	0
1.000	1.35	$B \times TM$	60	53,000	-0.17	10	28	3
1.000	1.35	$B \times TM$	50	113,000	-0.25	26	233	80
1.000	1.35	$B \times TM$	40	240,000	-0.37	44	1,344	792
1.000	1.35	$B \times TM$	30	860,000	-0.57	21	890	552
1.000	1.35	$B \times TM$	20	4,832,000	-0.78	8	682	430
1.000	1.35	$B \times TM$	15	14,773,000	-0.85	21	3,032	2,379
1.000	1.35	$B \times TM$	10	95,486,000	-0.94	3	753	512

Finally, we investigate the relationship between the shape of the distribution as characterized by $f(s)$ and the *RP* for a fixed premium size and constant $\langle ILR \rangle$. Table 6 shows, for example, that if a set of risks were initially priced as if they had the loss ratio distribution corresponding to Table M's group 60, but they turned out to actually have the distribution of group 73 (i.e., $53,000 accounts turn out behaving like $18,000 accounts), the returned premiums would be 6.8 percent of standard premium more than originally intended. The expected *RP* for group 60 is 10 percent; whereas, one should have expected 16.84 percent. Table 6 makes use of Equation 2.24 when f_0 is not equal to f_1.

TABLE 6

RETURNED PREMIUM (*RP*) AS A FUNCTION OF THE AGGREGATE LOSS DISTRIBUTION AS IDENTIFIED BY THE TABLE M GROUP NUMBER
[Insurance Charge Based Upon Group 60 For All Cases]

Group	*LCF*	*G*	*H*	*ILR*	*RP*
57	1.000	1.35	$B \times TM$	60.0%	8.43%
58	1.000	1.35	$B \times TM$	60.0	8.93
59	1.000	1.35	$B \times TM$	60.0	9.50
60	1.000	1.35	$B \times TM$	60.0	10.00
61	1.000	1.35	$B \times TM$	60.0	10.56
62	1.000	1.35	$B \times TM$	60.0	11.13
63	1.000	1.35	$B \times TM$	60.0	11.63
64	1.000	1.35	$B \times TM$	60.0	12.13
65	1.000	1.35	$B \times TM$	60.0	12.64
66	1.000	1.35	$B \times TM$	60.0	13.20
67	1.000	1.35	$B \times TM$	60.0	13.64
68	1.000	1.35	$B \times TM$	60.0	14.08
69	1.000	1.35	$B \times TM$	60.0	14.64
70	1.000	1.35	$B \times TM$	60.0	15.24
71	1.000	1.35	$B \times TM$	60.0	15.77
72	1.000	1.35	$B \times TM$	60.0	16.34
73	1.000	1.35	$B \times TM$	60.0	16.84

3. CONCLUDING REMARKS

While individual risk retrospective rating plans can be very responsive to individual risk experience for risks of any size, the responsiveness of the aggregate returned premium (*RP***S*) to changes in the aggregate loss ratio (<*ILR*>) for a portfolio of risks is rather weak for all but the so called "jumbo" accounts.

Risks typically written under the NCCI retrospective rating plans have standard premiums less than $1,000,000. Even with a plan maximum as high as 135 percent of the standard premium, the response (slope) ranged from a low of -.12 for risks that were near the lower limit of retrospective rating eligibility to a high of -.57 for risks that have almost $1,000,000 of standard premium.

The NCCI plans are significantly more responsive for the larger risks. Many jurisdictions now permit rating these "jumbo" accounts using retrospective rating formulas that do not strictly adhere to the NCCI parameters. In particular, the NCCI tabular expense provisions and NCCI expected loss ratios need not be used. Obviously, if one is free to select an extremely high maximum and free to load all of the expenses via a loss conversion factor, then even greater response could be expected.

This freedom is not available for the smaller accounts. Because of the ability to achieve very high responsiveness for "jumbo" accounts, and their ability to dominate any empirical study of industry-wide responsiveness, we developed our simulation for the evaluation of responsiveness for portfolios that consist of smaller policies. The discussion that follows is, therefore, confined to portfolios consisting of small and medium size policies.

For these risks, an unanticipated rate deficiency (such as one mandated by a regulator), or a uniform increase in all loss ratios for some other reason, can be expected to change the <*ILR*> without changing the distribution, *f(s)*, of *ILR*s around the *ILR*. When this occurs, only a small fraction of the loss ratio increase is reflected by a lower aggregate *RP*. As a result of this, mandatory retros for the residual market, while restoring some equity between risks, cannot be expected to compensate for uniform deterioration in the loss experience. Even retros with a high maximum (e.g., 150 percent of standard premium) provide little in the way of a safety valve.

If, in addition to missing the target <*ILR*>, the effects of inflation on Table M expected loss groups are not adequately reflected, additional *RP* will be generated, thereby increasing the "bottom line" loss. This was illustrated in Table 6.

If one could quantify the relationship between the reported <*ILR*> and its *f(s)* as losses mature (i.e., how the *ILR* distribution the retro parameters). If the <*ILR*> could be developed to its ultimate value, and the resulting $f(s)$ was known, then the ultimate *RP* could be estimated. This would be an enhancement to the method presented in Berry's 1980 paper [1]. The research that would be necessary to determine this functional relationship, $f(s)_{<ILR>,\, maturity}$, is well beyond the scope of this paper, but presents us with a challenge for further research.

Another application of Equation 1.24 is in the establishment of safety margins in retrospective rating. If the distribution of possible <*ILR*>s about the mean <*ILR*> is known (this is not the same distribution as f, which involves *ILR*s about <*ILR*>), then the expected profit could be calculated for a particular retro formula. For each <*ILR*> there would be an *RP* and these *RP*s could be averaged using the <*ILR*> distribution. By varying the insurance charge, I, the probability of achieving a profit of less than some number, α, could be reduced below some selected value, β. The details of this investigation are the subject of a future paper that relates the expected return on equity for a portfolio of retrospectively rated risks to this modified insurance charge.

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[7] Stafford, John R., *Retrospective Rating*, J&M Publications, Palatine, Illinois, 1981.

APPENDIX A

Section 1 presented a theoretical relationship between changes in the aggregate incurred loss ratio for a group of individual risks and the corresponding change in the retrospectively determined return premium (additional premium is, simply, negative return premium). This appendix presents a qualitative graphical interpretation of Equation 1.24 as well as a numerical example.

Figure A-1 presents the expected distribution of loss ratios (to standard premium) for a set of 400 a priori identical risks that are to be rated retrospectively. Without providing details concerning the retrospective rating formula, we assume that the plan minimum causes all risks with loss ratios that are less than 30 percent (all risks to the left of the *min retro prem* line on the graph) to pay the minimum retrospective premium. All risks with loss ratios that are greater than 100 percent (risks that lie to the right of the *max retro prem* line on the graph) pay the maximum retrospective premium. Risks with loss ratios between 30 percent and 100 percent are charged a retrospective premium that depends on their respective losses. The three terms of Equation 1.24 deal with the three regions of the graph.

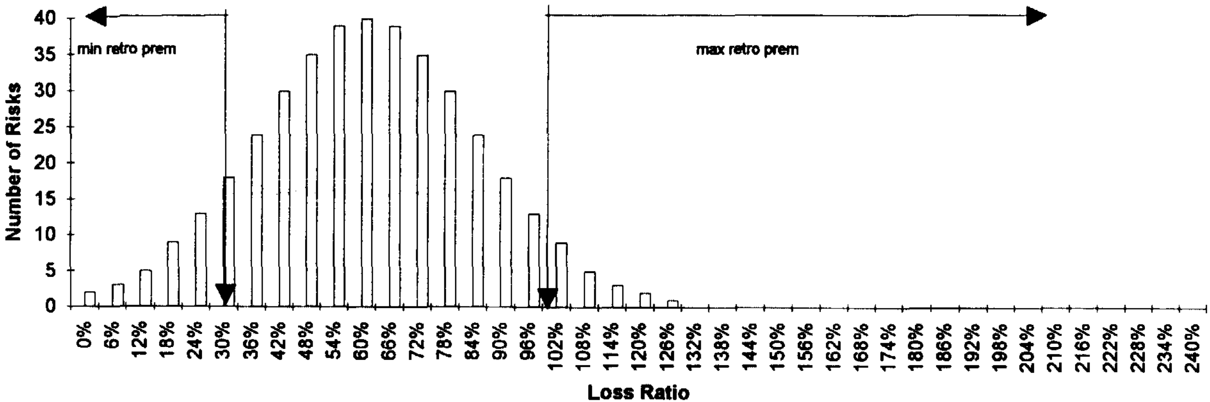
If every loss is 50 percent greater than expected (i.e., $g = 1.5$), then every risk in the first graph is shifted to the right, as shown in Figure A-2. For a given retrospective rating formula, the minimum retro premium and maximum retro premium lines remain unchanged by the difference between the expected and actual distribution.

The first term of Equation 1.24 assumes that every additional dollar of loss will result in a reduction in the aggregate premium that is returned. In particular, each additional dollar of loss is multiplied by the loss conversion factor, c, and the tax multiplier, TM, to determine the reduction in returned premium. The first term reflects the linear responsiveness of the retrospective rating formula. If every risk were to lie between the minimum and maximum lines, then the first term would accurately describe the entire situation.

FIGURE A-1

THE EXPECTED DISTRIBUTION

DISTRIBUTION OF RISKS BY LOSS RATIO



Risks with loss ratios that are less than the minimum pay the retrospective premium that corresponds to the minimum, regardless of the actual loss ratio.

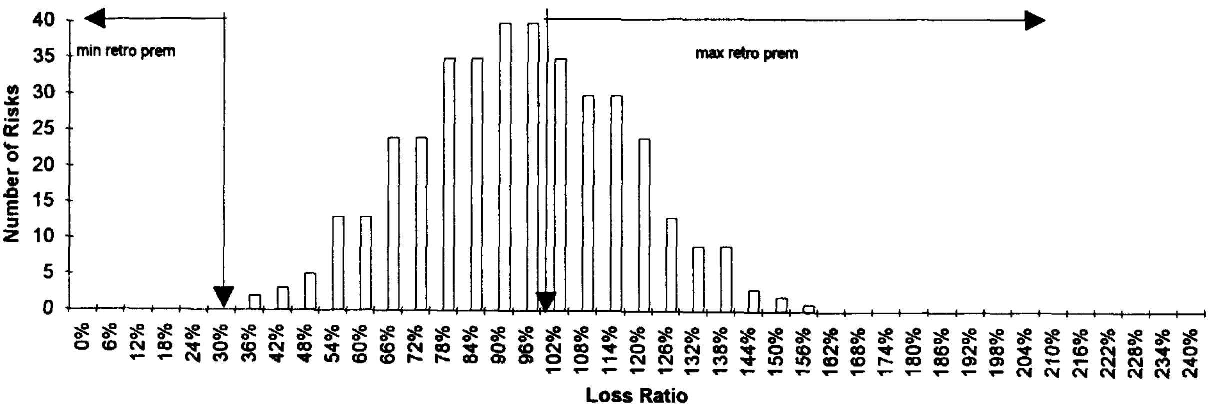
Risks with loss ratios between the minimum and maximum pay the retrospective premium indicated by the retrospective rating formula.

Risks with loss ratios that are greater than the maximum pay the retrospective premium that corresponds to the maximum, regardless of the actual loss ratio.

FIGURE A-2

THE CORRESPONDING DISTRIBUTION WITH $g = 1.5$

DISTRIBUTION OF RISKS BY LOSS RATIO



Risks with loss ratios that are less than the minimum pay the retrospective premium that corresponds to the minimum, regardless of the actual loss ratio.
Risks with loss ratios between the minimum and maximum pay the retrospective premium indicated by the retrospective rating formula.
Risks with loss ratios that are greater than the maximum pay the retrospective premium that corresponds to the maximum, regardless of the actual loss ratio.

Those risks that lie to the right of the maximum premium line would produce no change in the aggregate retrospective premium. The second term in Equation 1.24 deals with those risks that were expected to lie between the two extremes but which actually lie to the right of the maximum line. The losses are g times as large as expected (which explains the factor of g). The old maximum, r_{max}, was expressed in terms of the expected loss ratio. In terms of the actual aggregate loss ratio, it is only $^1\!/_g$ as large. In other words, the new situation is the same as if the old distribution had been realized, but the maximum retrospective premium had been shifted to the left (which explains the argument of X_1). The net effect of the additional risks in the right hand tail is to mitigate the decrease in the aggregate returned retrospective premium.

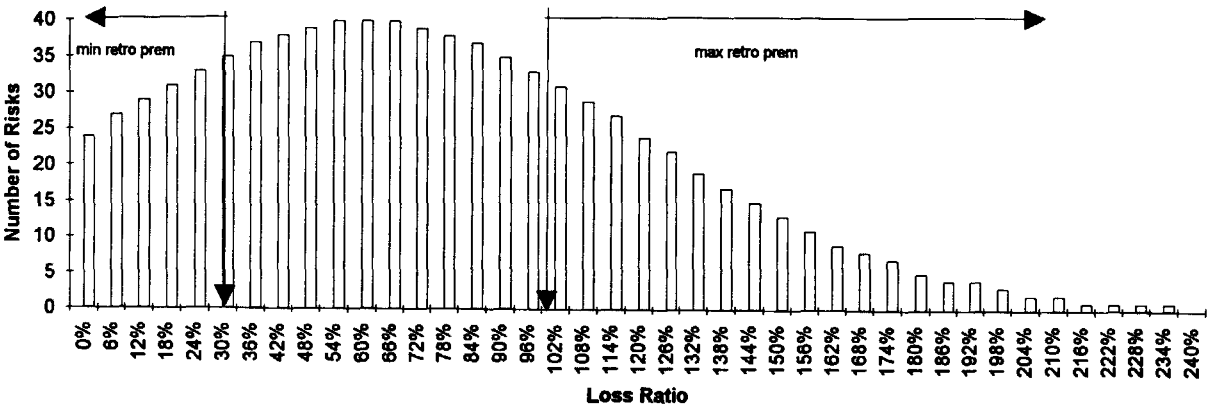
An offsetting effect occurs at the left side of the distribution. Here, some of the risks that were expected to pay the minimum retrospective premium now cross the line and become loss sensitive. The third term in Equation 1.24 represents the correction for the additional premium (reduction in the aggregate returned premium) resulting from those risks that cross the minimum line.

For any particular g, the magnitude of the two correction terms depends on the shape of the loss ratio distribution and relative location of the minimum and maximum premium lines. If we assume that the distribution in Figures A-1 and A-2 is typical of a large account, then the distribution shown in Figures A-3 and A-4 could represent a smaller account with its higher expected variance. (Smaller accounts can be expected to have higher probabilities for extreme loss ratios.) As with Figure A-1, the mean loss ratio of Figure A-3 is 60 percent. The same 50 percent increase in losses (Figure A-4) pins a much larger percentage of the individual risks to the maximum premium, so the retrospective rating formula is less responsive to the shift. Jumbo accounts, on the other hand, would be expected to have very compact loss ratio distributions. (Their loss ratios do not vary much from year to year.) With a fairly high maximum premium, one would expect most of the risks to remain between the two extremes, which would cause the first term in Equation 1.24 to dominate.

FIGURE A-3

THE EXPECTED DISTRIBUTION

DISTRIBUTION OF RISKS BY LOSS RATIO

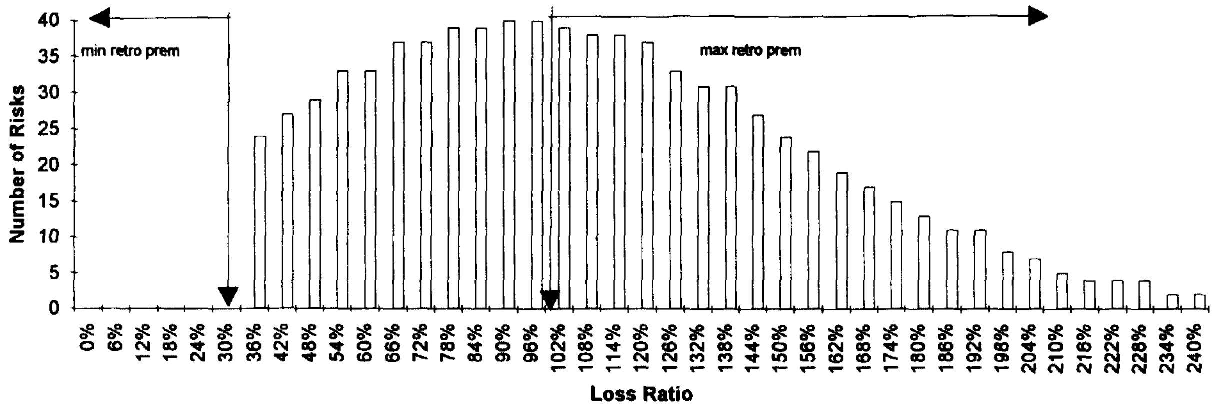


Risks with loss ratios that are less than the minimum pay the retrospective premium that corresponds to the minimum, regardless of the actual loss ratio.
Risks with loss ratios between the minimum and maximum pay the retrospective premium indicated by the retrospective rating formula.
Risks with loss ratios that are greater than the maximum pay the retrospective premium that corresponds to the maximum, regardless of the actual loss ratio.

FIGURE A-4

THE CORRESPONDING DISTRIBUTION WITH $g = 1.5$

DISTRIBUTION OF RISKS BY LOSS RATIO



Risks with loss ratios that are less than the minimum pay the retrospective premium that corresponds to the minimum, regardless of the actual loss ratio.
Risks with loss ratios between the minimum and maximum pay the retrospective premium indicated by the retrospective rating formula.
Risks with loss ratios that are greater than the maximum pay the retrospective premium that corresponds to the maximum, regardless of the actual loss ratio.

To provide a numerical example of Equation 1.24, consider the rather flat (hypothetical) distribution of loss ratios displayed in Figure A-5 and Figure A-6. The distribution appears to change shape only because we have grouped the loss ratios into bins that are five percent wide, and the 30 percent increase causes some of the groupings to change. The essential features are identical with those of the previous four graphs. A significant feature is the large spike that is expected to lie between the two extremes, but which actually lies to the right of the maximum premium line.

FIGURE A-5

THE EXPECTED DISTRIBUTION

DISTRIBUTION OF RISKS BY LOSS RATIO

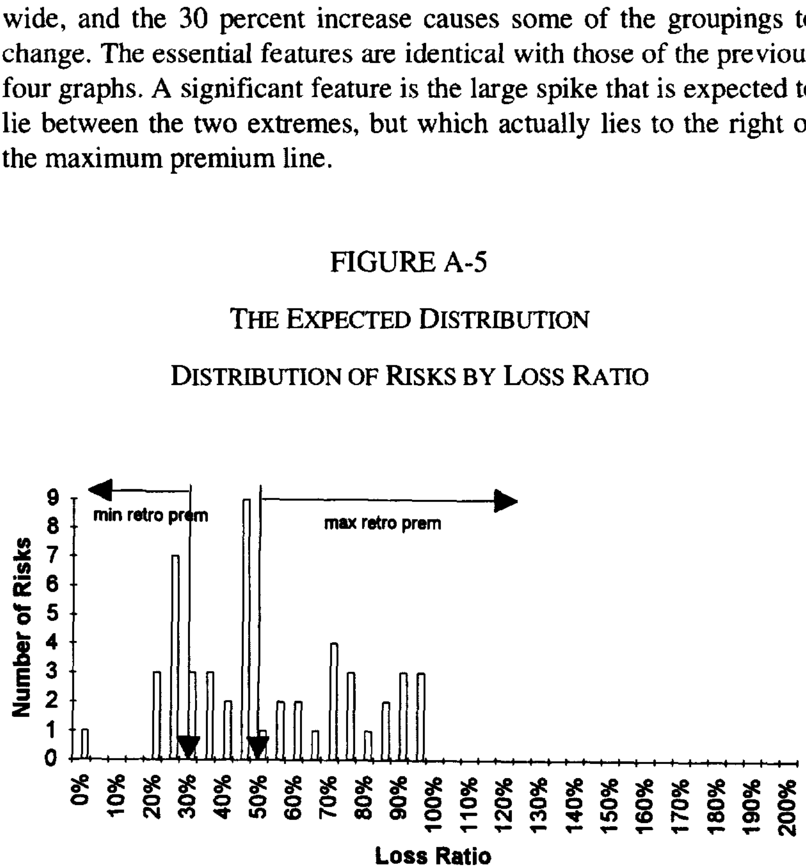


FIGURE A-6

THE CORRESPONDING DISTRIBUTION WITH $g = 1.3$

DISTRIBUTION OF RISKS BY LOSS RATIO

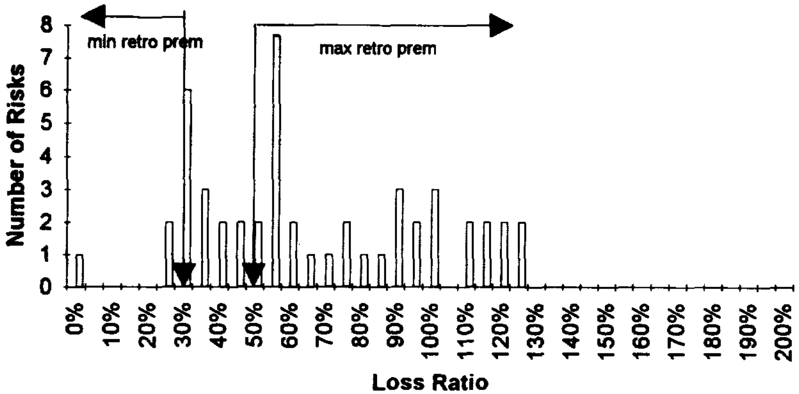


Exhibit A-1 introduces the numerical data corresponding to this group of 50 a priori identical risks. The expected average loss is equal to \$491.96. Individual policy loss ratios are distributed about the mean in the arbitrary (and perhaps a bit unrealistic) distribution that is displayed in Figure A-5. The individual policy standard premium was arbitrarily selected to be \$922.63. While the standard premium was selected at a level that provides for the expected incurred losses, the incurred expenses (including taxes), a reasonable profit, and a margin from which to pay a net retrospective premium return, the details behind the calculation need not be known in order to apply the results of Section 1. A knowledge of the aggregate premium returned under the established retrospective rating plan is required.

The retrospective premium for each of the 50 retrospectively rated risks will be determined by means of Equations A.1 and A.2:

$$R = (e*S + c*I*S + c*L)*TM, \tag{A.1}$$

subject to

$$H*S \leq R \leq G*S, \tag{A.2}$$

where

$$(e*S + c*I*S) = \$382.60;$$

$$c = 1.120;$$

$$TM = 1.031;$$

$$H*S = \$738.10;$$

$$G*S = \$968.76.$$

From Exhibit A-1 we see that the average retrospectively rated premium for the 50 risks is $873.39, which may or may not be in balance with the guaranteed cost rates. The retro plan will be in balance *if and only if* the average premium discount is equal to 5.3 percent of the standard premium, the average amount of returned premium under the retro plan specified above. Whether or not the original retro plan is in balance, the relations derived in Section 1 hold. For that reason, we will not provide any support for the expense and insurance charge (e and I) components of the rating formula. Exhibit A-1 provides the necessary information: For this set of risks with their common a priori loss ratio distribution, an aggregate incurred loss ratio equal to 53.3 percent of standard premium produces an aggregate retrospective *return* premium equal to 5.3 percent of standard premium.

A direct substitution of the aggregate average loss, \$491.96, into the retrospective rating formula given by Equations A.1 and A.2 produces a retrospective premium equal to \$962.54, or an *additional* premium equal to \$39.91 (i.e., a -4.3% *RP*). The reason why 5.3 percent of standard premium was returned when the aggregate average loss produces 4.3 percent additional premium lies in the way in which losses for risks 30-50 are treated in the formula. While these losses are fully reflected in the aggregate average loss, only the first \$497.35 of loss is reflected in the retrospective premium. This is, precisely, the capping effect which leads to the requirement of an insurance charge. To see how this capping limits the responsiveness of the plan, we apply the equations derived in Section 1.

To apply the equations derived in Section 1, we must know $X(r_{max})$, $Y(r_{min})$, $X(r_{max}/g)$, and $Y(r_{min}/g)$. The maximum premium, \$968.76, corresponds to a loss of \$497.35 which implies that $r_{max} = 1.01(=497.35/491.96)$. The minimum premium, \$738.10 corresponds to a loss of \$297.60 which implies that $r_{min} = 0.60(=297.60/491.96)$. Exhibit A-1 displays $X(1.01)$ and $Y(0.60)$ for the 50 risks, where the discrete forms of Equations 1.6 and 1.7 have been used:

$$X(1.01) = \sum_{1}^{50} \text{Max}(0, L_i - 497.35)/491.96 = .1944, \qquad \text{(A.3)}$$

and

$$Y(0.60) = \sum_{1}^{50} \text{Max}(0, 297.60 - L_i)/491.96 = .0378. \qquad \text{(A.4)}$$

If the same retrospective rating formula were to be applied to a different set of risks that have an expected loss of \$639.55 (130 percent of the original group's expected loss), which are similarly distributed about the mean, *f(s)* will be unchanged. (Remember that *s* measures each loss against the mean, so shifting the mean leaves *s* unchanged.) The original distribution can, therefore, be used to determine

$$X(r_{max}/g) = X(1.01/1.30) = X(0.78) = .3113, \qquad (A.5)$$

and

$$Y(r_{min}/g) = Y(0.60/1.30) = Y(0.46) = .0116, \qquad (A.6)$$

as shown in the last two columns of Exhibit A-1.

Had the new $f(s)$ been different, Equations A.5 and A.6 would have been calculated using the new distributions.

The second set of risks, which are displayed in Exhibit A-2, has an incurred loss ratio equal to 69.3 percent of standard premium (i.e., 30 percent higher than that of the original group of policies). Using Equation 1.24 we can predict the corresponding aggregate retrospectively determined return premium, which is observed (see bottom of Exhibit A-2) to be 1.3 percent of standard premium.

From Equation 1.24,

$$\begin{aligned} RP_0 + \Delta RP = 5.3\% & \\ & - (1.12)(69.3 - 53.3)(1.031) \\ & + (1.12)(53.3)(1.3 * .3113 - .1944)(1.031) \\ & - (1.12)(53.3)(1.3 * .0116 - .0378)(1.031) \\ & = 5.3\% - 18.5\% + 13.0\% - (-1.4\%) \\ & = 1.2\% \end{aligned}$$

is approximately equal to the aggregate retrospective returned premium. The error (1.2 percent vs. 1.3 percent) is due to rounding errors introduced by the discrete nature of the distribution.

The rather weak response of this retrospective rating formula (a 4 point decrease in the return premium corresponding to a 16 point increase in the aggregate incurred loss ratio) was due to the effects of capping. Not only were the increased losses in the previously capped 20 risks not reflected, but a portion of the increased loss from risks 19-29 has been capped away.

We must emphasize that the distribution of risks used in this example was selected to accentuate the effect of capping and to illustrate the method, not to produce a realistic model of the $f(s)$ for a set of retrospectively rated risks.

EXHIBIT A-1
THE EXPECTED DISTRIBUTION

Risk #	Loss $	S	$(e+cI)S$	c^*L	R	RP	$X(1.01)$	$Y(0.60)$	$X(0.78)$	$Y(0.46)$
1	0.00	922.63	382.60	0.00	738.10	184.53	0.00	297.90	0.00	229.15
2	200.00	922.63	382.60	224.00	738.10	184.53	0.00	97.90	0.00	29.15
3	210.00	922.63	382.60	235.20	738.10	184.53	0.00	87.90	0.00	19.15
4	221.00	922.63	382.60	247.52	738.10	184.53	0.00	76.90	0.00	8.15
5	232.00	922.63	382.60	259.84	738.10	184.53	0.00	65.90	0.00	0.00
6	244.00	922.63	382.60	273.28	738.10	184.53	0.00	53.90	0.00	0.00
7	244.00	922.63	382.60	273.28	738.10	184.53	0.00	53.90	0.00	0.00
8	244.00	922.63	382.60	273.28	738.10	184.53	0.00	53.90	0.00	0.00
9	244.00	922.63	382.60	273.28	738.10	184.53	0.00	53.90	0.00	0.00
10	256.00	922.63	382.60	286.72	738.10	184.53	0.00	41.90	0.00	0.00
11	269.00	922.63	382.60	301.28	738.10	184.53	0.00	28.90	0.00	0.00
12	282.00	922.63	382.60	315.84	738.10	184.53	0.00	15.90	0.00	0.00
13	296.00	922.63	382.60	331.52	738.10	184.53	0.00	1.90	0.00	0.00
14	311.00	922.63	382.60	348.32	753.58	169.05	0.00	0.00	0.00	0.00
15	327.00	922.63	382.60	366.24	772.05	150.58	0.00	0.00	0.00	0.00
16	343.00	922.63	382.60	384.16	790.53	132.10	0.00	0.00	0.00	0.00
17	360.00	922.63	382.60	403.20	810.16	112.47	0.00	0.00	0.00	0.00
18	378.00	922.63	382.60	423.36	830.94	91.69	0.00	0.00	0.00	0.00
19	397.00	922.63	382.60	444.64	852.88	69.75	0.00	0.00	13.27	0.00
20	417.00	922.63	382.60	467.04	875.98	46.65	0.00	0.00	33.27	0.00
21	418.00	922.63	382.60	468.16	877.13	45.50	0.00	0.00	34.27	0.00
22	418.00	922.63	382.60	468.16	877.13	45.50	0.00	0.00	34.27	0.00
23	418.00	922.63	382.60	468.16	877.13	45.50	0.00	0.00	34.27	0.00
24	418.00	922.63	382.60	468.16	877.13	45.50	0.00	0.00	34.27	0.00
25	418.00	922.63	382.60	468.16	877.13	45.50	0.00	0.00	34.27	0.00
26	418.00	922.63	382.60	468.16	877.13	45.50	0.00	0.00	34.27	0.00
27	439.00	922.63	382.60	491.68	901.38	21.25	0.00	0.00	55.27	0.00
28	461.00	922.63	382.60	516.32	926.79	-4.16	0.00	0.00	77.27	0.00
29	484.00	922.63	382.60	542.08	953.35	-30.72	0.00	0.00	100.27	0.00
30	508.00	922.63	382.60	568.96	968.76	-46.13	10.38	0.00	124.27	0.00
31	533.00	922.63	382.60	596.96	968.76	-46.13	35.38	0.00	149.27	0.00
32	560.00	922.63	382.60	627.20	968.76	-46.13	62.38	0.00	176.27	0.00
33	588.00	922.63	382.60	658.56	968.76	-46.13	90.38	0.00	204.27	0.00
34	617.00	922.63	382.60	691.04	968.76	-46.13	119.38	0.00	233.27	0.00
35	648.00	922.63	382.60	725.76	968.76	-46.13	150.38	0.00	264.27	0.00
36	661.00	922.63	382.60	740.32	968.76	-46.13	163.38	0.00	277.27	0.00
37	674.00	922.63	382.60	754.88	968.76	-46.13	176.38	0.00	290.27	0.00
38	687.00	922.63	382.60	769.44	968.76	-46.13	189.38	0.00	303.27	0.00
39	701.00	922.63	382.60	785.12	968.76	-46.13	203.38	0.00	317.27	0.00
40	715.00	922.63	382.60	800.80	968.76	-46.13	217.38	0.00	331.27	0.00
41	729.00	922.63	382.60	816.48	968.76	-46.13	231.38	0.00	345.27	0.00
42	744.00	922.63	382.60	833.28	968.76	-46.13	246.38	0.00	360.27	0.00
43	800.00	922.63	382.60	896.00	968.76	-46.13	302.38	0.00	416.27	0.00
44	816.00	922.63	382.60	913.92	968.76	-46.13	318.38	0.00	432.27	0.00
45	832.00	922.63	382.60	931.84	968.76	-46.13	334.38	0.00	448.27	0.00
46	849.00	922.63	382.60	950.88	968.76	-46.13	351.38	0.00	465.27	0.00
47	866.00	922.63	382.60	969.92	968.76	-46.13	368.38	0.00	482.27	0.00
48	883.00	922.63	382.60	988.96	968.76	-46.13	385.38	0.00	499.27	0.00
49	901.00	922.63	382.60	1,009.12	968.76	-46.13	403.38	0.00	517.27	0.00
50	919.00	922.63	382.60	1,029.28	968.76	-46.13	421.38	0.00	535.27	0.00
Total	24,598.00	46,131.50	19,130.00	27,549.76	43,669.71	2,461.79	4,781.03	930.70	7,657.68	285.62
Average	491.96	922.63	382.60	551.00	873.39	49.24	95.62	18.61	153.15	5.71
%Std Prem	53.3%	100.0%	41.5%	59.7%	94.7%	5.3%				
%Avg Loss							19.44%	3.78%	31.13%	1.16%

EXHIBIT A-2

THE CORRESPONDING DISTRIBUTION WITH $g = 1.3$

Risk #	Loss $	S	$(e + cI)S$	$c*L$	R	RP
1	0.00	922.63	382.60	0.00	738.10	184.53
2	260.00	922.63	382.60	291.20	738.10	184.53
3	273.00	922.63	382.60	305.76	738.10	184.53
4	287.30	922.63	382.60	321.78	738.10	184.53
5	301.60	922.63	382.60	337.79	742.72	179.91
6	317.20	922.63	382.60	355.26	760.73	161.90
7	317.20	922.63	382.60	355.26	760.73	161.90
8	317.20	922.63	382.60	355.26	760.73	161.90
9	317.20	922.63	382.60	355.26	760.73	161.90
10	332.80	922.63	382.60	372.74	778.76	143.87
11	349.70	922.63	382.60	391.66	798.26	124.37
12	366.60	922.63	382.60	410.59	817.78	104.85
13	384.80	922.63	382.60	430.98	838.80	83.83
14	404.30	922.63	382.60	452.82	861.32	61.31
15	425.10	922.63	382.60	476.11	885.33	37.30
16	445.90	922.63	382.60	499.41	909.35	13.28
17	468.00	922.63	382.60	524.16	934.87	-12.24
18	491.40	922.63	382.60	550.37	961.89	-39.26
19	516.10	922.63	382.60	578.03	968.76	-46.13
20	542.10	922.63	382.60	607.15	968.76	-46.13
21	543.40	922.63	382.60	608.61	968.76	-46.13
22	543.40	922.63	382.60	608.61	968.76	-46.13
23	543.40	922.63	382.60	608.61	968.76	-46.13
24	543.40	922.63	382.60	608.61	968.76	-46.13
25	543.40	922.63	382.60	608.61	968.76	-46.13
26	543.40	922.63	382.60	608.61	968.76	-46.13
27	570.70	922.63	382.60	639.18	968.76	-46.13
28	599.30	922.63	382.60	671.22	968.76	-46.13
29	629.20	922.63	382.60	704.70	968.76	-46.13
30	660.40	922.63	382.60	739.65	968.76	-46.13
31	692.90	922.63	382.60	776.05	968.76	-46.13
32	728.00	922.63	382.60	815.36	968.76	-46.13
33	764.40	922.63	382.60	856.13	968.76	-46.13
34	802.10	922.63	382.60	898.35	968.76	-46.13
35	842.40	922.63	382.60	943.49	968.76	-46.13
36	859.30	922.63	382.60	962.42	968.76	-46.13
37	876.20	922.63	382.60	981.34	968.76	-46.13
38	893.10	922.63	382.60	1,000.27	968.76	-46.13
39	911.30	922.63	382.60	1,020.66	968.76	-46.13
40	929.50	922.63	382.60	1,041.04	968.76	-46.13
41	947.70	922.63	382.60	1,061.42	968.76	-46.13
42	967.20	922.63	382.60	1,083.26	968.76	-46.13
43	1,040.00	922.63	382.60	1,164.80	968.76	-46.13
44	1,060.80	922.63	382.60	1,188.10	968.76	-46.13
45	1,081.60	922.63	382.60	1,211.39	968.76	-46.13
46	1,103.70	922.63	382.60	1,236.14	968.76	-46.13
47	1,125.80	922.63	382.60	1,260.90	968.76	-46.13
48	1,147.90	922.63	382.60	1,285.65	968.76	-46.13
49	1,171.30	922.63	382.60	1,311.86	968.76	-46.13
50	1,194.70	922.63	382.60	1,338.06	968.76	-46.13
Total	31,977.40	46,131.50	19,130.00	35,814.69	45,524.74	606.76
Average	639.55	922.63	382.60	716.29	910.49	12.14
% Std Prem	69.3%	100.0%	41.5%	77.6%	98.7%	1.3%

APPENDIX B

Once a particular retrospective rating plan (formula and limits) has been specified, an aggregate incurred loss ratio and corresponding aggregate retrospective premium could be determined for a set of risks with a known incurred loss ratio distribution by means of simulation. Using the cumulative density function of the distribution and a random number generator, individual risk *ILR*s would be selected and then subjected to the retrospective rating formula. After a sufficiently large number of repetitions, an aggregate (*<ILR>*, *RP*) pair could be generated. In Section 1, we showed that the same result could be found if two functions of the *ILR* distribution, the charge and savings, are known. While not a simulation in the usual sense, we shall refer to points that are generated by means of Equation 1.24 as the results of a simulation.

For all of the simulations, we assumed that the particular set of risks can be described by one of the *ILR* distributions that underlie the NCCI's Table M; and that their standard premiums are such that 26 percent of the standard premium is used to meet expenses (excluding premium taxes, but including a provision for profit); 60 percent of the standard premium is needed for the expected aggregate losses; and that premium taxes give rise to a tax multiplier that is equal to 1.046. These assumptions imply that 10 percent of the standard premium is available for an aggregate retrospective premium return. Given these assumptions, the point (.60, .10) is common to all of our simulations, regardless of the individual retrospective rating plan *LCF*, *G* or *H*. For each simulation, Table M was used to establish an insurance charge that contemplated (.60, .10) as its target.

A total of 17 simulation points were generated for each retrospective rating plan. The points had *ILR*s that began with a low of 51.84 percent and ran to a high of 113.28 percent with each successive *ILR* being 5 percent higher than the previous one. With (.60, .10) being a "given," Equation 1.24 was used to generate the *RP* component of each other aggregate (*ILR*, *RP*) pair.

Exhibit B-1 displays the results of a simulation for the following retrospective rating formula:

$$R = [.260 + 1.000*.265 + 1.000*L]*1.046, \tag{B.1}$$

subject to

$$.70*S \leq R \leq 1.40*S, \tag{B.2}$$

with all risks assumed to have the *ILR* distribution that underlies Table M loss group 60. The net insurance charge, 0.265, is the one that results from imposing the requirement that the retrospective rating plan be in balance with the corresponding guaranteed cost rates. In terms of r, the minimum loss ratio reflected in the retrospective rating is .24 times the expected loss ratio, and the maximum loss ratio is 1.360 times the expected *ILR*.

To this set of simulation points we fit three curves,

1. Linear Model: $RP = A + B*ILR,$ (B.3a)

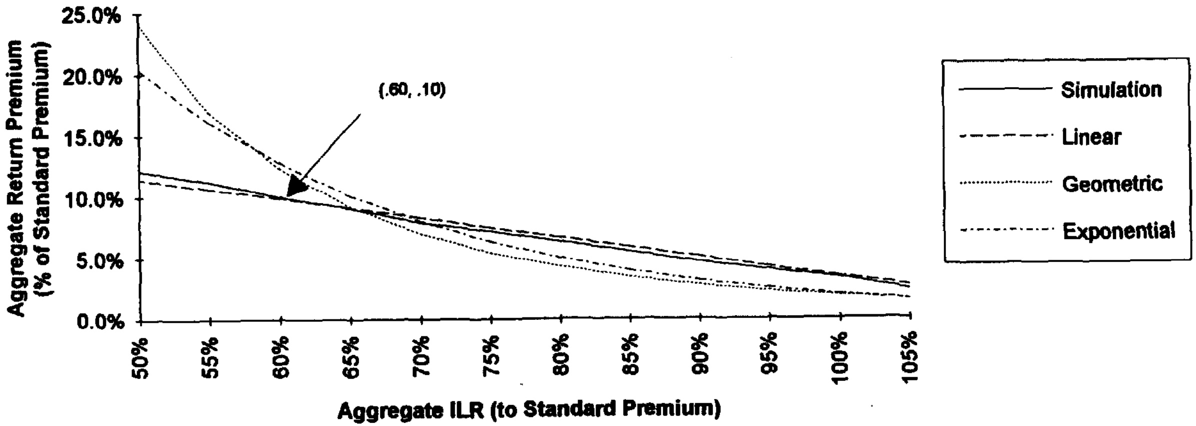
2. Geometric Model: $RP = A*(ILR^B)$, and (B.3b)

3. Exponential Model: $RP = A*e^{B*ILR}$. (B.3c)

For each of these, we determined the t statistics, for A and B, and the mean squared error (MSE) of the model using the 17 points. Because the MSEs for each model were so small, we have multiplied them by 1,000,000 (for example, the Geometric Model MSE of 0.000111 is, therefore, displayed as 111). Figure B-1 displays a graph of the simulation points and the three model curves.

FIGURE B-1

AGGREGATE *ILR* VS. AGGREGATE RETURN PREMIUM



As with the example that is displayed in Exhibit B-1 all of the simulations were described by models which had coefficients that were significant at the 99.95 percent confidence level. Under these circumstances, one would usually select the model that produced a curve with the least MSE.

With the exception of the simulations displayed in Table 5, all of the retrospective rating plans were based upon an expense and profit provision ratio equal to 26.0 percent of standard premium. There was nothing special about its selection. It lies about midway between the NCCI retrospective rating stock and non-stock company expense provisions for risks with standard premiums near $53,000. (See NCCI Tables XIV-A and XIV-B.) The 60 percent *ILR* was selected because it is typical of the NCCI expected loss ratios that are to be used for retrospective rating.

Table 5 involved risks of various sizes. As a result, we felt that the NCCI expense graduations should be reflected. For each premium size, we established the $(e + c*I)$ term using the appropriate expense provision, e, from the NCCI stock company expense table and the insurance charge corresponding to the appropriate Table M grouping (using the July 1, 1991 NCCI expected loss ranges).

While one could argue that the expected loss ratios for a group of risks that use identical manual rates should reflect a size of risk dependency, we adopted a common expected loss ratio for all of the risks. This is consistent with the way in which the NCCI retrospective rating plan is applied.

Intuitively, one would expect to select a curve with a negative first derivative (i.e., an increase in the aggregate *ILR* results in a decrease in the aggregate *RP*), but with a positive second derivative (i.e., as more and more individual risks "max" out, additional increases in the aggregate *ILR* have less of a decreasing effect on the aggregate *RP*). As long as B is negative, all three models possess negative first derivatives. Only the linear model fails to exhibit the intuitively required positive second derivative. The linear model has performed better than the other two (using the minimum MSE criterion) more often than not for the various plans considered here. Because the range of *ILR*s is expected to encompass almost any realistic situation, the danger associated with extrapolating too steep a curve (one that doesn't "pull up" for high values of the *ILR*) was considered minimal. The lack of an intuitively correct second derivative for the linear model was not considered to be a serious defect, and is largely outweighed by the simple interpretation of its slope (the *B* coefficient) as the fraction of the *ILR* increase that impacts the returned premium.

Depending on the particular application, one might wish to use the actual simulation points, the best fitting curve, or the better fitting curve which satisfies the intuitive requirements.

EXHIBIT B-1

ILR	Simulation	Linear	Geometric	Exponential	
51.84 %	11.79%	11.34 %	14.68%	13.46%	
54.42	11.44	10.91	13.12	12.44	
57.12	10.67	10.46	11.72	11.46	
60.00	10.00	9.98	10.46	10.51	<==Target
63.00	9.30	9.48	9.34	9.59	
66.18	8.74	8.95	8.33	8.71	
69.48	8.16	8.40	7.44	7.88	
72.96	7.46	7.81	6.65	7.09	
76.62	6.93	7.20	5.93	6.35	
80.46	6.28	6.56	5.30	5.65	
84.48	5.58	5.89	4.73	5.00	
88.68	5.10	5.19	4.23	4.40	
93.12	4.37	4.45	3.77	3.85	
97.80	3.65	3.67	3.37	3.34	
102.72	3.07	2.85	3.01	2.87	
107.88	2.20	1.99	2.68	2.46	
113.28	1.48	1.09	2.40	2.09	

			Value	*t*-Statistic
Linear Model	$A + B*ILR$	$A =$	0.1998386	64.726797
		$B =$	-0.1667903	-43.773508
		$MSE =$	8	
Geometric Model	$A*(ILR**B)$	$A =$	0.0319978	-51.081205
		$B =$	-2.3187566	-12.325123
		$MSE =$	111	
Exponential Model	$A*Exp(B*ILR)$	$A =$	0.6484224	-3.2121784
		$B =$	-3.0334045	-18.224868
		$MSE =$	43	

DISCUSSION BY HOWARD C. MAHLER

Robert Bender develops an equation for the relationship between the aggregate incurred loss ratio and the aggregate retrospective return premium. This discussion will illustrate this relationship using data for groups of actual insureds.

The data examined in each case are workers' compensation Unit Statistical Plan data at third report from one state.[1] It should be noted that, commonly, retrospective rating involves exposure in more than one state and/or exposure to lines of insurance in addition to workers' compensation. Also, retrospective rating plans commonly remain open beyond third report.[2] Thus the results shown here merely illustrate types of expected behavior, rather than the particular behavior that will occur in individual situations.

In each case, a particular retrospective rating plan was examined for a group of risks.[3] The retrospective premium was calculated for each risk based on its losses. The losses used were the reported losses multiplied by a factor chosen to adjust the overall loss ratio to a given value.[4] For example, if the reported loss ratio for the group of risks was 50 percent and the desired overall loss ratio was 60 percent, then each risk's losses were multiplied by $^{60}\!/_{50} = 1.2$. Thus we retain the shape of the reported loss ratio distribution, but adjust the mean by

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[1] The data available was for 1988 at third report for all Massachusetts workers' compensation risks. Subsets of the data are examined by size of risk and by voluntary versus assigned risks.

[2] For workers' compensation incurred losses (paid losses plus case reserves), there is generally upward development on average beyond third report. Not only would this raise the mean incurred loss ratio, it would also change the distribution of loss ratios around the mean. The coefficient of variation usually increases; i.e., the distribution gets more dispersed.

[3] No attempt was made to select retrospective rating plans that are in balance. (Each plan will be in balance for some incurred loss ratio, but that is not the focus of Bender's paper.)

[4] This could have been accomplished by multiplying the reported standard premium rather than the reported losses by a factor.

adjusting the overall scale. This adjustment to the loss ratios was made in order to yield average loss ratios ranging from 30 percent to 120 percent, to see how the total retrospective premium responds to changing loss ratios.

Exhibit 1 shows the results for a retrospective rating plan applied to all assigned risks with $150,000 or more in standard premium. The particular plan parameters are those used in the Massachusetts Assigned Risk Rating Program (MARRP). It should be noted that this plan was specifically designed *not* to be in balance, but rather to generate extra revenue for the assigned risk pool (in contrast to guaranteed cost policies).[5] However, this issue is beyond the scope of this review.[6]

As can be seen in Exhibit 1, the responsiveness of the MARRP depends on the incurred loss ratio. Bender defines the "slope" as the change in retrospective return premium per change in incurred loss ratio.[7] The larger the magnitude of the slope, the more responsive the plan.[8] The slopes for MARRP range from about $-\frac{1}{3}$ to $-\frac{3}{4}$ depending on the loss ratio.

For a given loss ratio distribution, the slope depends chiefly upon the "swing limits" of the plan. For those risks between the maximum and minimum premiums, the slope is minus the tax multiplier (*TM*) times the loss conversion factor (*LCF*). For risks either above the maximum or below the minimum premium, the slope is zero. The

[5] No consideration has been given here to potential collection problems that may result with a mandatory assigned risk retrospective rating program such as MARRP, which was in effect during 1993.

[6] This subject is discussed, for example, in William R. Gillam's discussion of David Skurnick's paper, "The California Table L," *PCAS* 1993.

[7] Since slope is defined in terms of return premiums, it is negative.

[8] It should be noted that not only are retrospective rating plans sensitive to individual loss experience. So is the standard premium which forms the starting point of retrospective rating. The standard premium includes the impact of the (prospective) experience rating plan. However, the standard premium is sensitive to prior years' losses, while the retrospective rating plan uses the loss experience on the policy to which it applies.

average slope for the whole set of risks examined is a weighted average of these two quantities. As the swing of the plan becomes wider due to higher maximums and lower minimums, more risks contribute $-TM \times LCF$ to the slope rather than zero. Thus, the wider the swing limits, the greater the magnitude of the slope.

The slope also increases in magnitude as either the tax multiplier or loss conversion factor is increased.

Exhibit 2 displays the results of a retrospective rating plan similar to that examined by Bender. The risks are voluntary risks with annual premium between $50,000 and $75,000. For these relatively small risks, the selected plan is relatively unresponsive. The slopes are very similar to those shown in Bender's analysis of a plan with (approximately) these parameters.

Part of the reason for the low responsiveness is the choice of plan parameters. However, part of the reason is that for these smaller risks (for retrospective rating), the loss ratio distribution is more dispersed.[9] Therefore, relatively few risks are between the maximum and minimum premiums.[10] If the loss ratio distribution were more compact, there would be generally more risks between the maximum and minimum premiums, resulting in a slope of larger magnitude.

This can be seen in Exhibit 3, where the same plan as in Exhibit 2 is examined, but for larger risks. This plan is more responsive for risks between $250,000 and $500,000 than for risks between $50,000 and $75,000.[11] This is due to the larger percentage of risks between the minimum and maximum premiums.

[9] In Bender's analysis, the dispersion of the loss ratio distribution is quantified via Table M.

[10] For a 60 percent loss ratio, only 15 percent of the risks are between the minimum and maximum premiums.

[11] This is not to imply that the same maximum, minimum, and basic premium would be appropriate for both sizes of risk. All retrospective rating plan parameters have been chosen solely for illustrative purposes.

For these same voluntary risks between \$250,000 and \$500,000, Exhibit 4 shows the results for a more responsive plan.

The retrospective premiums for the plans examined in Exhibits 1 through 4 are graphed in Figures 1 through 4, respectively. Similarly, Figures 5 through 8, respectively, graph the percent of policies at the minimum and the maximum premiums. One can see how the percentage of risks between the minimum and the maximum varies by loss ratio, as well as between the different examples. Generally, the larger this percentage, the more responsive the plan.

Conclusion

The general ideas in Bender's paper have been illustrated utilizing a particular set of actual data. The examples provided in Bender's paper were relatively unresponsive retrospective rating plans due to the size of the risks and particular plans he was considering.

The methodology in Bender's paper is particularly useful when there is a lack of sufficient data to allow the type of calculations performed in this discussion. The results of using the methodology should not be very sensitive to the precise details of how the table of insurance charges, Table M, has been constructed. Provided Table M is consistent with a reasonable overall estimate of the number of risks at the maximum and minimum premiums, the Bender methodology will provide good estimates of the responsiveness of the retrospective plans.

EXHIBIT 1

MARRP Plan Parameters

Incurred Loss Ratio (Third Report)	Percent of Risks at Maximum Premium	Percent of Risks at Minimum Premium	Ratio of Retrospective Premium to Standard Premium	Slope*
120%	46%	17%	138.7%	
110	42	18	135.2	-37%
100	40	19	131.3	-42
90	35	21	126.8	-49
80	30	24	121.6	-55
70	24	27	115.8	-62
60	18	32	109.2	-70
50	12	38	101.8	-76
40	9	43	94.1	-77
30	5	55	86.5	

Maximum Premium = 175%
Minimum Premium = 75%
Tax Multiplier = 1.15
Loss Conversion Factor = 1.1
Basic Premium Factor = 35%

Data: All assigned risks with $150,000 or more in Massachusetts workers' compensation standard premium (907 risks).

*Per Bender, the slope is the change in retrospective *return* premium per change in incurred loss ratio.

EXHIBIT 2

SMALL VOLUNTARY RISKS

Incurred Loss Ratio (Third Report)	Percent of Risks at Maximum Premium	Percent of Risks at Minimum Premium	Ratio of Retrospective Premium to Standard Premium	Slope*
120%	32%	57%	100.6%	
110	31	59	99.4	-12%
100	30	61	98.3	-12
90	28	62	97.0	-14
80	25	63	95.5	-16
70	23	65	93.8	-16
60	20	65	92.3	-16
50	18	68	90.6	-23
40	15	71	87.8	-33
30	11	77	84.0	

Maximum Premium = 135%
Minimum Premium = 75%
Tax Multiplier = 1.05
Loss Conversion Factor = 1.1
Basic Premium Factor = 30%

Data: All voluntary risks with between $50,000 and $75,000 in Massachusetts workers' compensation standard premium (519 risks).

*Per Bender, the slope is the change in retrospective *return* premium per change in incurred loss ratio.

EXHIBIT 3

LARGE VOLUNTARY RISKS

Incurred Loss Ratio (Third Report)	Percent of Risks at Maximum Premium	Percent of Risks at Minimum Premium	Ratio of Retrospective Premium to Standard Premium	Slope*
120%	51%	22%	113.4%	
110	49	25	111.6	-21%
100	46	25	109.3	-24
90	43	29	106.8	-27
80	37	32	103.9	-34
70	30	34	100.0	-41
60	24	40	95.8	-44
50	17	49	91.2	-49
40	10	56	86.0	-52
30	6	69	80.8	

Maximum Premium = 135%
Minimum Premium = 75%
Tax Multiplier = 1.05
Loss Conversion Factor = 1.1
Basic Premium Factor = 30%

Data: All voluntary risks with between $250,000 and $500,000 in Massachusetts workers' compensation standard premium (264 risks).

*Per Bender, the slope is the change in retrospective *return* premium per change in incurred loss ratio.

EXHIBIT 4

MORE RESPONSIVE PLAN

Incurred Loss Ratio (Third Report)	Percent of Risks at Maximum Premium	Percent of Risks at Minimum Premium	Ratio of Retrospective Premium to Standard Premium	Slope*
120%	46%	16%	114.2%	
110	43	16	111.3	-31%
100	39	17	108.0	-37
90	34	18	103.9	-45
80	29	18	99.0	-52
70	23	19	93.5	-59
60	17	25	87.2	-67
50	12	28	80.1	-76
40	8	33	72.0	-82
30	5	47	63.8	

Maximum Premium = 150%
Minimum Premium = 50%
Tax Multiplier = 1.05
Loss Conversion Factor = 1.1
Basic Premium Factor = 25%

Data: All voluntary risks with between $250,000 and $500,000 in Massachusetts workers' compensation standard premium (264 risks).

*Per Bender, the slope is the change in retrospective *return* premium per change in incurred loss ratio.

FIGURE 1

PLAN PER EXHIBIT 1, ASSIGNED RISKS: OVER $150,000
175 MAX AND 75 MIN

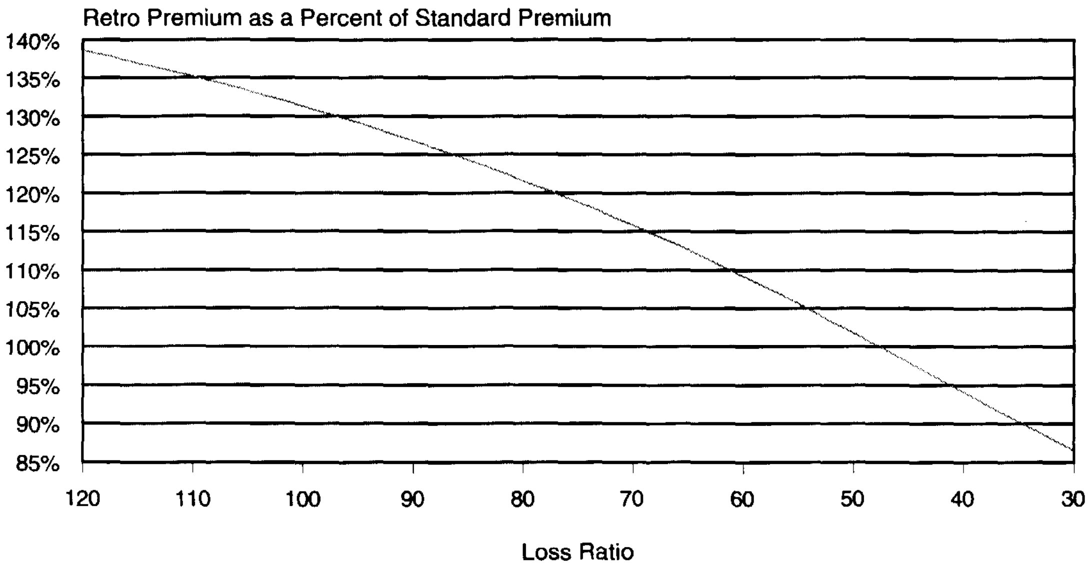


FIGURE 2

PLAN PER EXHIBIT 2, VOLUNTARY RISKS: $50,000 TO $75,000
135 MAX AND 75 MIN

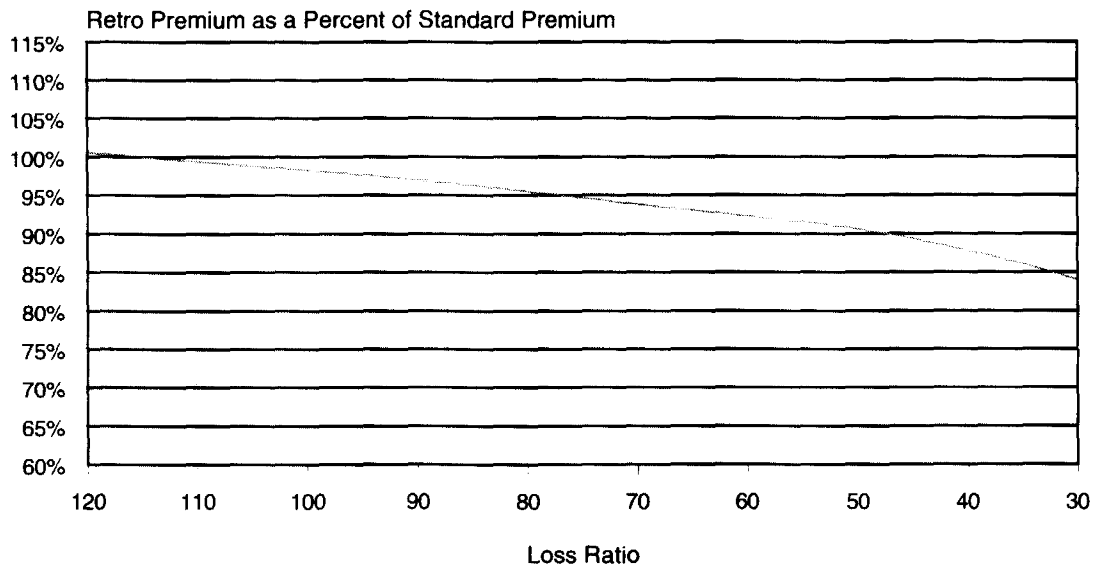


FIGURE 3

PLAN PER EXHIBIT 3, VOLUNTARY RISKS: $250,000 TO $500,000
135 MAX AND 75 MIN

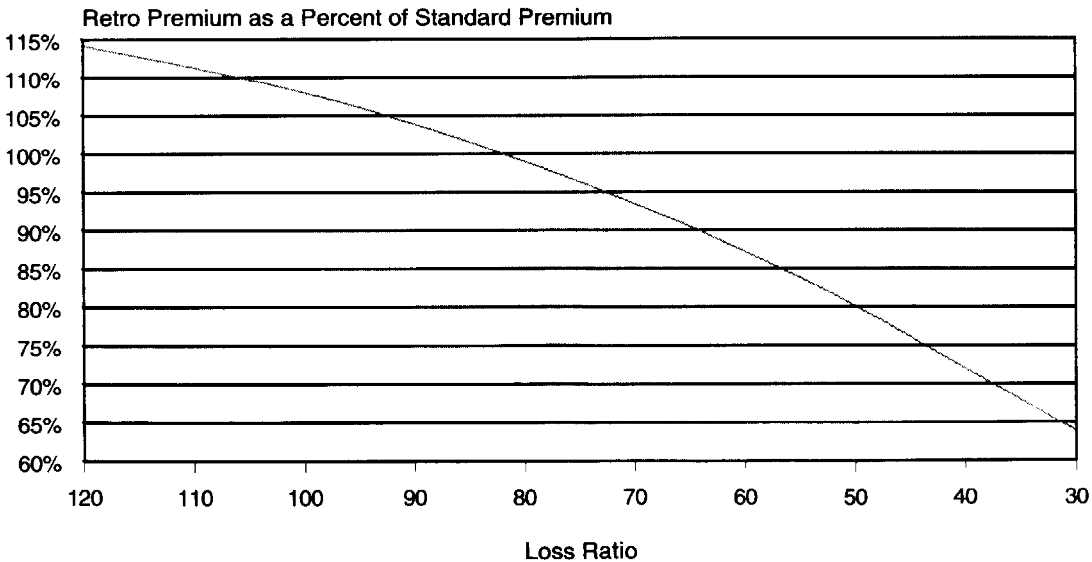


FIGURE 4

PLAN PER EXHIBIT 4, VOLUNTARY RISKS: \$250,000 TO \$500,000
150 MAX AND 50 MIN

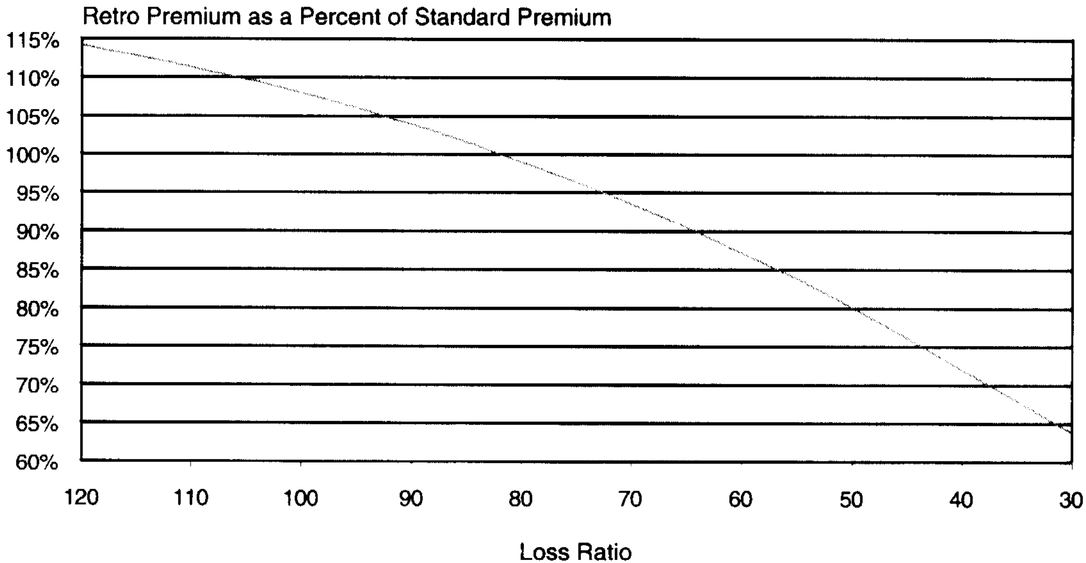


FIGURE 5

PLAN PER EXHIBIT 1, ASSIGNED RISKS: OVER $150,000
175 MAX AND 75 MIN

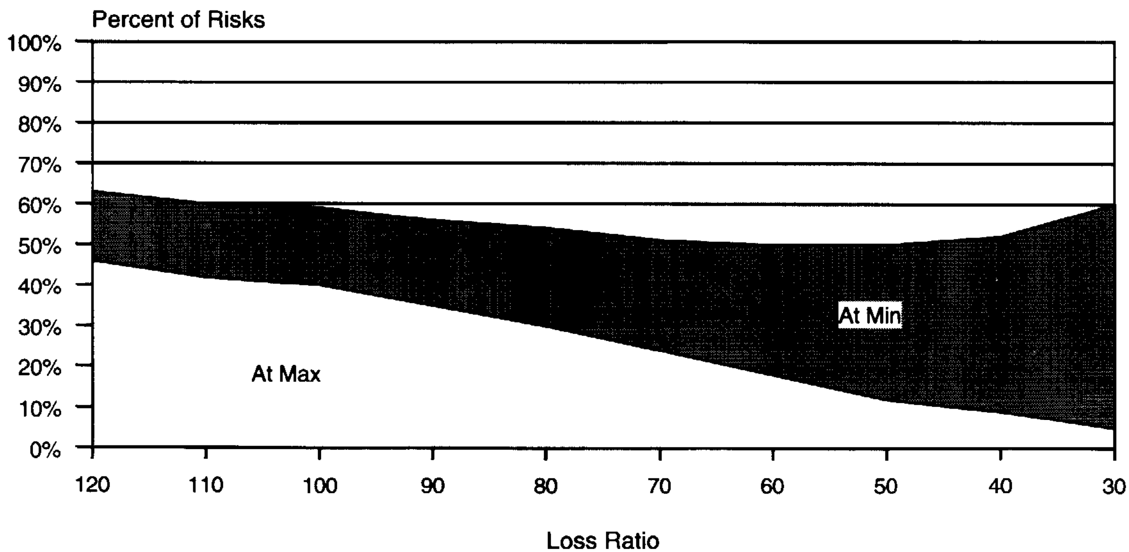


FIGURE 6

PLAN PER EXHIBIT 2, VOLUNTARY RISKS: $50,000 TO $75,000
135 MAX AND 75 MIN

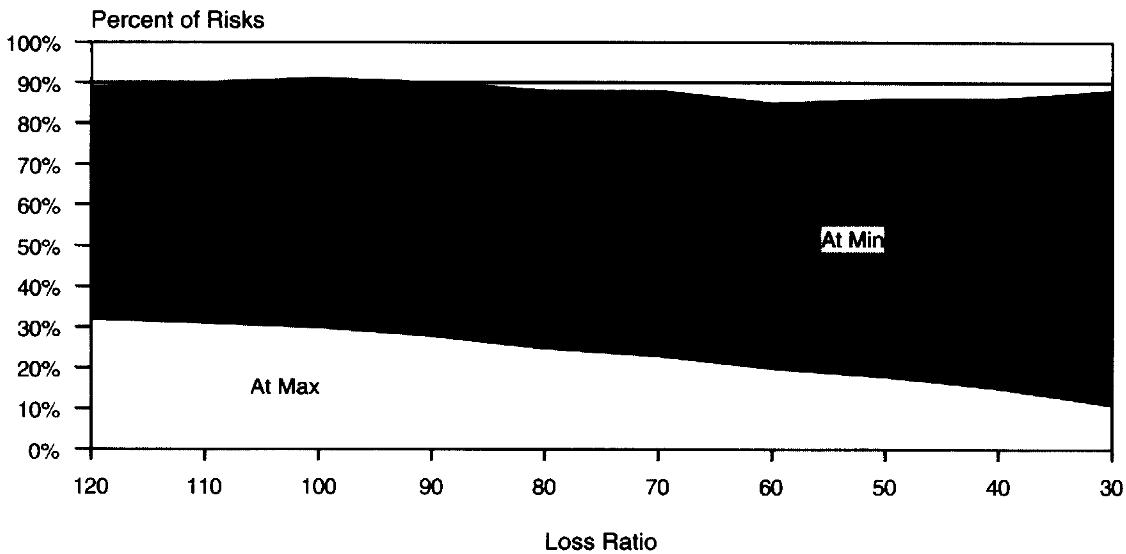


FIGURE 7

PLAN PER EXHIBIT 3, VOLUNTARY RISKS: $250,000 TO $500,000
135 MAX AND 75 MIN

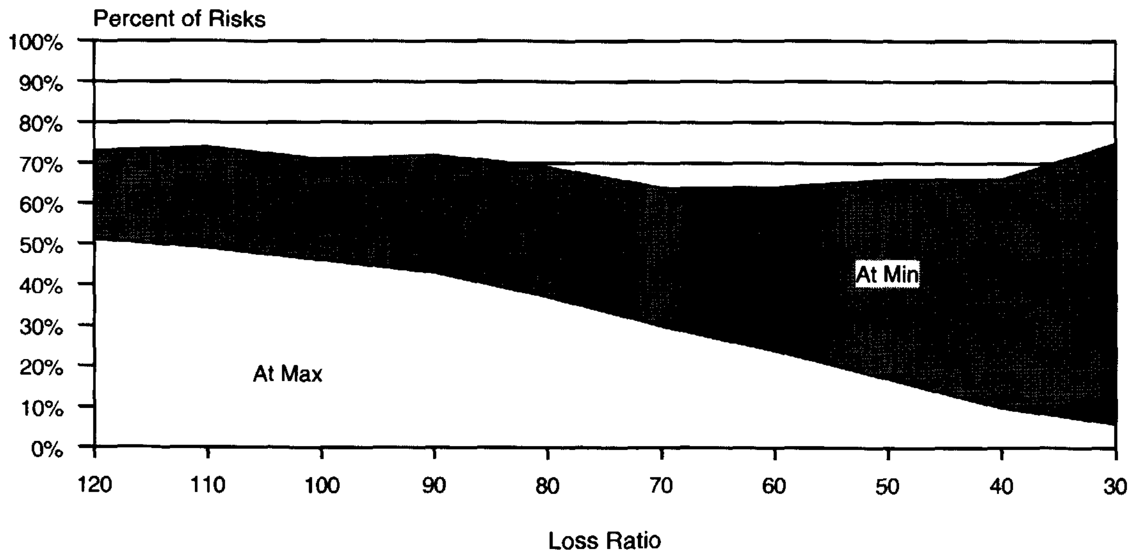
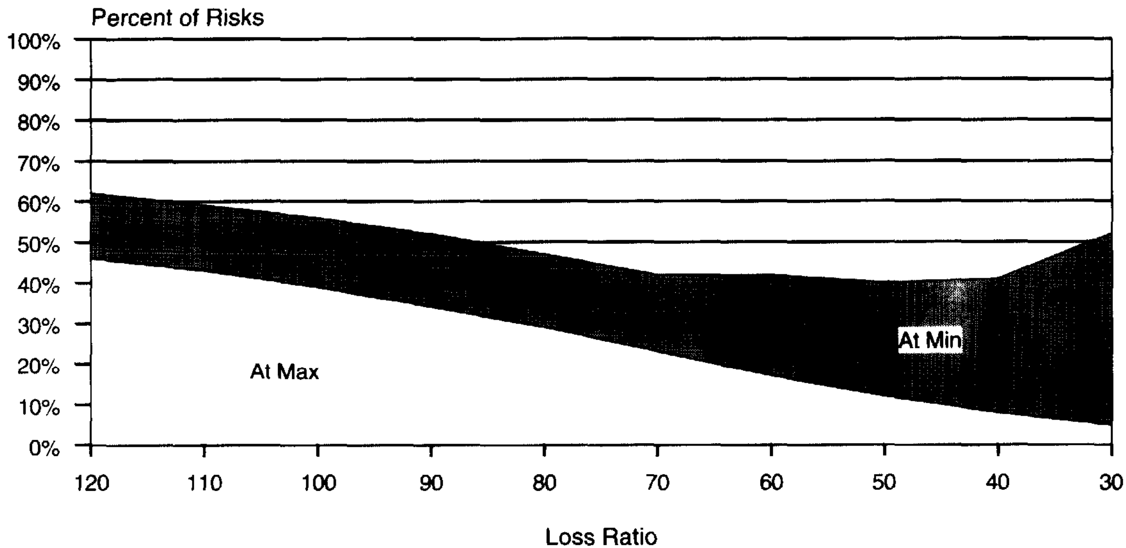


FIGURE 8

PLAN PER EXHIBIT 4, VOLUNTARY RISKS: $250,000 TO $500,000
150 MAX AND 50 MIN



QUANTIFYING THE UNCERTAINTY IN CLAIM SEVERITY ESTIMATES FOR AN EXCESS LAYER WHEN USING THE SINGLE PARAMETER PARETO

GLENN G. MEYERS, PH.D.

Abstract

This paper addresses the question: How valuable is a sample of excess claims in determining the expected claim severity in an excess layer of insurance?

An established procedure to estimate this expected claim severity is to first fit a model distribution to claim size data and then, using the fitted distribution, estimate the expected claim severity in the given excess layer. One of the more popular models used is the single parameter Pareto. This paper provides a means of quantifying the uncertainty in these excess claim severity estimates when using the single parameter Pareto. This approach requires one to incorporate prior opinions about the distribution of the Pareto parameter using Bayes' Theorem.

1. INTRODUCTION

Ever since Robert Miccolis's [2] classic paper on increased limits ratemaking was published, it has been an established procedure among members of the Casualty Actuarial Society to estimate the expected claim severity in an excess layer of insurance by first fitting a model distribution function to claim severity data and, using the fitted distribution, to estimate the expected claim severity in the given excess layer. One of the more popular models used is the single parameter Pareto. Its properties have been discussed on many occasions and the reader can consult the *Proceedings* for a very readable account by Stephen Philbrick [3].

An often stated concern in excess limits pricing is the uncertainty of the estimates of the excess claim severity. The purpose of this paper is to describe a Bayesian method of quantifying the uncertainty in excess claim severity estimates. This method is very easy to apply in the case of the single parameter Pareto.

2. A REVIEW OF THE SINGLE PARAMETER PARETO

The single parameter Pareto describes claims that are above a given truncation point, k. The cumulative distribution function is given by:

$$F(x) = 1 - \left(\frac{k}{x}\right)^q \quad \text{for } x \geq k \,. \tag{2.1}$$

The probability density function is given by:

$$f(x) = \frac{qk^q}{x^{q+1}} \quad \text{for } x \geq k \,. \tag{2.2}$$

Let $x_1, x_2, \ldots, x_n$ be a sample of n claims that are larger than k. The likelihood function, $L(q)$, is given by:

$$L(q) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n} \frac{qk^q}{x_i^{q+1}} \,.$$

Solving for the $\hat{q}$ that maximizes the likelihood function yields:

$$\hat{q} = \frac{n}{\sum_{i=1}^{n} \ln(x_i) - n\ln(k)} \,. \tag{2.3}$$

3. THE CONDITIONAL DISTRIBUTION OF $\hat{q}$

Let us temporarily assume that q is known. The purpose of this section is to describe the distribution of $\hat{q}$ in terms of q, with the final result being given in Equation 3.7 below.

We first note that:

$$\mathrm{E}[\ln(x)] = qk^q \int_k^{\infty} \frac{\ln(x)}{x^{q+1}}\, dx = \ln(k) + \frac{1}{q}\,. \tag{3.1}$$

From Equations 2.3 and 3.1, we get:

$$\mathrm{E}\left[\frac{1}{\hat{q}}\right] = \frac{1}{q}\,. \tag{3.2}$$

We next note that:

$$\mathrm{E}[\ln(x)^2] = qk^q \int_k^{\infty} \frac{\ln(x)^2}{x^{q+1}}\, dx = \ln(k)^2 + \frac{2\ln(k)}{q} + \frac{2}{q^2}\,. \tag{3.3}$$

From Equations 3.1 and 3.3, we see that:

$$\mathrm{Var}\,[\ln(x)] = \mathrm{E}[\ln(x)^2] - \mathrm{E}[\ln(x)]^2 = \frac{1}{q^2}\,. \tag{3.4}$$

Thus from Equations 2.3 and 3.4, we get:

$$\mathrm{Var}\left[\frac{1}{\hat{q}}\right] = \mathrm{Var}\left[\frac{\sum_{i=1}^{n} \ln(x_i) - n\ln(k)}{n}\right] = \frac{1}{nq^2}\,. \tag{3.5}$$

Now the central limit theorem states that the distribution of

$$\sum_{i=1}^{n} \ln(x_i)$$

will have an approximately normal distribution for sufficiently large n.

Thus, for known q, $1/\hat{q}$ has an approximately normal distribution with mean $1/q$ and variance $1/nq^2$. The conditional distribution of $1/\hat{q}$ given q is:

$$c(1/\hat{q}|q) = \sqrt{\frac{n}{2\pi}}\, q e^{-\left(\frac{1}{\hat{q}} - \frac{1}{q}\right)^2 nq^2/2}. \tag{3.6}$$

Now the distribution of $\hat{q}$ given q is:

$$c(\hat{q}|q) = c\,(1/\hat{q}|q)\left|\frac{d}{d\hat{q}}\left(\frac{1}{\hat{q}}\right)\right| = \sqrt{\frac{n}{2\pi}}\,\frac{q}{\hat{q}^2}\,e^{-\left(\frac{1}{\hat{q}} - \frac{1}{q}\right)^2 nq^2/2}$$

$$= \sqrt{\frac{n}{2\pi}}\,\frac{q}{\hat{q}^2}\,e^{-\frac{n\,(q-\hat{q})^2}{2\hat{q}^2}}. \tag{3.7}$$

4. BAYESIAN ESTIMATION

The treatment above considers q as the known quantity and $\hat{q}$ as the random variable. In practice, $\hat{q}$ is known and q is unknown. However, in many instances, we will have some prior knowledge or beliefs about the distribution of q. In other instances, we may have very little prior knowledge of the distribution of q. Our task is to use our knowledge of $\hat{q}$ to refine our knowledge about the distribution of q. To accomplish this, we use Bayes' Theorem.

We first consider the discrete case where q can take on values q_0, $q_1, q_2, \ldots, q_m$. Let the prior probabilities be given by $\Pr(q = q_i) = p_i$. By Bayes' Theorem, the posterior probability function of q_i is given by:

$$b(q_i|\hat{q}) = \frac{c(\hat{q}|q_i)p_i}{\sum_{j=1}^{m} c(\hat{q}|q_j)p_j}. \tag{4.1}$$

For the continuous case, the posterior probability density for q is given by:

$$b(q|\hat{q}) = \frac{c(\hat{q}|q)p(q)}{\int_0^\infty c(\hat{q}|q)p(q)dq}, \tag{4.2}$$

where $p(q)$ is the prior probability density of q.

Now it is a common practice for Bayesian statisticians to express the conditional, the prior, and the posterior distributions in simplest terms by ignoring all coefficients that do not depend upon q in the probability or density functions. The distributions, with the coefficients removed, are referred to as weight functions. In keeping with this practice, we replace c with v, b with w, and p with r and write:

$$c(\hat{q}|q) \propto v(\hat{q}|q) \equiv q e^{-\frac{n(q-\hat{q})^2}{2\hat{q}^2}} \tag{4.3}$$

in place of Equation 3.7,

$$b(q_i|\hat{q}) \propto w(q_i|\hat{q}) \equiv v(\hat{q}|q_i)r_i \tag{4.4}$$

in place of Equation 4.1, and

$$b(q|\hat{q}) \propto w(q|\hat{q}) \equiv v(\hat{q}|q)r(q) \tag{4.5}$$

in place of Equation 4.2.

It is often necessary to determine the constant of proportionality for Equations 4.4 and 4.5. This is usually done after the fact by finding the constant, T, which forces the total probability to be equal to 1. That is:

$$T = \frac{1}{\sum_{j=1}^{m} v(\hat{q}|q_j)r_j} \quad \text{or} \quad T = \frac{1}{\int_0^{\infty} v(\hat{q}|q)r(q)dq}. \tag{4.6}$$

An advantage of this practice is that it is no longer necessary to require that $r(q)$ be a proper distribution. The function $r(q)$ becomes a weighting function that can sum to anything, including infinity. The only requirement is that the sums in Equation 4.6 be finite. Prior distributions that sum to infinity are called improper, or diffuse, priors. They are useful when it is felt that there is little or no prior knowledge.

A rather interesting example can be constructed for the single parameter Pareto with the diffuse prior $p(q) \propto r(q) = 1/q$. We have

$$\begin{aligned} w(q|\hat{q}) &= v(\hat{q}|q)r(q) \\ &= qe^{\frac{-n\,(q-\hat{q})^2}{2\hat{q}^2}} \cdot \frac{1}{q} \\ &= e^{-\frac{n(q-\hat{q})^2}{2\hat{q}^2}}. \end{aligned} \tag{4.7}$$

By comparing Equation 4.7 with the standard normal distribution

$$\varphi(x) \propto e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

we see that the posterior distribution $b(q|\hat{q})$ is normal with mean $\hat{q}$ and variance $\hat{q}^2/n$.

This result should be compared to a standard "non-Bayesian" treatment. The distribution of $\hat{q}$, for a given q, is asymptotically normal with mean q and variance q^2/n. It has become common practice, using the methods demonstrated by Hogg and Klugman,[1] to express a confidence interval for q in terms of an approximately normal distribution with mean $\hat{q}$ and variance $\hat{q}^2/n$. This is admittedly an approximation. For the single parameter Pareto, the above Bayesian result provides a set of assumptions that make this approximation more meaningful.

5. THE DISTRIBUTION OF EXCESS CLAIM SEVERITY ESTIMATES

Obtaining the posterior distribution of q is only an intermediate step toward obtaining the posterior distribution of excess claim severity estimates. We now turn to the completion of this task.

In what follows, we will use a discrete prior distribution. This makes the procedure for getting the posterior distribution easy to set up on a spreadsheet program. The steps for constructing such a spreadsheet program are shown in Table 1 with the results given in various exhibits.

All examples in this paper assume that a maximum likelihood estimate of 1.75 has been obtained using data with a value of k equal to \$100,000. The task is to estimate the expected severity for a layer between \$1,000,000 and \$5,000,000.

It is important to note that the expected severity estimates in this paper will be conditional on the claim being greater than \$100,000. To use these results in practice, one must also consider the number of claims above \$100,000.

The prior means have little meaning if the prior distribution is improper and the means do not exist. The posterior mean and standard deviation of the q_is are given by:

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[1] See, for example, Section 4 of Chapter 3 in *Loss Distributions*, by Hogg and Klugman [1]. Examples 1 and 4 in this section are very pertinent to this discussion.

TABLE 1

SPREADSHEET DEFINITIONS

Column	Description
q_i	Values of q_i that have a specified start and end. These are divided into m equally spaced intervals. (We use $m = 30$ for the examples in this paper.)
r_i	Prior weights for q_i .
$v(\hat{q}\vert q_i)$	Conditional weights for $\hat{q}$ given q_i, as given by Equation 4.3.
$w(q_i\vert\hat{q})$	Posterior weights for q_i, which equal the product of the prior two columns as given by Equation 4.4.
$b(q_i\vert\hat{q})$	Posterior probabilities for q_i, which equal: $w(q_i\vert\hat{q})/\sum_{j=1}^{m} w\,(q_j\vert\hat{q})$.
$B(q_i\vert\hat{q})$	Cumulative posterior probabilities for q_i, which equal the sum of the posterior probabilities of q_j for $j \leq i$.
$E[X\vert q_i]$	Layer average severities given q_i, i.e., the expected severities for a given layer. For the single parameter Pareto, the layer average severities between retention R and limit L are given by the formula:

$$\int_R^L (1-F(x))dx = \begin{cases} \dfrac{k^{q_i}}{q-1}\left(\dfrac{1}{R^{q_i-1}} - \dfrac{1}{L^{q_i-1}}\right) & \text{for } q_i \neq 1 \\ k(\ln(L) - \ln(R)) & \text{for } q_i = 1 \end{cases} \tag{5.1}$$

$$\mathrm{E}[q_i|\hat{q}] = \sum_{i=1}^{m} q_i b(q_i|\hat{q}) \text{ and } \mathrm{Std}[q_i|\hat{q}] = \sqrt{\sum_{i=q}^{m} q_i^2\, b(q_i|\hat{q}) - \mathrm{E}[q_i|\hat{q}]^2}\ .$$

The posterior mean and standard deviation of the $\mathrm{E}[X|q_i]$s are given by:

$$\mathrm{E}[\mathrm{E}[X|\hat{q}]] = \sum_{i=1}^{m} \mathrm{E}[X|q_i] b(q_i|\hat{q}) \text{ and}$$

$$\mathrm{Std}\,[\mathrm{E}[X|\hat{q}]] = \sqrt{\sum_{i=1}^{m} \mathrm{E}[X|q_i]^2 b(q_i|\hat{q}) - \mathrm{E}[\mathrm{E}[X|\hat{q}]]^2}\ .$$

It is a rare event for a q_i or an $\mathrm{E}[X|q_i]$ to hit exactly on a t^{th} or a $(1-t)^{th}$ percentile, so we adopt the following convention for this paper.[2] The t^{th} percentile of q_i is the last q_i before the cumulative probability exceeds t. Similarly, the $(1-t)^{th}$ percentile of q_i is the last q_i before the cumulative probability exceeds $(1-t)$. We would proceed similarly for $\mathrm{E}[X|q_i]$ except that $\mathrm{E}[X|q_i]$ is a decreasing function of q_i. So in this case we replace t with $1-t$ for the t^{th} percentile and $(1-t)$ with t for the $(1-t)^{th}$ percentile. In the examples, we use $t = 2.5$ percent to calculate a 95 percent confidence interval.

For the sake of comparison, we also provide the "classical" estimates based on the estimate, $\hat{q}$, and the estimate of $\hat{q}$ put into Equation 5.1.

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[2] It is likely that various textbooks will define percentiles differently than is done here. A possible alternative would be to interpolate between the q_is. The motivation here is that this definition is easy to implement with the typical spreadsheet program and the final decision made as the result of the confidence interval is unlikely to be affected by the choice of confidence interval definition.

It is often helpful to describe the posterior distributions of q_i and $E[X|q_i]$ graphically. This is straightforward for the posterior distribution of q_i. One simply plots q_i on the horizontal axis and $b(q_i|\hat{q})$ on the vertical axis. An additional consideration for plotting $E[X|q_i]$ is that the values of $E[X|q_i]$ are not evenly spaced. If we want the graph to have approximately the same shape as the corresponding continuous posterior distribution, we must plot $E[X|q_i]$ on the horizontal axis and

$$\frac{B(q_{i+1}|\hat{q}) - B(q_{i-1}|\hat{q})}{E[X|q_{i-1}] - E[X|q_{i+1}]} \approx B'(E[X|q_i])$$

on the vertical axis. The plots corresponding to Exhibit 1 are on Figures 1 and 2.

FIGURE 1

POSTERIOR DISTRIBUTION OF q

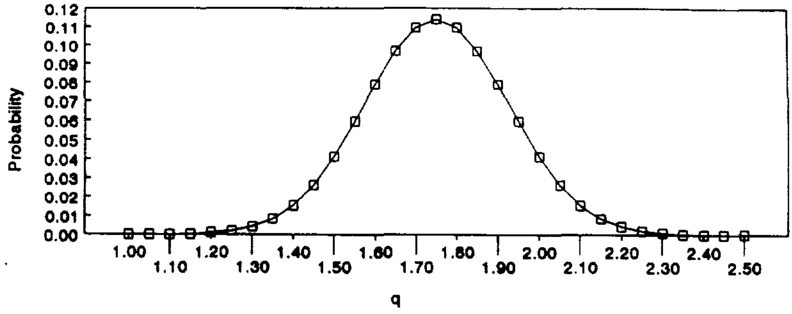
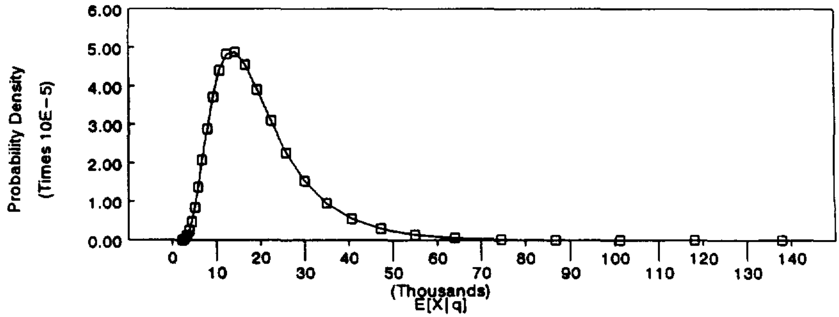


FIGURE 2

POSTERIOR DISTRIBUTION OF E[x|q]



6. THE EFFECT OF SAMPLE SIZE

What is noticeable about the example shown in Exhibit 1 is the large width of the confidence interval and the difference between the posterior mean and the classical estimate of the expected severity. Exhibits 2 and 3 show what happens when the sample size is increased to 1,000, and then 10,000 claims. Table 2 takes the key numbers from these exhibits.

TABLE 2

Source Exhibit	Sample Size	Classical Estimate	Posterior Mean	Posterior Std Dev	Approximate 95% Confidence Interval Low	High
1	100	16,619	19,062	10,640	6,922	54,964
2	1,000	16,619	16,847	2,769	12,755	23,711
3	10,000	16,619	16,642	859	15,213	18,703

Here we see that one must have a sample size of 10,000 to get the length of the confidence band down to 21 percent of the posterior mean claim severity.

Table 2 does point out a possible danger inherent in using the maximum likelihood estimate, $\hat{q}$, directly in Equation 5.1. If one truly believes that the prior distribution of q is proportional to $1/q$, then the classical estimate can produce a significant understatement of the posterior mean. This is especially true for small sample sizes.

7. ON THE CHOICE OF A PRIOR DISTRIBUTION

The bias for small sample sizes noted above may be a function of the prior distribution. In this section, we explore the implications of using different prior distributions.

In our first example, shown on Exhibit 1, we chose a prior distribution for q that was proportional to $1/q$. This had the effect of giving more weight to the smaller qs. Exhibit 4 shows the effect of choosing a prior distribution for q that is proportional to q. This has the effect of giving more weight to the larger qs. The summary of results for each exhibit is provided in Table 3.

TABLE 3

SUMMARY OF RESULTS

	Source Exhibit	Classical Estimate	Posterior Mean	Posterior Std Dev	Approximate 95% Confidence Interval Low	High
q_i	1	1.750	1.750	0.175	1.350	2.050
$E[X \mid q_i]$	1	16,619	19,062	10,640	6,922	54,964
q_i	4	1.750	1.785	0.173	1.400	2.050
$E[X \mid q_i]$	4	16,619	17,154	9,432	6,922	47,245

Perhaps the most notable observation is that the understatement of the posterior mean by the classical estimate of $E[X|q_i]$ is reduced with the second prior distribution. But we would caution against choosing a prior *for this reason.* The reason for choosing a prior distribution should be based on one's beliefs about the distribution of q.

Figures 3 and 4 provide graphical comparisons of the results of Exhibits 1 and 4.

A common sentiment of practitioners is: "I am extremely lucky to get 100 claims to analyze. Yet I can't go to my company and say: 'On the basis of (for example) Exhibit 1, my recommended value for the expected severity is $19,000, but it could reasonably be as low as $7,000 or as high as $55,000.' "

Many practitioners are, at least intuitively, aware of the large potential variability of the results and frequently override any outlying estimates citing "judgment." As the following example shows, it is possible to blend the maximum likelihood estimate and one's prior judgment by choosing an appropriate prior distribution.

FIGURE 3

POSTERIOR DISTRIBUTION OF q

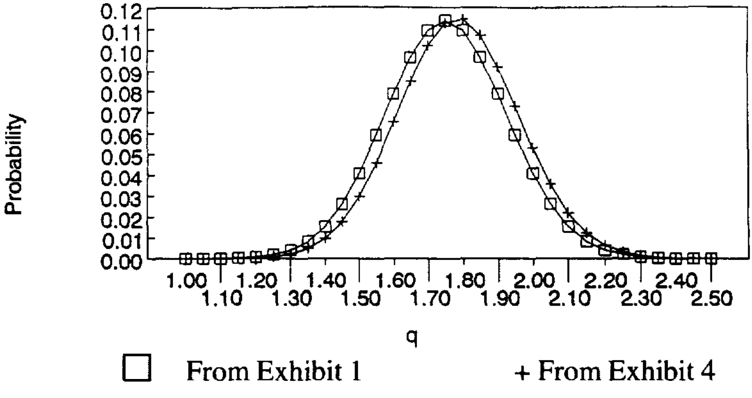
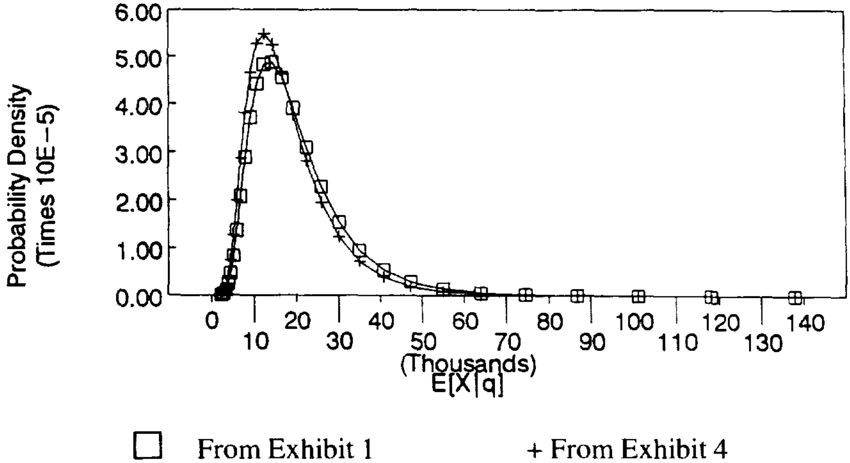


FIGURE 4

POSTERIOR DISTRIBUTION OF E[X | q]



Let us assume that one is willing to accept that the expected severity could be as low as \$17,500, or as high as \$25,000. Thus we allow q to be no lower than 1.612 or no higher than 1.732 (give or take some rounding error in q). Let us further assume that one feels that lower qs should be given more weight and selects a prior proportional to $1/q$. The resulting posterior distributions are in Exhibit 5.

Now, since we have a priori bounds on the range of the q_is, it makes sense to talk about prior means. The prior means of the q_is and the E[X|q_i]s are 1.671 and \$21,099, respectively. This should be compared with the posterior means of 1.675 and \$20,851, respectively. It should also be noted that the approximate 95 percent confidence intervals are pretty much the same as the a priori ranges of the q_is and the E[X|q_i]s. It would appear that, at least in this example, the information added by the 100 claims has a relatively minor impact on our estimated claim severity for the \$1,000,000 to \$5,000,000 layer.

This example also makes the point that it is possible to observe a $\hat{q}$ that is outside the prior range of q. However, the posterior mean of q, given $\hat{q}$, is within the prior range of q.

8. OTHER DISTRIBUTIONAL MODELS

In this section we indicate how one can proceed if a distributional model other than the single parameter Pareto is to be used. Let $\boldsymbol{\theta} = \{\theta_j\}$ be the parameter vector for the chosen model, $f(x|\boldsymbol{\theta})$, and let $\hat{\boldsymbol{\theta}} = \{\hat{\theta}_j\}$ be the maximum likelihood estimate of $\boldsymbol{\theta}$.

The procedure described in Section 5 will work with the following modifications.

1. The parameter q_i must be replaced with the vector $\boldsymbol{\theta}_i$.
2. Prior weights have to be assigned to each $\boldsymbol{\theta}_i$.
3. The conditional weights $v(\hat{\boldsymbol{\theta}}_i|\boldsymbol{\theta}_i)$ must be derived. Depending upon the distributional form selected, it may be possible to derive the weights directly as was done in Section 3. If this fails, there is an alternative approximation. As described by Hogg and Klugman,[3] the conditional distribution of $\hat{\boldsymbol{\theta}}$, given $\boldsymbol{\theta}$, is asymptotically a multivariate normal distribution with mean $\boldsymbol{\theta}$ and covariance matrix $\boldsymbol{\Sigma}^{-1}$, where $\boldsymbol{\Sigma} = \{a_{jk}\}$, and:

$$a_{jk} = -n\mathrm{E}\left[\frac{\partial^2 \ln(f(x|\boldsymbol{\theta}))}{\partial\theta_j \partial\theta_k}\right].$$

Then $$v(\hat{\boldsymbol{\theta}}|\boldsymbol{\theta}) = |\boldsymbol{\Sigma}| e^{-(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta})^T \boldsymbol{\Sigma} (\hat{\boldsymbol{\theta}}-\boldsymbol{\theta})/2}.$$

[3]Section 4 of Chapter 3 of *Loss Distributions* [1].

4. The formula for $E[X|\theta_i]$ will depend upon the distributional form of $f(x|\theta_i)$.

A bit of soul searching may be necessary to come up with a prior distribution for the parameter vector θ. One suggestion would be to place a prior distribution on $E[X|\theta]$ and translate the results into a prior distribution for θ. We came close to doing this in Exhibit 5 by choosing qs that restrict the expected severity between \$17,500 and \$25,000.

A complaint often heard is that one should be just as concerned about the model uncertainty as with parameter uncertainty. To address this complaint, one can put any number of models into this procedure, as long as prior probabilities for each model are assigned.

The problems associated with other severity models may indeed be formidable. We are fortunate to have a simple and realistic model like the single parameter Pareto to provide us with a blueprint.

9. A CONCLUDING REMARK

As noted in the Introduction, it is currently a common practice to use a fitted claim severity distribution to estimate the expected claim severity for an excess layer of insurance. These fits are often obtained with sample sizes containing fewer than 100 claims.

These estimates take a prominent role in insurance (and reinsurance) price negotiations. Insurance buyers will often readily accept estimates based on "their own data." One expects a buyer with a relatively low estimate to cite this as evidence that they deserve a break in their rates, while those buyers with relatively high estimates are in much weaker negotiating positions. While there may be significant differences between insurance buyers, the examples given above illustrate the dangers of drawing such a conclusion based solely on a fitted distribution. Good prior information should also play an important role in these negotiations.

While many practitioners recognize this, they are often under pressure to recognize "real data" supplied by the (re)insured. This paper provides a way to recognize the data and integrate it with prior information.

REFERENCES

[1] Hogg, Robert V., and Stuart A. Klugman, *Loss Distributions*, John Wiley & Sons, 1984.

[2] Miccolis, Robert S., "On the Theory of Increased Limits and Excess of Loss Pricing," *PCAS* LXIV, 1977, pp. 27-59.

[3] Philbrick, Stephen W., "A Practical Guide to the Single Parameter Pareto Distribution," *PCAS* LXXII, 1985, pp. 44-123.

EXHIBIT 1

Prior Distribution			Given			Observed	
q_0	q_{30}	r_i	k	Retention	Limit	$\hat{q}$	n
1.000	2.500	$1/q_i$	100,000	1,000,000	5,000,000	1.75	100

q_i	r_i	$v(\hat{q}\mid q_i)$	$w(q_i\mid \hat{q})$	$b(q_i\mid \hat{q})$	$B(q_i\mid \hat{q})$	$E[X\mid q_i]$
1.000	1.000	0.0001	0.0001	0.0000	0.0000	160,944
1.050	0.952	0.0004	0.0003	0.0000	0.0000	137,822
1.100	0.909	0.0011	0.0010	0.0001	0.0002	118,085
1.150	0.870	0.0032	0.0028	0.0003	0.0005	101,229
1.200	0.833	0.0086	0.0072	0.0008	0.0013	86,826
1.250	0.800	0.0211	0.0169	0.0019	0.0032	74,512
1.300	0.769	0.0477	0.0367	0.0042	0.0074	63,979
1.350	0.741	0.0990	0.0734	0.0084	0.0158	54,964
1.400	0.714	0.1895	0.1353	0.0154	0.0312	47,245
1.450	0.690	0.3336	0.2301	0.0262	0.0574	40,631
1.500	0.667	0.5407	0.3604	0.0411	0.0985	34,961
1.550	0.645	0.8067	0.5205	0.0593	0.1578	30,099
1.600	0.625	1.1081	0.6926	0.0789	0.2368	25,926
1.650	0.606	1.4015	0.8494	0.0968	0.3336	22,343
1.700	0.588	1.6320	0.9600	0.1094	0.4430	19,265
1.750	0.571	1.7500	1.0000	0.1140	0.5570	16,619
1.800	0.556	1.7280	0.9600	0.1094	0.6664	14,344
1.850	0.541	1.5713	0.8494	0.0968	0.7632	12,387
1.900	0.526	1.3159	0.6926	0.0789	0.8422	10,702
1.950	0.513	1.0149	0.5205	0.0593	0.9015	9,251
2.000	0.500	0.7209	0.3604	0.0411	0.9426	8,000
2.050	0.488	0.4716	0.2301	0.0262	0.9688	6,922
2.100	0.476	0.2842	0.1353	0.0154	0.9842	5,992
2.150	0.465	0.1577	0.0734	0.0084	0.9926	5,189
2.200	0.455	0.0806	0.0367	0.0042	0.9968	4,496
2.250	0.444	0.0380	0.0169	0.0019	0.9987	3,897
2.300	0.435	0.0165	0.0072	0.0008	0.9995	3,380
2.350	0.426	0.0066	0.0028	0.0003	0.9998	2,932
2.400	0.417	0.0024	0.0010	0.0001	1.0000	2,545
2.450	0.408	0.0008	0.0003	0.0000	1.0000	2,210
2.500	0.400	0.0003	0.0001	0.0000	1.0000	1,920

SUMMARY OF RESULTS

	Classical Estimate	Posterior Mean	Posterior Std Dev	Approximate 95% Confidence Interval Low	High
For q_i	1.750	1.750	0.175	1.350	2.050
For $E[X\mid q_i]$	16,619	19,062	10,640	6,922	54,964

EXHIBIT 2

Prior Distribution				Given		Observed	
q_0	q_{30}	r_i	k	Retention	Limit	$\hat{q}$	n
1.540	1.990	$1/q_i$	100,000	1,000,000	5,000,000	1.75	1,000

q_i	r_i	$v(\hat{q}\vert q_i)$	$w(q_i\vert\hat{q})$	$b(q_i\vert\hat{q})$	$B(q_i\vert\hat{q})$	$E[X\vert q_i]$
1.540	0.649	0.0011	0.0007	0.0001	0.0001	31,012
1.555	0.643	0.0031	0.0020	0.0002	0.0003	29,652
1.570	0.637	0.0079	0.0050	0.0005	0.0008	28,353
1.585	0.631	0.0186	0.0117	0.0013	0.0021	27,111
1.600	0.625	0.0406	0.0254	0.0027	0.0049	25,926
1.615	0.619	0.0824	0.0510	0.0055	0.0104	24,793
1.630	0.613	0.1553	0.0953	0.0103	0.0207	23,711
1.645	0.608	0.2719	0.1653	0.0179	0.0386	22,677
1.660	0.602	0.4424	0.2665	0.0288	0.0674	21,689
1.675	0.597	0.6686	0.3992	0.0432	0.1105	20,745
1.690	0.592	0.9389	0.5556	0.0601	0.1706	19,844
1.705	0.587	1.2250	0.7185	0.0777	0.2483	18,982
1.720	0.581	1.4850	0.8633	0.0934	0.3417	18,158
1.735	0.576	1.6724	0.9639	0.1042	0.4459	17,371
1.750	0.571	1.7500	1.0000	0.1081	0.5541	16,619
1.765	0.567	1.7013	0.9639	0.1042	0.6583	15,901
1.780	0.562	1.5368	0.8633	0.0934	0.7517	15,213
1.795	0.557	1.2897	0.7185	0.0777	0.8293	14,557
1.810	0.552	1.0056	0.5556	0.0601	0.8894	13,929
1.825	0.548	0.7285	0.3992	0.0432	0.9326	13,329
1.840	0.543	0.4903	0.2665	0.0288	0.9614	12,755
1.855	0.539	0.3066	0.1653	0.0179	0.9793	12,207
1.870	0.535	0.1782	0.0953	0.0103	0.9896	11,683
1.885	0.531	0.0962	0.0510	0.0055	0.9951	11,181
1.900	0.526	0.0482	0.0254	0.0027	0.9979	10,702
1.915	0.522	0.0225	0.0117	0.0013	0.9991	10,244
1.930	0.518	0.0097	0.0050	0.0005	0.9997	9,805
1.945	0.514	0.0039	0.0020	0.0002	0.9999	9,386
1.960	0.510	0.0015	0.0007	0.0001	1.0000	8,985
1.975	0.506	0.0005	0.0003	0.0000	1.0000	8,602
1.990	0.503	0.0002	0.0001	0.0000	1.0000	8,235

SUMMARY OF RESULTS

	Classical Estimate	Posterior Mean	Posterior Std Dev	Approximate 95% Confidence Interval Low	High
For q_i	1.750	1.750	0.055	1.630	1.840
For $E[X\vert q_i]$	16,619	16,847	2,769	12,755	23,711

EXHIBIT 3

Prior Distribution			Given			Observed	
q_0	q_{30}	r_i	k	Retention	Limit	$\hat{q}$	n
1.670	1.820	$1/q_i$	100,000	1,000,000	5,000,000	1.75	10,000

q_i	r_i	$v(\hat{q}\vert q_i)$	$w(q_i\vert\hat{q})$	$b(q_i\vert\hat{q})$	$B(q_i\vert\hat{q})$	$E[X\vert q_i]$
1.670	0.599	0.0000	0.0000	0.0000	0.0000	21,055
1.675	0.597	0.0002	0.0001	0.0000	0.0000	20,745
1.680	0.595	0.0006	0.0003	0.0000	0.0001	20,440
1.685	0.593	0.0017	0.0010	0.0001	0.0002	20,140
1.690	0.592	0.0047	0.0028	0.0003	0.0005	19,844
1.695	0.590	0.0121	0.0072	0.0008	0.0013	19,552
1.700	0.588	0.0287	0.0169	0.0019	0.0032	19,265
1.705	0.587	0.0625	0.0367	0.0042	0.0074	18,982
1.710	0.585	0.1255	0.0734	0.0084	0.0158	18,703
1.715	0.583	0.2321	0.1353	0.0154	0.0312	18,429
1.720	0.581	0.3957	0.2301	0.0262	0.0574	18,158
1.725	0.580	0.6218	0.3604	0.0411	0.0985	17,892
1.730	0.578	0.9004	0.5205	0.0593	0.1578	17,630
1.735	0.576	1.2016	0.6926	0.0789	0.2368	17,371
1.740	0.575	1.4779	0.8494	0.0968	0.3336	17,117
1.745	0.573	1.6752	0.9600	0.1094	0.4430	16,866
1.750	0.571	1.7500	1.0000	0.1140	0.5570	16,619
1.755	0.570	1.6848	0.9600	0.1094	0.6664	16,376
1.760	0.568	1.4949	0.8494	0.0968	0.7632	16,137
1.765	0.567	1.2224	0.6926	0.0789	0.8422	15,901
1.770	0.565	0.9212	0.5205	0.0593	0.9015	15,668
1.775	0.563	0.6398	0.3604	0.0411	0.9426	15,439
1.780	0.562	0.4095	0.2301	0.0262	0.9688	15,213
1.785	0.560	0.2416	0.1353	0.0154	0.9842	14,991
1.790	0.559	0.1313	0.0734	0.0084	0.9926	14,772
1.795	0.557	0.0658	0.0367	0.0042	0.9968	14,557
1.800	0.556	0.0304	0.0169	0.0019	0.9987	14,344
1.805	0.554	0.0129	0.0072	0.0008	0.9995	14,135
1.810	0.552	0.0051	0.0028	0.0003	0.9998	13,929
1.815	0.551	0.0018	0.0010	0.0001	1.0000	13,726
1.820	0.549	0.0006	0.0003	0.0000	1.0000	13,526

SUMMARY OF RESULTS

	Classical Estimate	Posterior Mean	Posterior Std Dev	Approximate 95% Confidence Interval Low	High
For q_i	1.750	1.750	0.017	1.710	1.780
For $E[X\vert q_i]$	16,619	16,642	859	15,213	18,703

EXHIBIT 4

Prior Distribution			Given			Observed	
q_0	q_{30}	r_i	k	Retention	Limit	$\hat{q}$	n
1.000	2.500	q_i	100,000	1,000,000	5,000,000	1.75	100

q_i	r_i	$v(\hat{q}\|q_i)$	$w(q_i\|\hat{q})$	$b(q_i\|\hat{q})$	$B(q_i\|\hat{q})$	$E[X\|q_i]$
1.000	1.000	0.0001	0.0001	0.0000	0.0000	160,944
1.050	1.050	0.0004	0.0004	0.0000	0.0000	137,822
1.100	1.100	0.0011	0.0012	0.0000	0.0001	118,085
1.150	1.150	0.0032	0.0037	0.0001	0.0002	101,229
1.200	1.200	0.0086	0.0103	0.0004	0.0006	86,826
1.250	1.250	0.0211	0.0264	0.0010	0.0016	74,512
1.300	1.300	0.0477	0.0620	0.0023	0.0038	63,979
1.350	1.350	0.0990	0.1337	0.0049	0.0088	54,964
1.400	1.400	0.1895	0.2653	0.0098	0.0185	47,245
1.450	1.450	0.3336	0.4837	0.0178	0.0364	40,631
1.500	1.500	0.5407	0.8110	0.0299	0.0662	34,961
1.550	1.550	0.8067	1.2504	0.0461	0.1123	30,099
1.600	1.600	1.1081	1.7730	0.0653	0.1777	25,926
1.650	1.650	1.4015	2.3124	0.0852	0.2629	22,343
1.700	1.700	1.6320	2.7744	0.1022	0.3651	19,265
1.750	1.750	1.7500	3.0625	0.1129	0.4780	16,619
1.800	1.800	1.7280	3.1104	0.1146	0.5926	14,344
1.850	1.850	1.5713	2.9070	0.1071	0.6997	12,387
1.900	1.900	1.3159	2.5002	0.0921	0.7919	10,702
1.950	1.950	1.0149	1.9790	0.0729	0.8648	9,251
2.000	2.000	0.7209	1.4418	0.0531	0.9179	8,000
2.050	2.050	0.4716	0.9669	0.0356	0.9535	6,922
2.100	2.100	0.2842	0.5968	0.0220	0.9755	5,992
2.150	2.150	0.1577	0.3392	0.0125	0.9880	5,189
2.200	2.200	0.0806	0.1774	0.0065	0.9946	4,496
2.250	2.250	0.0380	0.0855	0.0031	0.9977	3,897
2.300	2.300	0.0165	0.0379	0.0014	0.9991	3,380
2.350	2.350	0.0066	0.0155	0.0006	0.9997	2,932
2.400	2.400	0.0024	0.0058	0.0002	0.9999	2,545
2.450	2.450	0.0008	0.0020	0.0001	1.0000	2,210
2.500	2.500	0.0003	0.0006	0.0000	1.0000	1,920

SUMMARY OF RESULTS

	Classical Estimate	Posterior Mean	Posterior Std Dev	Approximate 95% Confidence Interval Low	High
For q_i	1.750	1.785	0.173	1.400	2.050
For $E[X\|q_i]$	16,619	17,154	9,432	6,922	47,245

EXHIBIT 5

Prior Distribution			Given			Observed	
q_0	q_{30}	r_i	k	Retention	Limit	$\hat{q}$	n
1.612	1.732	$1/q_i$	100,000	1,000,000	5,000,000	1.75	100

q_i	r_i	$v(\hat{q}\vert q_i)$	$w(q_i\vert\hat{q})$	$b(q_i\vert\hat{q})$	$B(q_i\vert\hat{q})$	$E[X\vert q_i]$
1.612	0.620	1.1812	0.7328	0.0265	0.0265	25,015
1.616	0.619	1.2054	0.7459	0.0270	0.0536	24,719
1.620	0.617	1.2294	0.7589	0.0275	0.0811	24,427
1.624	0.616	1.2532	0.7717	0.0280	0.1090	24,138
1.628	0.614	1.2768	0.7843	0.0284	0.1374	23,852
1.632	0.613	1.3001	0.7967	0.0289	0.1663	23,570
1.636	0.611	1.3232	0.8088	0.0293	0.1956	23,292
1.640	0.610	1.3460	0.8207	0.0297	0.2253	23,016
1.644	0.608	1.3685	0.8324	0.0302	0.2555	22,744
1.648	0.607	1.3906	0.8438	0.0306	0.2860	22,476
1.652	0.605	1.4123	0.8549	0.0310	0.3170	22,210
1.656	0.604	1.4335	0.8657	0.0314	0.3484	21,948
1.660	0.602	1.4544	0.8761	0.0317	0.3801	21,689
1.664	0.601	1.4747	0.8863	0.0321	0.4122	21,433
1.668	0.600	1.4946	0.8960	0.0325	0.4447	21,180
1.672	0.598	1.5139	0.9054	0.0328	0.4775	20,931
1.676	0.597	1.5327	0.9145	0.0331	0.5106	20,684
1.680	0.595	1.5508	0.9231	0.0334	0.5440	20,440
1.684	0.594	1.5684	0.9314	0.0337	0.5778	20,199
1.688	0.592	1.5853	0.9392	0.0340	0.6118	19,961
1.692	0.591	1.6016	0.9466	0.0343	0.6461	19,726
1.696	0.590	1.6171	0.9535	0.0345	0.6806	19,494
1.700	0.588	1.6320	0.9600	0.0348	0.7154	19,265
1.704	0.587	1.6461	0.9660	0.0350	0.7504	19,038
1.708	0.585	1.6595	0.9716	0.0352	0.7856	18,814
1.712	0.584	1.6721	0.9767	0.0354	0.8210	18,593
1.716	0.583	1.6839	0.9813	0.0355	0.8565	18,374
1.720	0.581	1.6949	0.9854	0.0357	0.8922	18,158
1.724	0.580	1.7051	0.9890	0.0358	0.9280	17,945
1.728	0.579	1.7144	0.9921	0.0359	0.9640	17,734
1.732	0.577	1.7229	0.9947	0.0360	1.0000	17,526

SUMMARY OF RESULTS

	Classical Estimate	Posterior Mean	Posterior Std Dev	Approximate 95% Confidence Interval Low	High
For q_i	1.750	1.675	0.035	1.612	1.728
For $E[X\vert q_i]$	16,619	20,851	2,198	17,734	25,015

DISCUSSION BY STUART A. KLUGMAN, PH.D.

While actuaries have had a Bayesian view of the world for decades, the adoption of methods that adhere strictly to the principles of modern Bayesian analysis has been slow. In his paper, Glenn Meyers shows that for a particular problem such an approach is not only feasible, but easy to complete. I am delighted that he has continued to take up the Bayesian cause, and with this note, I hope to provide just two extensions. One is to demonstrate that Meyers employed an approximation that was not needed for the particular prior distribution. The other is to provide an example that will confirm that his suggestions are indeed not limited to the Pareto distribution nor to one-parameter distributions.

To be fair to Meyers, and to continue his promotion of Bayesian methods as a practical solution to estimation problems, I will employ his definition of "practical:" that solutions can be obtained via simple spreadsheet calculations.

1. EXACT BAYESIAN CALCULATIONS

There have been a number of reasons for the slow adoption of exact Bayesian methods. One excellent discussion is Efron [3]. Aside from philosophical issues, there is a major computational one. Begin by defining the customary Bayesian estimation problem:

$$x = \text{data}$$

$$\theta = \text{parameter}$$

$$p(\theta) = \text{prior density}$$

$$f(x|\theta) = \text{model density}$$

$$f(\theta|x) = \text{posterior density}$$

$$t(\theta) = \text{quantity of interest}$$

$f(t|x)$ = posterior density of the quantity of interest.

In Meyers's paper, $\theta = q$, and t is the layer average severity, while the model density is the likelihood function.

One standard Bayesian estimate of a parameter is the posterior mean. For continuous models, the formula is

$$E(\theta|x) = \frac{\int \theta f(x|\theta)p(\theta)d\theta}{\int f(x|\theta)p(\theta)d\theta}. \tag{1.1}$$

The estimate of the quantity of interest is

$$E[t(\theta)|x] = \frac{\int t(\theta)f(x|\theta)p(\theta)d\theta}{\int f(x|\theta)p(\theta)d\theta}. \tag{1.2}$$

Thus any Bayesian estimation problem using the posterior mean reduces to evaluating a (possibly) multi-dimensional integral. The number and efficiency of methods to do so have greatly increased in the past decade. Four methods (extensions of one-dimensional numerical integration methods, Gauss-Hermite, Tierney-Kadane, Monte Carlo, empirical Bayes) are outlined in Klugman [4]. Recently two additional methods have been developed: the Gibbs sampler (Casella and George [2]) and sampling-resampling (Smith and Gelfand [5]). All of the methods require a large number of calculations and clearly do not meet our present standard of being spreadsheet-friendly.

Meyers offers the only alternative that requires a limited amount of calculation: Replace the customary continuous prior distribution with a discrete one. The integrals then become sums and are easy to calculate. The question that remains is whether additional approximations are needed in order to complete the posterior calculations.

2. EXACT CALCULATIONS FOR THE SINGLE PARAMETER PARETO DISTRIBUTION

For the specific problem addressed by Meyers we have

$$p(q) \propto q^{\alpha-1}e^{-\beta q} \tag{2.1}$$

$$f(x|q) = k^{nq}q^{n}(\Pi x_i)^{-q-1}. \tag{2.2}$$

The prior distribution is a Gamma distribution when α and β are both positive, and reduces to Meyers's noninformative prior when they are both zero. The posterior distribution is

$$f(q|x) \propto k^{nq}q^{n+\alpha-1}(\Pi x_i)^{-q-1}e^{-\beta q} \propto q^{n+\alpha-1}e^{-(\gamma+\beta)q} \tag{2.3}$$

where $\gamma = -n\ln(k) + \Sigma\ln(x_i)$. This is just another Gamma distribution and so the posterior mean is

$$\mathrm{E}(q|x) = \frac{\gamma}{\gamma+\beta}\hat{q} + \frac{\beta}{\gamma+\beta}\frac{\alpha}{\beta}, \tag{2.4}$$

the usual weighted average of the maximum likelihood estimator and the prior mean. When $\alpha = \beta = 0$, the posterior mean is $\hat{q}$ as in Meyers (so once again the "WYSIWYG" estimator is obtained) but without resorting to the Normal approximation. The posterior variance is

$$\mathrm{Var}\,(q|x) = \frac{n+\alpha}{(\gamma+\beta)^2}. \tag{2.5}$$

With $\alpha = \beta = 0$ it is $\hat{q}^2/n$, also in agreement with Meyers.

The difference comes when other features are desired or when numerical approximations are needed. The other feature desired in Meyers's paper is the layer average severity. The required integral for the posterior mean of the layer average severity is

$$\mathrm{E}(t|q) = \int_0^\infty \frac{k^q}{q-1}\left(\frac{1}{R^{q-1}} - \frac{1}{L^{q-1}}\right)\frac{(\gamma+\beta)^{n+\alpha}}{\Gamma(n+\alpha)}\, q^{n+\alpha-1}e^{-(\gamma+\beta)q}dq\,, \quad (2.6)$$

which cannot be integrated analytically. A simple discretization (the composite trapezoid rule) should approximate this integral. For Meyers's example, the values are $n = 100$, $\gamma = 57.143$, $R = 1{,}000{,}000$, $L = 5{,}000{,}000$, $k = 100{,}000$, $\alpha = 0$, and $\beta = 0$. The approximate integral, evaluating q every 0.05 from 0 to 3, produced a posterior mean of 18,971 and a posterior standard deviation of 9,989. These cannot be compared with Meyers's paper as he did not solve this example.

The above calculations took advantage of the fact that the Gamma prior distribution turned out to be conjugate for the single parameter Pareto likelihood. (That is, the posterior turned out to have the same density type as the prior. The major advantage is that the constants needed to make Equation 2.3 an equality can be found without integrating.) This will seldom be the case for actuarial examples. To continue Meyers's example, we can use the prior that appears in his Exhibit 1. Exhibit 1 of this discussion provides the equivalent results using Meyers's discrete prior but retaining the exact likelihood function. The results are similar to the exact calculation done previously with a posterior mean of 18,972 and standard deviation of 9,989. These numbers are similar to those obtained by Meyers, but were not expected to be exactly the same.

3. EXTENSIONS TO MULTI-PARAMETER PROBLEMS

Avoiding the Normal approximation for the distribution of maximum likelihood estimators—in fact, avoiding maximum likelihood estimators of q altogether— may make extensions of Meyers's analysis easier. The major difficulty is that any sums must now be taken over a relatively large number of values. This is because, for example, two-dimensional approximate integration requires the square of the number of function evaluations as compared with a similar one-dimensional approximation. Rather than produce general formulas, the example used previously is extended to the case of a Lognormal distribution. Suppose a sample of size 100 was taken and the sufficient statistics (the only numbers other than the parameter values needed to compute the likelihood function) were

$$\sum \ln(x_i) = 1000, \text{ and } \sum [\ln(x_i)]^2 = 10{,}300.$$

Thus, the maximum likelihood estimates of the Lognormal parameters are $\hat{\mu} = 10$, and $\hat{\sigma}^2 = 3$. This leads to a maximum likelihood estimate of the layer average severity of 48,770. The noninformative prior distribution selected is the standard one (Berger [1], pp. 83-87) for the normal distribution: $p(\mu,\sigma) \propto 1/\sigma$. This implies a uniform (over the entire real line) prior on μ that is independent of the prior on σ. Possible values were restricted to the range $8 \le \mu \le 12$ and $1.0 \le \sigma \le 2.5$. The other relevant functions are:

$$f(x|\mu,\sigma) \propto \sigma^{-n} e^{-\frac{1}{2}\sum\left(\frac{\ln x_i - \mu}{\sigma}\right)^2}$$

$$= \sigma^{-n} e^{-\frac{10{,}300 - 2{,}000\mu + 100\mu^2}{2\sigma^2}}, \qquad (3.1)$$

and

$$t(\mu,\sigma) = e^{\mu+\sigma^2}\left[\Phi\left(\frac{\ln(5{,}000{,}000) - \mu}{\sigma} - \sigma\right) - \Phi\left(\frac{\ln(1{,}000{,}000) - \mu}{\sigma} - \sigma\right)\right]$$

$$+ 4{,}000{,}000 - (5{,}000{,}000)\,\Phi\left(\frac{\ln 5{,}000{,}000 - \mu}{\sigma}\right)$$

$$+ 1{,}000{,}000\,\Phi\left(\frac{\ln(1{,}000{,}000) - \mu}{\sigma}\right). \qquad (3.2)$$

For the calculations, the ranges on μ and σ were split into 30 equally spaced intervals. This led to 961 function evaluations, of which 13 are displayed in Exhibit 2. The relevant posterior quantities appear at the end of the exhibit.

4. CONCLUSIONS

Through his paper, Glenn Meyers has reminded us that Bayesian calcuations can be relatively simple, and that they provide quantities of great interest to actuaries (mainly the standard deviation, and perhaps the complete distribution of the quantity to be estimated). This discussion points out that there may be simpler ways to do the calculations and that two-dimensional calculations are indeed feasible as Meyers indicated.

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[4] Klugman, Stuart A., *Bayesian Statistics in Actuarial Science with an Emphasis on Credibility*, Boston, Kluwer, 1992.

[5] Smith, Adrian F.M. and Alan E. Gelfand, "Bayesian Statistics Without Tears: A Sampling-Resampling Perspective," *The American Statistician*, 46, 1992, pp. 84-88.

EXHIBIT 1

SINGLE PARAMETER PARETO—POSTERIOR ANALYSIS

q	t	b(post)	B(postcdf)	$q*b$	$q*q*b$	$t*b$	$t*t*b$
1.000	160944	0.0000	0.0000	0.00000	0.00000	0.07	10512
1.050	137822	0.0000	0.0000	0.00000	0.00000	0.40	554478
1.100	118085	0.0000	0.0000	0.00002	0.00002	1.98	233847
1.150	101229	0.0001	0.0001	0.00009	0.00010	7.95	804496
1.200	86826	0.0003	0.0004	0.00037	0.00044	26.46	2297361
1.250	74512	0.0010	0.0014	0.00125	0.00156	74.21	5529579
1.300	63979	0.0028	0.0042	0.00361	0.00469	177.72	11370313
1.350	54964	0.0067	0.0109	0.00903	0.01220	367.78	20214671
1.400	47245	0.0141	0.0249	0.01970	0.02758	664.74	31405343
1.450	40631	0.0261	0.0510	0.03781	0.05482	1059.42	43045108
1.500	34961	0.0430	0.0940	0.06443	0.09664	1501.62	52498447
1.550	30099	0.0634	0.1573	0.09824	0.15226	1907.56	57414828
1.600	25926	0.0844	0.2417	0.13498	0.21596	2187.06	56700811
1.650	22343	0.1019	0.3436	0.16819	0.27752	2277.50	50885512
1.700	19265	0.1125	0.4561	0.19120	0.32502	2166.60	41739159
1.750	16619	0.1139	0.5700	0.19931	0.34880	1892.83	31457522
1.800	14344	0.1064	0.6764	0.19149	0.34468	1525.98	21889173
1.850	12387	0.0921	0.7684	0.17030	0.31506	1140.29	14124672
1.900	10702	0.0741	0.8425	0.14079	0.26750	793.00	8486676
1.950	9251	0.0557	0.8982	0.10860	0.21177	515.19	4765789
2.000	8000	0.0392	0.9374	0.07844	0.15688	313.75	2510000
2.050	6922	0.0260	0.9634	0.05322	0.10910	179.69	1243787
2.100	5992	0.0162	0.9796	0.03402	0.07145	97.07	581624
2.150	5189	0.0096	0.9892	0.02055	0.04419	49.60	257374
2.200	4496	0.0053	0.9945	0.01176	0.02587	24.03	108049
2.250	3897	0.0028	0.9973	0.00639	0.01438	11.07	43138
2.300	3380	0.0014	0.9988	0.00331	0.00760	4.86	16415
2.350	2932	0.0007	0.9995	0.00163	0.00383	2.03	5966
2.400	2545	0.0003	0.9998	0.00077	0.00185	0.82	2075
2.450	2210	0.0001	0.9999	0.00035	0.00085	0.31	692
2.500	1920	0.0001	1.0000	0.00017	0.00038	0.12	222

	mle	post mean	post sd	low 95	high 95
For q	1.750	1.7500	0.1749	1.400	2.100
For E(x)	16,619	18,972	9,989	5,992	47,245

EXHIBIT 2

LOGNORMAL POSTERIOR ANALYSIS

μ	σ	prior	t	model	posterior
8	1	1	0	9.930E-153	3.173E-108
8	1.75	.57143	320	1.155E-74	2.108E-30
8	2.5	.4	14,390	7.683E-65	9.818E-21
9.867	1.75	.57143	12,154	1.989E-46	3.631E-2
10	1	1	20	7.175E-66	2.292E-21
10	1.7	.58824	12,174	2.593E-46	4.874E-2
10	1.75	.57143	15,172	2.658E-46	4.854E-2
10	1.8	.55556	18,616	2.326E-46	4.128E-2
10	2.5	.4	114,619	6.066E-51	7.753E-7
10.133	1.75	.57143	18,845	1.989E-46	3.631E-2
12	1	1	20,411	9.930E-153	3.173E-108
12	1.75	.57143	234,939	1.155E-65	2.108E-30
12	2.5	.4	560,653	7.683E-65	9.818E-21

$E(\mu)$	10.000
StdDev(μ)	.17586
$E(\sigma)$	1.7541
StdDev(σ)	.12609
$E(t)$	17,452
StdDev(t)	10,811

Note: The entries in the "model" column are the evaluation of Equation 3.1. The entries in the "posterior" column are the entries in the "model" column divided by σ and then multiplied by a constant to make them sum to one.

RESIDUALS AND INFLUENCE IN REGRESSION

EDMUND S. SCANLON

Abstract

The purpose of this paper is to cover some techniques in statistics that are important for testing the appropriateness of a fitted regression equation. These techniques, which are often used by statisticians, are not completely covered in the Proceedings. *Specifically, the areas discussed are:*

- *Elimination of the Constant in the Regression Equation*
- *Regression Diagnostics*
- *Analysis of Residuals*

1. INTRODUCTION

Estimating the parameters of a regression equation entails more than simply fitting a line to a set of data. During the estimation process, it is important to determine if the underlying assumptions are met and whether the equation accurately models the studied process.

The purpose of this paper is to discuss some aspects of regression that are important in testing the appropriateness of the fitted regression equation. These aspects, some of which are briefly covered in the present Syllabus are: the constant term in the multiple regression equation, regression diagnostics, and the analysis of residuals. It should be noted that the material described is contained in the references listed at the end of this paper.

2. THE CONSTANT TERM

At least two papers in the *Proceedings* [(1),(5)] suggest removing the constant term in the regression equation under some circumstances. The circumstances described in the papers seem to be reasonable causes for removal of the constant term. For example, one reason outlined is that the constant does not *explain* any of the change in the dependent variable. Thus, the constant term should be carefully scrutinized and perhaps removed. Another suggested reason to remove the constant is to achieve a regression model which is intuitively sensible. Unfortunately, the removal of the constant, especially when it is statistically significant, tends to impair the accuracy of the model. Hence, something should be added to compensate for the removal of the constant.

Both papers seem to imply that the constant should be eliminated if the corresponding t-statistic is insignificant (i.e., $|t| < 2$). Or, if the t-statistic is significant, one should try to search for another independent variable in an attempt to reduce the significance of the constant term.

These procedures are unreliable for three reasons:

1. Although it is sometimes not clearly stated in statistics texts, the *objective* in the traditional statistical test is to decide whether or not to reject the null hypothesis. Acceptance of the null hypothesis is not the issue. For example, in analysis-of-variance (ANOVA), the null hypothesis (H_0) is that $\mu_1 = \mu_2 = \ldots = \mu_n$. If the F-statistic is significant, one may reject the null hypothesis. Even in the absence of a high F-statistic, the null hypothesis is difficult to believe; all one can say is that the hypothesis cannot be proved false (i.e., fail to reject H_0).

2. Eliminating the constant gives the origin (which can be considered one observation) an undue amount of leverage on the fitted regression equation (the subject of leverage is discussed in more detail in the next section). As a result of this elimination, the regression line is forced through a particular point, the origin. In other words, the origin, as an observation, is given special treatment because it is not subject to the least squares constraint (i.e., minimize the sum of squares). Thus, the origin has more influence on the regression model than a typical observation.

3. An additional independent variable may not be easily found.

This is not to say that one should never eliminate the constant in the regression equation. The point is that this elimination should not be considered lightly. We will illustrate this with a numerical example in the next section.

3. REGRESSION DIAGNOSTICS

When analyzing the appropriateness of a regression equation, most statisticians review the data to see which observations are "influencing" the estimation of the regression equation coefficients. More generally, the statistician wants to identify subsets of the data that have disproportional influence on the estimated regression model. As discussed by Belsley et al. [2], these influential subsets can come from a number of sources:

1. improperly recorded data,
2. errors that are inherent in the data, and
3. outliers that are legitimately extreme observations.

Belsley et al. indicate some interesting situations that might be subject to detection by diagnostics. Exhibit 1 summarizes these situations. Part 1 displays the ideal situation: All the data is essentially grouped together. In Part 2, the point labeled z is an aberration or outlier; but, since it is near $\overline{x}$ there is no adverse effect on the slope. However, the estimate of the intercept will obviously be influenced. Part 3 also displays a data set with an outlier. However, the outlier in this example is consistent with the slope indicated by the remaining data. Because of this consistency, adding the outlier to the regression calculation reduces the variance of the parameter estimates (i.e., improves the quality of the regression). Generally, if the variance of the independent variable is small, slope estimates will be unreliable. Part 4 is a problem situation, since the outlier essentially defines the slope. In the absence of the outlier, the slope might be anything. The outlier has extreme influence on the slope. Part 5 is a case where there are two outliers that are both influential and whose effects are complementary. Such a situation may call for use of one of the following procedures:

1. deleting the observations,
2. downweighting (i.e., giving less weight to the observations),
3. reformulating the model (e.g., adding or deleting independent variables); or
4. where possible, using more observations.

Part 6 displays a situation where deletion of either outlier has little effect on the regression outcome because neither outlier exerts much influence upon the regression parameters. Parts 5 and 6 highlight the need to examine the effects of general subsets of data.

To demonstrate some of these situations more clearly, we consider the numerical data and regression results on Exhibit 2, Part 1. This data set, which is plotted on Part 2, is similar to the pattern displayed on Exhibit 1, Part 1. The regression results, as expected from the uniformity of the plot, indicate a very good fit.

In order to illustrate the Exhibit 1, Part 2 situation, the point (7, 14.3) was added to the base data set. The regression results and the plot of the data are on Exhibit 2, Parts 3 and 4, respectively. It is interesting to note how the additional observation influenced the parameter estimates. The constant changed from .702 to 1.171. However, the slope estimate change was negligible (.808 to .784).

The Exhibit 1, Part 3 case can be demonstrated by adding the point (17, 14.3) in lieu of (7, 14.3). This new outlier is consistent with the remaining data (i.e., it lies on the path of the line indicated by the base data set). The regression results and the plot for this revised data set are displayed on Exhibit 2, Parts 5 and 6, respectively. The results indicate minimal change in the parameter estimates. Hence, (17, 14.3) does not have significant influence on the regression model. However, adding the point (17, 14.3) decreased the variance of the parameter estimates. It should also be noted that the standard error of the residual associated with this outlier is relatively smaller than the standard errors associated with all the other observations. The magnitude of the standard error for a residual relative to the other standard errors is an indication of the leverage of a point (i.e., the *potential* of the point to influence the calculation of the regression equation). Leverage depends on whether an observation is an outlier with respect to the x axis.

To demonstrate another example of leverage, the point (17, 10) was used instead of (17, 14.3) and a new regression equation was calculated. The regression results and the plot can be found on Exhibit 2, Parts 7 and 8, respectively. The contrast between the two outliers, (7, 14.3) and (17, 10), is interesting. The outlier (7, 14.3), an outlier with respect to the y axis, is about 8 units away from where it "should be." The other outlier, (17, 10), an outlier with respect to both the x and y axes, is only about 4 units away from where it "should be." However, the influence of (17, 10) on the parameter estimates is much greater than that of (7, 14.3). As mentioned earlier, the (7, 14.3) outlier influenced only the estimate of the constant. There was a negligible change in the estimate of the slope.

The estimates of the parameters under the varying data sets are summarized in the following table:

	Base Set	Set with (7, 14.3)	Set with (17, 10)
Intercept	.702	1.171	1.394
Slope	.808	.784	.705

As indicated by the table, the point (17, 10) influences both the intercept and slope estimates to a much greater extent than (7, 14.3).

At this time we return to the question of eliminating the constant. It is interesting to note the situation of removing the constant term when fitting a regression line to the base data set (Exhibit 2, Part 1). The regression analysis of the base data set indicates that the t-statistic for the constant term is not "statistically significant." However, removing the constant term from the regression equation influences the slope estimate considerably. The coefficient of the independent variable is now .883 as compared to .808 .

The preceding examples indicate that when there are two or fewer independent variables, scatter plots such as Exhibit 1 can quickly reveal any outliers. However, when there are more than two independent variables, scatter plots may not reveal multivariate outliers that are separated from the bulk of the data. What follows is a discussion of some diagnostic statistics that are useful in detecting such outliers.

There are a number of different statistics used by statisticians to detect outliers in the data. One such statistic is Cook's D_i (or Cook's Distance) statistic. The statistic is named after the statistician R. D. Cook. Cook's D_i measures the *influence* of the i^{th} observation. It is based on the difference between two estimators (one estimator includes the i^{th} observation in the data; the other excludes the i^{th} observation). Using matrix notation, Cook's D_i is defined as follows:

$$D_i = \{\hat{\beta} - \hat{\beta}(i)\}^T X^T X \{\hat{\beta} - \hat{\beta}(i)\}/ps^2,$$

where:

- X is the n by p matrix that contains the values of the independent variables (i.e., n different values of the $(p\text{–}1)$ independent variables together with a first column that is equal to unity, representing the constant); this is the same X that is used in the familiar multiple regression equation $Y = X\beta + e$ (see, for example, Miller and Wichern [6, supplement 5B]);
- X^T is the transpose of X;
- $\hat{\beta}$ is the usual least squares estimator vector of p by 1 dimensions;
- $\hat{\beta}(i)$ is the least squares estimator after the i^{th} data point has been omitted from the data, also p by 1 dimensions;
- p is the number of independent variables plus one;

s^2 is the estimate of variance provided by residual mean square error from using the full data set;

$\{\hat{\beta}-\hat{\beta}(i)\}$ is the difference between the two p by 1 vectors, also p by 1.

A large D_i represents an influential observation; that is, an observation that has more than the average influence on the estimation of the parameters. Presently, there is no formal definition of a "large D_i." However, there are some general rules that statisticians follow. First, if $D_i > 1$, then the observation should probably be considered influential. Second, if all D_is are below 1, a value considerably greater than the other values should be considered influential. Once a point with a large D_i has been identified, the actuary would want to examine the point to be certain that such an observation is typical and not an aberration. With Cook's D_i, the actuary can review all outliers and decide whether or not to eliminate an observation. This process is somewhat analogous to the reserving actuary eliminating high/low loss development link ratios from an average.

The preceding formula for Cook's D_i is rather cumbersome. Fortunately, it is standard output for most statistical software packages.

The so-called *hat statistic*, h_{ii}, is another tool that is helpful in determining which observations have significant *leverage*. Using matrix notation, the hat matrix (which contains the hat statistics) can be derived from the usual regression equations:

$$Y = X\beta + e\,,$$

$$\hat{Y} = X\hat{\beta}\,;$$

$$\text{Since } \hat{\beta} = (X^TX)^{-1}\,X^TY\,,$$

$$\hat{Y} = X(X^TX)^{-1}\,X^TY\,,$$

$$= HY,$$

where H, the n by n hat matrix, is defined as:

$$H = X(X^TX)^{-1}X^T.$$

H is called the hat matrix because it transforms the vector of observed responses, Y, into the vector of fitted responses, $\hat{Y}$. From this, the vector of residuals can be defined as:

$$\begin{aligned}\hat{e} &= Y - \hat{Y} \\ &= Y - X(X^TX)^{-1} X^T Y \\ &= [I - H]Y.\end{aligned}$$

It is shown in Weisberg [8] that:

$$\mathrm{E}(\hat{e}_i) = 0, \text{ and } \mathrm{Var}(\hat{e}_i) = \sigma^2(1-h_{ii}),$$

where the hat statistic, h_{ii}, is the i^{th} diagonal element of H. This is in contrast to the errors, e_i, for which the variance is constant for all i. Incidentally, the variance for $\hat{y}_i$ is $\sigma^2 h_{ii}$.

Hence, cases with large values of h_{ii} will have small values of $\mathrm{Var}(\hat{e}_i)$. As h_{ii} approaches unity, the variance of the i^{th} residual approaches zero. In other words, as h_{ii} approaches unity, $\hat{y}_i$ (the estimate) approaches the observed value, y_i. This is why h_{ii} is called the leverage of the i^{th} observation. The effect of the i^{th} observation on the regression is more likely to be large if h_{ii} is large. Similar to Cook's D_i, the hat matrix is standard output from common statistical software packages.

How large is a "large" h_{ii} ? This issue is addressed by Belsley et al. They show that, if the explanatory variables are independently distributed as the multivariate Gaussian, it is possible to compute the exact distribution of certain functions of the h_{ii}s. Specifically, $(n-p)[h_{ii}-(1/n)]/(1-h_{ii})(p-1)$ is shown to be distributed as F with $p-1$ and $n-p$ degrees of freedom. For $p>10$ and $n-p>50$, the 95% value for F is less than 2. Hence, $2p/n$ is roughly a good cutoff (twice the balanced average h_{ii}).

At this point, it is appropriate to discuss the difference between *influence* and *leverage*. The leverage of an observation was just defined as h_{ii}. Note that this is independent of the dependent variable. Hence, the definition of leverage ignores the role played by y_i (the observation). Influence, on the other hand, is defined as follows:

> A case is said to be *influential* if appreciable changes in the fitted regression coefficients occur when it is removed from the data [7].

Another way to look at the difference between influence and leverage is as follows. The h_{ii}s are indicating how well the independent data is "spread out." Exhibit 1, Part 4 displays data that contain an observation that has leverage. The amount of leverage would be reduced if there were more observations with larger independent values near that point's independent value.

Cook's D_i actually indicates how much of the leverage is being exerted by the observation on the estimation of the coefficients. Therefore, Cook's D_i is more helpful in analyzing a regression model.

It is interesting to examine the relationship between h_{ii} and D_i to understand the difference between influence and leverage. Weisberg [8] derives the following relationship:

$$D_i = \{e_i / s\,(1-h_{ii})^{1/2}\}^2\, h_{ii} / p\,(1-h_{ii}).$$

This formula is helpful in a number of ways. First, it shows that Cook's D_i can be calculated from data output of the full regression without the need to recompute estimates excluding observations. Second, it displays the relationship of Cook's D_i, the studentized residuals, and the measure of leverage. Third, it shows explicitly that the hat diagonal describes only the potential for influence. D_i will be large only if both h_{ii} and the associated residual are large.

This clarification is important because the two terms (influence and leverage) are sometimes used synonymously. For example, Cook and Weisberg [3] mention authors who interpret h_{ii} as the amount of leverage *or* influence.

Thus far, this discussion has focused on "single row" diagnostics. As mentioned earlier, Exhibit 1, Part 6 indicates the need for multiple row diagnostics; for example, in situations where one outlier masks the effect of another outlier. Such techniques exist and the interested reader is referred to Belsley et al [2].

To illuminate the use of these diagnostics, a multiple regression model is fitted to some pure premium data in Exhibit 3. The model used is similar to some work performed by the Insurance Services Office; namely, the alternative trend models.

In this example, pure premium (PP) is the dependent variable, while the Consumer Price Index (CPI) and the change in the Gross National Product (GNP) are the independent variables. It should be noted that the values of these variables are realistic, but fabricated for the example. Exhibit 3, Part 1 displays results from fitting a regression model to 10 observations. The model fitted is as follows:

$$\text{Pure Premium} = b_0 + b_1\text{CPI} + b_2\text{GNP}.$$

Including all 10 observations in the calculation produces an excellent fit, as indicated by the adjusted R^2. Nevertheless, the residuals do become relatively larger as the pure premium increases. The h_{ii} values indicate that the 1st and the 9th observations have a considerable amount of leverage. The D_1 value indicates that the 1st observation is not influencing the model's coefficients. However, the D_9 value does indicate that the 9th observation has significant influence. In order to improve the model, consideration should be given to removing the 9th observation. Exhibit 3, Part 2 displays output excluding this observation. These results can be summarized as follows:

1. The adjusted R^2 improved slightly.
2. The residuals are now more stable than before.
3. The values of D_i are stable.

4. ANALYSIS OF RESIDUALS

Analysis of residuals is touched upon in some of the present *Syllabus* readings. As indicated in the readings, residuals can be helpful in determining if two required regression assumptions have been violated; namely, the error terms must be independent and the variance must be constant for all observations. Violations of these assumptions are associated with the terms autocorrelation and heteroscedasticity, respectively.

Heteroscedasticity, or non-constant variance, is typically detected by using a so-called residual plot [6]. If the plot of residuals is shaped like a cone (see Exhibit 4), it is likely that heteroscedasticity exists. These residual plots are also helpful in determining whether the regression equation needs an additional independent term.

It is important to note (as mentioned in the previous section) that the variance of $\hat{e}_i$ is not a constant for all i. As a matter of fact, it would be unusual for all the $\hat{e}_i$'s to have the same variance. That is, it is possible to have a pattern similar to Exhibit 4 simply because the h_{ii}s are not constant.

Improved diagnostics can be achieved by dividing the residuals by an estimate of the standard error. Specifically, the residual, $\hat{e}_i$, should be divided by $s(1-h_{ii})^{1/2}$ for all i. These scaled residuals, also known as the studentized residuals, will all have a common variance, if the model is correct. The studentized residuals can then be used, graphically, to test for heteroscedasticity.

An additional point which is not emphasized in many books is the reason the residuals are plotted against $\hat{Y}$ and not Y. The reason is that the residuals and the actual observations are correlated, but the residuals and the fitted values are not.

This can be shown [4] by calculating the sample correlation coefficients between the residuals and the actual and fitted observations. First, the sample correlation coefficient between e and Y, r_{ey}, is calculated as follows:

$$r_{ey} = \{\Sigma(e_i-\overline{e})(Y_i-\overline{Y})\}/\{\Sigma(e_i-\overline{e})^2\,\Sigma(Y_i-\overline{Y})^2\}^{1/2}.$$

The numerator,

$$\begin{aligned}
\Sigma(e_i-\overline{e})(Y_i-\overline{Y}) &= \Sigma e_i(Y_i-\overline{Y}) && \text{since } \overline{e}=0, \text{ if a constant is in the model}\\
&= \Sigma e_i Y_i && (\overline{e}=0)\\
&= e^T Y && \text{in matrix notation}\\
&= e^T e && \text{because } e^T e = Y^T(I-H)^T(I-H)Y\\
& && \phantom{\text{because } e^T e} = Y^T(I-H)Y\\
& && \phantom{\text{because } e^T e} = e^T Y\\
&= \text{Residual sum of squares.}
\end{aligned}$$

Therefore,

$$r_{ey} = \{\text{ Residual sum of squares / Total sum of squares}\}^{1/2} = (1-R^2)^{1/2}.$$

The calculation of $r_{e\hat{y}}$ is similar to the above,

$$\Sigma(e_i-\overline{e})(\hat{Y}-\overline{Y}) = \Sigma e_i\hat{Y}_i = e^T\hat{Y} = Y^T(I-H)^T HY = 0.$$

Hence, $r_{e\hat{y}} = 0$.

5. CONCLUSIONS

The field of statistics is a tremendous resource that, except for a theoretical foundation, goes untapped by casualty actuaries. I hope this paper adds modestly to the knowledge of some actuarial practitioners and inspires other such summaries.

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EXHIBIT 1
Part 1

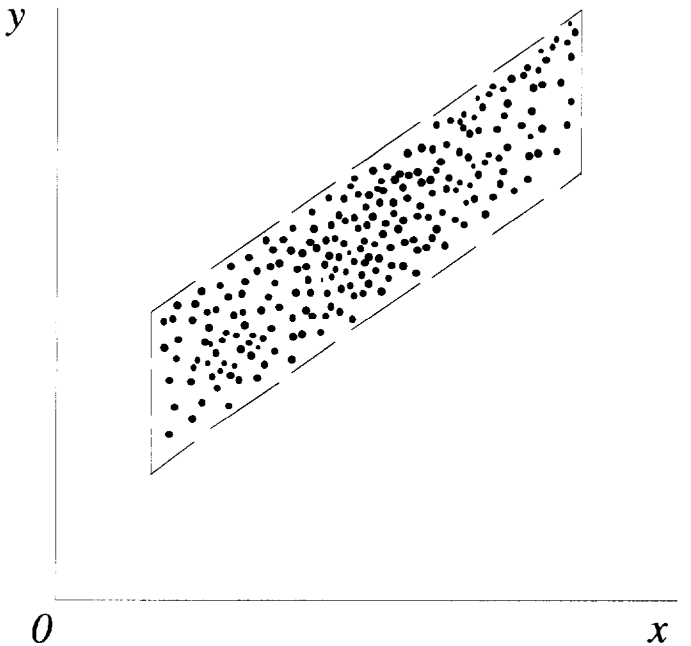
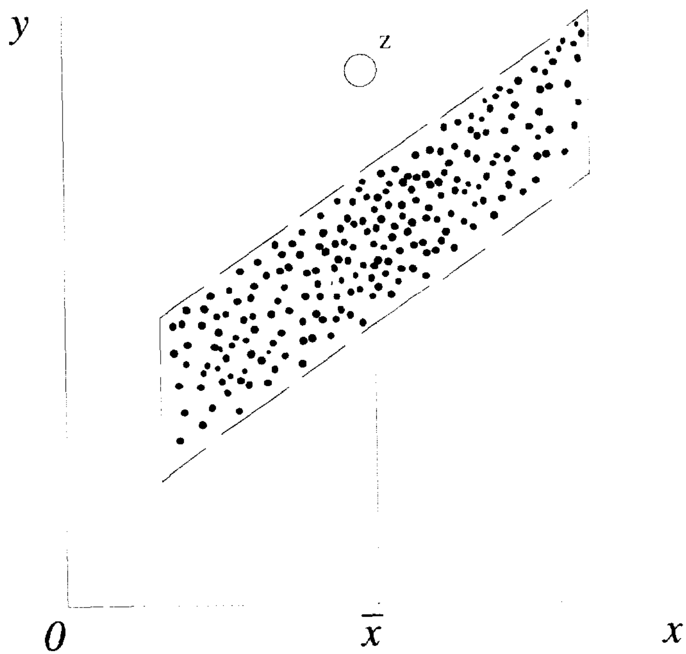


EXHIBIT 1

Part 2



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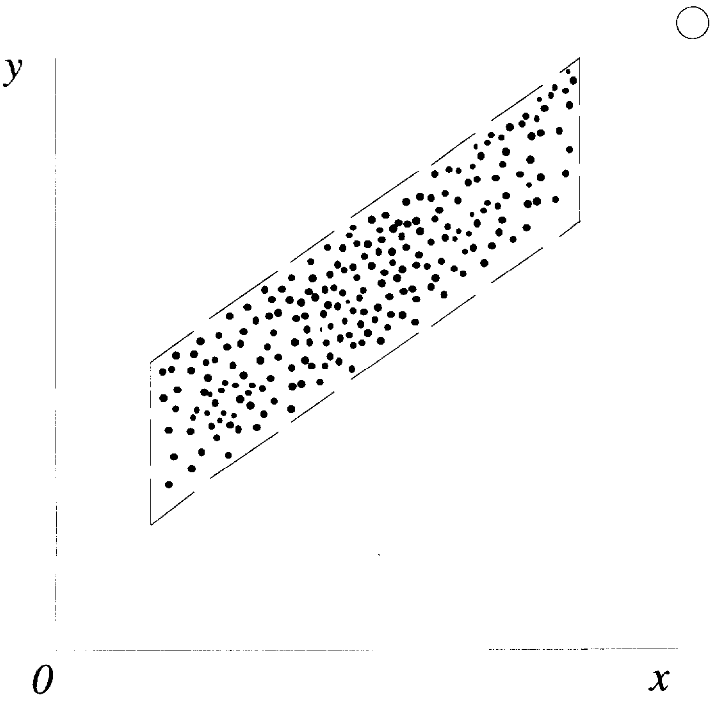
Part 3



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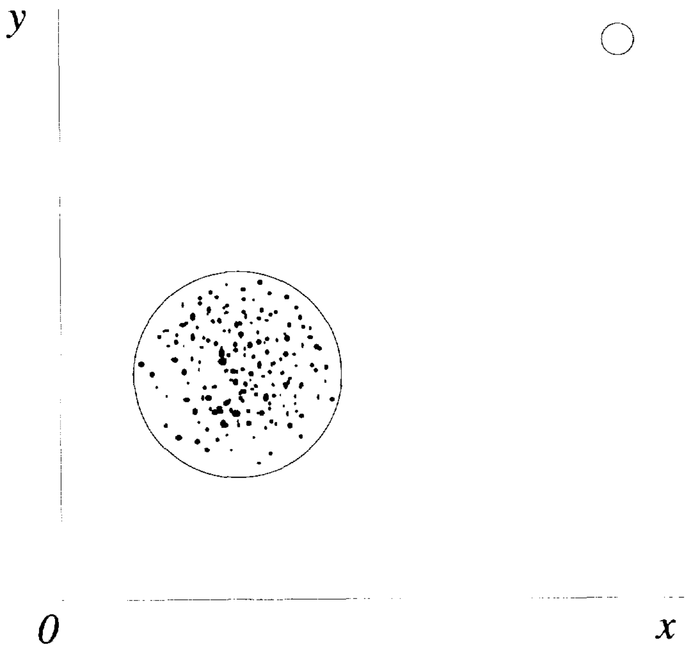
Part 4



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Part 5

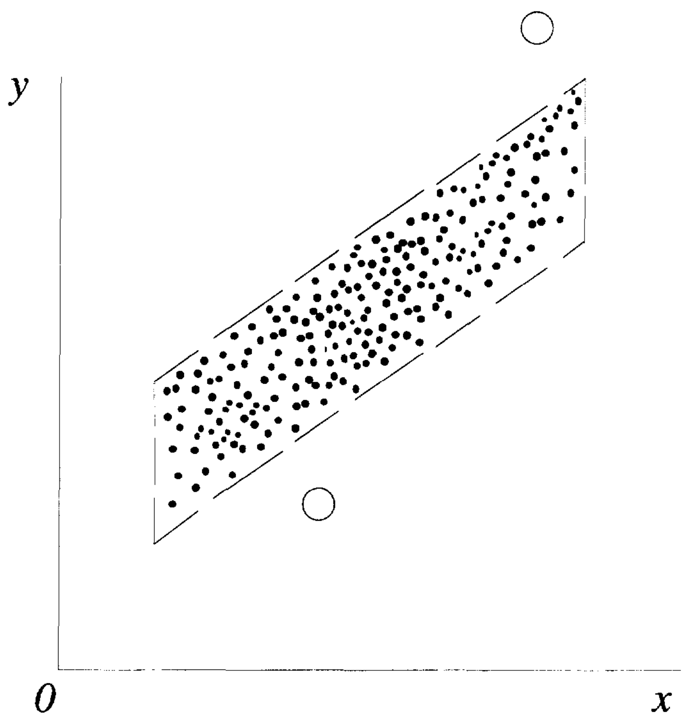

EXHIBIT 1
Part 6

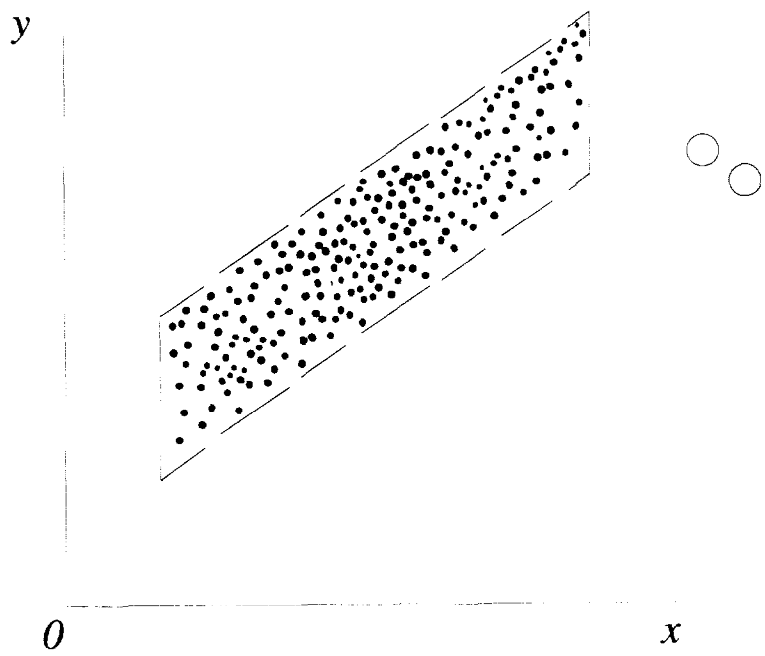

EXHIBIT 2
Part 1

REGRESSION ANALYSIS

	Parameter Estimates	Standard Error	t Statistic	Probability > $\|t\|$
Intercept	0.7016	0.5387	1.302	0.2046
Coefficient	0.8079	0.0628	12.870	0.0001

Adjusted R-Squared: 0.8636

X	Y	Predicted Value Y	Std. Err. of Pred.	Residual	Std. Err. of Residual
2.50	2.10	2.7214	0.3985	-0.6214	1.0050
3.00	2.30	3.1254	0.3721	-0.8254	1.0151
3.00	3.30	3.1254	0.3721	0.1746	1.0151
3.00	4.30	3.1254	0.3721	1.1746	1.0151
4.00	3.30	3.9333	0.3220	-0.6333	1.0321
4.00	5.40	3.9333	0.3220	1.4667	1.0321
5.00	4.30	4.7413	0.2771	-0.4413	1.0450
5.50	5.60	5.1453	0.2574	0.4547	1.0500
6.00	7.30	5.5492	0.2403	1.7508	1.0541
6.50	4.40	5.9532	0.2262	-1.5532	1.0572
7.00	6.00	6.3572	0.2158	-0.3572	1.0594
7.00	6.50	6.3572	0.2158	0.1428	1.0594
7.50	7.50	6.7611	0.2097	0.7389	1.0606
8.20	6.30	7.3267	0.2088	-1.0267	1.0608
9.00	6.60	7.9731	0.2189	-1.3731	1.0587
9.00	8.50	7.9731	0.2189	0.5269	1.0587
9.00	9.00	7.9731	0.2189	1.0269	1.0587
9.50	6.50	8.3770	0.2306	-1.8770	1.0562
10.00	9.30	8.7810	0.2458	0.5190	1.0528
10.50	8.00	9.1850	0.2639	-1.1850	1.0484
11.00	8.50	9.5890	0.2843	-1.0890	1.0431
11.00	10.60	9.5890	0.2843	1.0110	1.0431
12.00	10.00	10.3969	0.3302	-0.3969	1.0295
12.00	11.50	10.3969	0.3302	1.1031	1.0295
12.50	12.00	10.8009	0.3552	1.1991	1.0211
13.00	10.00	11.2048	0.3810	-1.2048	1.0117
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EXHIBIT 2
Part 2

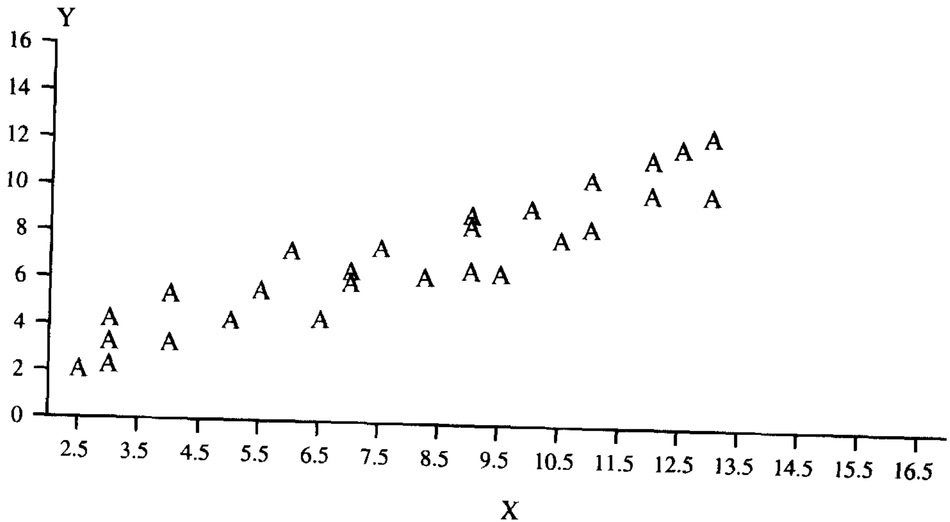


EXHIBIT 2
Part 3

REGRESSION ANALYSIS

	Parameter Estimates	Standard Error	t Statistic	Probability > $\|t\|$
Intercept	1.1709	0.9196	1.273	0.2142
Coefficient	0.7840	0.1078	7.276	0.0001

Adjusted R-Squared: 0.6579

X	Y	Predicted Value Y	Std. Err. of Pred.	Residual	Std. Err. of Residual
2.50	2.10	3.1319	0.6784	-1.0319	1.7312
3.00	2.30	3.5241	0.6329	-1.2241	1.7484
3.00	3.30	3.5241	0.6329	-0.2241	1.7484
3.00	4.30	3.5241	0.6329	0.7759	1.7484
4.00	3.30	4.3085	0.5465	-1.0085	1.7773
4.00	5.40	4.3085	0.5465	1.0915	1.7773
5.00	4.30	5.0929	0.4691	-0.7929	1.7992
5.50	5.60	5.4851	0.4353	0.1149	1.8077
6.00	7.30	5.8772	0.4058	1.4228	1.8146
6.50	4.40	6.2694	0.3817	-1.8694	1.8198
7.00	6.00	6.6616	0.3640	-0.6616	1.8234
7.00	6.50	6.6616	0.3640	-0.1616	1.8234
7.50	7.50	7.0538	0.3538	0.4462	1.8254
8.20	6.30	7.6029	0.3531	-1.3029	1.8256
9.00	6.60	8.2304	0.3715	-1.6304	1.8219
9.00	8.50	8.2304	0.3715	0.2696	1.8219
9.00	9.00	8.2304	0.3715	0.7696	1.8219
9.50	6.50	8.6226	0.3923	-2.1226	1.8175
10.00	9.30	9.0148	0.4191	0.2852	1.8116
10.50	8.00	9.4070	0.4507	-1.4070	1.8039
11.00	8.50	9.7992	0.4863	-1.2992	1.7947
11.00	10.60	9.7992	0.4863	0.8008	1.7947
12.00	10.00	10.5836	0.5662	-0.5836	1.7711
12.00	11.50	10.5836	0.5662	0.9164	1.7711
12.50	12.00	10.9757	0.6094	1.0243	1.7567
13.00	10.00	11.3679	0.6542	-1.3679	1.7405
13.00	12.50	11.3679	0.6542	1.1321	1.7405
7.00	14.30	6.6616	0.3640	7.6384	1.8234

EXHIBIT 2
Part 4

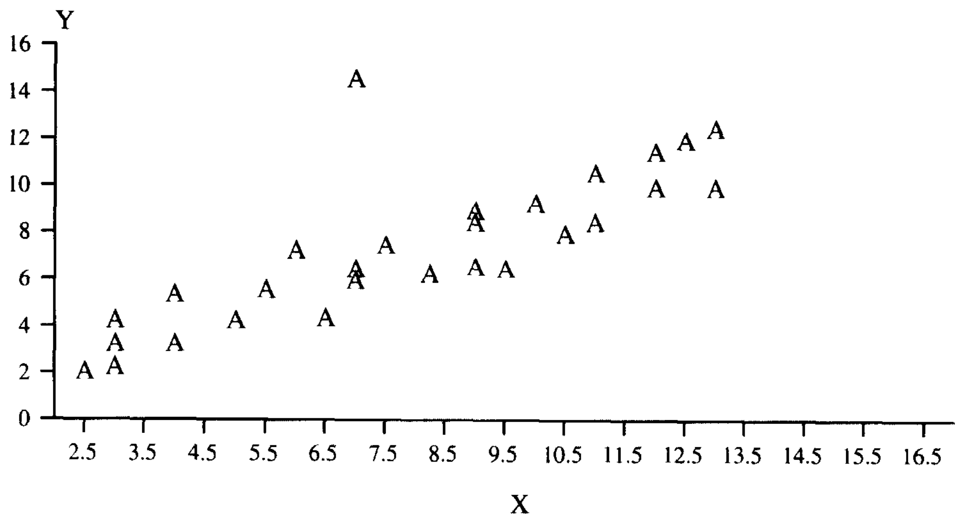


EXHIBIT 2
Part 5

REGRESSION ANALYSIS

	Parameter Estimates	Standard Error	t Statistic	Probability > $\|t\|$
Intercept	0.7229	0.4930	1.466	0.1546
Coefficient	0.8048	0.0547	14.713	0.0001

Adjusted R-Squared: 0.8887

X	Y	Predicted Value Y	Std. Err. of Pred.	Residual	Std. Err. of Residual
2.50	2.10	2.7348	0.3723	-0.6348	0.9929
3.00	2.30	3.1372	0.3496	-0.8372	1.0011
3.00	3.30	3.1372	0.3496	0.1628	1.0011
3.00	4.30	3.1372	0.3496	1.1628	1.0011
4.00	3.30	3.9420	0.3064	-0.6420	1.0152
4.00	5.40	3.9420	0.3064	1.4580	1.0152
5.00	4.30	4.7467	0.2674	-0.4467	1.0261
5.50	5.60	5.1491	0.2502	0.4509	1.0305
6.00	7.30	5.5515	0.2348	1.7485	1.0341
6.50	4.40	5.9539	0.2218	-1.5539	1.0369
7.00	6.00	6.3562	0.2115	-0.3562	1.0391
7.00	6.50	6.3562	0.2115	0.1438	1.0391
7.50	7.50	6.7586	0.2044	0.7414	1.0405
8.20	6.30	7.3220	0.2004	-1.0220	1.0413
9.00	6.60	7.9658	0.2047	-1.3658	1.0404
9.00	8.50	7.9658	0.2047	0.5342	1.0404
9.00	9.00	7.9658	0.2047	1.0342	1.0404
9.50	6.50	8.3681	0.2119	-1.8681	1.0390
10.00	9.30	8.7705	0.2223	0.5295	1.0368
10.50	8.00	9.1729	0.2354	-1.1729	1.0339
11.00	8.50	9.5753	0.2509	-1.0753	1.0303
11.00	10.60	9.5753	0.2509	1.0247	1.0303
12.00	10.00	10.3801	0.2871	-0.3801	1.0208
12.00	11.50	10.3801	0.2871	1.1199	1.0208
12.50	12.00	10.7824	0.3073	1.2176	1.0149
13.00	10.00	11.1848	0.3285	-1.1848	1.0082
13.00	12.50	11.1848	0.3285	1.3152	1.0082
17.00	14.30	14.4039	0.5192	-0.1039	0.9246

EXHIBIT 2
Part 6

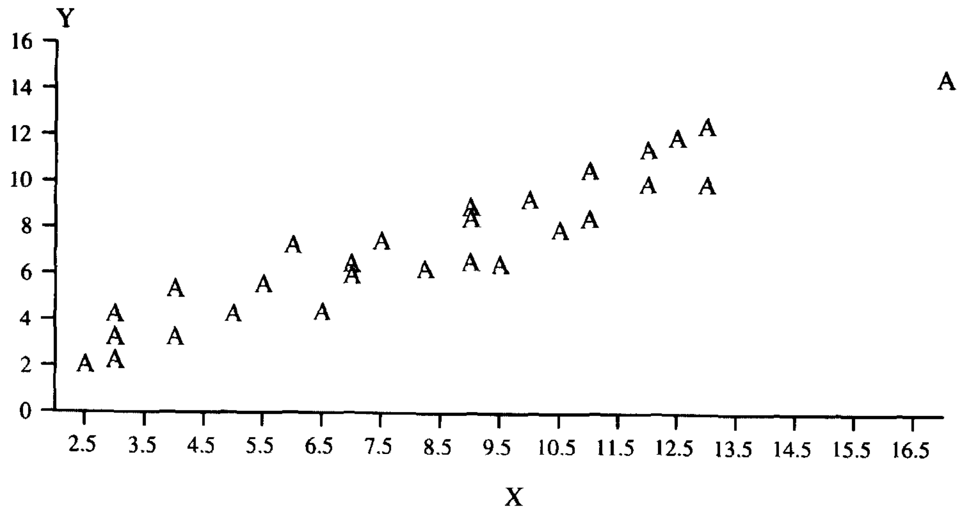


EXHIBIT 2
Part 7

REGRESSION ANALYSIS

	Parameter Estimates	Standard Error	t Statistic	Probability > $\|t\|$
Intercept	1.3944	0.6061	2.3006	0.0297
Coefficient	0.7046	0.0672	10.4851	0.0001

Adjusted R-Squared: 0.8013

X	Y	Predicted Value Y	Std. Err. of Pred.	Residual	Std. Err. of Residual
2.50	2.10	3.1560	0.4577	-1.0560	1.2206
3.00	2.30	3.5083	0.4298	-1.2083	1.2307
3.00	3.30	3.5083	0.4298	-0.2083	1.2307
3.00	4.30	3.5083	0.4298	1.2417	1.2307
4.00	3.30	4.2129	0.3767	-0.9129	1.2480
4.00	5.40	4.2129	0.3767	1.1871	1.2480
5.00	4.30	4.9175	0.3288	-0.6175	1.2615
5.50	5.60	5.2698	0.3076	0.3302	1.2668
6.00	7.30	5.6221	0.2887	1.6779	1.2713
6.50	4.40	5.9745	0.2727	-1.5745	1.2748
7.00	6.00	6.3268	0.2601	-0.3268	1.2774
7.00	6.50	6.3268	0.2601	0.1732	1.2774
7.50	7.50	6.6791	0.2513	0.8209	1.2792
8.20	6.30	7.1723	0.2464	-0.8723	1.2801
9.00	6.60	7.7360	0.2516	-1.1360	1.2791
9.00	8.50	7.7360	0.2516	0.7640	1.2791
9.00	9.00	7.7360	0.2516	1.2640	1.2791
9.50	6.50	8.0883	0.2605	-1.5883	1.2773
10.00	9.30	8.4406	0.2733	0.8594	1.2747
10.50	8.00	8.7930	0.2895	-0.7930	1.2711
11.00	8.50	9.1453	0.3084	-0.6453	1.2666
11.00	10.60	9.1453	0.3084	1.4547	1.2666
12.00	10.00	9.8499	0.3530	0.1501	1.2549
12.00	11.50	9.8499	0.3530	1.6501	1.2549
12.50	12.00	10.2022	0.3778	1.7978	1.2477
13.00	10.00	10.5545	0.4038	-0.5545	1.2395
13.00	12.50	10.5545	0.4038	1.9455	1.2395
17.00	10.00	13.3730	0.6383	-3.3730	1.1367

EXHIBIT 2
Part 8

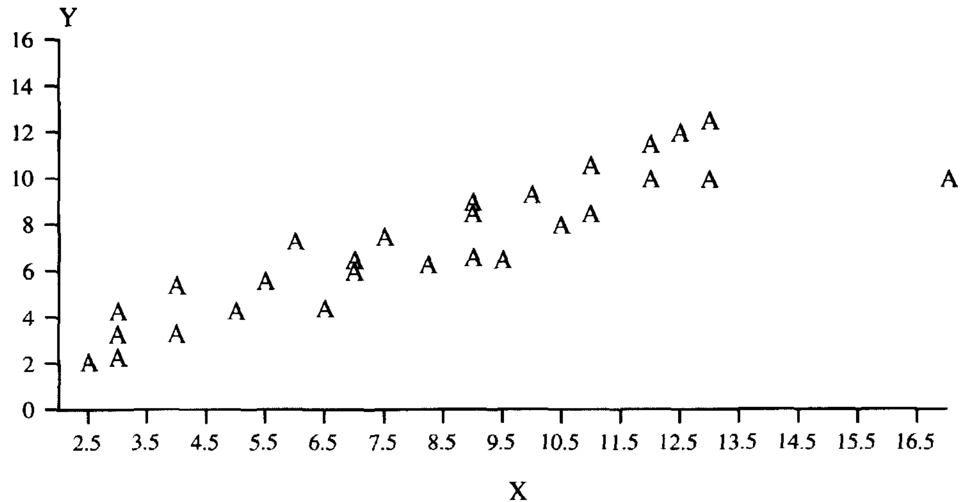


EXHIBIT 3
Part 1

ANALYSIS OF REGRESSION MODEL

	Parameter Estimates	Standard Error	*t* Statistic	Probability > |*t*|
Intercept	-54.262	7.051	-7.696	0.0001
GNP	2.570	0.449	5.724	0.0007
CPI	1.024	0.037	27.676	0.0001

Adjusted R-Squared: 0.9916

Obs	PP	GNP	CPI	Fitted PP	Residual
1	105.0	3.00	148	105.0	-0.0446
2	111.0	3.07	153	110.3	0.6540
3	114.0	2.97	157	114.2	-0.1863
4	118.0	4.14	158	118.2	-0.2171
5	126.0	4.00	166	126.1	-0.0518
6	128.0	3.42	170	128.7	-0.6586
7	134.0	2.67	177	133.9	0.0985
8	134.0	1.78	178	132.6	1.3612
9	131.0	1.09	180	132.9	-1.9144
10	138.0	1.50	183	137.0	0.9591

	Residual Plot						
Obs	-2	-1	0	1	2	Hat Diagonal	Cook's *D*
1						0.4818	0.001
2				*		0.2808	0.073
3						0.1939	0.003
4						0.3069	0.009
5						0.3664	0.001
6		*				0.2345	0.055
7						0.2108	0.001
8				**		0.2147	0.203
9		****				0.3979	1.269
10				**		0.3125	0.192

EXHIBIT 3
Part 2

ANALYSIS OF REGRESSION MODEL

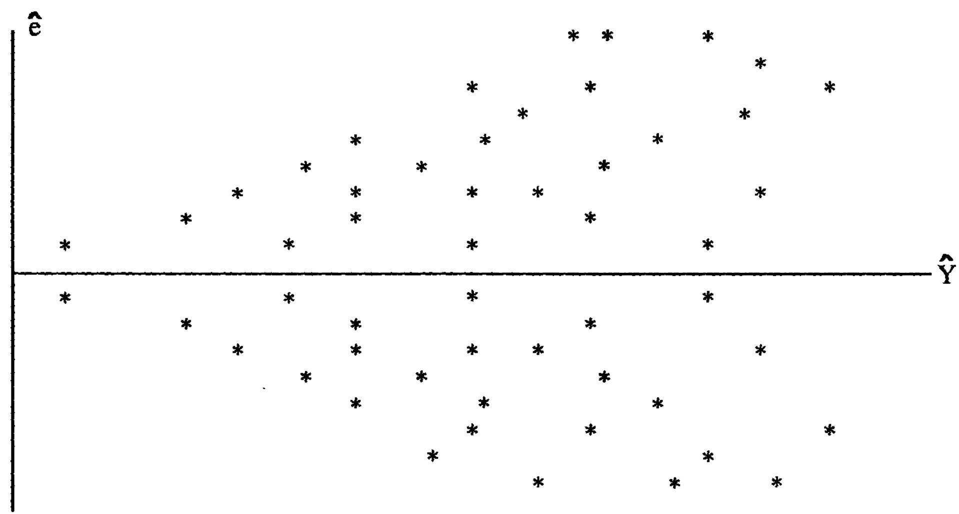
	Parameter Estimates	Standard Error	t Statistic	Probability > $\|t\|$
Intercept	-51.960	3.234	-16.067	0.0001
GNP	1.987	0.232	8.565	0.0001
CPI	1.022	0.017	60.118	0.0001

Adjusted R-Squared: 0.9984

Obs	PP	GNP	CPI	Fitted PP	Residual
1	105.0	3.00	148	105.3	-0.2673
2	111.0	3.07	153	110.5	0.4832
3	114.0	2.97	157	114.4	-0.4064
4	118.0	4.14	158	117.8	0.2471
5	126.0	4.00	166	125.7	0.3487
6	128.0	3.42	170	128.6	-0.5874
7	134.0	2.67	177	134.3	-0.2519
8	134.0	1.78	178	133.5	0.4941
9	138.0	1.50	183	138.1	-0.0600

```
              Residual Plot
Obs     -2   -1    0    1    2    Hat Diagonal   Cook's D
 1                                   0.4899       0.206
 2     |           |**         |     0.2856       0.200
 3     |          *|           |     0.2018       0.080
 4     |           |*          |     0.3423       0.074
 5     |           |*          |     0.3928       0.198
 6     |         **|           |     0.2353       0.212
 7     |          *|           |     0.2310       0.038
 8     |           |**         |     0.3382       0.288
 9     |           |           |     0.4831       0.010
```

EXHIBIT 4



UNBIASED LOSS DEVELOPMENT FACTORS

DANIEL M. MURPHY

Abstract

Casualty Actuarial Society literature is inconclusive regarding whether the loss development technique is biased or unbiased, or which of the traditional methods of estimating link ratios is best. This paper frames the development process in a least squares regression model so that those questions can be answered for link ratio estimators commonly used in practice, and for two new average development factor formulas. As a byproduct, formulas for variances of point estimates of ultimate loss and loss reserves are derived that reflect both parameter risk and process risk. An approach to measuring confidence intervals is proposed. A consolidated industry workers' compensation triangle is analyzed to demonstrate the concepts and techniques. The results of a simulation study suggest that in some situations the alternative average loss development factor (LDF) formulas may outperform the traditional estimators, and that the performance of the incurred loss development technique can approach that of the Bornhuetter-Ferguson and Stanard-Bühlmann techniques.

1. INTRODUCTION

Three common methods of estimating link ratios are the Simple Average Development (SAD) method—the arithmetic average of the link ratios; the Weighted Average Development (WAD) method—the sum of losses at the end of the development period divided by the sum of the losses at the beginning; and the Geometric Average Development (GAD) method—the n^{th} root of the product of n link ratios. Casualty actuarial literature is inconclusive regarding which method is "best" or indeed whether the methods are biased or unbiased. See, for example, Stanard [9] and Robertson's discussion [7]. The purpose of this paper is to present a mathematical framework for evaluating the accuracy of these methods; to suggest alternatives; and to unearth valuable information about the variance of the estimates of developed ultimate loss.

It is assumed that the actuary has exhausted all adjustments for systematic or operational reasons why a development triangle may appear as it does, and the only concern left is how to deal with the remaining noise. Although the paper uses accident year to refer to the rows of the triangle, the theory also applies to policy year and report year triangles.

2. POINT ESTIMATES

When we say that we expect the value of incurred losses as of, say, 24 months to equal the incurred value as of 12 months times a link ratio, it is possible that what we really mean is this: the value of incurred losses as of 24 months is a random variable whose expected value is conditional on the 12 month incurred value, and equals that 12 month value times an unknown constant. Symbolically,

$$y = bx + e,$$

where x and y are the current and next evaluations, respectively; b is the unknown constant development factor, called the age-to-age factor or link ratio; and e represents random noise. The first step in developing losses is estimating the link ratios.

Expected Value of the Link Ratio

Let us first generalize, and suppose that the relationship between x and y is fully linear rather than strictly multiplicative. The more general model is

Model I $\quad y = a + bx + e.$

$E(e) = 0$; $\mathrm{Var}(e)$ is constant across accident years; and the e's are uncorrelated between accident years and are independent of x.

This model is clearly a regression of 24-month losses y on 12-month losses x. Although x is *a priori* a random variable, once an evaluation is made it is treated as a constant for the purpose of loss development. More precisely, the model says that the expected value of the random variable y conditional on the random variable x is linear in x: $E(y \mid x) = a + bx$. With this understanding of the relationship between x and y, all classical results of least squares regression may be brought to bear on the theory of loss development. See, for example, Scheffé [8]. For the remainder of this paper, all expectations are conditional on the current evaluation.

The well known Gauss-Markoff Theorem says that the Best Linear Unbiased Estimates (BLUE) of a and b are the least squares estimates, denoted $\hat{a}$ and $\hat{b}$:

$$\hat{b} = \frac{\Sigma (x - \overline{x})\, y}{\Sigma (x - \overline{x})^2}$$

and

$$\hat{a} = \overline{y} - \hat{b}\,\overline{x} \quad .$$

This model will be referred to as the Least Squares Linear (LSL) model.

Section 5 presents an argument that claim count development may follow the LSL model, supported by the simulation study of Appendix B. However, if one believes the y-intercept should truly be zero, perhaps the model to use is

Model II $\qquad y = bx + e.$

E $(e) = 0$; Var(e) is constant across accident years; and the e's are uncorrelated between accident years and are independent of x.

This model would not be appropriate if there were a significant probability that y should not equal zero when x does.

It is well known that the BLUE estimator for b under Model II is

$$\hat{b} = \frac{\Sigma xy}{\Sigma x^2}. \tag{2.1}$$

This model will be referred to as the Least Squares Multiplicative (LSM) model.

Can the LSL or LSM assumptions be revised to say something about the more common development factor averages? Take the assumption of constant variance across accident years. Triangles of incurred or paid dollars under the force of trend may not conform to this assumption. On-leveling the loss triangle may try to adjust for such heteroskedasticity, but may introduce unwelcome side effects as well. A model that speaks directly to the issue of non-constant variances is:

Model III $\qquad y = bx + xe.$

E $(e) = 0$; Var(e) is constant across accident years; and the e's are uncorrelated between accident years and independent of x.

This model differs from Model II in that it explicitly postulates a dependent relationship between the current evaluation, x, and the error term, xe. Divide both sides of this equation by x. This model also says that the ratio of consecutive evaluations is constant across accident years. In other words, it is the development percent, not the development dollars, and the random deviation in that percent that behave consistently from one accident year to the next.

This model's BLUE for b is the simple average development (SAD) factor, denoted b_{SAD}. This is easy to see. Transform Model III as follows:

Model III′ $\quad y/x = b + e$

or $\quad u = bv + e,$

where v is identically equal to unity. Formula 2.1 says that

$$\hat{b} = \frac{\Sigma vu}{\Sigma v^2} = \frac{\sum \frac{y}{x}}{\Sigma 1} = \frac{1}{n} \sum \frac{y}{x}$$

which is b_{SAD}.

One may object that the proportionality of the error term to the full value of x overemphasizes the true relationship. It may seem more plausible that the variance of y, or the square of the error term, is proportional to x. The model[1] that describes this relationship is:

Model IV $\quad y = bx + \sqrt{x}e.$

E $(e) = 0$; Var (e) is constant across accident years; and the e's are uncorrelated between accident years and independent of x.

This model's BLUE for b is the weighted average development (WAD) factor, denoted b_{WAD}. This is also easy to see. Transforming

[1] This model was inspired by Dr. Thomas Mack at the presentation of his 1993 Theory of Risk prize paper "Measuring the Variability of Chain Ladder Reserve Estimates."

Model IV by dividing both sides by $\sqrt{x}$ turns it into a simple regression of $u = y/\sqrt{x}$ onto $v = \sqrt{x}$. Formula (2.1) becomes:

$$\hat{b} = \frac{\Sigma uv}{\Sigma v^2} = \frac{\sum \frac{y}{\sqrt{x}}\sqrt{x}}{\Sigma x} = \frac{\Sigma y}{\Sigma x},$$

which is b_{WAD}. Thus, the weighted average is the best estimator if the variance of the development error is proportional to the beginning evaluation.

A fifth model that can also adjust for trend is:

Model V $\quad y = bxe.$

E $(e) = 1$; Var(e) is constant across accident years; and the e's are uncorrelated between accident years and independent of x.

This model says that random noise shocks the development process multiplicatively, and may be appropriate in those situations in which the random error in the percentage development is itself expected to be skewed. The BLUE for b under Model V is the geometric average development (GAD) factor, denoted b_{GAD}. Indeed, transform Model V by taking the logarithm of both sides:

$$\ln y = \ln b + \ln x + \ln e$$

or

$$\ln y - \ln x = \ln b + \ln e$$

which is of the form

$$u = b'v + e'$$

where $b' = \ln b$, $v = 1$, and E $(e') = 0$. Then Formula (2.1) simplifies to:

$$\hat{b}' = \frac{\Sigma uv}{\Sigma v^2} = \frac{1}{n}\Sigma u = \frac{1}{n}\Sigma(\ln y - \ln x) = \frac{1}{n}\Sigma \ln\frac{y}{x}.$$

Therefore, the least squares estimator of the "untransformed" parameter b is:

$$\hat{b} = e^{\hat{b}'} = \exp\left(\frac{1}{n}\sum \ln\frac{y}{x}\right) = \left(\exp\sum \ln\frac{y}{x}\right)^{1/n} = \left(\Pi\frac{y}{x}\right)^{1/n}$$

which is b_{GAD}.

For the remainder of the paper, the Linear model will refer to LSL. The Multiplicative models will refer to Models II to V—LSM, SAD, WAD, and GAD—unless otherwise noted.

Estimate of the Next Evaluation

The following point estimates of the expected value of incurred losses as of the next evaluation given the current evaluation are unbiased under the assumptions of their respective models:[2]

Linear	Multiplicative
$\hat{y} = \hat{a} + \hat{b}x$	$\hat{y} = \hat{b}x$.

Estimated Ultimate Loss: A Single Accident Year

The Chain Ladder Method states that if b_1 is a link ratio from 12 to 24 months, b_2 is a link ratio from 24 to 36 months, etc., and if U is the number of links required to reach ultimate, then $B_U = b_1\, b_2 \ldots b_U$ is the (to-ultimate) loss development factor (LDF). The implicit assumption is that future development is independent of prior development. This assumption may not hold in practice when, for example,

[2] Theorem 1 in Appendix C proves this for the linear model. The proof for the multiplicative models is similar.

management issues orders for a one-time-only strengthening in case reserves.

This all-important Chain Ladder Independence Assumption (CLIA) says that the relationship between consecutive evaluations does not depend on the relationship between any other pair of consecutive evaluations. In mathematical terms, the random variable corresponding to losses evaluated at one point in time *conditional on the previous evaluation* is independent of any other evaluation *conditional on its previous evaluation*. A direct result of this assumption is the fact that an unbiased estimate of a loss development factor is the product of the unbiased link ratio estimates; symbolically, $\hat{B}_U = \hat{b}_1 \hat{b}_2 \dots \hat{b}_U$.

The very simplicity of the closed form LDF is one of the beauties of the multiplicative chain ladder method. But a closed form, to-ultimate expression is not necessary, and quite cumbersome for the more general LSL approach. Instead, this paper proposes the use of a recursive formula. A recursive estimate of developing ultimate loss illuminates the missing portion of the triangle (clarifying the communication of the analysis to management and clients), enables the actuary to switch models mid-chain, and is straightforward to program, even in a spreadsheet. Perhaps the most compelling reason, however, is that a recursive estimate is invaluable for calculating variances of predicted losses. (See Section 3.)

The mathematical theory for developing recursive estimates of ultimate loss conditional on the current evaluation proceeds as follows. Consider a single fixed accident year. Let x_0 denote the (known) current evaluation and let $x_n \mid x_0$ denote the random variable corresponding to the n^{th} subsequent (unknown) evaluation conditional on the current evaluation. The goal is to find an unbiased estimator for $x_n \mid x_0$.

By definition, an unbiased estimate of $x_n \mid x_0$ is one which estimates $\mu_n = E(x_n \mid x_0)$. Let $\hat{\mu}_n$ denote such an estimate of μ_n. Theorem 2 (Appendix C) proves that the $\hat{\mu}_n$ defined according to the recursive formulas in Table 2.1 are unbiased under the assumptions of their respective models.

TABLE 2.1

POINT ESTIMATE — $\hat{\mu}_n$

FUTURE VALUE OF A SINGLE ACCIDENT YEAR
n TIME PERIODS IN THE FUTURE

Model	$n = 1$	$n > 1$
Linear	$\hat{a}_1 + \hat{b}_1 x_0$	$\hat{a}_n + \hat{b}_n \hat{\mu}_{n-1}$
Multiplicative	$\hat{b}_1 x_0$	$\hat{b}_n \hat{\mu}_{n-1}$

An unbiased estimate of ultimate loss conditional on the current evaluation is therefore $\hat{\mu}_U$.

Estimated Total Ultimate Loss: Multiple Accident Years

An estimate of total ultimate loss for more than one accident year combined could be obtained by simply adding up the separate accident year $\hat{\mu}_U$'s. However, a recursive expression is preferred primarily for the purpose of calculating variances because development estimates of ultimate loss for different accident years are not independent.

Notation quickly obscures the derivation, but the idea of a recursive estimate of total ultimate loss for multiple accident years is this. Start at the bottom left corner of the triangle and develop the youngest accident year to the next age. Then, add that estimate to the current evaluation of the second youngest accident year, and develop the sum to the next age. Continue recursively. An unbiased estimate of total losses at ultimate will be the final sum.

The formulas are developed as follows. To keep the indices from becoming too convoluted, index the rows of the triangle in reverse order so that the youngest accident year is the zero row, the next youngest is row 1, and so on. Next, index the columns so that the 12 month column is the zero column, the 24 month column is column 1, etc. A full triangle of $N + 1$ accident years appears in Figure 1. Let

$$S_n = \sum_{i=0}^{n-1} x_{i,n} \mid x_{i,i}$$

denote the sum of the accident years' future evaluations conditional on the accident years' current evaluations, and set $M_n = \mathrm{E}\,(S_n)$. We are looking for an unbiased estimate $\hat{M}_n$ of M_n. Recursive formulas for $\hat{M}_n$ are given in Table 2.2. (See Theorem 9 in Appendix C.)

FIGURE 1

NOTATION FOR THE KNOWN AND UNKNOWN PORTIONS OF A LOSS TRIANGLE

	Age of Accident Year									
A/Y	0	1	2		n-1	n	n+1		N-1	N
N	$x_{N,0}$	$x_{N,1}$	$x_{N,2}$	...	$x_{N,n-1}$	$x_{N,n}$	$x_{N,n+1}$	...	$x_{N,N-1}$	$x_{N,N}$
N-1	$x_{N-1,0}$	$x_{N-1,1}$	$x_{N-1,2}$	...	$x_{N-1,n-1}$	$x_{N-1,n}$	$x_{N-1,n+1}$	...	$x_{N-1,N-1}$	$x_{N-1,N} \mid x_{N-1,N-1}$
N-2	$x_{N-2,0}$	$x_{N-2,1}$	$x_{N-2,2}$	...	$x_{N-2,n-1}$	$x_{N-2,n}$	$x_{N-2,n+1}$	...	$x_{N-2,N-1} \mid x_{N-2,N-2}$	$x_{N-2,N} \mid x_{N-2,N-2}$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
n	$x_{n,0}$	$x_{n,1}$	$x_{n,2}$	...	$x_{n,n-1}$	$x_{n,n}$	$x_{n,n+1} \mid x_{n,n}$	...	$x_{n,N-1} \mid x_{n,n}$	$x_{n,N} \mid x_{n,n}$
n-1	$x_{n-1,0}$	$x_{n-1,1}$	$x_{n-1,2}$	...	$x_{n-1,n-1}$	$x_{n-1,n} \mid x_{n-1,n-1}$	$x_{n-1,n+1} \mid x_{n-1,n-1}$	...	$x_{n-1,N-1} \mid x_{n-1,n-1}$	$x_{n-1,N} \mid x_{n-1,n-1}$
n-2	$x_{n-2,0}$	$x_{n-2,1}$	$x_{n-2,2}$	...	$x_{n-2,n-1} \mid x_{n-2,n-2}$	$x_{n-2,n} \mid x_{n-2,n-2}$	$x_{n-2,n+1} \mid x_{n-2,n-2}$	...	$x_{n-2,N-1} \mid x_{n-2,n-2}$	$x_{n-2,N} \mid x_{n-2,n-2}$
	$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
1	$x_{1,0}$	$x_{1,1}$	$x_{1,2} \mid x_{1,1}$	...	$x_{1,n-1} \mid x_{1,1}$	$x_{1,n} \mid x_{1,1}$	$x_{1,n+1} \mid x_{1,1}$	...	$x_{1,N-1} \mid x_{1,1}$	$x_{1,N} \mid x_{1,1}$
0	$x_{0,0}$	$x_{0,1} \mid x_{0,0}$	$x_{0,2} \mid x_{0,0}$	...	$x_{0,n-1} \mid x_{0,0}$	$x_{0,n} \mid x_{0,0}$	$x_{0,n+1} \mid x_{0,0}$	...	$x_{0,N-1} \mid x_{0,0}$	$x_{0,N} \mid x_{0,0}$

TABLE 2.2

POINT ESTIMATE — $\hat{M}_n$

TOTAL FUTURE VALUE OF MULTIPLE ACCIDENT YEARS
n TIME PERIODS IN THE FUTURE

Model	$n = 1$	$n > 1$
Linear	$\hat{a}_1 + \hat{b}_1 x_{0,0}$	$n\hat{a}_n + \hat{b}_n (\hat{M}_{n-1} + x_{n-1,n-1})$
Multiplicative	$\hat{b}_1 x_{0,0}$	$\hat{b}_n (\hat{M}_{n-1} + x_{n-1,n-1})$

Estimated Reserves for Outstanding Loss

Assuming paid dollars to date are not expected to be adjusted significantly,[3] an unbiased estimate of outstanding loss for a single accident year is $\hat{\mu}_U$ – paid to date. For multiple accident years, an unbiased estimate is $\hat{M}_U$ – total paid to date.

3. VARIANCE

The least squares point estimators of development factors, ultimate losses, or reserves are functions of random variables. As such, they are themselves random variables with their own inherent variances. Estimates of these variances will be addressed in turn.

Variance of the Link Ratio Estimators

For the LSL or LSM models, the formula for the variance of the link ratio estimator is a straightforward result of least squares theory. For the other models, one must first transform the data so that the model takes on the usual regression form (i.e., the error term does not involve x).[4] Once the regression theory yields up the estimate of

[3]Which is not true if salvage, subrogation, or deductible recoveries could be significant.

[4]Model III, for example.

$\text{Var}(\hat{b})$, one applies that to the original, "untransformed" data in the formulas for estimated future losses (below).

We will adopt the convention that a "hat" (^) over a quantity denotes an unbiased estimate of that quantity. Unbiased estimates of the variances of the link ratio estimators are given in Table 3.1. These formulas can be found in many statistics texts. See Miller and Wichern [6], for example.

TABLE 3.1
ESTIMATES OF THE VARIANCES OF THE LINK RATIO ESTIMATORS

Model	$\hat{\text{V}}\text{ar}(\hat{a})$	$\hat{\text{V}}\text{ar}(\hat{b})$
LSL	$\dfrac{\Sigma x^2}{n\Sigma(x-\bar{x})^2}\hat{\sigma}^2$	$\dfrac{\hat{\sigma}^2}{\Sigma(x-\bar{x})^2}$
LSM	n/a	$\dfrac{\hat{\sigma}^2}{\Sigma x^2}$

The average "x value" $\bar{x}=\frac{1}{I}\sum x_i$ is the average of the known evaluations of prior accident years as of the age of the link ratio being estimated; I is the number of accident years used in the average. The unbiased estimate $\hat{\sigma}^2$ of the variance σ^2 of the error term e, sometimes denoted s^2, is the Mean Square Error (MSE) of the link ratio regression. The MSE, or its square root s (sometimes referred to as the standard error of the y estimate), can be found in regression software output. Most regression software will also calculate $\hat{\text{V}}\text{ar}(\hat{b})$, or its square root (sometimes referred to as the standard error of the coefficient).

Variance of Estimated Ultimate Loss: A Single Accident Year

It is time to make an important distinction. The point estimate of ultimate loss $\hat{\mu}_U$ from Section 2 above is an estimate of the expected value of the (conditional on x_0) ultimate loss x_U. Actual ultimate loss will vary from its expected value in accordance with its inherent variation about its developed mean μ_U . As a result, the risk that actual ultimate loss will differ from the prediction $\hat{\mu}_U$ is comprised of two components.

The first component, Parameter Risk, is the variance in the estimate of the expected value of $x_U \mid x_0$. The second component, Process Risk,[5] is the inherent variability of ultimate loss about its conditional mean μ_U. Symbolically, if (conditional on x_0) ultimate loss for a given accident year is expressed as the sum of its (conditional) mean plus a random error term ε_U,

$$x_U \mid x_0 = \mu_U + \varepsilon_U,$$

then the variance in the prediction of ultimate loss $pred_U$ is

$$\begin{aligned} \mathrm{Var}(pred_U) &= \mathrm{Var}(\hat{\mu}_U) + \mathrm{Var}(\varepsilon_U) \\ &= \text{Parameter Risk} + \text{Process Risk} \\ &= \text{Total Risk}. \end{aligned}$$

Tables 3.2 and 3.3 give recursive formulas for estimates of Parameter Risk and Process Risk, respectively.[6]

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[5] This Process Risk is the conditional variance of developing losses about the conditional mean. As pertains to triangles of incurred loss dollars, it includes the unconditional *a priori* process risk of the loss distribution (mitigated by the knowledge of losses emerged to date), the random variation of the claims occurrence and reporting patterns, and the random variation within case reserves.

[6] The Parameter Risk formulas are derived in Theorem 6. The Process Risk formulas are derived in Theorem 7A for LSL and LSM, Theorem 7B for WAD, and Theorem 7C for SAD. See Appendix C.

TABLE 3.2
PARAMETER RISK ESTIMATE — $\widehat{\mathrm{Var}}\,(\hat{\mu}_n)$
A SINGLE ACCIDENT YEAR

Model	$n = 1$	$n > 1$
Linear	$\dfrac{\hat{\sigma}_1^2}{I_1} + (x_0 - \bar{x}_0)^2\, \widehat{\mathrm{Var}}(\hat{b}_1)$	$\dfrac{\hat{\sigma}_n^2}{I_n} + (\hat{\mu}_{n-1} - \bar{x}_{n-1})^2\, \widehat{\mathrm{Var}}(\hat{b}_n) +$ $\hat{b}_n^2\, \widehat{\mathrm{Var}}(\hat{\mu}_{n-1}) + \widehat{\mathrm{Var}}(\hat{b}_n)\, \widehat{\mathrm{Var}}(\hat{\mu}_{n-1})$
Multiplicative	$x_0^2\, \widehat{\mathrm{Var}}(\hat{b}_1)$	$\hat{\mu}_{n-1}^2\, \widehat{\mathrm{Var}}(\hat{b}_n) +$ $\hat{b}_n^2\, \widehat{\mathrm{Var}}(\hat{\mu}_{n-1}) + \widehat{\mathrm{Var}}(\hat{b}_n)\, \widehat{\mathrm{Var}}(\hat{\mu}_{n-1})$

The average "x value"

$$\bar{x}_{n-1} = \frac{1}{I_n} \sum_{I=0}^{N} x_{i,n-1}$$

is the average of the known evaluations of prior accident years as of age $n - 1$; I_n is the number of data points in the regression estimate of development from age $n - 1$ to age n. Each of the other quantities in Table 3.2 come from the loss triangle, from x_0, from Section 2, from the regression output ($\hat{\sigma}^2$, $\widehat{\mathrm{Var}}(\hat{b})$), or from the prior recursion step ($\widehat{\mathrm{Var}}(\hat{\mu}_{n-1})$). The Multiplicative models refer to LSM, WAD, and SAD, but not GAD.[7]

[7] The regression calculation on the logarithm-transformed data will provide an estimate of the variance of the transformed parameter b', but there is no easy translation to an estimate of the variance of the original parameter b. The best way to work with the GAD model is in its transformed state. See Section 4 and Theorem 8 of Appendix C. Similarly, Tables 3.3, 3.4, and 3.5 exclude mention of the GAD model.

TABLE 3.3
PROCESS RISK ESTIMATE — $\hat{\mathrm{Var}}(x_n \mid x_0)$
A SINGLE ACCIDENT YEAR

Model	$n=1$	$n>1$
LSL, LSM	$\hat{\sigma}_1^2$	$\hat{\sigma}_n^2 + \hat{b}_n^2\,\hat{\mathrm{Var}}(x_{n-1} \mid x_0)$
WAD	$x_0\,\hat{\sigma}_1^2$	$\hat{\mu}_{n-1}\hat{\sigma}_n^2 + \hat{b}_n^2\,\hat{\mathrm{Var}}(x_{n-1} \mid x_0)$
SAD	$x_2^2\,\hat{\sigma}_1^2$	$(\hat{\mu}_{n-1}^2 + \hat{\mathrm{Var}}(x_{n-1} \mid x_0))\sigma_n^2 + \hat{b}_n^2\,\hat{\mathrm{Var}}(x_{n-1} \mid x_0)$

Each of the quantities in Table 3.3 come from the loss triangle, Section 2, the regression output, or the prior recursion step.

Note that ultimate loss is not ultimate until the final claim is closed. Suppose it takes C development periods, $C > U$, to close out the accident year. Then the estimate of ultimate loss is not of $x_U \mid x_0$ but of $x_C \mid x_0$. Although the point estimate would be the same at age C as at age U, the variances will not be the same. Even if b_n is not significantly different from unity for $n > U$, whereby parameter risk halts at age U, process risk continues to build up, so recursive estimates of $\mathrm{Var}(x_n \mid x_0)$ should be carried out beyond $n = U$.

Variance of Estimated Ultimate Loss: Multiple Accident Years

Actual total ultimate loss S_U for multiple (open) accident years will vary from the estimate $\hat{M}_U$ as a result of two sources of uncertainty: Parameter Risk—the variance in the estimate of M_U—and Process Risk—the inherent variance of S_U about its developed mean M_U. Symbolically, if we express total ultimate loss for multiple accident years (conditional on the current evaluation of all accident years) as the sum of its mean M_U plus a random error term E_U,

$$S_U = M_U + E_U,$$

then the variance in the prediction of total ultimate loss $pred_U$ is

$$\begin{aligned}\mathrm{Var}(pred_U) &= \mathrm{Var}(\hat{M}_U) + \mathrm{Var}(E_U)\\ &= \text{Parameter Risk} + \text{Process Risk}\\ &= \text{Total Risk}.\end{aligned}$$

Tables 3.4 and 3.5 give recursive formulas for estimates of Parameter Risk and Process Risk, respectively.[8]

TABLE 3.4
PARAMETER RISK ESTIMATE — $\hat{\mathrm{V}}\mathrm{ar}\,(\hat{M}_n)$
MULTIPLE ACCIDENT YEARS

Model	$n = 1$	$n > 1$
Linear	$\frac{\hat{\sigma}_1^2}{I_1} + (x_0 - \bar{x}_0)^2\,\hat{\mathrm{V}}\mathrm{ar}(\hat{b}_1)$	$n^2\frac{\hat{\sigma}_n^2}{I_n} + (\hat{M}_{n-1} + x_{n-1,n-1} - n\bar{x}_{n-1})^2\,\hat{\mathrm{V}}\mathrm{ar}(\hat{b}_n)$ $+ \hat{b}_n^2\,\hat{\mathrm{V}}\mathrm{ar}\,(\hat{M}_{n-1}) + \hat{\mathrm{V}}\mathrm{ar}\,(\hat{b}_n)\,\hat{\mathrm{V}}\mathrm{ar}\,(\hat{M}_{n-1})$
Multiplicative	$x_0^2\,\hat{\mathrm{V}}\mathrm{ar}(\hat{b}_1)$	$(\hat{M}_{n-1} + x_{n,n})^2\,\hat{\mathrm{V}}\mathrm{ar}\,(\hat{b}_n) +$ $\hat{b}_n^2\,\hat{\mathrm{V}}\mathrm{ar}\,(\hat{M}_{n-1}) + \hat{\mathrm{V}}\mathrm{ar}\,(\hat{b}_n)\hat{\mathrm{V}}\mathrm{ar}\,(\hat{M}_{n-1})$

TABLE 3.5
PROCESS RISK—$\hat{\mathrm{V}}\mathrm{ar}\,(S_n)$
MULTIPLE ACCIDENT YEARS

Model	$n = 1$	$n > 1$
LSL, LSM	$\hat{\sigma}_1^2$	$n\hat{\sigma}_n^2 + \hat{b}_n^2\,\hat{\mathrm{V}}\mathrm{ar}\,(S_{n-1})$
WAD	$x_0\,\hat{\sigma}_1^2$	$(\hat{M}_{n-1} + x_{n-1,n-1})\,\hat{\sigma}_n^2 + \hat{b}_n^2\,\hat{\mathrm{V}}\mathrm{ar}\,(S_{n-1})$
SAD	$x_2^2\,\hat{\sigma}_1^2$	$\left(x_{n-1,n-1}^2 + \sum_{i=0}^{n-2}\hat{\mu}_{i,n-1}^2 + \hat{\mathrm{V}}\mathrm{ar}\,(S_{n-1})\right)\sigma_n^2$ $+ b_n^2\,\mathrm{Var}\,(S_{n-1})$

[8]Theorems 6 and 7 of Appendix C.

Variance of Estimated Outstanding Losses: Single or Multiple Accident Years

Assume paid losses are constant at any given evaluation. Then the variance of loss reserves equals the variance of ultimate losses.

4. CONFIDENCE INTERVALS

Confidence intervals are phrased in terms of probabilities, so this discussion can no longer avoid making assumptions about the probability distribution of the error terms, e_n. The traditional assumption is that they are normally distributed or, under GAD, lognormally distributed.

Confidence Intervals Around the Link Ratios

Let α be the probability measurement of the width of the confidence interval. Table 4.1 gives two-sided $100\,\alpha\%$ confidence intervals around the true LSL link ratios (a_n, b_n), where $t_\alpha(df)$ denotes Student's t distribution with df degrees of freedom and where I_n is the number of data points used in the estimate of the n^{th} link ratio. The degrees of freedom under the linear model are $I_n - 2$ because two parameters are estimated; $df = I_n - 1$ under the multiplicative models because only the single parameter b_n need be estimated.

TABLE 4.1
$100\,\alpha\%$ CONFIDENCE INTERVALS AROUND THE LINK RATIO PARAMETERS

Model	a_n	b_n
Linear	$\hat{a}_n \pm t_{\alpha/2}(I_n - 2)\sqrt{\hat{\mathrm{V}}\mathrm{ar}(\hat{a}^n)}$	$\hat{b}_n \pm t_{\alpha/2}(I_n - 2)\sqrt{\hat{\mathrm{V}}\mathrm{ar}(\hat{b}^n)}$
Multiplicative	n/a	$\hat{b}_n \pm t_{\alpha/2}(I - 1)\sqrt{\hat{\mathrm{V}}\mathrm{ar}(\hat{b}^n)}$

These formulas could be used, for example, to test the hypothesis that a_n is not significantly different from zero or that b_n is not significantly different from unity. If the first hypothesis were true, then a multiplicative model may be preferred over the more general linear model. The second test of hypothesis would give an objective means of selecting U.

Near the tail of the triangle, the degrees of freedom drop prohibitively. Inferences about the link ratios become less precise. If it can be assumed that beyond a certain age the variances of the residuals in the development model are identical (i.e., $\sigma_i^2 = \sigma_j^2$ for all i and j greater than some value), then a single estimate of that MSE can be obtained by solving for all link ratios simultaneously.[9]

Confidence Intervals Around Estimated Ultimate Loss

This section is motivated by the GAD model because all results are exact.[10] Under the transformed GAD model (and assuming identically distributed e_n's),

$$\ln(x_n) = \ln(b_n) + \ln(x_{n-1}) + \ln(e),$$

or

$$x_n' = b_n' + x'_{n-1} + e'.$$

[9]With a moderately-sized 5 x 5 triangle the two-tailed 90 percentile t-value is only 18% greater than the smallest possible 90 percentile t-value, namely the 90 percentile point on the standard normal curve. This can be especially important for the small triangles that consultants or companies underwriting new products are wont to see. For an example of this, see the case study in Appendix A.

[10]See Theorem 8 in Appendix C. The multiplicative chain ladder method makes the probability distribution of the error term of the compound process rather intractable. The logarithmic transformation turns the GAD compound multiplicative process into a compound additive process in which case regression theory yields exact results.

The point estimate of ultimate transformed loss for a single accident year is:

$$pred' = \hat{\mu}_C' = \hat{\mu}_U' = x_0' + \sum_{j=1}^{u} \hat{b}_j' .$$

An unbiased estimate of the Total Error = Parameter Error + Process Error of the (transformed) prediction is:

$$\hat{V}\text{ar}\,(pred') = \left(C + \sum_{j=1}^{U} \frac{1}{I_j} \right) \hat{\sigma}'^2 .$$

Therefore, assuming one only wants to limit the downside risk, a one-sided $100\alpha\%$ confidence interval for ultimate loss is:

$$\hat{\mu}_C' - t_\alpha(df)\sqrt{\hat{V}\text{ar}\,(pred')}$$

where df equals the number of data points in the multiple regression less the number of estimated link ratios, u. Finally, the corresponding $100\alpha\%$ confidence interval around the "untransformed" prediction of ultimate loss is:

$$e^{\hat{\mu}'_C - t_a(df)\sqrt{\hat{V}\text{ar}\,(pred')}} .$$

With this motivation, an approximate $100\alpha\%$ one-sided confidence interval around a recursive ultimate loss prediction using any of the models is:

$$pred - t_a\,(df)\sqrt{\hat{V}\text{ar}\,(pred)},$$

where df equals the total number of data points used in all link ratio estimates less the total number of estimated parameters. Two-sided confidence intervals are similarly defined, using $\pm\, t_{\alpha/2}\,(df)$. If df is large enough, $t_\alpha(df)$ may be replaced by z_α, the standard normal point, without significant loss of accuracy. This is often done in practice, particularly in time series analysis, even when *df* is not particularly large. The *t* distribution is preferred, however, because the thinner tails of the standard normal will understate the radius of the confidence interval. For another perspective on this subject, see Gardner [3].

Confidence Intervals Around Reserves

Confidence intervals around reserves are obtained by subtracting paid dollars from the endpoints of the confidence intervals around ultimate loss, because if:

$$\alpha = \text{Prob}\left\{\text{lower bound} \le \text{ultimate loss} \le \text{upper bound}\right\},$$

then as well,

$$\alpha = \text{Prob}\left\{\text{lower bound} - \text{paid} \le \text{outstanding loss} \le \text{upper bound} - \text{paid}\right\}.$$

5. AN ARGUMENT IN SUPPORT OF A NON-ZERO CONSTANT TERM

When the current evaluation is zero but the next evaluation is not expected to be, the loss development method is abandoned. Three alternatives might be Bornhuetter-Ferguson, Stanard-Bühlmann, or a variation on frequency-severity. LSL might be a fourth possibility.

Consider the development of reported claim counts. Let *exposure* be the true ultimate number of claims for a given accident year. Assume that the reporting pattern is the same for all claims. That is, if p_n is the probability that a claim is reported before the end of the n^{th} year, then the p_n's are independent and identically distributed for all claims. Based on these assumptions, it is not difficult to show that if x_n is the cumulative number of reported claims as of the n^{th} evaluation then

$$E\,(x_n \mid x_{n-1}) = exposure\,\frac{p_n - p_{n-1}}{1 - p_{n-1}} + \frac{1 - p_n}{1 - p_{n-1}}\,x_{n-1} \tag{5.1}$$

which is of the form $a_n + b_n x_{n-1}$. Clearly the constant term a_n is non-zero until all claims are reported.

Equation 5.1 becomes surprisingly simple when the reporting pattern is exponential, as might be expected from a Poisson frequency process. In that case the LSL coefficients (a_n, b_n) are identical for every age n. This fact can be put to good use for small claim count triangles, as demonstrated in Appendix B.

The constant term a_n of Equation 5.1 is proportional to *exposure*. The slope factor b_n does not depend on *exposure* but only on the reporting pattern (the p's). Therefore, an increase in *exposure* from one accident year to the next will result in an upward, parallel shift in the development pattern. Claim count triangles, therefore, can be expected to display development samples randomly distributed about not a *single* regression line but *multiple parallel* regression lines.

Equation 5.1 may also be used as a paradigm for loss dollars, where trend may provide an upward force on *exposure*.

6. CONCLUSION

The traditional methods of calculating average development factors are the least squares estimators of an appropriately framed mathematical model. The conclusion is that link ratio averages are unbiased if the development process conforms to the specified model. If the independence assumption of the chain ladder method holds as well, the loss development method is unbiased.

A happy byproduct of the least squares perspective is that formulas for the variances of estimated ultimate loss and reserves drop right out. The formulas are particularly easy to apply if ultimate loss by accident year is estimated through an iterative procedure, rather than through a single, closed-form expression. Confidence intervals around ultimate loss and reserves can be estimated easily, although the suggested approach yields only approximate results (with a special case exception).

The simulation study in Appendix B suggests that, in some situations, the performance of the more general linear model may exceed that of the multiplicative models and may even rival that of the non-linear Bornhuetter-Ferguson and Stanard-Bühlmann methods.

Some questions for further research come to mind. Can the formulas for parameter error be used in conjunction with the collective risk model? Is there a simple way to estimate the correlation between paid and incurred triangles, and how can that information be used to derive optimal, variance-minimizing weights for making final selections from the paid and incurred development estimates? Can the theory be used to find credibility formulas for averaging link ratios from small triangles with link ratios from larger triangles? Finally, can the Chain Ladder Independence Assumption be relaxed, to allow, say, for higher-than-expected development in one period to be followed by less-than-expected development the next?

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APPENDIX A

A CASE STUDY OF INDUSTRYWIDE WORKERS' COMPENSATION

The methods of this paper are applied to the consolidated industry workers' compensation incurred loss triangle as of December 31, 1991 [1]. The data and link ratios are displayed in Exhibits A-1 and A-2. Bulk plus IBNR reserves are removed from the incurred loss and ALAE triangles of Schedule P-Part 2. We will use the loss development method based on five-year weighted average (WAD) link ratios to estimate total ultimate loss for accident years 1982 through 1991. Then we will calculate the variance of that estimate, and use it to estimate the confidence level of industry reserves for those years.

Per the text, to estimate variances for the WAD method we must first transform the data by taking the square root of all "current evaluations" x, then dividing all "future evaluations" y by $\sqrt{x}$. We will model the data in two parts: 1) for the 12:24 month link ratios, and 2) for all other link ratios simultaneously. We shall see that there are justifiable statistical reasons for splitting the triangle this way. In addition it helps demonstrate the methodology.

Exhibit A-3 runs the regression for the 12:24 month link ratios. The original data evaluated as of 12 and 24 months for the five most recent accident years—1986 through 1990—are shown, as well as the transformed data. Using a popular spreadsheet package, the regression was run on the transformed data. The regression output indicates a good fit ($R^2 = 95\%$). Note that the "x coefficient" agrees with the average link ratio in Exhibit A-2; the variance of that estimated parameter is $0.01487^2 = 0.00022$. The MSE is 13.6272, which drives not only the variance of that estimated link ratio parameter but also the process error in the development of losses from age 12 to age 24.

For setting up the multiple regression solution of the remaining link ratios—24:36 months through 108:120 months—refer to Exhibit A-4. We first build the y vector by stacking the "next" evaluations of those link ratios on top of each other. Then we create the x matrix by placing the "current" evaluation in the same row as the corresponding y value. For each successive age of development, the x values are placed in successive columns. The transformed data are shown in Exhibit A-5, and the regression output is shown in Exhibit A-6. The R^2 value is extremely high. The MSE is much lower (0.3545) than it was for 12:24 development, which suggests that it was indeed prudent to split up the triangle into two regressions. Again, note that the x coefficients correspond to the original five-year weighted averages in Exhibit A-2.

These parameters and variances are almost all that is needed to complete the triangle in Exhibit A-8. In fact, these factors will square the triangle to 120 months, but not to ultimate. Since Part 2 of Schedule P does not include a tail factor, we will estimate a tail from Part 1 as follows.

For the five oldest accident years, we will compare developed 120-month losses (actuals for accident year 1982) with ultimate losses per industry estimates as reported in Schedule P-Part 1. Under the assumption that industry ultimate losses for those relatively mature years are reasonably accurate, we will use the weighted average of that ratio as the 120:ultimate tail factor. This weighted average is subject to random variation, so we will use the techniques of the paper to estimate the MSE and variance of that tail factor estimate. This is done in Exhibit A-7.

Exhibit A-8 shows the completed triangle, followed by the variance calculations using the formulas of Tables 3.4 and 3.5. For example, the Table 2.2 recursive formula calculates the 48-month future value M_3 of accident year 1989 through 1991 losses in total as $72{,}731 = (47{,}611+21{,}624) \times 1.05051$. The Table 3.4 recursive formula calculates the Parameter Risk of that estimate as:

$$103{,}328 = (47{,}611 + 21{,}624)^2 \times 0.00216^2 + 1.05051^2 \times 73{,}370 + 0.00216^2 \times 73{,}370.$$

The Table 3.5 formula calculates the Process Risk of the projection as:

$$323{,}963 = (47{,}611 + 21{,}624) \times 0.3545 + 1.05051^2 \times 271{,}435.$$

The estimates of ultimate loss using this procedure are compared with the consolidated industry estimates in Exhibit A-9. Total projected ultimate loss and ALAE using the five year weighted averages of the link ratios, and the tail factor as estimated above, is (in millions) \$191,509. The industry carried ultimate is \$188,251, or about 1.7% less than indicated, a seemingly small difference. However, the standard deviation of the projection is only \$1,840. So the carried ultimate is about 1.77 standard deviations less than the projection. Therefore, using the Student t distribution with 30 degrees of freedom,[11] the estimated one-sided confidence level for industry reserves is about 4%.

[11]Add up the *df*'s in Exhibits A-3, A-6, and A-7.

EXHIBIT A-1

CONSOLIDATED INDUSTRY WORKERS' COMPENSATION REPORTED INCURRED LOSSES AND ALLOCATED EXPENSES BY AGE (EXCLUDING BULK + IBNR) ($ 000,000 OMITTED)

Accident Year	Age 12	24	36	48	60	72	84	96	108	120
1982	6,174	8,061	8,639	8,951	9,207	9,363	9,464	9,559	9,634	9,725
1983	6,891	9,117	9,682	10,136	10,464	10,651	10,774	10,893	11,025	
1984	8,048	10,761	11,937	12,656	13,023	13,285	13,449	13,615		
1985	8,796	12,050	13,287	14,060	14,572	14,835	15,109			
1986	9,450	13,086	14,552	15,334	15,797	16,144				
1987	10,953	15,074	16,699	17,485	17,961					
1988	12,776	17,600	19,519	20,299						
1989	13,600	19,677	21,624							
1990	14,890	21,268								
1991	15,497									

Source: Best's *Aggregates & Averages*, 1992 Edition.

EXHIBIT A-2
CONSOLIDATED INDUSTRY WORKERS' COMPENSATION LINK RATIOS

Accident Year	Development Period (Months) 12:24	24:36	36:48	48:60	60:72	72:84	84:96	96:108	108:120
1982	1.30566	1.07167	1.03614	1.02859	1.01694	1.01086	1.00995	1.00785	1.00949
1983	1.32298	1.06201	1.04683	1.03238	1.01788	1.01153	1.01108	1.01214	
1984	1.33712	1.10933	1.06023	1.02896	1.02013	1.01240	1.01234		
1985	1.36995	1.10269	1.05812	1.03641	1.01807	1.01851			
1986	1.38472	1.11204	1.05372	1.03020	1.02194				
1987	1.37619	1.10786	1.04703	1.02722					
1988	1.37757	1.10906	1.03996						
1989	1.44687	1.09892							
1990	1.42837								
Five Year Weighted Average									
	1.40597	1.10576	1.05051	1.03080	1.01927	1.01379	1.01127	1.01014	1.00949

EXHIBIT A-3

ESTIMATING THE 12:24 MONTH PARAMETER USING REGRESSION

Accident Year	y 24 months	x 12 months	$y/\sqrt{x}$ 24 months	$\sqrt{x}$ 12 months
1986	13,086	9,450	134.61	97.21
1987	15,074	10,953	144.03	104.66
1988	17,600	12,776	155.71	113.03
1989	19,677	13,600	168.73	116.62
1990	21,268	14,890	174.30	122.02

Regression Output:

Constant	0	
Std Err of y Est	3.6915	MSE = 13.6272
R Squared	95.03%	
Number of Observations	5	
Degrees of Freedom	4	

x Coefficient	1.40597
Std Err of Coef.	0.01487

EXHIBIT A-4

ESTIMATING THE 24:36 THROUGH 108:120 MONTH PARAMETERS USING REGRESSION
STEP 1: ARRANGING THE DATA BEFORE TRANSFORMATION ($ 000,000 OMITTED)

Accident		x							
Year	y	24 mos	36 mos	48 mos	60 mos	72 mos	84 mos	96 mos	108 mos
1985	13,287	12,050							
1986	14,552	13,086							
1987	16,699	15,074							
1988	19,519	17,600							
1989	21,624	19,677							
1984	12,656		11,937						
1985	14,060		13,287						
1986	15,334		14,552						
1987	17,485		16,699						
1988	20,299		19,519						
1983	10,464			10,136					
1984	13,023			12,656					
1985	14,572			14,060					
1986	15,797			15,334					
1987	17,961			17,485					
1982	9,363				9,207				
1983	10,651				10,464				
1984	13,285				13,023				
1985	14,835				14,572				
1986	16,144				15,797				
1982	9,464					9,363			
1983	10,774					10,651			
1984	13,449					13,285			
1985	15,109					14,835			
1982	9,559						9,464		
1983	10,893						10,774		
1984	13,615						13,449		
1982	9,634							9,559	
1983	11,025							10,893	
1982	9,725								9,634

EXHIBIT A-5
ESTIMATING THE 24:36 TO 108:120 MONTH PARAMETERS USING REGRESSION
STEP 2: TRANSFORMING THE DATA FOR THE REGRESSION

Accident		$\sqrt{x}$							
Year	$y/\sqrt{x}$	24 mos	36 mos	48 mos	60 mos	72 mos	84 mos	96 mos	108 mos
1985	121.04	109.77							
1986	127.21	114.39							
1987	136.02	122.77							
1988	147.13	132.66							
1989	154.15	140.27							
1984	115.84		109.26						
1985	121.97		115.27						
1986	127.11		120.63						
1987	135.30		129.23						
1988	145.29		139.71						
1983	103.94			100.68					
1984	115.76			112.50					
1985	122.89			118.57					
1986	127.57			123.83					
1987	135.83			132.23					
1982	97.58				95.95				
1983	104.12				102.29				
1984	116.41				114.12				
1985	122.89				120.71				
1986	128.44				125.69				
1982	97.81					96.76			
1983	104.39					103.20			
1984	116.69					115.26			
1985	124.05					121.80			
1982	98.25						97.28		
1983	104.95						103.80		
1984	117.40						115.97		
1982	98.54							97.77	
1983	105.64							104.37	
1982	99.08								98.15

EXHIBIT A-6
ESTIMATING THE 24:36 TO 108:120 MONTH PARAMETERS USING REGRESSION
STEP 3: RUNNING THE REGRESSION

Regression Output:		
Constant	0	
Std Err of y Est	0.5954	MSE =0.3545
R Squared	99.9%	
Number of Observations	30	
Degrees of Freedom	22	

	24:36	36:48	48:60	60:72	72:84	84:96	96:108	108:120
x Coefficient	1.10576	1.05051	1.03080	1.01927	1.01379	1.01127	1.01014	1.00949
Std Err of Coef.	0.00214	0.00216	0.00226	0.00237	0.00271	0.00324	0.00416	0.00607

EXHIBIT A-7

ESTIMATING THE TAIL FACTOR USING REGRESSION

Accident Year	Developed Losses to Age 120 (y)	Carried Ultimate (x)	Tail Factor
1982	9,725	9,966	1.02482
1983	11,130	11,355	1.02019
1984	13,884	14,081	1.01422
1985	15,581	15,720	1.00889
1986	16,877	17,141	1.01561
Wtd Avg	67,197	68,263	1.01586

Regression Matrix

Accident Year	$y/\sqrt{x}$	$\sqrt{x}$
1982	101.06	98.615
1983	107.63	105.500
1984	119.51	117.830
1985	125.93	124.820
1986	131.94	129.910

Regression Output:

Constant	0	
Std Err of y Est	0.6680	MSE = 0.4462
R Squared	99.7%	
Number of Observations	5	
Degrees of Freedom	4	

x Coefficient(s)	1.01586
Std Err of Coef.	0.00258

EXHIBIT A-8
CONSOLIDATED INDUSTRY WORKERS' COMPENSATION
COMPLETED LOSS DEVELOPMENT TRIANGLE
($ 000,000 OMITTED)

Accident	Age										
Year	12	24	36	48	60	72	84	96	108	120	Ultimate
1982	6,174	8,061	8,639	8,951	9,207	9,363	9,464	9,559	9,634	9,725	9,879
1983	6,891	9,117	9,682	10,136	10,464	10,651	10,774	10,893	11,025	11,130	11,307
1984	8,048	10,761	11,937	12,656	13,023	13,285	13,449	13,615	13,753	13,884	14,104
1985	8,796	12,050	13,287	14,060	14,572	14,835	15,109	15,280	15,435	15,581	15,828
1986	9,450	13,086	14,552	15,334	15,797	16,144	16,366	16,551	16,719	16,877	17,145
1987	10,953	15,074	16,699	17,485	17,961	18,307	18,559	18,768	18,959	19,138	19,442
1988	12,776	17,600	19,519	20,299	20,924	21,328	21,622	21,865	22,087	22,296	22,650
1989	13,600	19,677	21,624	22,716	23,415	23,866	24,196	24,468	24,716	24,951	25,346
1990	14,890	21,268	23,518	24,706	25,467	25,957	26,315	26,612	26,881	27,136	27,567
1991	15,497	21,789	24,093	25,310	26,089	26,592	26,959	27,263	27,539	27,800	28,241

n	1	2	3	4	5	6	7	8	9	10
M_n	21,789	47,611	72,731	95,896	116,050	134,017	150,806	166,088	178,794	191,509
Parameter Risk	53,070	73,370	103,328	153,825	232,678	367,838	610,182	1,091,197	2,266,302	2,574,752
Process Risk	211,184	271,435	323,963	377,671	433,552	493,096	557,499	627,671	703,340	810,521
Total Risk	264,254	344,805	427,291	531,496	666,231	860,934	1,167,681	1,718,868	2,969,642	3,385,272
Standard Deviation	514	587	654	729	816	928	1,081	1,311	1,723	1,840

EXHIBIT A-9

CONSOLIDATED INDUSTRY WORKERS' COMPENSATION
ESTIMATED REDUNDANCY/(DEFICIENCY) IN CARRIED RESERVES AND ASSOCIATED LEVEL OF CONFIDENCE
ACCIDENT YEARS 1982-1991
($ 000,000 OMITTED)

Accident Year	Estimated Ultimate	Carried Ultimate	Redundancy/ (Deficiency)
1982	9,879	9,966	87
1983	11,307	11,355	48
1984	14,104	14,081	(23)
1985	15,828	15,720	(109)
1986	17,145	17,141	(4)
1987	19,442	19,304	(138)
1988	22,650	22,217	(433)
1989	25,346	24,645	(702)
1990	27,567	26,710	(856)
1991	28,241	27,112	(1,129)
Total	191,509	188,251	(3,258)

Standard Deviation	1,840
Degrees of Freedom	30
Deficiency Ratio to Standard Deviation	-1.77
Approximate Confidence Level	4%

APPENDIX B

COMPARING THE MODELS USING SIMULATION

In the 1985 *Proceedings*, Mr. James Stanard published the results of a simulation study of the accuracy of four simple methods of estimating ultimate losses using a 5x5 incurred loss triangle. For the exposure tested[12] it was demonstrated that WAD loss development was clearly inferior to three additive methods, Bornhuetter-Ferguson (BF), Stanard-Bühlmann (SB)[13], and a little-used method called the Additive Model (ADD), because it had greater average bias and a larger variance. The three additive methods differ from the multiplicative methods in that they adjust incurred losses to date by an estimated dollar increase to reach ultimate, whereas the multiplicative methods adjust by an estimated percentage increase. ADD's estimated increase is a straightforward calculation of differences in column means, $\bar{y}-\bar{x}$. BF and SB estimated increases are more complicated functions of the data.

Stanard's simulation was replicated here to test additionally the accuracy of LSM, LSL, SAD, and GAD. The model does not attempt to predict "beyond the triangle," which is to say that the methods project incurred losses to the most mature age available in the triangle, namely the age of the first accident year. In the discussion below, "ultimate loss" refers to case incurred loss as of the most mature available age.

The LSL method was modified to use LSM in those instances when the development factors were "obviously wrong," defined to be

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[12]Normally distributed frequency with mean = 40 and standard deviation $=\sqrt{40}$ claims per year, uniform occurrence date during the year, lognormal severity with mean = \$10,400 and standard deviation = \$34,800, exponential report lag with mean = 18 months, exponential payment lag with mean = 12 months, and case reserve error proportional to a random factor equal to a lognormal random variable with mean = 1 and variance = 2, and to a systematic factor equal to the impact of trend between the date the reserve is set and the date the claim is paid.

[13]Mr. Stanard called this the "Adjustment to Total Known Losses" method, a.k.a. the "Cape Cod Method."

when either the slope or the constant term was negative. In real-life situations, this rudimentary adjustment for outliers can be expected to be improved upon with more discerning application of actuarial judgment. The reason this modification was necessary is due to the fact that a model that fits data well does not necessarily predict very well. As an extreme example, LSL provides an exact fit to the sample data for the penultimate link ratio (two equations, two unknowns), but the coefficients so determined reveal nothing about the random processes that might cause another accident year to behave differently. It is not possible to identify every conceivable factor that could explain the otherwise "unexplained" variance of a model. Such unidentified variables are reflected through the averaging process of statistical analysis: as the number of data points minus the number of parameters (the definition of degrees of freedom) increases, the model captures more of the unexplained factors and becomes a better predictor.

In Exhibits B-1 through B-4, the average bias and standard deviation of the first accident year are zero because, as stated above, the simulation defines "ultimate" to be the current age of that accident year.

Exhibit B-1: Claim Counts Only

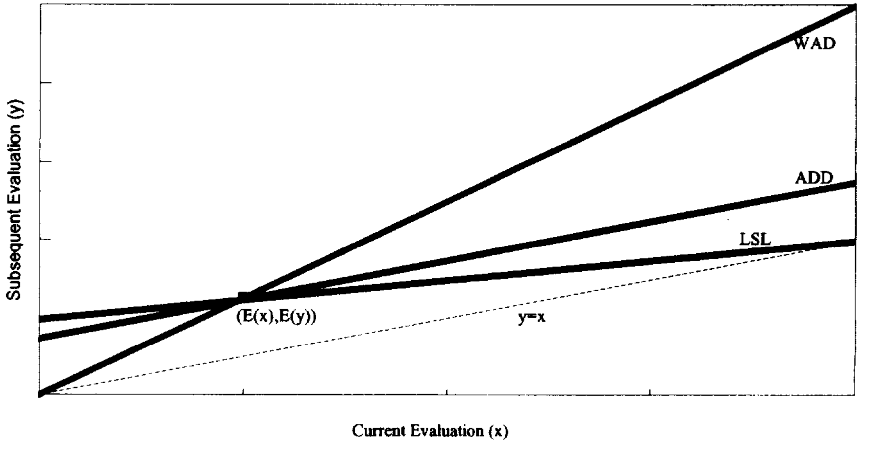
In this case, 5,000 claim count triangles were simulated; the "actual ultimate" as of the last column was simulated; accident year ultimates were predicted using the separate methods; and averages and standard deviations of the prediction errors were calculated.

LSL is the best performer, as measured by the standard deviation of the accident-year-total projection. The additive models—ADD, SB, and BF—are not far behind. Of the multiplicative estimators, LSM has the smallest bias and the smallest variance for every accident year. WAD is almost as accurate.

Why should these results not be surprising? Consider first the average bias. In Figure B-1 is graphed the relationship between incurred counts at 12 months, x, with incurred losses at 24 months, y, which we know from Section 5 of the text must be a linear relationship with a positive constant term. The ADD and WAD estimates are also shown. All relationships are shown in their idealized states where LSL is collinear with the true relationship and where the point $(\bar{x}, \bar{y})$ coincides with its expectation $(E(x), E(y))$. Note that the ADD model is parallel to the line $y = x$ because it adds the same amount for every value of x. The conditional (on x) bias is the signed, vertical distance from the estimated relationship to the true relationship. As is clear from Figure B-1, WAD and ADD can be expected to overstate y for $x > E(x)$ and understate y for $x < E(x)$. The weighted average of the conditional bias across all values of x, weighted by the probability density $f(x)$, is simulated by the average bias that appears in Exhibit B-1.

Ideally, this weighted average of the bias across all values of x should be expected to be zero, which it is for the Additive Model. ADD estimates $E(y) - E(x)$ using $\bar{y} - \bar{x}$ calculated from prior acci-

FIGURE B-1
IDEALIZED DEVELOPMENT ESTIMATORS
NO TREND



dent years. Since the environment in the first scenario—exposure, frequency, trend, etc.—does not change by accident year, the average of 5,000 simulated samples of this dollar difference across all possible values of x should get close to the true average dollar difference by the law of large numbers, so the average bias should get close to zero. For the multiplicative estimators, the average bias will probably not be zero. Take the WAD method for example. Clearly there is a positive probability (albeit small) that $\bar{x} = 0$, so the expected value of the WAD link ratio $\bar{y} / \bar{x}$ is infinity. The average of 5,000 simulations of this ratio attempts to estimate that infinite expected value, so it should not be surprising that WAD usually overstates development—and the greater the probability that $\bar{x} = 0$. the greater the overstatement.[14]

The average bias of the BF and SB methods should be greater than zero as well because the LDFs on which they rely are themselves overstated more often than not. The average LSM bias is a more complicated function of the probability distribution of x because the LSM link ratio involves x terms in the numerator and squared x terms in the denominator. The average bias appears to shift as an accident year matures. The LSL method as modified herein has residual average bias because it incorporates the biased LSM method when it detects outliers. It also seems to be the case that the bias of the estimated 4:5 year link ratio is driving the cumulative bias for the immature years.

Figure B-1 illustrates the difference between a model that is unbiased for each possible value of x, LSL, and a model which is "unbiased" only in the average, ADD. To reiterate, the purely multiplicative and purely additive estimators will understate expected development when the current evaluation is less than expected and overstate expected development when the current evaluation is greater than expected.

[14]This argument can be made more rigorous. The condition that the probability of the sample average of x be greater than zero is a sufficient but not necessary condition that $E\,(b_{WAD}) = \infty$. For a general, heuristic argument that WAD yields biased estimates, see Stanard [8].

Next, consider the variance. In simplified terms, the average bias statistic allows expected overstatements to cancel out expected understatements. This is not the case for the variance statistic. In Figure B-1 it is clear that, ideally, the ADD estimate of y will be closer to the true conditional expected value of y (the idealized LSL line) than will the WAD estimate for virtually all values of x. Thus, the variance of ADD should be less than the variance of WAD. The variance of LSL should be the smallest of all. However, LSL estimates twice as many parameters than do ADD and LSM, so it needs a larger sample size to do a comparable job. For the relatively small and thin triangles simulated here, a pure unmodified LSL estimate flops around like a fish out of water—the price it must pay to be unbiased for all values of x. In other words, in actual practice, the variance of an LSL method unmodified for outliers and applied to a triangle with few degrees of freedom will probably be horrendous. What is perhaps remarkable is the degree to which the rudimentary adjustment adopted here tames the LSL method.

Finally, let's look at what would happen if we estimated the LSL parameters under the assumption that all link ratio coefficients (a_n, b_n) are equal. We know from the previous section that this is true because the reporting pattern is exponential. The results of this model are:

SIMULATION RESULTS WHEN
ALL LINK RATIO PARAMETERS ARE ASSUMED EQUAL

A/Y	Average Bias	Std Dev Bias	Average % Bias	Std Dev % Bias	Age-Age Bias	Age-Age % Bias
1	0.000	0.000	0.000	0.000		
2	0.025	1.275	0.001	0.034	1.035	1.001
3	0.006	1.669	0.001	0.044	(0.019)	0.000
4	(0.034)	1.850	0.000	0.049	(0.040)	(0.001)
5	(0.006)	1.815	0.001	0.049	0.028	0.001
Total	(0.010)	5.064	0.000	0.027		

This model is the beneficiary of more degrees of freedom (eight—two parameters estimated from ten data points for each iteration) and as a result has the smallest average bias and variance yet. These results lead to a somewhat counter-intuitive conclusion: Information about development across immature ages sheds light on future development across mature ages. For example, the immature development just experienced by the young accident year 4 from age 1 to age 2 is a valuable data point in the estimate of the upcoming development of the old accident year 2 from age 4 to age 5. This should not be viewed simply as a bit of mathematical prestidigitation but as an example of the efficiencies that can be achieved if simplifying assumptions—even as innocuous as exponential reporting—can be justified.

Exhibit B-2: Random Severity, No Trend

In this case, 5,000 triangles of aggregate, trend-free incurred losses were simulated and the same calculations were performed.

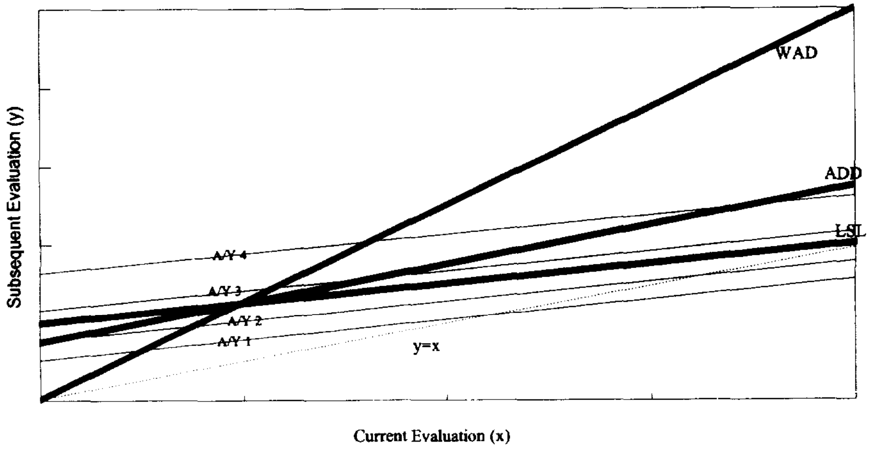
Rarely does the property/casualty actuary experience loss triangles devoid of trend, so this model is of limited interest. The introduction of uncertainty via the case reserves makes it more likely that negative development will appear, in which case LSL reverts to LSM. As a result, the additive models overtake LSL in accuracy.

Exhibit B-3: Random Severity, 8% Severity Trend Per Year

This is where it gets interesting. This could be considered the typical situation in which an actuary compiles a loss triangle that includes trend and calculates loss development factors. In this case, the environment is changing. The trending process follows the Unified Inflation Model (Butsic and Balcarek, [2]) with $\alpha = \frac{1}{2}$, which is to say that half of the impact of inflation is a function of the occurrence date and half is a function of the transaction date (e.g., evaluating the case incurred or paying the claim).

At first, one might think that a multiplicative estimator would have had a better chance of catching the trend than would an additive estimator, but such does not appear to be the case. Consider Figure B-2 which graphs expected 12-24 month development for the first four accident years. Trend has pushed the true development line upward at an 8% clip, illustrated by four thin lines. The LSL model tries to estimate the average of the development lines, the WAD estimator tries to pass through the average ($\bar{x}, \bar{y}$) midpoint of all accident years combined, and the additive estimators try to find the line parallel to the line $y=x$ which also passes through the average midpoint. Again, ADD will probably be closer than WAD to the average LSL line for every value of x for each accident year. The upward trend makes it more likely that the estimated LSL intercept will be less than zero,

FIGURE B-2
IDEALIZED DEVELOPMENT ESTIMATORS
WITH TREND



which makes it more likely that LSL reverts to LSM, so the modified LSL's variance gets closer yet to the variance of LSM.

Exhibit B-4: Random Severity, 8% Trend, On-Level Triangle

In this case, rows of the triangle were trended to the level of the most recent accident year assuming that the research department is perfect in its estimate of past trend. For most of the models, the total bias decreases from that of the not-on-level scenario while the total variance increases. LSM and WAD are virtually unchanged, GAD and SAD are exactly unchanged (of course), and the nonlinear estimates move in opposite directions.

For the most part, working with the on-level triangle does seem to improve the accuracy of estimated ultimate loss, but perhaps not to the degree one might hope. It would be interesting to see if working with separate claim count and on-level severity triangles would successfully decompose the random effects and further improve the predictions.

EXHIBIT B-1
Part 1

CLAIM COUNTS ONLY

	A/Y	Average Bias	Std Dev Bias	Average % Bias	Std Dev % Bias	Age-Age Bias	Age-Age % Bias
LSL							
	1	0.000	0.000	0.000	0.000		
	2	0.116	2.000	0.003	0.053	0.116	0.003
	3	0.153	2.772	0.004	0.073	0.037	0.001
	4	0.101	3.166	0.003	0.083	(0.052)	(0.001)
	5	0.080	3.780	0.003	0.100	(0.021)	0.000
	Total	0.451	8.251	0.002	0.043		
ADD							
	1	0.000	0.000	0.000	0.000		
	2	0.059	1.868	0.002	0.049	0.059	0.002
	3	0.075	2.847	0.002	0.075	0.016	0.000
	4	0.047	3.644	0.002	0.096	(0.028)	0.000
	5	0.096	3.692	0.003	0.097	0.049	0.001
	Total	0.277	8.407	0.001	0.044		
LSM							
	1	0.000	0.000	0.000	0.000		
	2	0.116	2.000	0.003	0.053	0.116	0.003
	3	0.143	3.321	0.004	0.087	0.027	0.001
	4	0.004	5.246	0.000	0.138	(0.139)	(0.004)
	5	(0.748)	10.536	(0.020)	0.277	(0.752)	(0.020)
	Total	(0.485)	14.009	(0.003)	0.074		
WAD							
	1	0.000	0.000	0.000	0.000		
	2	0.116	2.000	0.003	0.053	0.116	0.003
	3	0.203	3.336	0.005	0.088	0.087	0.002
	4	0.281	5.308	0.007	0.139	0.078	0.002
	5	0.888	11.101	0.023	0.292	0.607	0.016
	Total	1.488	14.520	0.008	0.076		

EXHIBIT B-1
Part 2

CLAIM COUNTS ONLY

	A/Y	Average Bias	Std Dev Bias	Average % Bias	Std Dev % Bias	Age-Age Bias	Age-Age % Bias
GAD							
	1	0.000	0.000	0.000	0.000		
	2	0.116	2.000	0.003	0.053	0.116	0.003
	3	0.234	3.345	0.006	0.088	0.118	0.003
	4	0.424	5.346	0.011	0.140	0.190	0.005
	5	1.873	11.585	0.049	0.305	1.449	0.038
	Total	2.647	14.943	0.014	0.079		
SAD							
	1	0.000	0.000	0.000	0.000		
	2	0.116	2.000	0.003	0.053	0.116	0.003
	3	0.265	3.354	0.007	0.088	0.149	0.004
	4	0.571	5.390	0.015	0.142	0.306	0.008
	5	2.958	12.268	0.078	0.322	2.387	0.062
	Total	3.910	15.530	0.021	0.082		
SB							
	1	0.000	0.000	0.000	0.000		
	2	0.102	1.940	0.003	0.051	0.102	0.003
	3	0.147	3.021	0.004	0.079	0.045	0.001
	4	0.137	3.997	0.004	0.105	(0.010)	0.000
	5	0.185	4.280	0.006	0.113	0.048	0.002
	Total	0.571	9.564	0.003	0.050		
BF							
	1	0.000	0.000	0.000	0.000		
	2	0.114	1.952	0.003	0.051	0.114	0.003
	3	0.184	3.064	0.005	0.081	0.070	0.002
	4	0.215	4.151	0.006	0.109	0.031	0.001
	5	0.338	5.164	0.010	0.136	0.123	0.004
	Total	0.851	10.626	0.004	0.056		

EXHIBIT B-2
Part 1

RANDOM SEVERITY, NO TREND

	A/Y	Average Bias	Std Dev Bias	Average % Bias	Std Dev % Bias	Age-Age Bias	Age-Age % Bias
LSL							
	1	0	0	0.000	0.000		
	2	9,206	193,945	0.026	0.302	9,206	0.026
	3	8,749	218,463	0.069	0.420	(458)	0.042
	4	30,028	429,112	0.138	0.650	21,279	0.065
	5	39,426	535,959	0.228	1.004	9,398	0.079
	Total	87,410	888,404	0.040	0.356		
LSM							
	1	0	0	0.000	0.000		
	2	9,206	193,945	0.026	0.302	9,206	0.026
	3	6,192	221,114	0.033	0.415	(3,015)	0.007
	4	24,331	477,371	0.052	0.742	18,140	0.018
	5	12,290	825,131	0.036	1,404	(12,042)	(0.015)
	Total	52,019	1,127,243	0.020	0.453		
WAD							
	1	0	0	0.000	0.000		
	2	9,206	193,945	0.026	0.302	9,206	0.026
	3	11,815	222,675	0.048	0.421	2,608	0.021
	4	51,641	515,997	0.119	0.807	39,826	0.068
	5	116,664	894,747	0.310	1.597	65,023	0.171
	Total	189,327	1,208,220	0.088	0.487		
GAD							
	1	0	0	0.000	0.000		
	2	9,206	193,945	0.026	0.302	9,206	0.026
	3	13,873	219,115	0.054	0.412	4,666	0.027
	4	61,706	484,892	0.147	0.763	47,833	0.088
	5	184,903	854,318	0.489	1.593	123,197	0.298
	Total	269,687	1,130,473	0.130	0.469		

EXHIBIT B-2
Part 2

RANDOM SEVERITY, NO TREND

	A/Y	Average Bias	Std Dev Bias	Average % Bias	Std Dev % Bias	Age-Age Bias	Age-Age % Bias
SAD							
	1	0	0	0.000	0.000		
	2	9,206	193,945	0.026	0.302	9,206	0.026
	3	20,621	227,597	0.072	0.440	11,415	0.045
	4	97,144	598,072	0.233	0.980	76,523	0.150
	5	405,202	1,241,904	1.063	2.516	308,058	0.673
	Total	532,174	1,552,136	0.255	0.640		
ADD							
	1	0	0	0.000	0.000		
	2	158	185,077	0.010	0.329	158	0.010
	3	(7,445)	196,201	0.023	0.472	(7,603)	0.013
	4	324	272,189	0.066	0.581	7,769	0.042
	5	(2,668)	271,443	0.140	0.680	(2,991)	0.069
	Total	(9,631)	596,942	(0.004)	0.255		
SB							
	1	0	0	0.000	0.000		
	2	6,126	184,062	0.026	0.304	6,126	0.026
	3	3,909	196,494	0.052	0.430	(2,217)	0.025
	4	15,414	291,195	0.097	0.575	11,506	0.043
	5	11,071	286,813	0.172	0.698	(4,344)	0.068
	Total	36,520	633,658	0.017	0.271		
BF							
	1	0	0	0.000	0.000		
	2	9,040	200,965	0.034	0.373	9,040	0.034
	3	10,750	221,175	0.073	0.525	1,710	0.038
	4	29,330	331,648	0.132	0.691	18,580	0.055
	5	37,124	374,743	0.225	0.886	7,794	0.082
	Total	86,244	820,177	0.040	0.342		

EXHIBIT B-3
Part 1

RANDOM SEVERITY, 8% TREND

	A/Y	Average Bias	Std Dev Bias	Average % Bias	Std Dev % Bias	Age-Age Bias	Age-Age % Bias
LSL							
	1	0	0	0.000	0.000		
	2	12,848	190,771	0.030	0.300	12,848	0.030
	3	11,815	318,796	0.061	0.469	(1,034)	0.030
	4	8,339	515,561	0.080	0.629	(3,475)	0.018
	5	(23,573)	731,012	0.075	0.944	(31,912)	(0.005)
	Total	9,430	1,181,752	0.002	0.367		
LSM							
	1	0	0	0.000	0.000		
	2	12,848	190,771	0.030	0.300	12,848	0.030
	3	16,307	328,599	0.043	0.475	3,458	0.013
	4	27,133	580,424	0.057	0.728	10,826	0.013
	5	8,411	1,111,762	0.035	1.360	(18,722)	(0.021)
	Total	64,698	1,504,280	0.021	0.472		
WAD							
	1	0	0	0.000	0.000		
	2	12,848	190,771	0.030	0.300	12,848	0.030
	3	23,423	333,524	0.057	0.477	10,575	0.026
	4	62,726	608,272	0.122	0.775	39,303	0.061
	5	169,257	1,272,791	0.310	1.620	106,531	0.168
	Total	268,255	1,659,744	0.098	0.527		
GAD							
	1	0	0	0.000	0.000		
	2	12,848	190,771	0.030	0.300	12,848	0.030
	3	26,050	331,370	0.062	0.466	13,201	0.031
	4	77,169	580,779	0.149	0.755	51,119	0.082
	5	277,757	1,295,202	0.495	1.717	200,588	0.301
	Total	393,824	1,619,314	0.148	0.534		

EXHIBIT B-3
Part 2

RANDOM SEVERITY, 8% TREND

	A/Y	Average Bias	Std Dev Bias	Average % Bias	Std Dev % Bias	Age-Age Bias	Age-Age % Bias
SAD							
	1	0	0	0.000	0.000		
	2	12,848	190,771	0.030	0.300	12,848	0.030
	3	35,174	346,105	0.080	0.497	22,326	0.049
	4	124,456	685,305	0.235	0.924	89,282	0.144
	5	647,473	4,098,366	1.107	4.508	523,017	0.706
	Total	819,951	4,291,335	0.299	1.164		
ADD							
	1	0	0	0.000	0.000		
	2	(2,249)	177,229	0.008	0.337	(2,249)	0.008
	3	(15,161)	262,260	0.009	0.461	(12,912)	0.001
	4	(35,576)	335,003	0.005	0.511	(20,414)	(0.004)
	5	(92,221)	399,076	(0.028)	0.551	(56,645)	(0.033)
	Total	(145,207)	757,285	(0.053)	0.249		
SB							
	1	0	0	0.000	0.000		
	2	10,229	177,339	0.036	0.323	10,229	0.036
	3	7,628	272,101	0.055	0.456	(2,601)	0.018
	4	(5,009)	357,093	0.057	0.530	(12,637)	0.002
	5	(62,946)	420,117	0.021	0.590	(57,936)	(0.034)
	Total	(50,098)	825,565	(0.018)	0.269		
BF							
	1	0	0	0.000	0.000		
	2	16,575	212,872	0.052	0.421	16,575	0.052
	3	23,046	310,265	0.091	0.589	6,471	0.037
	4	25,574	422,741	0.114	0.668	2,529	0.021
	5	(9,528)	534,249	0.101	0.780	(35,103)	(0.012)
	Total	55,667	1,113,743	0.020	0.357		

EXHIBIT B-4
Part 1

RANDOM SEVERITY, 8% TREND, ESTIMATES BASED ON ON-LEVEL (AT 8%) TRIANGLE

	A/Y	Average Bias	Std Dev Bias	Average % Bias	Std Dev % Bias	Age-Age Bias	Age-Age % Bias
LSL							
	1	0	0	0.000	0.000		
	2	12,848	190,771	0.030	0.300	12,848	0.030
	3	19,663	321,503	0.080	0.479	6,815	0.049
	4	38,827	508,047	0.147	0.637	19,164	0.062
	5	44,325	695,596	0.216	0.928	5,498	0.060
	Total	115,663	1,148,516	0.045	0.357		
LSM							
	1	0	0	0.000	0.000		
	2	12,848	190,771	0.030	0.300	12,848	0.030
	3	16,069	326,583	0.043	0.473	3,220	0.013
	4	26,536	577,658	0.055	0.725	10,467	0.012
	5	3,262	1,070,100	0.027	1.316	(23,274)	(0.027)
	Total	58,715	1,459,667	0.019	0.460		
WAD							
	1	0	0	0.000	0.000		
	2	12,848	190,771	0.030	0.300	12,848	0.030
	3	23,310	332,453	0.057	0.476	10,461	0.026
	4	62,521	607,521	0.121	0.774	39,211	0.061
	5	166,470	1,251,178	0.305	1.598	103,950	0.164
	Total	265,149	1,635,365	0.097	0.520		
GAD							
	1	0	0	0.000	0.000		
	2	12,848	190,771	0.030	0.300	12,848	0.030
	3	26,050	331,370	0.062	0.466	13,201	0.031
	4	77,169	580,779	0.149	0.755	51,119	0.082
	5	277,757	1,295,202	0.495	1.717	200,588	0.301
	Total	393,824	1,619,314	0.148	0.534		

EXHIBIT B-4
Part 2

RANDOM SEVERITY, 8% TREND, ESTIMATES BASED ON ON-LEVEL (AT 8%) TRIANGLE

	A/Y	Average Bias	Std Dev Bias	Average % Bias	Std Dev % Bias	Age-Age Bias	Age-Age % Bias
SAD							
	1	0	0	0.000	0.000		
	2	12,848	190,771	0.030	0.300	12,848	0.030
	3	35,174	346,105	0.080	0.497	22,326	0.049
	4	124,456	685,305	0.235	0.924	89,282	0.144
	5	647,473	4,098,366	1.107	4.508	523,017	0.706
	Total	819,951	4,291,335	0.299	1.164		
ADD							
	1	0	0	0.000	0.000		
	2	(205)	182,866	0.014	0.358	(205)	0.014
	3	(4,949)	272,965	0.033	0.505	(4,744)	0.019
	4	(3,371)	352,774	0.074	0.577	1,578	0.040
	5	(7,726)	422,975	0.140	0.664	(4,335)	0.061
	Total	(16,251)	833,130	(0.003)	0.277		
SB							
	1	0	0	0.000	0.000		
	2	8,650	175,543	0.032	0.316	8,650	0.032
	3	10,927	275,491	0.063	0.471	2,277	0.030
	4	17,818	368,370	0.106	0.570	6,891	0.040
	5	12,875	440,455	0.173	0.684	(4,943)	0.061
	Total	50,271	870,120	0.021	0.284		
BF							
	1	0	0	0.000	0.000		
	2	12,243	199,536	0.041	0.382	12,243	0.041
	3	20,320	303,669	0.084	0.567	8,078	0.041
	4	38,157	423,818	0.142	0.679	17,837	0.054
	5	51,227	547,415	0.223	0.842	13,070	0.071
	Total	121,946	1,110,267	0.046	0.356		

APPENDIX C

THEOREMS

Theorem 1: Under the assumptions of Model I, $y_{LSL} = a_{LSL} + b_{LSL}x$ is an unbiased estimator of y; i.e., $E(y_{LSL}) = E(y)$. Under the assumptions of Model II, $y_{LSM} = b_{LSM}\,x$ is an unbiased estimator of y.

Proof: Model I assumes that $E(y) = a + bx$. Since all expectations are conditional on x and since a_{LSL} and b_{LSL} are unbiased, we have

$$\begin{aligned} E(y_{LSL}) &= E(a_{LSL} + b_{LSL}x) \\ &= E(a_{LSL}) + E(b_{LSL}x) \\ &= E(a_{LSL}) + E(b_{LSL})x \\ &= a + bx \\ &= E(y). \end{aligned}$$

The proof for LSM is similar.

Lemma 1: Under LSL, $E(x_n \mid x_0) = a_n + b_n E(x_{n-1} \mid x_0)$. Under LSM, $E(x_n \mid x_0) = b_n E(x_{n-1} \mid x_0)$.

Proof 1: The proof will be given for LSL. The proof for LSM is similar.

First,

$$f(x_n|x_0) = \frac{f(x_n, x_0)}{f(x_0)}$$

$$= \frac{\int_{x_{n-1}} f(x_n, x_{n-1}, x_0)\, dx_{n-1}}{f(x_0)} .$$

Next, the "Multiplication Rule" of conditional density functions (Hogg and Craig [4, p. 64]) states that

$$f(x_n, x_{n-1}, x_0) = f(x_n \mid (x_{n-1}, x_0))\, f(x_{n-1} \mid x_0)\, f(x_0).$$

Therefore,

$$f(x_n \mid x_0) = \frac{\int_{x_{n-1}} f(x_n \mid (x_{n-1}, x_0))\, f(x_{n-1} \mid x_0)\, f(x_0)\, dx_{n-1}}{f(x_0)}$$

$$= \int_{x_{n-1}} f(x_n \mid (x_{n-1}, x_0))\, f(x_{n-1} \mid x_0)\, dx_{n-1}\,.$$

By the CLIA, the random variable $x_n \mid x_{n-1}$ is independent of x_0. Therefore $f(x_n \mid (x_{n-1}, x_0))$ does not depend on x_0, so $f(x_n \mid (x_{n-1}, x_0)) = f(x_n \mid x_{n-1})$. The rest of the proof hinges on our ability to interchange the order of integration. We will make whatever assumptions are necessary about the form of the density functions to justify that step. Then

$$\begin{aligned}
\mathrm{E}\,(x_n \mid x_0) &= \int_{x_n} x_n f(x_n \mid x_0) dx_n \\
&= \int_{x_n} x_n \left(\int_{x_{n-1}} f(x_n \mid (x_{n-1}, x_0))\, f(x_{n-1} \mid x_0)\, dx_{n-1} \right) dx_n \\
&= \int_{x_{n-1}} \left(\int_{x_n} x_n f(x_n \mid (x_{n-1}, x_0))\, dx_n \right) f(x_{n-1} \mid x_0)\, dx_{n-1} \qquad \text{(C.1)} \\
&= \int_{x_{n-1}} \left(\int_{x_n} x_n f(x_n \mid x_{n-1}) dx_n \right) f(x_{n-1} \mid x_0)\, dx_{n-1}
\end{aligned}$$

$$= \int_{x_{n-1}} (a_n + b_n\, x_{n-1}) f(x_{n-1} | x_0)\, dx_{n-1}$$

$$= a_n + b_n \int_{x_{n-1}} x_{n-1} f(x_{n-1} \mid x_0)\, dx_{n-1}$$

$$= a_n + b_n\, \mathrm{E}\,(x_{n-1} \mid x_0)\,.$$

Proof 2: Recall the well-known identity $\mathrm{E}\,(X) = \mathrm{E}_Y\,[\mathrm{E}\,(X|Y)]$ (Hossack, et al, [5, p. 63]). Consider the following variation reiterated in Equation C.1 above:

$$\mathrm{E}\,(x_n \mid x_0) = \mathrm{E}_{x_{n-1} \mid x_0}\,[\mathrm{E}\,(x_n \mid (x_{n-1}, x_0))]\,.$$

For LSL we have:

$$\begin{aligned} \mathrm{E}\,(x_n \mid x_0) &= \mathrm{E}_{x_{n-1}|x_0}\,[\mathrm{E}\,(x_n \mid (x_{n-1}, x_0))] \\ &= \mathrm{E}_{x_{n-1}|x_0}\,[\mathrm{E}\,(x_n \mid x_{n-1})] \quad \text{by CLIA} \\ &= \mathrm{E}_{x_{n-1}|x_0}\,[a_n + b_n x_{n-1}] \\ &= a_n + b_n\, \mathrm{E}\,(x_{n-1} \mid x_0)\,. \end{aligned}$$

Theorem 2: $\mathrm{E}\,(\hat{\mu}_n \mid x_0) = \mathrm{E}\,(x_n \mid x_0)$.

Proof: By induction on n. The proof will be given for LSL; the proof for LSM is similar.

For $n = 1$, the theorem is simply a restatement of Theorem 1.

Assume that $\mathrm{E}\,(\hat{\mu}_{n-1} \mid x_0) = \mathrm{E}\,(x_{n-1} \mid x_0)$. We have that $\hat{\mu}_n = \hat{a}_n + \hat{b}_n \hat{\mu}_{n-1}$ where $\hat{a}_n$ and $\hat{b}_n$ are functions of the random variables $x_n | x_{n-1}$, and $\hat{\mu}_{n-1}$ is a function of the random variables $x_{n-1} \mid x_{n-2}, \ldots, x_1 \mid x_0$, and x_0. The CLIA implies that $x_n | x_{n-1}$ is independent of $x_{n-1}|x_{n-2}, \ldots, x_1|x_0$, and x_0, so $\hat{a}_n$ and $\hat{b}_n$ are independent of $\hat{\mu}_{n-1}$. Therefore,

$$
\begin{aligned}
E(\hat{\mu}_n|x_0) &= E(\hat{a}_n|x_0) + E(\hat{b}_n|x_0)E(\hat{\mu}_{n-1}|x_0) \quad \text{where } \hat{b}_n \text{ and } \hat{\mu}_{n-1} \text{ are independent} \\
&= E_{x_{n-1}|x_0}[E(\hat{a}_n|(x_{n-1}, x_0))] + E_{x_{n-1}|x_0}[E(\hat{b}_n|(x_{n-1}, x_0))]\, E(\hat{\mu}_{n-1}|x_0) \\
&= E_{x_{n-1}|x_0}[E(\hat{a}_n|(x_{n-1})] + E_{x_{n-1}|x_0}[E(\hat{b}_n|(x_{n-1})]\, E(\hat{\mu}_{n-1}|x_0) \\
&= E_{x_{n-1}|x_0}[a_n] + E_{x_{n-1}|x_0}[b_n]\, [E(\hat{\mu}_{n-1}|x_0) \\
&= a_n + b_n E(\hat{\mu}_{n-1}|x_0) \\
&= a_n + b_n E(x_{n-1}|x_0) \quad \text{by the induction hypothesis} \\
&= E(x_n|x_0) \quad \text{by Lemma 1.}
\end{aligned}
$$

Theorem 3:

	Linear	Multiplicative
For $n = 1$:	$\mathrm{Var}(\hat{\mu}_1) = \dfrac{\sigma_1^2}{I_1} + (x_0 - \bar{x}_0)^2\, \mathrm{Var}(\hat{b}_1)$	$\mathrm{Var}(\hat{\mu}_1) = x_0^2\, \mathrm{Var}(\hat{b}_1)$
For $n > 1$:	$\mathrm{Var}(\hat{\mu}_n) = \dfrac{\sigma_n^2}{I_n} + (\mu_{n-1} - \bar{x}_{n-1})^2 \mathrm{Var}(\hat{b}_n) + b_n^2\, \mathrm{Var}(\hat{\mu}_{n-1}) + \mathrm{Var}(\hat{b}_n)\, \mathrm{Var}(\hat{\mu}_{n-1})$	$\mathrm{Var}(\hat{\mu}_n) = \mu_{n-1}^2 \mathrm{Var}(\hat{b}_n) + b_n^2\, \mathrm{Var}(\hat{\mu}_{n-1}) + \mathrm{Var}(\hat{b}_n)\mathrm{Var}(\hat{\mu}_{n-1})$

Proof: We will prove the multiplicative case first. We saw in Theorem 6 that $\hat{b}_n$ and $\hat{\mu}_{n-1}$ are independent random variables. The formula (Hogg and Craig, [4, p. 178, problem 4.92]) for the variance of the product of two independent random variables x and y is:

$$\mathrm{Var}(xy) = \sigma_x^2\sigma_y^2 + \mu_x^2\sigma_y^2 + \mu_y^2\sigma_x^2 .$$

This proves the assertion because $\hat{b}_n$ is unbiased.

For the linear case,

$$\mathrm{Var}(\hat{\mu}_n) = \mathrm{Var}(\hat{a}_n) + 2\mathrm{Cov}(\hat{a}_n, \hat{b}_n\hat{\mu}_{n-1}) + \mathrm{Var}(\hat{b}_n\hat{\mu}_{n-1}) .$$

It is well known (Miller and Wichern, [6, p. 202]) that the random variables $\bar{x}_n$ and $\hat{b}_n$ are uncorrelated when $\hat{b}_n$ is determined by least squares. Since all expectations are conditional, we have that

$$\begin{aligned}\mathrm{Var}(\hat{a}_n) &= \mathrm{Var}(\bar{x}_n - \bar{x}_{n-1}\hat{b}_n)\\ &= \mathrm{Var}(\bar{x}_n) + \bar{x}_{n-1}^2\mathrm{Var}(\hat{b}_n)\\ &= \frac{\sigma_n^2}{I_n} + \bar{x}_{n-1}^2\mathrm{Var}(\hat{b}_n). \qquad (C.2)\end{aligned}$$

Next,

$$\begin{aligned}\mathrm{Cov}(\hat{a}_n, \hat{b}_n\hat{\mu}_{n-1}) &= \mathrm{E}(\hat{\mu}_{n-1})\mathrm{Cov}(\hat{a}_n, \hat{b}_n) \quad \text{where } \hat{\mu}_{n-1} \text{ is independent of } \hat{a}_n \text{ and } \hat{b}_n\\ &= \mu_{n-1}\mathrm{Cov}(\hat{a}_n, \hat{b}_n)\end{aligned}$$

and

$$\begin{aligned}\mathrm{Cov}(\hat{a}_n, \hat{b}_n) &= \mathrm{Cov}(\bar{x}_n - \bar{x}_{n-1}\hat{b}_n, \hat{b}_n)\\ &= \mathrm{Cov}(-\bar{x}_{n-1}\hat{b}_n, \hat{b}_n)\\ &= -\bar{x}_{n-1}\mathrm{Var}(\hat{b}_n). \qquad (C.3)\end{aligned}$$

Putting these together with the formula for $\mathrm{Var}(\hat{b}_n\hat{\mu}_{n-1})$ from the multiplicative derivation above we have:

$$\begin{aligned}\operatorname{Var}(\hat{\mu}_n) &= \frac{\sigma_n^2}{I_n} + \bar{x}_{n-1}^2 \operatorname{Var}(\hat{b}_n) - 2\mu_{n-1}\bar{x}_{n-1}\operatorname{Var}(\hat{b}_n)\\ &\quad + \hat{\mu}_{n-1}^2 \operatorname{Var}(\hat{b}_n) + \hat{b}_n^2 \operatorname{Var}(\hat{\mu}_{n-1}) + \operatorname{Var}(\hat{\mu}_{n-1})\\ &= \frac{\sigma_n^2}{I_n} + (\mu_{n-1} - \bar{x}_{n-1})^2 \operatorname{Var}(\hat{b}_n)\\ &\quad + b_n^2 \operatorname{Var}(\hat{\mu}_{n-1}) + \operatorname{Var}(\hat{b}_n)\operatorname{Var}(\hat{\mu}_{n-1}).\end{aligned}$$

Theorem 4A: Under LSL and LSM,

$$\operatorname{Var}(x_n|x_0) = \sigma_n^2 + b_n^2 \operatorname{Var}(x_{n-1}|x_0).$$

Therefore, an estimate of the process risk can be had by plugging in estimates of σ_n^2, b_n^2 and the estimate of process risk from the prior recursion step.

Proof:

$$\begin{aligned}\operatorname{Var}(x_n|x_0) &= \mathrm{E}_{x_{n-1}|x_0}[\operatorname{Var}(x_n|(x_{n-1}, x_0))] + \operatorname{Var}_{x_{n-1}|x_0}[\mathrm{E}(x_n|x_{n-1}, x_0))]\\ &= \mathrm{E}_{x_{n-1}|x_0}[\operatorname{Var}(x_n|x_{n-1})] + \operatorname{Var}_{x_{n-1}|x_0}[\mathrm{E}(x_n|x_{n-1})] \qquad \text{by CLIA}\\ &= \mathrm{E}_{x_{n-1}|x_0}(\sigma_n^2) + \operatorname{Var}_{x_{n-1}|x_0}(a_n + b_n x_{n-1}) \qquad \text{under LSL}\\ &= \sigma_n^2 + b_n^2 \operatorname{Var}(x_{n-1}|x_0) \qquad \text{under LSL or LSM.}\end{aligned}$$

Theorem 4B: For the WAD method, an estimate of the process variance of the prediction of the next evaluation for a single accident year is:

for $n = 1$,

$$\hat{\mathrm{Var}}\,(x_n|x_0) = x_0\,\hat{\sigma}_1^2$$

and for $n > 1$,

$$\hat{\mathrm{Var}}\,(x_n|x_0) = \hat{\mu}_{n-1}\hat{\sigma}_n^2 + \hat{b}_n^2\,\hat{\mathrm{Var}}\,(x_{n-1}|x_0)\,.$$

Proof: For $n = 1$, the WAD model states that

$$x_1 = x_0 b_1 + \sqrt{x_0 e_1}\,,$$

where the variance of the random variable e_1 is σ_1^2. Therefore, the variance of x_1 given x_0 equals the variance of the error term $\sqrt{x_0\, e_1}$, or $x_0\,\sigma_1^2$. An estimate of this process risk can be had by plugging in the estimate $\hat{\sigma}_1^2$ of σ_1^2 and the actual value of x_0.

For $n > 1$,

$$\begin{aligned}
\mathrm{Var}\,(x_n|x_0) &= \mathrm{E}_{x_{n-1}|x_0}[\mathrm{Var}\,(x_n|(x_{n-1}, x_0))] + \mathrm{Var}_{x_{n-1}|x_0}[\mathrm{E}\,(x_n|(x_{n-1}, x_0))]\\
&= \mathrm{E}_{x_{n-1}|x_0}\,[\mathrm{Var}\,(x_n|x_{n-1})] + \mathrm{Var}_{x_{n-1}|x_0}[\mathrm{E}\,(x_n|x_{n-1})] \qquad \text{by CLIA}\\
&= \mathrm{E}_{x_{n-1}|x_0}(x_{n-1}\sigma_n^2) + \mathrm{Var}_{x_{n-1}|x_0}(b_n x_{n-1}) \qquad \text{under WAD}\\
&= \mathrm{E}\,(x_{n-1}|x_0)\sigma_n^2 + b_n^2\,\mathrm{Var}\,(x_{n-1}|x_0).
\end{aligned}$$

Estimates of this quantity can be had by plugging in estimates of the individual parameters: $\hat{\sigma}_n^2$ for σ_n^2, the point estimate of μ_{n-1}, $\hat{b}_n$, for b_n, and the parameter risk estimate from the previous recursion step for $\mathrm{Var}\,(x_{n-1}|x_0)$.

Theorem 4C: For the SAD method, an estimate of the process variance of the prediction of the next evaluation for a single accident year is:

for $n = 1$,

$$\hat{\mathrm{V}}\mathrm{ar}\,(x_n | x_0) = x_0^2\, \hat{\sigma}_1^2,$$

and for $n > 1$,

$$\hat{\mathrm{V}}\mathrm{ar}\,(x_n | x_0) = \hat{\mu}_{n-1}^2\ \hat{\sigma}_n^2 + \hat{b}_n^2\ \hat{\mathrm{V}}\mathrm{ar}\,(x_{n-1} | x_0)\ .$$

Proof: For $n = 1$, the SAD model states that:

$$x_1 = x_0 b_1 + x_0 e_1\ ,$$

where the variance of the random variable e_1 is σ_1^2. Therefore, the variance of x_1 given x_0 equals the variance of the error term $x_0 e_1$, or $x_0^2 \sigma_1^2$. An estimate of this process risk can be had by plugging in the estimate $\hat{\sigma}_1^2$ of σ_1^2 and the actual value of x_0.

For $n > 1$,

$$\begin{aligned}
\mathrm{Var}\,(x_n | x_0) &= \mathrm{E}_{x_{n-1}|x_0}\,[\mathrm{Var}\,(x_n | (x_{n-1}, x_0))] + \mathrm{Var}_{x_{n-1}|x_0}[\mathrm{E}\,(x_n | (x_{n-1}, x_0))] \\
&= \mathrm{E}_{x_{n-1}|x_0}\,[\mathrm{Var}\,(x_n | x_{n-1})] + \mathrm{Var}_{x_{n-1}|x_0}[\mathrm{E}\,(x_n | x_{n-1})] \qquad \text{by CLIA} \\
&= \mathrm{E}_{x_{n-1}|x_0}(x_{n-1}^2 \sigma_n^2) + \mathrm{Var}_{x_{n-1}|x_0}(b_n x_{n-1}) \qquad \text{under SAD} \\
&= \mathrm{E}\,(x_{n-1}^2 | x_0)\,\sigma_n^2 + b_n^2\ \mathrm{Var}\,(x_{n-1} | x_0) \\
&= (\mu_{n-1}^2 + \mathrm{Var}(x_{n-1} | x_0))\,\sigma_n^2 + b_n^2\ \mathrm{Var}\,(x_{n-1} | x_0).
\end{aligned}$$

Estimates of this quantity can be had by plugging in estimates of the individual parameters: $\hat{\sigma}_n^2$ for σ_n^2, the point estimate of μ_{n-1}, $\hat{b}_n$, for b_n, and the parameter risk estimate from the previous recursion step for $\mathrm{Var}\,(x_{n-1} | x_0)$.

Lemma 2: $\mathrm{E}\,(S_n) = na_n + b_n\,(\mathrm{E}\,(S_{n-1}) + x_{n-1,\,n-1})$.

Proof:

$$
\begin{aligned}
E(S_n) &= E\left(\sum_{i=0}^{n-1} x_{i,n}|x_{i,i}\right) \\
&= \sum_{i=0}^{n-1} E(x_{i,n}|x_{i,i}) \\
&= \sum_{i=0}^{n-1} E_{x_{i,n-1}|x_{i,i}}[E(x_{i,n}|(x_{i,n-1}, x_{i,i}))] \\
&= \sum_{i=0}^{n-1} E_{x_{i,n-1}|x_{i,i}}[E(x_{i,n}|(x_{i,n-1})] \qquad \text{by CLIA} \\
&= \sum_{i=0}^{n-1} E_{x_{n-1}|x_0}(a_n + b_n\, x_{i,n-1}) \\
&= na_n + b_n\left(\sum_{i=0}^{n-2} E(x_{i,n-1}|x_{i,i}) + x_{n-1,n-1}\right) \\
&= na_n + b_n\,(E(S_{n-1}) + x_{n-1,n-1}).
\end{aligned}
$$

Theorem 5: Let $XD_n = (x_{0,0}, x_{1,1}, \ldots, x_{n-1,n-1})$ denote the current diagonal of the triangle for the n youngest accident years. Then

$$E(\hat{M}_n|XD_n) = E(S_n)\,.$$

Proof: By induction on n. The proof will be given for LSL; the proof for LSM is similar. For $n = 1$, we know that:

$$\begin{aligned}
\mathrm{E}\,(\hat{M}_1|XD_1) &= \mathrm{E}\,(\hat{\mu}_{0,1}|x_{0,0}) \\
&= \mathrm{E}\,(x_{0,1}|x_{0,0}) \qquad \text{by Theorem 2} \\
&= \mathrm{E}\,(S_1).
\end{aligned}$$

Now, assume

$$\mathrm{E}\,(\hat{M}_{n-1}|XD_{n-1}) = \mathrm{E}\,(S_{n-1}).$$

Under LSL,

$$\hat{M}_n = n\hat{a}_n + \hat{b}_n\,(\hat{M}_{n-1} + x_{n-1,n-1})$$

where $\hat{a}_n$ and $\hat{b}_n$ are functions of the random variables $x_{i,n}|\,x_{i,n-1}$, $i \geq n$, and $\hat{M}_{n-1}$ is a function of random variables $x_{i,j}|\,x_{i,j-1}$ and of $x_{j,j}$ for $j < n$ and $i > n$. By the CLIA, $\hat{a}_n$ and $\hat{b}_n$ are independent of $\hat{M}_{n-1}$. Therefore:

$$\begin{aligned}
\mathrm{E}\,(\hat{M}_n \mid XD_n) &= \mathrm{E}\,(n\hat{a}_n + \hat{b}_n\,(\hat{M}_{n-1} + x_{n-1,n-1}) \mid XD_n) \\
&= \mathrm{E}\,(n\hat{a}_n \mid XD_n) + \mathrm{E}\,(\hat{b}_n \mid XD_n)\,\mathrm{E}\,(\hat{M}_{n-1} + x_{n-1,n-1} \mid XD_n) \\
&= na_n + b_n(\mathrm{E}\,(\hat{M}_{n-1} \mid XD_{n-1}) + x_{n-1,n-1}) \\
&= na_n + b_n(\mathrm{E}\,(S_{n-1}) + x_{n-1,n-1}) \qquad \text{by the induction hypothesis} \\
&= \mathrm{E}\,(S_n) \qquad \text{by Lemma 2.}
\end{aligned}$$

Theorem 6: Parameter Risk

	Linear	Multiplicative
For $n = 1$:		
	$\text{Var}(\hat{M}_1) = \frac{\sigma_1^2}{I_1} + (x_{0,0} - \bar{x}_0)^2\,\text{Var}(\hat{b}_1)$	$\text{Var}(\hat{M}_1) = x_{0,0}^2\text{Var}(\hat{b}_1)$
For $n > 1$:		
	$\text{Var}(\hat{M}_n) = n^2\frac{\sigma_n^2}{I_n} + (M_{n-1} + x_{n-1,n-1} - n\bar{x}_{n-1})^2\,\text{Var}(\hat{b}_n) + b_n^2\,\text{Var}(\hat{M}_{n-1}) + \text{Var}(\hat{b}_n)\text{Var}(\hat{M}_{n-1})$	$\text{Var}(\hat{M}_n) = (M_{n-1} + x_{n,n})^2\text{Var}(\hat{b}_n) + b_n^2\,\text{Var}(\hat{M}_{n-1}) + \text{Var}(\hat{b}_n)\,\text{Var}(\hat{M}_{n-1})$

Proof: We will prove the multiplicative case first. Since $\hat{M}_n = \hat{b}_n\,(\hat{M}_{n-1} + x_{n-1,n-1})$, the proof is immediate by virtue of the formula for the variance of the product of two independent random variables, once we note that:

$$\text{Var}\,(\hat{M}_{n-1} + x_{n-1,n-1}) = \text{Var}\,(\hat{M}_{n-1})$$

because $x_{n-1,\,n-1}$ can be treated as a constant with respect to this conditional variance.

For the linear case,

$$\begin{aligned}\text{Var}\,(\hat{M}_n) &= \text{Var}\,(n\hat{a}_n) + \hat{b}_n(\hat{M}_{n-1} + x_{n-1,n-1})) \\ &= \text{Var}\,(n\hat{a}_n) + 2\text{Cov}\,(n\hat{a}_n, \hat{b}_n(\hat{M}_{n-1} + x_{n-1,\,n-1})) + \\ &\qquad \text{Var}\,(\hat{b}_n(\hat{M}_{n-1} + x_{n-1,\,n-1})).\end{aligned}$$

In the proof of Theorem 3 we saw that (Equation C.2)

$$\text{Var}\,(\hat{a}_n) = \frac{\sigma_n^2}{I_n} + \bar{x}_{n-1}^{\,2}\text{Var}\,(\hat{b}_n),$$

and that (Equation C.3)

$$\text{Cov}\,(\hat{a}_n,\hat{b}_n) = -\bar{x}_{n-1}\text{Var}\,(\hat{b}_n)\,.$$

Since $\hat{M}_{n-1}$ is independent of $\hat{a}_n$ and $\hat{b}_n$ and since all expectations are conditional on the current diagonal,

$$\text{Cov}\,(n\hat{a}_n,\hat{b}_n(\hat{M}_{n-1}+x_{n-1,n-1})) = n\text{E}\,(\hat{M}_{n-1}+x_{n-1,n-1})\text{Cov}\,(\hat{a}_n,\hat{b}_n)\,.$$

Therefore

$$\text{Var}\,(\hat{M}_n) = n^2\left(\frac{\sigma_n^2}{I_n}+\bar{x}^{\,2}_{n-1}\text{Var}\,(\hat{b}_n)\right) - 2n\text{E}\,(\hat{M}_{n-1}+x_{n-1,n-1})\,\bar{x}_{n-1}\text{Var}\,(\hat{b}_n)$$

$$+\,(\text{M}_{n-1}+x_{n-1,n-1})^2\,\text{Var}\,(\hat{b}_n)+b_n^2\,\text{Var}\,(\hat{M}_{n-1})+\text{Var}\,(\hat{b}_n)\,\text{Var}\,(\hat{M}_{n-1})$$

$$= n_2\frac{\sigma_n^2}{I_n}+(M_{n-1}+x_{n-1,n-1}-n\bar{x}_{n-1})^2\,\text{Var}(\hat{b}_n)$$

$$+\,b_n^2\,\text{Var}\,(\hat{M}_{n-1})+\text{Var}\,(\hat{b}_n)\,\text{Var}\,(\hat{M}_{n-1}).$$

Theorem 7A: Process Risk for the LSL and LSM models

For $n = 1$: $\text{Var}\,(S_1) = \sigma_1^2$;

for $n > 1$: $\text{Var}\,(S_n) = n\sigma_n^2 + b_n^2\,\text{Var}\,(S_{n-1})$.

Proof: For $n = 1$, S_1 is just the first future value of the youngest accident year conditional on its current value; i.e., $S_1 = x_{0,1}|x_{0,0}$. Therefore, $\text{Var}\,(S_1) = \text{Var}\,(x_{0,1}|x_{0,0}) = \sigma_1^2$ by definition of σ_1.

For $n > 1$, let X_{n-1} denote the vector of random variables $(x_{0,0}, \ldots, x_{n-2,n-1})$ corresponding to the unknown future evaluations of the $n-1$ youngest accident years as of age $n-1$. It is understood that all expectations are conditional on the current diagonal. First, recall that $S_n = \sum_{i=0}^{n-1} x_{i,n} \mid x_{i,i}$.

Next, note that

$$\mathrm{Var}(S_n) = \mathrm{E}_{x_{n-1}}[\mathrm{Var}(S_n|X_{n-1})] + \mathrm{Var}_{x_{n-1}}[\mathrm{E}(S_n|X_{n-1})]\,. \qquad \text{(C.4)}$$

For the first term,

$$\mathrm{Var}(S_n|X_{n-1}) = \mathrm{Var}\left(\sum_{i=0}^{n-1} x_{i,n}|x_{i,n-1}\right)$$

$$= \sum_{i=0}^{n-1} \mathrm{Var}(x_{i,n}|x_{i,n-1}) \text{ because accident years are independent}$$

$$= n\sigma_n^2$$

because σ_n^2 is constant across accident years.

For the second term of Equation C.4,

$$\mathrm{E}(S_n|X_{n-1}) = \mathrm{E}(a_n + b_n(S_{n-1} + x_{n-1,n-1})) \quad \text{where } a_n = 0 \text{ for LSM}$$

$$= \mathrm{E}(a_n + b_n x_{n-1,n-1} + b_n S_{n-1}).$$

Therefore,

$$\mathrm{Var}_{x_{n-1}}[\mathrm{E}(S_n|X_{n-1})] = \mathrm{Var}_{x_{n-1}}(b_n S_{n-1})$$

because a_n, b_n, and $x_{n-1,n-1}$ are constants

$$= b_n^2 \,\mathrm{Var}(S_{n-1}).$$

Putting the two terms together, we have:

$$\operatorname{Var}(S_n) = n\sigma_n^2 + b_n^2 \operatorname{Var}(S_{n-1}).$$

An unbiased estimate of this quantity can be had by plugging in unbiased estimates of σ_n^2 and b_n^2, and the Process Risk estimate from the prior recursion step.

Theorem 7B: Process Risk for the WAD model

For $n = 1$: $\operatorname{Var}(S_1) = x_{0,0}\sigma_1^2$;

for $n > 1$: $\operatorname{Var}(S_n) = (\mathrm{M}_{n-1} + x_{n-1,n-1})\,\sigma_n^2 + b_n^2 \operatorname{Var}(S_{n-1})$.

Proof: The $n = 1$ case is just Theorem 4B. For $n > 1$, the proof follows that of Theorem 7A, with one difference; namely, $\operatorname{Var}(x_{i,n}|x_{i,n-1}) = x_{i,n-1}\sigma_n^2$. So the first term of Equation C.4 is:

$$\begin{aligned}
\mathrm{E}_{X_{n-1}}[\operatorname{Var}(S_n|X_{n-1})] &= \mathrm{E}_{X_{n-1}}\left[\sum_{i=0}^{n-1} x_{i,n-1}\sigma_n^2\right] \\
&= \sigma_n^2\,\mathrm{E}\left[\sum_{i=0}^{n-1} x_{i,n-1} + x_{n-1,n-1}\right] \\
&= \sigma_n^2\,(M_{n-1} + x_{n-1,n-1})
\end{aligned}$$

by definition of M_{n-1}. Since the second term of Equation C.4 simplifies to the same quantity as in Theorem 7A, this theorem is proved.

Theorem 7C: Process Risk for the SAD model

For $n = 1$: $\operatorname{Var}(S_1) = x_{0,0}^2\,\sigma_1^2$;

for $n > 1$: $\operatorname{Var}(S_n) = (x_{n-1,n-1}^2 + \sum_{i=0}^{n-2}\mu_{i,n-1}^2 + \operatorname{Var}(S_{n-1}))\,\sigma_n^2 + b_n^2 \operatorname{Var}(S_{n-1})$.

Proof: The $n = 1$ case is just Theorem 4C. For $n > 1$, we have only to derive the first term of Equation C.4 in the proof of Theorem 7A. For SAD, $\mathrm{Var}(x_{i,n}|x_{i,n-1}) = x_{i,n-1}^2\,\sigma_n^2$, so for $i < n-1$,

$$\mathrm{E}_{x_{i,n-1}}[\mathrm{Var}(x_{i,n}|x_{i,n-1})] = \sigma_n^2\,\mathrm{E}\,(x_{i,n-1}^2)$$

$$= \sigma_n^2\,[\mathrm{E}^2\,(x_{i,n-1}) + \mathrm{Var}\,(x_{i,n-1})]$$

$$= \sigma_n^2[\mu_{i,n-1}^2 + \mathrm{Var}(x_{i,n-1})].$$

Therefore

$$\mathrm{E}_{x_{n-1}}\,[\mathrm{Var}\,(S_n|X_{n-1})] = \mathrm{E}_{x_{n-1}}\,[\mathrm{Var}\left(\sum_{i=0}^{n-1} x_{i,n}|x_{i,n-1}\right)]$$

by definition of S_n

$$= \left(\sum_{i=0}^{n-2}\mathrm{E}_{x_{i,n-1}}(\mathrm{Var}\,(x_{i,n}|x_{i,n-1}))\right) + \mathrm{Var}\,(x_{n-1,n}|x_{n-1,n-1})$$

because accident years are independent

$$= \sigma_n^2\left(\sum_{i=0}^{n-2}\mu_{i,n-1}^2 + \sum_{i=0}^{n-2}\mathrm{Var}\,(x_{i,n-1})\right) + x_{n-1,n-1}^2\sigma_n^2$$

$$= \sigma_n^2\left(\sum_{i=0}^{n-2}\mu_{i,n-1}^2 + \mathrm{Var}\left(\sum_{i=0}^{n-2}x_{i,n-1}\right)\right) + x_{n-1,n-1}^2\sigma_n^2$$

$$= \sigma_n^2 \left(\sum_{i=0}^{n-2} \mu_{i,n-1}^2 + \mathrm{Var}(S_{n-1}) \right) + x_{n-1,n-1}^2 \sigma_n^2 .$$

This proves the theorem.

Theorem 8: Under the transformed GAD model:

$$x'_n = b'_n + x'_{n-1} + e'_n$$

where we assume that $\sigma_j'^2 = \mathrm{Var}(e'_j)$ are identical for every j, the estimate of the variance of the prediction of ultimate (transformed) loss

$$\hat{\mu}'_u = x'_0 + \sum_{j=1}^{u} \hat{b}'_j$$

is

$$\left(c + \sum_{j=1}^{u} \frac{1}{I_j} \right) \hat{\sigma}'^2$$

where $\hat{\sigma}'^2$ denotes the MSE of the simultaneous solution of the link ratios of the transformed model.

Proof: Since we assume equal variances by development age, we may solve for all parameters b_j simultaneously with the equation:

$$\begin{pmatrix} x'_{n,1} - x'_{n,0} \\ x_{n-1,1} - x'_{n-1,0} \\ \cdot \\ x'_{1,1} - x'_{1,0} \\ x'_{n,2} - x'_{n,1} \\ \cdot \\ x'_{2,2} - x'_{2,1} \\ \cdot \\ x'_{n,n-1} - x'_{n,n-2} \\ x'_{n-1,n-1} - x'_{n-1,n-2} \\ x'_{n,n} - x'_{n,n-1} \end{pmatrix} = \begin{pmatrix} 1\,0 \ldots 0\,0 \\ 1\,0 \ldots 0\,0 \\ \cdot \\ 1\,0 \ldots 0\,0 \\ 0\,1 \ldots 0\,0 \\ \cdot \\ 0\,1 \ldots 0\,0 \\ \cdot \\ 0\,0 \ldots 1\,0 \\ 0\,0 \ldots 1\,0 \\ 0\,0 \ldots 0\,1 \end{pmatrix} \times \begin{pmatrix} b'_1 \\ b'_2 \\ \cdot \\ b'_{n-1} \\ b'_n \end{pmatrix} + \begin{pmatrix} e'_1 \\ e'_1 \\ \cdot \\ e'_1 \\ e'_2 \\ \cdot \\ e'_2 \\ \cdot \\ e'_{n-1} \\ e'_{n-1} \\ e'_n \end{pmatrix},$$

or, in more concise format, $\boldsymbol{Y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{E}$. It is well known that the least squares estimator of $\boldsymbol{\beta}$ is $\hat{\boldsymbol{\beta}} = (\boldsymbol{X'X})^{-1}\boldsymbol{X'Y}$ and that the variance-covariance matrix of this estimator is $(\boldsymbol{X'X})^{-1}\sigma'^2$. In this case, it is clear by inspection that $\boldsymbol{X'X}$ is a diagonal matrix whose j^{th} entry equals I_j, the number of data points in the estimate of the j^{th} link ratio, and whose off-diagonal elements are zero. Thus, $\text{Var}(\hat{b}'_j) = \dfrac{\sigma'^2}{I_j}$ and $\text{Cov}(\hat{b}'_i, \hat{b}'_j) = 0$ for $i \neq j$. Therefore, the Parameter Risk

$$\text{Var}\left(C + \sum_{j=1}^{U} \hat{b}'_j \right)$$

is exactly equal to:

$$\sigma'^2 \sum_{j=1}^{U} \frac{1}{I_j}.$$

The Process Risk is equal to:

$$\sum_{j=1}^{C} \mathrm{Var}\,(e'_n) = C\,\sigma'^2 \,.$$

These variances are estimated by substituting the estimate $\hat{\sigma}'^2$ for σ'^2.

ADDRESS TO NEW MEMBERS—MAY 16, 1994

KEVIN M. RYAN

My task today is not to congratulate you—others have properly done that and will continue to do that. Rather, my task today is to encourage you, to encourage you to be more creative and more imaginative than I expect you otherwise will be. I offer this advice with the hope that it will add a small measure to your collective success. It is offered in the spirit of friendly advice.

It is much like the story of the man who after a hard, lonesome, and troubled journey made his way to an inn and gratefully sat at a table. The waiter asked him what he wanted, and he replied, "Something to eat and a friendly word." With that the man ordered the meatloaf. When the waiter returned he told the man, "And now for the friendly word. Don't eat the meatloaf." In that spirit, I offer some friendly words.

You and I share, among other things, a plague from admirers and detractors alike. This plague consists of the definitions of what an actuary is and does. Most often these are comical; sometimes they are not. I will bore you with none of them. What I would like to dwell on is the nature of the work we do—to address what may appear on the surface to be a trite question: Is actuarial work an art or a science?

We know artists exist even when no one buys their paintings. Does actuarial work exist if we do not have a user of our services? I suppose so. Certainly all art and science have this in common. They exist even when they do not have users, buyers or appreciators. Lyndon Johnson, when asked whether he was so superstitious as to believe that the horseshoe nailed on his office wall would bring him luck, replied that he was not superstitious at all, but that he understood the horseshoe brought good luck whether you believed it did or not.

I would contend that actuarial science is in the broadest sense both an art and a science. To have you accept that assumption, I must add definitions as to what art and science are. I define art as the conscious use of skill and creative imagination in creating an aesthetic object, and science as the knowledge covering general truths or the operations of general laws. Combining the two, I will define actuarial work as the conscious use of skill, creative imagination, and knowledge of the general laws regarding financial uncertainty. It is the inclusion of art in the definition of actuarial work that I wish to stress.

Oscar Wilde said, "The moment an artist takes notice of what other people want, and tries to supply the demand, he ceases to be an artist." I cannot argue that you must not be responsive to the demands of the marketplace, but the demands you do respond to are limited by the responsibility to your profession. To profess otherwise is the disaster of dishonesty, the shame of unprofessional conduct.

But certainly it is up to you to fashion how you will approach your life's work. There is the story told of a young philosopher who went into the mountains of Tibet to speak to the elderly seer. He asked the sage, "Master, what is life?" The sage closed his eyes in thought for a few moments then replied, "Life is the smell of a fresh new rose." "But Master," said the young philosopher, "in the Andes an elderly Inca told me that life was a sharp stone." "That's his life," replied the sage.

In stressing art, I run the risk that you will not appreciate actuarial work as science. Although you will limit yourselves and your professional life if you do not appreciate it as art, if you do not bring to the process creativity and imagination, you will be less than a complete actuary if you do not continue to broaden and enhance your scientific knowledge. Because you deal with uncertainty, do not be misled into thinking that it is not science. Quantifying uncertainty is certainly science, and you must continue to develop and nurture that knowledge.

Lastly, I remind you that getting something done is an accomplishment; getting something done right is an achievement. For your good and the good of us all, I pray you achieve much.

MINUTES OF THE 1994 SPRING MEETING

May 15-18, 1994

THE MARRIOTT COPLEY PLACE, BOSTON, MASSACHUSETTS

Sunday, May 14, 1994

The Board of Directors held their regular quarterly meeting from noon to 5:00 p.m.

Registration was held from 4:00 p.m. to 6:00 p.m.

From 5:30 p.m. to 6:30 p.m., there was a special presentation to new Associates and their guests. The session included an introduction to the standards of professional conduct and the CAS committee structure.

A welcome reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, May 15, 1994

Registration continued from 7:00 a.m. to 8:30 a.m.

CAS President Irene Bass recognized special invitees and guests, including James R. Kehoe, President of the Society of Actuaries in Ireland; Christopher D. Daykin, Governing Actuary of the United Kingdom; Barnet N. Berin, President-Elect of the Society of Actuaries; and Dr. Thomas Mack of Munich Reinsurance in Germany, who is also the recipient of the first ever Charles A. Hachemeister Prize. Bass also recognized Past Presidents of the CAS who were in attendance at the meeting, including Phillip N. Ben-Zvi (1985), Ronald L. Bornhuetter (1975), Charles A. Bryan (1990), David P. Flynn (1992), Michael Fusco (1989), David G. Hartman (1987), Frederick W. Kilbourne (1982), W. James MacGinnite (1979), George D. Morison (1976), Thomas E. Murrin (1963-1964), Kevin M. Ryan (1988), Jerome A. Scheibl (1980), LeRoy J. Simon (1971), and Michael L. Toothman (1991).

The ceremony for new Fellows and new Associates was held from 8:10 a.m. to 8:30 a.m. John Kollar, John Purple, and Dave Hafling announced the 149 new Associates and the 17 new Fellows. The names of these individuals follow:

NEW FELLOWS

Richard R. Anderson	Warren A. Klawitter	Robert L. Miller
Benoit Carrier	Gilbert M. Korthals	Donald D. Palmer
Stephen R. DiCenso	Paul W. Lavrey	Karen L. Pehrson
Shawn F. Doherty	John J. Limpert	Tom A. Smolen
George Fescos	Paul R. Livingstone	Beth M. Wolfe
Allan A. Kerin	Cassandra M. McGill	

NEW ASSOCIATES

Mark A. Addiego	Julie S. Chadowski	Christine A. Gennett
Elise M. Ahearn	Daoguang E. Chen	Joyce G. Hallaway
Timothy P. Aman	John S. Chittenden	William D. Hansen
Michael J. Andring	Kuei-Hsia R. Chu	Steven T. Harr
William M. Atkinson	Rita E. Ciccariello	Lise A. Hasegawa
Lewis V. Augustine	Laura R. Claude	Amy J. Himmelberger
Robert S. Ballmer, II	J. Paul Cochran	Thomas A. Huberty
Jack Barnett	Frank S. Conde	Sandra L. Hunt
Rose D. Barrett	Pamela A. Conlin	Fong-Yee J. Jao
Martin J. Beaulieu	Francis L. Decker, IV	June V. Jarvis
Brian P. Beckman	Kurt S. Dickmann	Charles N. Kasmer
Richard Belleau	Andrew J. Doll	Mark J. Kaufman
Cynthia A. Bentley	John P. Doucette	Louis K. Korth
LaVerne J. Biskner, III	Robert G. Downs	Mary D. Kroggel
Suzanne E. Black	Bernard Dupont	Cheung S. Kwan
Michael G. Blake	David M. Elkins	Mylene J. Labelle
Gina L. Blakeney-Smith	Martin A. Epstein	Bertrand J. LaChance
Erik R. Bouvin	Dianne L. Estrada	Blair W. Laddusaw
Robert E. Brancel	Michael A. Falcone	Elaine Lajeunesse
Christopher G. Brunetti	Karen M. Fenrich	Lewis Y. Lee
Mark E. Burgess	Stephen A. Finch	Julie Lemieux-Roy
Mark W. Callahan	Daniel B. Finn	Paul B. LeStourgeon
Robert N. Campbell	Brian C. Fischer	Kenneth A. Levine
Daniel G. Carr	Douglas E. Franklin	Aaron S. Levine
Julia C. Causbie	Kirsten A. Frantom	Frank K. Ling
Maureen A. Cavanaugh	Cynthia J. Friess	Andrew M. Lloyd
Francis D. Cerasoli	Nathalie Gamache	Ronald P. Lowe, Jr.

Robert G. Lowery
Christopher J. Luker
Barbara S. Mahoney
Robert G. Mallison, Jr.
Gabriel O. Maravankin
Robert F. Maton
Emma Macasieb McCaffrey
Charles L. McGuire, III
David W. McLaughry
Kathleen A. McMonigle
Robert F. Megens
Daniel J. Merk
Timothy Messier
Stephen J. Mildenhall
Scott M. Miller
Gregory A. Moore
Robert J. Moser
Mark A. O'Brien
Denise R. Olson
John E. Pannell
Wende A. Pemrick
Robert L. Penick
Beverly L. Phillips
Mark A. Piske
Gregory J. Poirier
Tracey S. Powers
Mark Priven
Arlie J. Proctor
Donald A. Riggins
Douglas S. Rivenburgh
Paul J. Rogness
David A. Russell
Sean W. Russell
Stephen P. Russell
Linda M. K. Saunders
Gerson Smith
Louis B. Spore
Douglas W. Stang
Laurence H. Stauffer
Judith L. Stolle
Ilene G. Stone
Collin J. Suttie
Jeanne E. Swanson
John P. Thorrick
Tony King Gwan Tio
Dom M. Tobey
Glenn A. Tobleman
Theresa A. Turnacioglu
Robert C. Turner, Jr.
Ching-Hom Rick Tzeng
Robert W. Van Epps
Jeffrey A. Van Kley
Mark D. van Zanden
Trent R. Vaughn
Robert J. Vogel
W. Olivia Wacker
Joseph W. Wallen
Lisa Marie Walsh
Alice M. Wang
Gregory S. Wanner
Michelle M. Wass
Geoffrey T. Werner
Tad E. Womack
Robert S. Yenke
Benny S. Yuen
George H. Zanjani
Joshua A. Zirin
Rita M. Zona

CAS President Irene K. Bass then introduced Kevin M. Ryan, who presented the Address to New Members. Alice Gannon, CAS Vice President of Programs and Communications, presented the highlights of the program.

David L. Miller, the Chairperson for the CAS Committee on the Review of Papers, summarized the five new papers, and mentioned that only four papers would be presented at this meeting. He asked that the presenting authors stand and be recognized.

The new *Proceedings* papers were:

1. "Aggregate Retrospective Premium Ratio as a Function of the Aggregate Incurred Loss Ratio"

   Author: Robert K. Bender
   Assistant Actuary, Kemper Reinsurance Company

2. "Residuals and Influence in Regression"

   Author: Edmund S. Scanlon
   Secretary, Home Insurance Company

3. "Quantifying the Uncertainty in Claim Severity Estimates for an Excess Layer When Using the Single Parameter Pareto"

   Author: Glenn Meyers
   Assistant Vice President and Actuary,
   Insurance Services Office, Inc.

4. "Unbiased Loss Development Factors"

   Author: Daniel M. Murphy
   Vice President and Chief Actuary,
   Argonaut Insurance Company

CAS President Irene K. Bass then began the presentation of awards. She gave some background information about the Charles A. Hachemeister Prize, which was being awarded for the first time at this meeting to Dr. Thomas Mack of Munich Re in Germany. Jim Hall, Chairperson of the International Relations Committee, later recognized Dr. Mack and his contributions to ASTIN and the property/casualty field.

Bass then announced that Yong Yao was the recipient of the Harold W. Schloss Memorial Scholarship Fund. He will be presented with a $500 scholarship.

Stephen P. Lowe spoke about the activities of the American Academy of Actuaries's Casualty Practice Council.

After calling for reviews of prior *Proceedings* papers, Bass introduced Linda Ruthardt, Commissioner of Insurance for the Massachusetts Division of Insurance, who gave the Welcoming Address from 9:15 a.m. to 9:30 a.m.

The business session was adjourned at 9:30 a.m.

After a refreshment break, Bass introduced the keynote speaker, Stuart A. Varney, who is an international business correspondent for

Cable News Network (CNN). His topic was Global Economics, and he answered many questions from the audience afterward.

The first general session was held from 11:00 a.m. to 12:30 p.m.

General Session—Catastrophe Insurance
Moderator: LeRoy J. Simon

Panelists: Donald Kramer
Tempest Reinsurance, Ltd.

Linda Chu Takayama
Hawaii Insurance Commissioner

Mark Weston
Ernst & Young

After a luncheon, the following concurrent sessions were held from 1:30 p.m. to 3:00 p.m.:

1. Catastrophe Exposures

Moderator: John J. Kollar
Vice President, Insurance Services Office, Inc.

Panelists: Dr. David Busch
Research Associate,
Program for the Study of Developed Shorelines,
Department of Geology, Duke University

Stuart B. Mathewson
Consulting Actuary, Tillinghast/Towers Perrin

2. Benchmarking Corporate Actuarial Departments

Moderator: Lee R. Steeneck
Vice President, General Reinsurance Corporation

Panelists: Phillip N. Ben-Zvi
Executive Partner, Coopers & Lybrand

Charles A. Bryan
Partner, Ernst & Young

3. Asset Risk and Returns

Panelists: Tony Kao
Director, Quantitative Research,
General Motors Investment Management Corporation

William Panning
Vice President, ITT/Hartford Insurance Group

4. Using Sampling Techniques to Solve Actuarial Problems

Moderator: Richard A. Derrig
Senior Vice President,
Automobile Insurers Bureau of Massachusetts

Panelists: Susan Groshong
Statistical Consultant, CNA Insurance Companies

Herb Weisberg
President, Correlation Research Inc.

5. CAS Actuarial Research Corner

Moderator: Robert S. Miccolis
Senior Vice President and Actuary,
Reliance Reinsurance Corporation

6. Questions and Answers with the CAS Board of Directors

Moderator: Allan M. Kaufman
CAS President-Elect,
Principal, Milliman & Robertson, Inc.

Panelists: Steven F. Goldberg
Senior Vice President,
United Services Automobile Association

Gary S. Patrik
Senior Vice President and Actuary,
North American Reinsurance Corporation

Susan T. Szkoda
Second Vice President and Actuary,
The Travelers Insurance Company

After a refreshment break from 3:00 p.m. to 3:30 p.m., the following concurrent sessions continued:

1. Catastrophe Exposures

   Moderator: John J. Kollar
   Vice President, Insurance Services Office, Inc.

   Panelists: Dr. David Busch
   Research Associate,
   Program for the Study of Developed Shorelines,
   Department of Geology, Duke University

   Stuart B. Mathewson
   Consulting Actuary, Tillinghast/Towers Perrin

2. Shareholder Value—Its Application to Property/Casualty Insurance Companies

   Panelists: Lee Barnes
   Davis International Banking Consultants

   Charles A. Bryan
   Partner, Ernst & Young

3. Lloyd's of London

   Panelists: Heidi E. Hutter
   Consulting Actuary,
   Hutter Management Consultants, Ltd.

   Tony Jones
   Actuary, Sturge Holdings, Ltd.

   John P. Ryan
   Consulting Actuary, Tillinghast/Towers Perrin

4. Regression Techniques for Small Samples and Rare Events

   Panelists: Richard A. Derrig
   Senior Vice President,
   Automobile Insurers Bureau of Massachusetts

   Nitin R. Patel
   Vice President, CYTEL Software Corporation

5. ASTIN Paper: "Which Stochastic Model is Underlying the Chain Ladder Method"

   Author: Dr. Thomas Mack
   Munich Reinsurance

6. CAS Syllabus Committee

   Moderator: Steven G. Lehmann
   Actuary,
   State Farm Mutual Automobile Insurance Company

   Panelists: Donna S. Munt
   Vice President,
   United Services Automobile Association

   Gail M. Ross
   Consulting Actuary, Tillinghast/Towers Perrin

An Officers' Reception for New Fellows and Guests was held from 5:30 p.m. to 6:30 p.m., and the General Reception for all members and their guests was held from 6:30 p.m. to 7:30 p.m.

Tuesday, May 17, 1994

Registration began at 7:30 a.m.

Two general sessions were held from 8:30 a.m. to 10:00 a.m.

1. Alternative Automobile Rating Mechanisms

   Moderator: Phillip N. Ben-Zvi
   Executive Partner, Coopers & Lybrand

   Panelists: Andrew Tobias
   Author,
   Founder of the Coalition for
   Common Sense Auto Insurance

   Richard G. Woll
   Senior Actuary
   Allstate Research and Planning Center

2. Environmental Liability/Superfund

   Moderator: Amy S. Bouska
   Consulting Actuary, Tillinghast/Towers Perrin

   Panelists: Lloyd S. Dixon
   Economist, Institute for Civil Justice, RAND

   Richard D. Smith
   President, Chubb Corporation

   Dennis E. Eckart
   Partner, Arter & Hadden

After a refreshment break, the following three *Proceedings* papers were presented, and four concurrent sessions were held from 10:30 a.m. to noon. The *Proceedings* papers that were presented are as follows:

1. "Quantifying the Uncertainty in Claim Severity Estimates for an Excess Layer When Using the Single Parameter Pareto"

   Author: Glenn Meyers
   Assistant Vice President and Actuary,
   Insurance Services Office, Inc.

2. "Residuals and Influence in Regression"

   Author: Edmund S. Scanlon
   Secretary, Home Insurance Company

3. "Unbiased Loss Development Factors"

   Author: Daniel M. Murphy
   Vice President and Chief Actuary,
   Argonaut Insurance Company

The concurrent sessions that were held follow:

1. Catastrophe Ratemaking

   Moderator: Nolan E. Asch
   Senior Vice President and Actuary,
   SCOR U.S. Corporation

Panelists: Daniel F. Gogol
Senior Vice President,
General Reinsurance Corporation

David H. Hays
Actuary, State Farm Fire and Casualty Company

2. Lloyd's of London

Panelists: Heidi E. Hutter
Consulting Actuary,
Hutter Management Consultants, Ltd.

Tony Jones
Actuary, Sturge Holdings, Ltd.

John P. Ryan
Consulting Actuary, Tillinghast/Towers Perrin

3. Environmental Liability Exposure

Moderator: Charles W. McConnell
Senior Vice President and Chief Actuary,
The Home Insurance Company

Panelists: Raja R. Bhagavatula
Consulting Actuary, Milliman & Robertson, Inc.

John Butler
Principal, Putnam, Hayes, Bartlett, Inc.

Susan K. Woerner
Corporate Actuary, Nationwide Insurance Company

4. ASB Standard of Practice—Rate of Return/Profit Provision

Moderator: Mark Whitman
Assistant Vice President and Actuary,
Insurance Services Office, Inc.

Panelists: Task Force Members

CAS Regional Affiliates met for lunch from noon to 2:00 p.m. Various CAS committees met from 1:00 p.m. to 5:00 p.m.

After a lunch break, four concurrent sessions were held from 1:30 p.m. to 3:00 p.m. They were:

1. Catastrophe Modeling Software

   Moderator: Albert J. Beer
   Senior Vice President, American Re-Insurance Company

   Panelists: Karen M. Clark
   President, Applied Insurance Research, Inc.

   G. Thompson Hutton
   President and Chief Executive Officer,
   Risk Management Solutions, Inc.

   Dr. Charles R. Scawthorn
   Vice President of Research and Development,
   EQE International

2. Quality Assurance for the Actuarial Work Product

   Moderator: Robert F. Conger
   Consulting Actuary, Tillinghast/Towers Perrin

   Panelists: Linda L. Bell
   Senior Vice President and Chief Actuary,
   ITT/Hartford Insurance Group

   Thomas S. Carpenter
   Senior Vice President and Chief Actuary,
   Arbella Mutual Insurance Company

   Michael F. McManus
   Vice President and Actuary,
   Chubb Group of Insurance Companies

3. The Appointed Actuary's Report to Management

   Moderator: Susan T. Szkoda
   Second Vice President and Actuary,
   The Travelers Insurance Company

   Panelists: David J. Oakden
   Consulting Actuary, Tillinghast/Towers Perrin

Robert W. Stein
Partner, Ernst & Young

4. Asset Risk and Returns

Panelists: Tony Kao
Director, Quantitative Research,
General Motors Investment Management Corporation

William Panning
Vice President, ITT/Hartford Insurance Group

All members and guests enjoyed an evening dinner at the Boston Museum of Science from 6:30 p.m. to 10:30 p.m.

Wednesday, May 18, 1994

One *Proceedings* paper was presented while concurrent sessions ran from 8:00 a.m. to 9:30 a.m.

Proceedings Paper:

"Aggregate Retrospective Premium Ratio as a Function of the Aggregate Incurred Loss Ratio"

Author: Robert K. Bender
Assistant Actuary, Kemper Reinsurance Company

Concurrent Sessions:

1. Catastrophe Ratemaking

Moderator: Nolan E. Asch
Senior Vice President and Actuary,
SCOR U.S. Corporation

Panelists: Daniel F. Gogol
Senior Vice President,
General Reinsurance Corporation

David H. Hays
Actuary, State Farm Fire and Casualty Company

2. Quality Assurance for the Actuarial Work Product

   Moderator: Robert F. Conger
   Consulting Actuary, Tillinghast/Towers Perrin

   Panelists: Linda L. Bell
   Senior Vice President and Chief Actuary,
   ITT/Hartford Insurance Group

   Thomas S. Carpenter
   Senior Vice President and Chief Actuary,
   Arbella Mutual Insurance Company

   Michael F. McManus
   Vice President and Actuary,
   Chubb Group of Insurance Companies

3. The Appointed Actuary's Report to Management

   Moderator: Susan T. Szkoda
   Second Vice President and Actuary,
   The Travelers Insurance Company

   Panelists: David J. Oakden
   Consulting Actuary, Tillinghast/Towers Perrin

   Robert W. Stein
   Partner, Ernst & Young

4. The Role of the Appointed Actuary: Modeling an Insurance Company's Financial Performance

   Moderator: Sholom Feldblum
   Assistant Vice President and Associate Actuary,
   Liberty Mutual Insurance Company

   Panelists: Christopher D. Daykin
   Government Actuary of the United Kingdom

   Stephen P. Lowe
   Vice President, Tillinghast/Towers Perrin

After a refreshment break, the following general session was held.

The Outlook for Health Care Reform

Moderator: Frederick W. Kilbourne
Independent Actuary, The Kilbourne Company

Panelists: Edmund F. Kelly
President and Chief Operating Officer,
Liberty Mutual Insurance Group

Michael S. Pritula
Partner, McKinsey & Company

Richard Victor
Director, Workers Compensation Research Institute

Debra T. Ballen
Senior Vice President-
Policy Development and Research,
American Insurance Association

CAS President Irene K. Bass presented the closing remarks and announced future CAS meetings.

May 1994 Attendees

The 1994 CAS Spring Meeting was attended by 252 Fellows, 191 Associates, and 186 Guests. The names of the Fellows and Associates in attendance follow:

FELLOWS

Terry J. Alfuth	Bruno P. Bauer	Gavin Christophe Blair
Manuel Almagro, Jr.	Edward J. Baum	LeRoy A. Boison, Jr.
Karen E. Amundsen	Albert J. Beer	Martin Bondy
Richard R. Anderson	Linda L. Bell	Ronald L. Bornhuetter
Nolan E. Asch	Phillip N. Ben-Zvi	Amy S. Bouska
Richard V. Atkinson	Robert K. Bender	J. Scott Bradley
Anthony J. Balchunas	Norman J. Bennett	James F. Brannigan
Karen H. Balko	Abbe Sohne Bensimon	Robert A. Brian
Katharine Barnes	Regina M. Berens	Robert S. Briere
W. Brian Barnes	John R. Bevan	Randall E. Brubaker
Donald T. Bashline	Raja R. Bhagavatula	Charles A. Bryan
Irene K. Bass	James E. Biller	Jeanne H. Camp

Ruy A. Cardoso
Kenneth E. Carlton, III
Thomas S. Carpenter
John D. Carponter
Benoit Carrier
James K. Christie
Allan Chuck
Joseph F. Cofield
Robert F. Conger
Eugene C. Connell
John B. Conners
Ann M. Conway
Alan C. Curry
Janice Z. Cutler
Robert A. Daino
Robert V. Deutsch
Stephen R. DiCenso
Shawn F. Doherty
Michael C. Dolan
John P. Donaldson
John P. Drennan
Howard M. Eagelfeld
Kenneth Easlon
Gary J. Egnasko
Valere M. Egnasko
Douglas D. Eland
Thomas J. Ellefson
James Ely
Glenn A. Evans
James A. Faber
Doreen S. Faga
Bill Faltas
Sholom Feldblum
George Fescos
Wayne H. Fisher
Beth E. Fitzgerald
Nancy G. Flannery
Kirk G. Fleming
David P. Flynn
Edward W. Ford
David C. Forker
E. Frederick Fossa
Michael Fusco
Alice H. Gannon
Robert W. Gardner
Thomas L. Ghezzi
Joseph A. Gilles
Owen M. Gleeson
Daniel C. Goddard
Steven F. Goldberg
Charles T. Goldie
James F. Golz
Karen Pachyn Gorvett
Timothy L. Graham
Patrick J. Grannan
Clyde H. Graves
Nancy A. Graves
Larry A. Haefner
David N. Hafling
James A. Hall, III
Alan J. Hapke
David C. Harrison
David H. Hays
E. LeRoy Heer
John Herzfeld
David R. Heyman
Kathleen M. Holler
Randall D. Holmberg
Ruth A. Howald
Heidi E. Hutter
Andrew P. Johnson
Eric J. Johnson
Wendy A. Johnson
Steven J. Johnston
Thomas S. Johnston
Brian A. Jones
Adrienne B. Kane
Allan M. Kaufman
Anne E. Kelly
C.K. Stan Khury
Frederick W. Kilbourne
Richard O. Kirste
Frederick O. Kist
Warren A. Klawitter
Charles D. Kline, Jr.
John Joseph Kollar
Gilbert M. Korthals
Gary I. Koupf
Gustave A. Krause
Rodney E. Kreps
Jeffrey L. Kucera
Andrew E. Kudera
Dean K. Lamb
Dennis L. Lange
Paul W. Lavrey
Merlin R. Lehman
Steven G. Lehmann
Joseph W. Levin
John J. Limpert
Orin M. Linden
Barry C. Lipton
Paul R. Livingstone
Stephen P. Lowe
W. James MacGinnitie
Brett A. MacKinnon
Christopher P. Maher
Howard C. Mahler
Mark J. Mahon
Stuart B. Mathewson
Robert W. Matthews
Charles W. McConnell
Cassandra M. McGill
Michael F. McManus
Glenn G. Meyers
Robert S. Miccolis
David L. Miller
David L. Miller

Michael J. Miller
Neil B. Miner
Karl G. Moller
Phillip S. Moore
Roy K. Morell
George D. Morison
Thomas G. Moylan
Evelyn Toni Mulder
Robert T. Muleski
Donna S. Munt
Peter J. Murdza, Jr.
Daniel M. Murphy
Thomas E. Murrin
Chris E. Nelson
Kenneth J. Nemlick
William Anthony Niemczyk
Kathleen C. Nomicos
Terrence M. O'Brien
Paul G. O'Connell
David J. Oakden
Robert G. Palm
Donald D. Palmer
Jacqueline Edith Pasley
Bruce Paterson
Gary S. Patrik
Susan J. Patschak
Karen L. Pehrson
Kai-Jaung Pei
Bernard A. Pelletier
Jill Petker
Stephen W. Philbrick
Herbert J. Phillips
Stuart Powers
John M. Purple
Andrew J. Rapoport
Scott E. Reddig
Ronald C. Retterath
Robert S. Roesch
Steven Carl Rominske
Gail M. Ross
Kevin M. Ryan
Donald D. Sandman
Edmund S. Scanlon
Jerome A. Scheibl
David C. Scholl
Joseph R. Schumi
Susanne Sclafane
Brian E. Scott
Kim A. Scott
Margaret E. Seiter
Marie Sellitti
Roy G. Shrum
Mark J. Silverman
Christy L. Simon
LeRoy J. Simon
David Skurnick
Lisa A. Slotznick
Michael B. Smith
Richard A. Smith
Tom A. Smolen
G. Clinton Sornberger
Joanne S. Spalla
Daniel L. Splitt
Sanford R. Squires
Lee R. Steeneck
Charles Walter Stewart
Stuart B. Suchoff
Christian Svendsgaard
Susan T. Szkoda
Catherine Harwood Taylor
John P. Tierney
Darlene P. Tom
Michael L. Toothman
Christopher J. Townsend
Nancy R. Treitel
Frank J. Tresco
Mary L. Turner
Oakley (Lee) E. Van Slyke
Jennifer A. Violette
Michael A. Visintainer
Joseph L. Volponi
William J. VonSeggern
James C. Votta
Michael C. Walsh
Thomas V. Warthen, III
Bernard L. Webb
Nina H. Webb
Thomas A. Weidman
Mark Whitman
Peter W. Wildman
David A. Withers
Susan K. Woerner
Beth M. Wolfe
Richard G. Woll
Arlene Frances Woodruff
Patrick B. Woods
Chung-Ye Yen
Richard P. Yocius
Heather E. Yow

ASSOCIATES

Mark A. Addiego
Elise M. Ahearn
Timothy P. Aman
William M. Atkinson
Lewis V. Augustine
Robert S. Ballmer, II
Jack Barnett
Rose D. Barrett
Martin J. Beaulieu
Brian P. Beckman
Cynthia A. Bentley
LaVerne J. Biskner, III

Suzanne E. Black
Gina L. Blakeney Smith
Gary Blumsohn
Erik R. Bouvin
Robert E. Brancel
Ward M. Brooks
Christopher G. Brunetti
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John S. Chittenden
Kuei-Hsia R. Chu
Rita E. Ciccariello
Laura R. Claude
Donald L. Closter
Michael A. Coca
J. Paul Cochran
Frank S. Conde
Pamela A. Conlin
Warren P. Cooper
Mary L. Corbett
Brian K. Cox
Daniel A. Crifo
Karen L. Davies
James R. Davis
Francis L. Decker IV
Gordon F. Diss
John P. Doucette
Robert G. Downs
David M. Elkins
Martin A. Epstein
Dianne L. Estrada
Michael A. Falcone
Karen M. Fenrich
Stephen A. Finch
Daniel B. Finn
Loy W. Fitz
Douglas E. Franklin
Kirsten A. Frantom
Cynthia J. Friess
Christine A. Gennett
Peter M. Gidos
Daniel F. Gogol
Donald E. Gould
Ewa Gutman
Holmes M. Gwynn
Joyce G. Hallaway
Leigh Joseph Halliwell
Sidney M. Hammer
William D. Hansen
Steven T. Harr
Lise A. Hasegawa
Joseph A. Herbers
Thomas G. Hess
Amy J. Himmelberger
Thomas A. Huberty
Sandra L. Hunt
Fong-Yee J. Jao
June V. Jarvis
James P. Jensen
Daniel J. Johnston
Stephen H. Kantor
Charles N. Kasmer
Mark J. Kaufman
Martin Kevin Kelly
Ann L. Kiefer
Louis K. Korth
Mary D. Kroggel
Cheung S. Kwan
Mylene J. Labelle
Bertrand J. LaChance
Elaine Lajeunesse
Lewis Y. Lee
Julie Lemieux-Roy
Paul B. LeStourgeon
Kenneth A. Levine
Andrew M. Lloyd
Robert G. Lowery
Christopher J. Luker
Barbara S. Mahoney
Laura Manley
Gabriel O. Maravankin
Suzanne Martin
Robert F. Maton
Emma Macasieb McCaffrey
Stephen J. McGee
Eugene McGovern
Charles L. McGuire, III
Donald R. McKay
David W. McLaughry
James P. McNichols
Robert F. Megens
Daniel J. Merk
Robert J. Meyer
Stephen J. Mildenhall
Stacy L. Mina
Gregory A. Moore
Francois L. Morissette
Robert J. Moser
Kevin J. Moynihan
Antoine A. Neghaiwi
Lynn Nielsen
Victor A. Njakou
Mark A. O'Brien
Denise R. Olson
John E. Pannell
Wende A. Pemrick
Robert L. Penick
Beverly L. Phillips
Mark A. Piske
Richard A. Plano

Mark Priven
Arlie J. Proctor
R. Stephen Pulis
Katherine D. Radin
James E. Rech
Donna J. Reed
Al J. Rhodes
Elizabeth M. Riczko
Tracey S. Ritter
Douglas S. Rivenburgh
Paul J. Rogness
James B. Rowland
David A. Russell
Sean W. Russell
Stephen P. Russell
Beverley K. Ryan
John F. Ryan
John P. Ryan
Linda M. K. Saunders
Peter R. Schwanke
Robert D. Share
Patricia E. Smolen
Louis B. Spore
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The 1994 CAS Spring Meeting was officially adjourned at 11:45 a.m. after the closing remarks.

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EXTENDED SERVICE CONTRACTS

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Abstract

Automobile extended service contracts (ESCs) have been in existence for many years. Due to the nature of the coverage, an insurer may not know the actual results for a particular book for some time after the book has been in place. This paper discusses this coverage and unique characteristics of ESCs that should be recognized when analyzing experience for an ESC program. The paper also discusses some approaches that have been used to address these problems and to derive reserve and rate estimates for ESC programs.

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1. INTRODUCTION

Automobile extended service contracts (ESCs) have been responsible for financial losses to more than one insurer. Although the impact of ESC programs on insurance industry profitability cannot be determined, some multiple line insurers have left the market after substantial ESC losses, and some specialty carriers have become insolvent. Witness, for example, the demise of Consumers Indemnity, a Washington based ESC writer, and the recent fortunes of both American Warranty and General Warranty, both very large ESC administrators. Because of the way ESC business is treated, data from published financial statements usually are not useful in identifying the effect of ESC business on particular insurers. However, the author is aware of several insurers that are no longer in this market, due largely to poor loss experience.

The cause of difficulties can usually be traced to inadequate pricing. Often though, misunderstanding key aspects of these contracts can be a major factor. In this paper, we discuss ESCs, identify areas that can lead to future financial problems, and describe some approaches for analyzing ESC experience.

2. BACKGROUND

Most people buying a new or used car from a dealer are presented the opportunity to purchase additional protection against mechanical breakdown of that car. This protection can be in the form of a policy purchased directly from an insurer with the dealer acting as an agent, or as a response to a direct mail appeal from an insurer. In a limited number of states, direct insurance is the only type of transaction allowed for such coverage. In this case of direct insurance, state insurance regulation including rate regulation, anti-rebate statutes, and agency licensing requirements usually applies.

A more common arrangement is a contract between the buyer and the dealer, often with an administrator or managing general agent providing administration of the program. The dealer then obtains insurance to cover the liability assumed under the contract or self-insures the risk. In this case, since state insurance laws often exclude service contracts and warranties, regulation usually applies to the transaction between the dealer and the insurer but not necessarily to the transaction between the car buyer and the dealer. In the latter transaction, the dealer knows the wholesale price for coverage received, and is free to set the retail price charged to the car buyer. The buyer can negotiate the price with the dealer, if the buyer is aware of the nature of this arrangement.

In either the direct insurance or service contract arrangement, the basic idea of protection is the same: In exchange for a sum of money, a promise is given to repair or replace covered parts that fail for specified causes during the term of coverage. This term is usually expressed in both time and mileage elapsed, although there are some contracts without any mileage limitation.

3. RESERVE CONSIDERATIONS FOR ESCs

Some believe, with justification, that this type of coverage is for physical damage to a vehicle, and, as such, loss reserves are not a significant item. This is often the case. Claims are usually reported quickly after they occur, repairs made soon after authorization is given, and payments made promptly. Thus the reserves for claims incurred, whether or not they have been reported, are often relatively small. Exceptions can occur, however, in cases where, because of processing features and possible batching of claims in a particular ESC program, there is a longer time lag between loss occurrence and final payment.

In this line of business, many program managers tend to rely on calendar year loss ratios calculated as losses incurred in the year divided by premiums earned in the year. The rationale is that, since loss reserves play a relatively minor role, calendar year experience is not materially affected by reserve movement. This is correct as far as it goes. Without consideration of the flow of premium, however, this reasoning can lead to disastrous results.

It is critically important in evaluating loss ratio results for an ESC program to recognize that losses generally cannot be expected to emerge uniformly during the life of a contract which can be in effect for several years. For this reason, the rate at which premiums are earned can have a significant impact on the loss ratios. Thus, when reviewing loss ratios for an ESC program it is important to know what "earning curve" is used to bring premiums into income. Until it is verified that the earning pattern accurately tracks loss emergence, loss ratios using premiums earned with that pattern should be suspect.

An approach that does not depend directly on the formula used to earn premiums would be to estimate ultimate losses for fixed groups of policies. If this approach is taken, the separation of the resulting unpaid amounts among the various reserve categories must still be considered for accounting purposes. For example, the ultimate losses for policy year 1990 as of December 31, 1992, will be composed of:

1. losses paid through December 31, 1992, on 1990 policies,
2. case reserves for open known claims as of December 31, 1992, if such reserves are set,
3. additional reserves for development of reserves on known claims including possible re-openings (development reserves),
4. reserves for claims that occurred before December 31, 1992, but are not yet reported (true IBNR reserves), and
5. amounts estimated to be paid for claims expected to arise after December 31, 1992 during the unexpired terms of current contracts.

Loss reserves would provide for items 2, 3 and 4. As noted above, claims usually are closed rather quickly (reducing case reserves), are usually reported quickly (reducing true IBNR), and are usually easy to evaluate (reducing development reserves), so true loss reserves are usually rather small.

The last item, the amount expected to be paid on unexpired portions of current contracts, is generally provided for in the unearned premium reserves. Unless the formula used to earn premiums matches the expected loss emergence, a mismatch between the unearned premium reserves and the amounts included in item 5 above can occur, even if the premiums charged are correct.

If premiums are earned more rapidly than losses are expected to emerge, and if incurred losses are compared to earned premiums, the resulting loss ratios will be understated at early ages. If, in addition, rates are inadequate and the program is growing, the "profitable" new business will offset the losses on the "unprofitable" old business, masking difficulties even further.

3. EXAMPLE OF TIMING MISMATCH

The following example, though hypothetical, does parallel the experience of more than one insurer with this type of business. For this example, assume that:

1. the insurer earns premium on a pro-rata basis over time,
2. all ESCs are on new cars for five years or 50,000 miles, whichever comes first, and
3. losses emerge during the life of a contract in the following pattern:

EXAMPLE LOSS EMERGENCE FOR ONE POLICY

Year	Percentage of Losses Incurred
1	5%
2	15
3	25
4	30
5	25

We used these assumptions, along with the simplifying one of uniform issuing of contracts through the year, with an assumed loss ratio of 150% to derive Exhibit 1. As can be seen from the resulting loss ratios, the mismatch between the emergence of losses and the premium earning is definitely misleading. The program starts out with a 38% loss ratio, and loss ratios do not exceed 100% until the end of the fourth year on a calendar year basis and not until the end of the fifth year on an inception-to-date basis. By this time the insurer is already committed to several years of very unprofitable business.

Though this example may seem somewhat extreme, the potential for mismatch exists in almost any earning formula used for this type of business. The actuary must be cautious in relying on loss ratios calculated as the ratio of incurred losses to earned premiums in ESC business, even if the incurred losses include proper provision for all claims that have already occurred.

If loss ratios based on earned premiums are to be used to make financial decisions regarding an ESC book, the match between loss emergence and earning should be checked. A pattern of increasing loss ratios over time, as shown on Part 3 of Exhibit 1, should give some warning that a mismatch may be occurring. However, the presence of newer policy years, contributing more to the earned premiums than to losses incurred, could mask that pattern, especially if there is growth in the business.

Rather, it would be better to look at a fixed group of policies and see how the earning formula has tracked with the historical emergence of losses. The following table shows the progression of the cumulative indicated loss ratios for policy year 1 from Exhibit 1:

LOSS RATIO EMERGENCE—POLICY YEAR 1

Calendar Year	Indicated Loss Ratio
1	38%
2	63
3	98
4	129
5	146
6	150

The increase shows up more quickly than in the inception-to-date or even calendar year loss ratios for the entire book. Any consistent pattern in the loss ratios for a fixed group of policies over time, either up or down, provides a warning that there may be a mismatch in timing between premium earning and loss emergence.

Several aspects of ESCs can influence the emergence of losses during the life of a contract. The following section deals with these characteristics and their potential effect on loss emergence.

4. CHARACTERISTICS OF ESCs

Several characteristics of ESCs make the analysis of ESC experience significantly different from that for many other types of insurance. First, the contracts themselves differ from many other insurance coverages. As noted above, the contract is often between an automobile dealer and an automobile purchaser, with insurance covering the dealer's liability assumed under the contract.

ESCs often run for many years and contract holders have limited rights to cancel coverage. Most ESCs come with mileage limitations, although unlimited mileage contracts have been issued. There will thus be contracts expiring before their time limit, as a result of exceeding the mileage limitation.

ESC coverages normally begin where manufacturer warranties end. They usually exclude anything covered under manufacturer warranties, and they sometimes provide coverage for items such as towing, car rental, and travel interruption expenses not covered under the original warranty. Generally very little loss is expected to be incurred by the ESC policies during the original manufacturer warranties. These warranties are usually at least one year or 12,000 miles and can be three years or 36,000 miles or even longer on many early 1990 models. Thus we would expect much less than one-fifth of all ESC losses to arise during the first year of a five-year policy.

Finally, most ESCs have a provision for the transfer of the contract in the event of the transfer of a covered car, after payment of a specified fee and after application to the insurer or dealer. Otherwise, coverage does not continue to the new owner. In most cases, contracts cannot be transferred if the car is sold to a dealer. It is therefore not unusual for cars to be sold without the contract being transferred to the new owner.

A second area of difference is in the nature of the hazard insured; that is, the cost of repair of certain covered parts that fail during the contract term and are not otherwise covered by manufacturer warranty. Thus, different manufacturer warranties will cause ESC losses on different vehicles to emerge differently. This will also cause proportionately less of the covered losses to emerge in the early stages of the contract, while the car is new and the manufacturer warranty is in effect, than in later stages as parts wear out and costs increase. Finally, we may even expect different makes, models, or even model years to experience different cost emergence patterns than others.

A third area of difference lies in the nature of the contract purchasers. The purchasers often have a choice of contract length and mileage limitations. Thus it is possible that selection will affect the characteristics of the contract holders of different contract terms and the rate at which mileage restrictions form the real limit on coverage. There may be other situations where the contract holders forget the coverage or sell the covered vehicle without transferring coverage. In addition, most ESCs require that the vehicle owner comply with certain service requirements. Different contract holders may have different attitudes toward such requirements.

These characteristics could lead one to conclude that, on the average, there is less exposure to loss at the end of the contract term measured by time than at the start. This is often the case for contracts sold on used cars; however, it is definitely not the case in most new car coverages. This line of reasoning ignores the fact that the more expensive claims tend to occur near the end of the policy term. In addition, the presence of manufacturer warranties tends to reduce costs in the early stages of new car contracts. The inescapable conclusion is that, in either new or used car coverages, losses cannot be expected to arise uniformly during the term of the contract.

5. AN AGGREGATE APPROACH TO LOSS ESTIMATION

Instead of concentrating on loss and unearned premium reserves separately, the actuary could take a unified approach in monitoring the profitability of an ESC program. Such an approach would focus on the ultimate forecast position of the program, rather than using earned premiums. Ultimate losses would be forecasted and compared with premiums to assess program profitability. In this way, the earning curve arises implicitly from the loss data and does not need to be specified beforehand. Separate analyses could then be performed to estimate the portion of the resulting estimated total unpaid amount attributable to claims that have already occurred. The remainder would provide an estimate of the amount necessary to fund for losses that have not yet occurred.

Usual actuarial projection methods making use of data triangles can also be used for forecasts in ESC programs. If losses are grouped by accident period (month, quarter, year), where an accident is defined to be the occurrence of a covered repair, the resulting projections will provide estimates for accidents that have occurred. These estimates can then be used to estimate the amount of loss and expense reserves necessary for claims that have occurred (items 2, 3 and 4 above). As mentioned above, we generally find the tail to be fairly short in these cases, often with 90% or more of losses paid within 12 months of the repair.

Many writers of ESC coverages have different coverage terms available with multiple choices of both length of time and mileage limitations. The lag from repair to payment, however, should not depend materially on those options but rather should relate to the operation of the individual ESC program. For this reason an actuary may be able to gain stability in projections by accident period by combining the data for several policy terms.

As with other lines of insurance, many factors can influence and change the lag from repair to final payment. One obvious factor is the structure of the particular program. Some programs require pre-authorization of repairs that exceed a certain amount, while others may require pre-approval on all repairs. Some programs may require frequent submission of claims from the dealer to the administrator, whereas the frequency may be much less in other programs. The administrator may also batch reportings to the insurer. Such batching affects the timing of information flow. Changes in these or other procedures can affect projections of ultimate losses on an accident period basis.

Another factor to be aware of is the presence of case reserves. In some cases the pre-authorized repair amounts are entered as incurred losses. As with other coverages, if such data are available, both incurred and paid loss forecasts are possible.

Projections using triangles organized by policy period will provide estimates of ultimate losses for all policies issued during a particular period. As noted above, this approach has the benefit of not relying on specific earning formulae to estimate the profitability of a book of ESC business. Rather, this approach uses the emergence patterns inherent in the program's own data. However, it brings with it all the difficulties inherent in estimating losses for longer tailed lines.

For five-year policies, a policy year will not have expired until six years from its start. There is an additional lag from when the last policy expires until the last payment is made, making the lag in the neighborhood of seven to eight years until all claims are settled. The percentage of losses emerging in early stages of development is further reduced by the presence of new car manufacturer warranties. It is not unusual for 2% to 5%, or even less, of the losses for a single five-year policy to emerge in its first year. Thus the experience for relatively green policy periods has the potential for substantial future development.

Some may prefer to analyze experience by model year. The benefit of such analysis is that it keeps the experience for similar vehicles together. It does extend the lag until a year is completely closed, since manufacturers may introduce next year's models relatively early in a year and have those cars in stock well into the next model year. It is conceivable that a model year 19xx could last from March 19xx-1 until March 19xx+1, or even longer.

The tail can be shortened a little by separately considering the lag from policy issue to claim occurrence versus the lag from claim occurrence to final settlement. The accident period development could be used first to develop policy period data to ultimate for claims already incurred. The resulting adjusted data would then have a maximum lag of one more period than the policy term. For example, adjusted policy year data for five-year policies would have a maximum six year lag from the beginning of the policy year until the policy year is fully closed. Similarly, for policy quarters, the maximum lag would be five and one-quarter years.

This two-step approach has another benefit. It separates lag characteristics that are under the direct control of the insurer or administrator (occurrence to settlement) from those that are less subject to their control (policy to occurrence). This latter pattern should be more dependent on the actual policy provisions, term, and mileage limitation and less dependent on specific characteristics of a particular ESC program and administrative structure. In this case, other data sources may also prove useful. If other sources are used, however, the actuary should consider the effects of potential differences in ESC provisions between programs.

Exhibits 2 through 7 provide an example of these concepts. These data are all hypothetical but present general characteristics of ESC programs. We assume that these data are for five-year contracts with the same mileage term.

Exhibit 2 shows accident year paid loss development, Exhibit 3 shows policy year paid loss development, and Exhibit 4 shows the distribution of paid losses by policy year and accident year. All these data are as of September 30, 1992, and the policy and accident years represent fiscal years ending September 30. Fiscal year was selected over calendar year due to the timing of new model roll-out by manufacturers, which typically takes place around October 1. As mentioned above, however, there are many exceptions to this general rule.

Exhibit 2 also shows the indicated development factors and resulting projections of ultimate losses by accident year. Given the relatively short tail inherent in these losses, development factor methods probably provide reasonably accurate forecasts of ultimate losses by accident year. The difference between these forecasts and the amounts paid to date can provide estimates of required total loss reserves by accident year. Although the tail is usually fairly short, this exhibit shows that true loss reserves cannot be completely ignored in these sample data.

Similar development factor projections are also shown in Exhibit 3 for losses sorted by fiscal policy year. In this case, the ultimate loss estimates include projections for future claims as well as for claims that have already occurred. Here, given the tail inherent in the development, development factor methods may not be sufficient to provide stable forecasts, especially in later policy years. Also shown in Exhibit 3 is an estimate for development after age 84 months. Though this represents time after all policies have expired, there is the potential for later development on payments. This estimate is based on projections from Exhibit 5.

The top portion of Exhibit 4 shows the distribution of loss payments as of September 30, 1992, by fiscal policy year and fiscal accident year. For example, of the $10,696,000 in payments to date for the policy year ending September 30, 1988, $43,000 arose from accidents occurring during the year ending September 30, 1988, $814,000 arose from accidents occurring during the year ending September 30, 1989, and so forth.

Since all of these amounts are valued as of September 30, 1992, the last diagonal represents accidents occurring during the year ending September 30, 1992, currently at 12 months of maturity. Similarly, the next older diagonal represents accidents occurring during the year ending September 30, 1991, currently at 24 months of maturity. We use the accident year development from Exhibit 2 to project these amounts to their estimated ultimate levels. These estimates are shown in the bottom portion of Exhibit 4. Here the 12 month factor is used to develop the losses along the last diagonal

$$41 = 29 \times 1.407,\ 1{,}373 = 976 \times 1.407, \text{ etc.},$$

the 24 month factor is used to develop losses along the next older diagonal

$$64 = 62 \times 1.035,\ 1{,}755 = 1{,}696 \times 1.035, \text{ etc.},$$

with similar calculations for the remaining estimates.

The amounts shown in the top portion of Exhibit 5 are the cumulative totals from the bottom portion of Exhibit 4. These amounts are estimates of the emergence of losses during the life of the particular contracts in contrast to the payment of losses during the life of the contracts as shown in Exhibit 3. We then use development factor methods to derive another set of ultimate loss estimates as shown in Exhibit 5.

As discussed in greater detail below, changes in manufacturer warranties can affect the development of losses for new car contracts. For this reason both Exhibits 3 and 5 show two sets of development factor selections. In this hypothetical case we assumed that changes in original manufacturer warranties were made for 1990 models. Thus, development for policy years ending September 30, 1990 and subsequent is expected to be different than that for earlier years. In these exhibits the different factors were judgmentally selected. Later in this paper we will describe some approaches that may assist in quantifying the effects of such changes.

Exhibit 6 summarizes the results of the projections from Exhibits 3 and 5. Also in that exhibit is a third forecast method that does not have the "leveraging" problem of development factor methods. This third method is akin to a Bornhuetter-Ferguson approach but uses adjusted and trended pure premiums based on development factor projections instead of loss ratios as its initial estimate. Column 3 shows the initial selections by policy year which are based on development factor projections shown in Columns 1 and 2. Column 5 is the pure premium indicated by these initial selections. The pure premiums in Column 6 are based on these initial pure premiums, taking into account both trend and an estimated 10% decrease because of changes in manufacturer warranties in 1990. Part 2 of Exhibit 6 shows the calculation of these smoothed pure premiums in more detail. We first adjust the pure premiums to a common warranty level, using the assumed 10%, and then trend the resulting pure premiums. Then we adjust the trended pure premiums to reflect the assumed effect of the warranty change.

The forecasts in Column 7 are the adjusted policy year/accident year losses from Exhibit 5 plus the product of expected losses [Column 6 x Column 4] and the proportion of losses expected to emerge in the future. This latter amount is [1 – 1/age-to-ultimate factor] using the development from Exhibit 5. These calculations are shown in more detail on Exhibit 6, Part 2.

The remainder of Exhibit 6, Part 1 shows the final selections, the resulting pure premiums and total unpaid losses by policy year. Also shown is the separation of that total unpaid amount between loss reserves and estimated unpaid amounts on unexpired terms of current policies.

Some contracts have provisions that allow car buyers to cancel for various reasons. It is also not unusual for new car contracts to be sold after the car purchase but before the expiration of the manufacturer's warranty. This latter situation is especially true for some insurers who market directly to the new car buyer after the sale. In this case, the effective date of the contract is often recorded as the date the car was put in service.

There can be development in premiums and contract counts over time. Analyses based on losses implicitly include this development. It should be recognized explicitly, however, in methods that consider average losses per contract or expected loss ratios. In this case, the actuary should consider the development of contracts and adjust the forecasts accordingly.

If we calculate loss ratios to monitor the experience in an ESC program, we can use these results to estimate the appropriate earning curves to use. For example, if the program has contingent commissions or some form of retrospective rating, we could use our earning curves to estimate earned premiums for a particular agent or dealer. Exhibit 7 shows the loss emergence implied by the analysis in Exhibits 2 through 6. However, because of the assumed changes in manufacturer warranties for 1990, we suggest using different emergence curves for 1989 and prior contracts versus 1990 and subsequent contracts for these specific calculations. In actual applications, the impact of changes in new car manufacturer warranties should be considered when reviewing earning curves, or equivalent development patterns. We include additional discussion of these adjustments in Section 8.

6. FORECASTS WITHOUT SUFFICIENT DATA

Often an insurer or administrator will not have sufficient experience to assemble complete development triangles needed for the analysis described above. Even with substantial experience available, changes in manufacturer warranties or in contract provisions may require adjustments before that experience can be used for projections.

An additional complication arises in ESC programs that have a large variety of available terms and mileage limitations. Some programs are designed as a cafeteria where a customer can choose among several mileage limitations within each of several time limitations. Though this is often cited as an advantageous sales feature, it further subdivides an already small data base. If there has not been a significant shift in the mix of mileages chosen within a particular time limitation, a combination of the mileages may provide a broader base upon which to make projections.

In the case of changes in manufacturer warranty or ESC provisions, the insurer or administrator may have sufficient data to recast past experience under the new manufacturer warranty or ESC provisions. This is the preferred approach.

In case sufficient data are not available, or if available data are too sparse, the actuary may need to develop estimates of future development from other sources. Currently, no central statistical organization collects and summarizes ESC data or provides other compilation services such as those performed by the Insurance Services Office (ISO) or the National Council on Compensation Insurance (NCCI). On the contrary, most administrators and many insurers hold their data very closely. Thus, modeling from other sources may be required.

One approach is to use Monte Carlo simulation to model the interaction of various aspects of ESCs to estimate the timing of loss emergence. The approach we discuss here concentrates on the loss emergence from policy issue to loss occurrence. This pattern should be less dependent on the activities of a particular insurer or administrator than the development of payments from occurrence to final payment. The latter lag could be estimated using data specific to the insurer or administrator.

The modeling approach described here considers the following aspects of the ESC under analysis:

1. contract term measured by time,
2. contract term measured by mileage,
3. treatment of transfers (vehicle re-sales) in contract,
4. cost of repairs by mileage,
5. inflation in repair and parts costs,
6. effect of manufacturer warranties on costs, and
7. effect of contract provisions on costs.

Exhibit 8 is a diagram that summarizes this approach. In this model we randomly select the mileage to be traveled by a particular car in each year of the contract. Based on the mileage driven in each year, we then estimate the total covered cost limited by various contract provisions. The modeling is then carried out for many cars, to determine relative loss emergence during the life of a contract. This relative emergence can then be used as a substitute for the factors derived in Exhibit 5.

As noted above, it is not unusual to have situations where hard data are not available to quantify various parameters of the simulation. In such cases, we must turn to publicly available sources of information, one of which is published information from the United States Department of Transportation (DOT).

Exhibits 9 through 11 present some such information that can be used in this exercise. Exhibit 9 presents distributions of annual mileage driven by year of ownership and shows that cars tend to be driven less as they age. Exhibit 10 presents data on vehicle retention patterns, and Exhibit 11 provides information regarding repair costs.

It would also seem reasonable that the mileage a particular car is driven in one year will not be independent of the mileage driven in other years. Thus we could make the selection of mileage in subsequent years dependent on the miles driven in earlier years.

There are many possible approaches to reflect this potential dependence. One is to select a Bayesian model wherein the mileage for an individual car follows some random distribution, with the parameters of the distribution being uncertain. Dependence from year to year can be reflected by similar selections of the uncertain parameters from year to year.

An inverse Burr distribution provides excellent fits to the annual mileage distributions shown in Exhibit 9. We then assume that the mileage for an individual car in a particular year has an inverse Weibull distribution; i.e., that such mileage has the cumulative density function:

$$F(x|\theta) = e^{-\theta x^{-\tau}}.$$

Here the parameter τ is assumed to be fixed and known, and the parameter θ is assumed to be unknown but has a Gamma distribution with probability density function given by:

$$g(\theta) = \frac{\lambda^{\alpha}\,\theta^{\alpha-1}\,e^{-\lambda\theta}}{\Gamma(\alpha)}.$$

In this case the posterior distribution of the annual mileage x is an inverse Burr distribution with parameters α, $\lambda^{-1/\tau}$, and τ. The proof of this is given in the appendix to this paper.

We model the annual mileage by first randomly selecting a probability level, p, for a particular simulated car. For each year in the life of the simulated car, we select the parameter θ as that value having p probability in the corresponding Gamma distribution. We select the mileage for that year using that value of θ as the parameter in the inverse Weibull distribution. This procedure maintains some dependence from one year to the next, in that the parameter θ is at the same probability level from one year to the next, but still maintains randomness in the mileage for individual cars.

We model the annual mileage by first randomly selecting a probability level, p, for a particular simulated car. For each year in the life of the simulated car, we select the parameter θ as that value having p probability in the corresponding Gamma distribution. We select the mileage for that year using that value of θ as the parameter in the

As mentioned above, if a car is sold to a party other than a car dealer, most service contracts provide for the transfer of the ESC with the payment of a fee, usually \$25, and the completion of the proper forms. However, many cars are sold without the necessary paperwork or are traded. Thus, sales can affect an ESC's exposure to loss, especially in the later years.

Exhibit 10 shows some retention data published by the DOT. Though this source is a bit old, it does show that vehicle sales can affect ESC loss development. Before these data are used, however, it should be noted that there may be selection in the purchase of ESCs. Those who buy an ESC may expect to own their cars longer. Direct use of the statistics in Exhibit 10 may tend to understate the level of losses in the latter stages of new car contracts. We caution that vehicle retention patterns may have changed significantly since the compilation of the data in Exhibit 10. Given current conditions, the amounts shown in Exhibit 10 should probably be considered as upper bounds for actual retention practices. To the extent that information specific to a particular program is available, it should be used to obtain better estimates of retention rates.

These two exhibits summarize information that can be used to estimate the "retention" of contracts, that is, to estimate the percentage limitations or were transferred. As mentioned above, however, losses generally cannot be expected to arise evenly over the life of ESCs. We still need to incorporate loss information in the model.

In addition to estimates of driving and ownership patterns, the DOT has also published data regarding the cost of owning and operating automobiles. Exhibit 11 presents a summary of the 1984 study. As can be seen in that exhibit, the cost per mile of the category "unscheduled repairs and maintenance" is not constant during the life of a car. It rises during the first 81,000 miles, then falls off to relatively low levels near the end.

The cost portion of our model combines the randomly generated total miles driven with this cost model to estimate total costs. For example, if one of the simulated cars were to travel 13,000 miles the first year and 9,500 miles the second, the first year costs, using the average from Exhibit 11, would be \$10.40 (13,000 × \$0.0008) and the cumulative costs through the second year, with a total of 22,500 miles, would be \$40.40 (14,500 × \$0.0008 + 8,000 × \$0.0036). Thus the indicated second year costs would be \$30.00. We then apply a selected inflation factor, derived at least in part from considering vehicle repair costs in the Consumer Price Index (CPI), to estimate losses to be paid in each year of a contract.

The estimates presented so far all assume that the coverage offered by an ESC matches the costs in "unscheduled repairs and maintenance" shown in Exhibit 11. There are several factors that can affect this assumption.

One factor is the particular ESC itself. Different ESCs have different exclusions of covered parts. If we assume that such exclusions affect the same proportion of costs at all mileage levels, then the data from Exhibit 11 can still be useful in estimating the *timing* of losses in contrast to the emergence of absolute dollar costs. If we expect that contract exclusions can have a substantial impact on these amounts, we could make adjustments before we simulate the results.

A more important influence on ESC costs is changes in new car manufacturer warranties. The data in Exhibit 11 were directed toward a 1984 model car. It is safe to assume that these are based on the existence of a new car warranty that covered virtually all failures in the first year or 12,000 miles, with little or no coverage after that. This was the predominant form of warranty at that time.

Currently, however, several different warranties exist. Almost every manufacturer offers "bumper-to-bumper" coverage for the first year or 12,000 miles, and most offer additional coverage on major components for a period after that. An example is Chrysler's "7/70" that extends coverage on the power train (portions of the engine, transmission, and differential or trans-axle) to the first seven years or 70,000 miles, after payment of a $100 deductible. General Motors' coverage for most 1992 models is three years or 36,000 miles on a "bumper-to-bumper" basis. As mentioned above, ESCs can still experience losses in this period.

In addition to variation in extended warranties among manufacturers, there is also variation within the same manufacturer. Often "high end" cars come with more complete extended manufacturer warranties than other cars from the same manufacturer. Even cars of the same make and model may have different manufacturer warranties. For example, Chrysler offered buyers of 1992 models a choice between the "7/70" option or "bumper-to-bumper" coverage for the first three years or 36,000 miles.

Estimates of loss emergence for particular manufacturers or vehicles could use loss estimates similar to those in Exhibit 11, after adjustment for changes in underlying manufacturer warranties. For example, if a manufacturer has a one year or 12,000 mile basic bumper-to-bumper warranty and a three year or 36,000 mile extended warranty on power train components, and if power train losses are assumed to be 60% of all losses, we could multiply losses between 12,000 and 36,000 miles in Exhibit 11 by 40% as an approximation of the effect of these changes.

In the rare situations where an ESC program covers only one manufacturer and when that manufacturer has modified the warranties on all its vehicles uniformly, this analysis may be sufficient.

Unfortunately, most programs cover vehicles from several, if not all, manufacturers, and different manufacturers have incorporated different changes in their underlying warranties at different times. Hence adjustment of emergence patterns for a more complex book of business tends to be much more complicated in practice.

Also complicating emergence patterns for ESCs is selection by the contract holders. The potential contract holder's perceptions of how he or she will use the car over the coming years may influence the choice of term and mileage limitation selected. For example, if the buyer plans to sell the car after five years, he or she will have little interest in six or seven year contracts. Similarly, if the buyer typically drives many miles per year, he or she would opt for high mileage limitations or even unlimited mileage coverage, if it is available.

Thus, different contract terms can have different underlying loss cost patterns, even if the underlying manufacturer warranties and ESC contracts are the same. This further complicates analysis for an immature program where such differences may not yet be apparent.

In addition, the emergence of losses for a program can be influenced by other factors such as the presence of "good" or "bad" models or model years as well as features unique to a particular ESC program. All of these factors should be considered when modeling in practice.

Exhibit 12 shows the results of the simulation of 50,000 cars using the unadjusted data from Exhibits 9 through 11 for a 5 year/100,000 mile contract, assuming that the Exhibit 11 data are for a basic manufacturer's warranty of one year/12,000 miles of bumper-to-bumper coverage with no extended manufacturer coverage. As can be seen, there is a relatively small portion of loss expected to emerge in the first year of the contract.

The results of this basic simulation are distributions of expected loss emergence in each year of a single contract. These estimates can be combined with assumptions or estimates regarding the writing of policies during a period of time (year, quarter) to derive estimates of loss emergence for individual policy periods. These loss emergence patterns can then be combined with estimates of lags from emergence to final payment to estimate payment lags for policy periods. An example of such a combination for a policy year is shown in Exhibit 13.

Exhibits 9 through 11 show published data that can be a source for estimated costs and mileage distributions for input to the simulation model. Of course, the closer the particular input assumptions are to the experience of the program, the better the model will estimate the emergence and cost patterns for the program. Even if the program is not fully mature, sufficient data may be available to refine the estimates from Exhibits 9 through 11.

Some may express concern regarding the costs of obtaining more detailed data relative to the benefits those data could provide. We have found that, in practice, the benefits of refined data usually outweigh the associated costs. As with other areas of practical actuarial work, this remains a valid consideration.

We caution that these estimates are *for example only*. In practice, actual loss emergence often differs from these model estimates. Thus these particular estimates should not be used without full verification that they are appropriate for the particular program.

7. INCORPORATION OF LIMITED PROGRAM DATA

It is not uncommon to have substantial development experience for a limited number of policy years. This, in some respects, is the worst of both worlds. There is too much real data to ignore but not enough to rely on completely.

In these cases we are able to test the appropriateness of the models against what is already present in the real data. Here again, it is very useful to separate the loss-to-payment lag from the policy-issue-to-loss-emergence lag. Since the first tends to be shorter and more dependent on individual insurer or administrator procedures, even relatively green programs have useful experience in this area.

Once these two lags are separated, the issue-to-emergence lags predicted by the model can then be compared with those actually present in the data, even if only for early emergence stages. If the two curves have similar shapes where real data are available, then we can make use of the emergence from the model to estimate the tail for immature policy periods.

If, however, there are differences, the reasons for those differences should be explored. It may be appropriate, after review, to adjust the emergence predicted by the model to reflect patterns apparent in the actual loss emergence.

In addition to these adjustments to the model emergence patterns, we can consider the appropriateness of the various model assumptions to the particular ESC program. As indicated in Section 6, the primary input data for the simulation model are:

1. the mileage distributions for each year in the life of the car;
2. the estimated costs of repair at various mileage points in the life of the car;
3. the estimated inflation between contract issuance and time of repair; and
4. the estimated rates of contract termination during the life of the contract.

Except for the inflation assumptions, Exhibits 9 through 11 provide examples of some of these estimates, though the data themselves may be somewhat dated.

For example, the average annual mileage for the distributions in Exhibit 9 roughly compares with the annual aggregates in Exhibit 11. However, use patterns change and average annual mileages which were appropriate for 1981 may not be reflective of current driving habits. In particular, a 1988 publication from the U.S. Department of Energy (DOE) titled "Household Vehicles Energy Consumption, 1988" indicates that during 1988, 1987 models averaged 13,400 miles, 1986 models averaged 12,600 miles, 1985 models averaged 12,100 miles, and so forth. This is a different annual mileage pattern than shown in Exhibit 11. In addition, the same 1988 DOE study indicated that the average number of miles driven per vehicle has increased from 9,399 in 1983 to 10,246 in 1988. The more recent information should be incorporated in forecasts of loss emergence.

Mileage distributions may also become important in quantifying selection by insureds between contracts of different terms. As noted above, it is possible that those selecting higher mileage contracts may expect to have a higher annual mileage than those selecting a lower mileage policy. To the extent that significant selection is expected, it may be beneficial to modify the mileage distributions used to model the loss emergence for different contract terms. In this case, we should increase the annual mileages used to model higher mileage contracts relative to those used to model lower mileage contracts.

Actual experience under an ESC program may also be useful in refining estimates of the cost curves used in the simulation model. Many ESC data bases capture mileage at time of repair. This can be very useful in estimating cost emergence.

It is usually a relatively simple task to sort loss payments into categories by mileage at time of repair. There is a difficulty in using these data directly. If the data are taken from policy periods that are not yet fully mature, we do not know the number of contracts expected to be exposed to potential loss in a particular mileage category. Thus, without some estimate of earned exposure we cannot estimate the true average cost for various mileage categories.

We again turn to our Monte Carlo simulation model to derive estimates of these earned exposures. In this case we will not concern ourselves with the costs but simply worry about the proportion of cars that can be expected to have various total mileages at various contract ages. Exhibit 14 is an example of such a distribution, derived from the Monte Carlo model using the assumptions from Exhibits 9 and 10.

The percentages shown in Exhibit 14 represent the estimated proportion of policies of a given age that will exceed the indicated mileage. We can use these estimates in conjunction with written exposures in a particular program to estimate the number of exposures generating losses in a particular mileage range.

An example of this calculation is shown in Exhibit 15. Part 2 displays the contract-years exposed to losses for the ages and mileage entries. These are the products of total contracts by contract age with the corresponding proportions in Exhibit 14. Thus, there are a total of 511,000 (200,000 + 150,000 + 100,000 + 50,000 + 10,000 + 1,000) contracts in this hypothetical program, as shown in Part 1 of Exhibit 15. The lower section of Part 1 shows the number of contracts exposed to losses in each year, given the simplifying assumption of uniform writing during a year. For example, by the end of 1992 all contracts issued from 1987 through 1991 generated a full year of exposure in their first year. With the simplifying assumption of even writings during the year, the 1992 contracts generated one-half of a year of exposure. Thus there were approximately 411,000 contracts contributing to losses in their first year. Similarly, 1987 through 1990 contracts and half, on average, of the 1991 contracts experienced second year exposure, for a total of 236,000, and so forth.

Given the percentages from Exhibit 14, all of these contracts could contribute to losses above 0 miles, but not all could contribute to losses above 6,000 miles. In fact, from Exhibit 14, an estimated 37.24% of the 411,000 first year exposures would contribute in this range (153,056 = .3724 × 411,000), 82.17% of the second year exposures (193,921 = .8217 × 236,000), and so forth. Part 2 thus provides estimates of the number of contracts having exposure in the various mileage bands.

Exhibit 16 provides an example of how these estimates can be used to obtain better estimates of costs per mile, even for an immature new car program. This exhibit shows hypothetical costs for repairs in various mileage intervals. In this case we have assumed that all costs have been adjusted to a common cost level before aggregation. The amounts in Column 3 are adjusted to reflect the cost per mile for an individual contract. We are assuming that these are five-year contracts; thus we divide the exposure count from Column 2 by 5 to calculate Column 3.

Though the costs themselves are hypothetical, the resulting cost-per-mile estimates in Column 3 do represent patterns that arise in practice. Note that the costs start quite low in early years. This is due primarily to the existence of manufacturer warranties covering losses in the first year or 12,000 miles. We could now use these averages in place of the estimates in Exhibit 11 in the Monte Carlo model to obtain a better picture of the loss emergence under a particular program.

8. OTHER USES FOR EMERGENCE MODEL FORECASTS

The primary value of these emergence models is that they can provide insight as to *relative* loss differences under various situations. One such application is in estimating the timing of loss emergence, as described in the previous section.

These models can also be useful in providing insight into the influence of various factors on the overall cost of ESCs. For example, we can use the model to estimate the relative cost difference between five year/50,000 and five year/100,000 mile contracts. This can be done by simply changing the mileage limitation in the model from 50,000 to 100,000. Better estimates of relative differences can be obtained by running the same random set of vehicles with both mileage limitations. Note that the resulting estimates implicitly assume that insureds for different terms will have the same inherent loss pattern. This ignores potential selection by insureds and should be recognized when reviewing results.

The model can also be used to estimate the impact of changes in manufacturer warranties on the costs covered by ESCs. In this case two different cost functions can be used on the same random set of vehicles. For example, we used the input assumptions from Exhibits 9 through 11 and an assumed cost inflation of 7% to derive Table 1, estimates for a five year/100,000 mile new-car ESC. In this case we assumed that the first manufacturer warranty was for one year/12,000 miles for all components, the second for three years/36,000 miles for all components, while the third was for one year/12,000 for all components with coverage for the power train for seven years/70,000 miles. In these calculations we assumed that power train repairs constituted 60% of total costs.

As the table shows, changes in the manufacturer warranty can have a noticeable effect on both the loss emergence and costs of ESCs. Both of the alternative warranties tend to lengthen the emergence curve even though they both reduce total ESC costs. This is of significance in practice. Unless adjusted, development methods based on older contracts with more limited manufacturer warranties may tend to understate losses on more recent contracts where manufacturer warranties cover more. Conversely, pure premium trends will be depressed by the introduction of longer manufacturer warranties.

TABLE 1

EXAMPLE EMERGENCE AND RELATIVE COSTS UNDER ALTERNATIVE MANUFACTURER WARRANTIES

	Manufacturer Warranty		
Contract Age	1/12	3/36	7/70
1	3.80%	2.60%	3.40%
2	20.30	15.80	16.80
3	49.40	44.50	43.70
4	78.40	76.40	75.00
5	100.00	100.00	100.00
Relative Cost	1.00	0.91	0.63

As noted above, these comparisons are based on the assumption that power train losses constitute a uniform 60% of all losses. It is likely that power train losses will experience a different emergence than non-power-train losses. We could use the methodology in Exhibit 16, applied separately to power train and non-power-train losses, to derive separate emergence curves to refine this rough assumption.

If the ESC program has only one make, we could use the revised curves to estimate the impact of changes in manufacturer warranties.

If, however, as in many ESC programs, there are many different underlying manufacturer warranties, then simply calculating separate cost-per-mile curves may not provide sufficient data to modify emergence curves for the program. In fact, the losses that would be used in Exhibit 16 are themselves reduced by existing manufacturer warranties, and these warranties themselves can change from one model year to the next.

Thus, the assumption that changes in warranties can be addressed by simple modifications of the cost-per-mile input data may not hold. In such a case we could develop separate cost curves for each major component of cost to a program. Such components could include those costs covered by the ESCs but not covered under the basic (often bumper-to-bumper) manufacturer warranty, those costs covered by the basic warranty but not covered by an extended (often power train) warranty, and those costs covered by the extended manufacturer warranty. Once these separate curves are estimated using the data for a particular program, we could refine estimates of the effects of changes in underlying warranties on the costs and emergence of losses in a program.

With sufficient data the approaches in Exhibits 14 through 16 may provide a means of separately identifying these separate cost curves *if credible data were available by loss component and mileage, and separately for different underlying manufacturer warranties.* Unfortunately sufficient data in this detail are seldom available. We must sometimes use curve fitting methods that consider the underlying mix of manufacturer warranties to estimate these components. Because of the complexity of this approach and the survey nature of this paper, it will not be discussed further here.

9. USED CAR COVERAGES

Although the above discussion focuses primarily on new car coverages, the same techniques can be applied to analyze the experience of used car programs. As opposed to new car coverages, used car contract terms are relatively short, running between one and three years. In addition, manufacturer warranties generally have less influence on experience for used cars than for new cars. This leads to a greater proportion of losses emerging in the earlier stages of used car contracts than in the later stages.

On the other hand, there may be greater moral hazard present in used car contracts than in those for new cars. The presence of ESCs on used cars can provide a dealer with incentives to recondition used cars at the cost of the ESC program. When analyzing experience for a particular program this possibility should be recognized; in addition, measures should be taken in the program to avoid such reconditioning.

10. LOSS RATIOS IN ESC PROGRAMS

As with many areas of insurance, loss ratios, calculated as incurred (or even paid) losses divided by earned premiums, are frequently used to monitor the profitability of an ESC program. Hopefully the foregoing discussion makes it clear that simple earning patterns probably do not provide sufficient match to expected loss emergence to be relied upon solely.

As mentioned above, we cannot expect losses to emerge uniformly during the life of an ESC. Except in the extremely rare case of unlimited contracts, mileage limitations, and to some extent ownership transfers, reduce the number of used car contracts able to generate losses in their later stages. For used car contracts we often expect that losses will emerge more quickly than pro-rata, and pro-rata earning in fact may provide a conservative basis on which to evaluate profitability.

On the other hand, losses for new car contracts can usually be expected to emerge more slowly than time limitations alone, at least in the early stages of the contract. This usually happens even in unlimited mileage contracts. Though the emergence curves used above are hypothetical, they do present general patterns that appear in new car contracts.

One thing is clear, however: Any formula that claims to apply to a broad range of contracts over a broad range of makes and model years is suspect. One would generally expect that more losses will emerge during the last year of a five year/100,000 mile contract than in the last year of a five year/50,000 mile contract, even though the two contracts experience similar loss experience in their first year or two, with all other variables held constant.

Similarly, one would expect different experience for similar contracts for different model years. In this case, changes in manufacturer warranties would influence the amount and timing of losses during the life of the contracts. Generally, one would expect that extending the manufacturer's warranty on a vehicle will lower the losses on a given ESC. However, this will also push proportionately more losses into the tail of the loss emergence curve. Thus, an earning formula that was appropriate before the introduction of extended manufacturer warranties may earn premiums too rapidly after such introduction.

If earned premiums are used to assess the profitability of an ESC program, we strongly recommend that they be calculated to match the expected flow of losses incurred and that this match be verified periodically. The methods used above can be used for the first calculation. Once they are calculated, emergence patterns should be tested regularly, if loss ratios are to be relied upon.

Probably the easiest way to periodically test the appropriateness of an earning curve is to test that curve against incurred losses (including IBNR) for a fixed policy period. If the resulting loss ratios show a consistent upward pattern as time progresses, we could suspect that the earning curve is pulling premiums into income faster than losses are emerging. Conversely, if the curve shows a consistent downward pattern as time progresses, then we could suspect conservative earning of premium. A match could be indicated by a loss ratio progression that seems to randomly move around a fixed level. However, it is possible that the emergence and earning patterns match over one interval, only to deviate over another. This too should be considered when reviewing loss ratios for an ESC program.

Loss ratios are often used in ratemaking applications, both for determining overall rate level change requirements and in estimating relativities among various classes. As with other lines of insurance, the selected pure premiums shown in Exhibit 6, or corresponding loss ratios, can be used to assess overall rate level adequacy.

We may also be able to use loss ratios to assist in determining the relative adequacy of class rates. One could use the earning curve determined from the aggregate book to estimate earned premium by class. We caution, however, that since classes are usually composed of similar vehicles, new car warranties may vary substantially by class. In this case the actuary could modify the earning curves to reflect the differences in manufacturer warranties and calculate loss ratios that should provide a better indication of relative loss potential among the various classes.

11. STRUCTURE OF ESC PROGRAMS

As indicated in Section 2, there are two common types of ESC programs. Other arrangements also exist. In some, a portion of the amounts collected by the dealer are put into a fund and an insurer provides coverage if that fund is depleted.

A common element in many ESC programs is a middleman. This role can be taken on by a managing general agent or a third party administrator. Usually this party supplies data processing, claims, marketing, and other services and sometimes determines rates and rate plans for the program in exchange for a fee. The fee can be a flat charge per contract, a percentage of insurance premiums, or even related to the loss experience of the program. This structure becomes important when evaluating permissible loss ratios. The structure also influences the amount required for loss reserves. The longer the pipeline between car buyer and insurer, the longer the expected lag can be between claim occurrence and final claim payment. This in turn would indicate proportionately larger loss reserve requirements.

12. HOW MUCH PREMIUM WAS CHARGED?

Given the many hands funds may flow through, the actuary must know the precise definition of premium. Not only does this impact the premium tax that the insurer pays, but it influences the permissible or expected loss ratio for the business. This expected loss ratio, along with additional loads, is then used to monitor rate level adequacy.

Briefly, the cash flow for an administered ESC program wherein the insurance transaction is between the dealer and the insurer may look like:

Car Buyer Payment

Dealer Costs, Profit

Remitted to Administrator

Administrator Costs, Profit

Remitted to Insurer

Insurer Costs, Profit

Fund for Losses

In this case it would not be unusual for the amount "Remitted to Insurer" to be considered premium. It is obvious that the insurer would then expect a relatively large portion of the premium to be available to fund losses. An expected or permissible loss ratio in this case could be in the neighborhood of 85%.

On the other hand, in a program where the car buyer is the insured, with the dealer acting as agent, the amount that the buyer pays may be considered premium. In this case, that premium should provide for commissions to the dealer, administrator fees, insurer costs, and profit. The permissible or expected loss ratio could be small, possibly as low as 30% or lower.

ESC programs are written in a highly competitive market. It is likely that auto dealers choose from more than one program. There is great incentive to sell the contracts with the lowest wholesale price to the car buyer in order to maximize dealer profit. In conjunction with potentially under-priced policies offered in some programs, this stiff competition makes it difficult to implement rate increases. On the other hand, dealers who have experienced the insolvency of one or more of their ESC carriers and now realize they are responsible for the repairs may be more selective in their choice of program.

13. OTHER IMPLICATIONS

Clearly, the rate at which premiums are earned impacts a company's financial position. *We are not advocating any formula or position as to how premiums are earned for financial statements.* An insurer should be aware, however, of the impact of an ESC program on its financial statements. If, after evaluation of an ESC program, an insurer finds that the total unpaid losses, including claims expected to arise from the unexpired terms of existing contracts, exceed the total of its loss and unearned premium reserves, it may need to post additional reserves.

Actuaries who prepare statutory opinions for companies with ESC exposure should be aware of the implications of this conclusion. All ESCs we have seen provide for the reimbursement of expenses for repair of a covered part that breaks down during the term of the contract, limited either by time or mileage. One may argue that the obligation to pay does not exist until a repair occurs and that the loss date is the date of the covered repair. If this position is taken, the loss and loss adjustment expense reserve would provide only for future payments for repairs that have already occurred. As a result, loss and loss adjustment expense reserves would include no provision for repairs that have not yet occurred.

We understand that additional reserves for deficiencies in the unearned premium reserve are required for statements prepared under generally accepted accounting principles (GAAP). However, under current statutory accounting standards, we understand that there is no requirement to book such deficiencies on statutory statements, nor are there explicit provisions for such deficiencies in current annual statements. In this case, an insurer may elect to include a write-in item or segregation of surplus to provide for such deficiencies.

If the amount of these indicated additional reserves is material in terms of a company's surplus, an actuary preparing a statutory statement of actuarial opinion on loss and loss adjustment expense reserves may face a dilemma. The loss and loss adjustment expense reserves may be adequately stated, but the actuary's analysis may imply that the financial solidity of the company is impaired due to future obligations under existing contracts. In addition, there does not appear to be a way to reflect this in statutory statements. Unfortunately, we do not have a solution to offer, but refer the actuary to the appropriate standards of practice.

This additional reserve may not be deductible for the purposes of federal income taxes until the losses are incurred. Thus, the insurer may be in the position of having to increase its reserves without the benefit of a corresponding tax deduction. Again, we do not have a solution to this dilemma, but raise it as a consideration in dealing with ESC programs.

This is not the only area where federal income tax laws come into play. In situations where the insurance policy is with the dealer, it may be considered as contractual liability and included under Other Liability in the annual statement. Thus the Internal Revenue Service (IRS) may require the use of Other Liability discount factors when calculating the deduction for incurred losses, even though the expected pay-out for loss reserves would generally be expected to be very short.

An exhaustive discussion of all aspects of an ESC program and their impact on an insurer is beyond the scope of this paper. The above discussion provides only a brief view of some of the hidden complexities of such a program.

14. CONCLUSION

An ESC program can provide a profitable book of business to an insurer. However, monitoring the profitability of that book presents unique problems. Contracts are often sold for multiple years with limited right to cancellation. Although the coverage generally has losses paid soon after the occurrence, the extended period of coverage heightens the role of the unearned premium reserve on the financial soundness of the program. Unlike most other insurance policies, losses cannot be expected to emerge uniformly throughout the term of the policy. Thus pro rata earning of premium usually does not provide a match between income and liabilities. This can result in inaccurate conclusions regarding the profitability of ESC programs. Insurers writing such programs should continually monitor the fit between premium earning and loss emergence, if loss ratios using earned premiums are to be used to monitor the profitability of an ESC program.

Probably the best way to assess profitability, however, is to compare forecasted total losses with premiums. In this way there are no assumptions regarding the timing of premium earning, and the actual loss experience can provide insight to the future emergence of losses for later policy periods.

The author is aware of only three other publications in the actuarial literature dealing with ESC programs, as presented in the attached bibliography. The concepts and approaches presented there, as well as those presented here, should be considered as starting points in the analysis of an ESC program. There remains much to be done in this area.

REFERENCES

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[2] Noonan, Simon J., "The Use of Simulation Techniques in Addressing Auto Warranty Pricing and Reserving Issues," *Casualty Actuarial Society Forum,* Special Edition, Ratemaking Call Papers, 1993, pp. 25-52.

[3] Schilling, Timothy L.,"The Challenge of Pricing Extended Warranties," *Pricing,* Casualty Actuarial Society Discussion Paper Program, 1990, pp. 369-393.

EXHIBIT 1
Part 1

EXAMPLE: LOSS RATIOS UNDER MISMATCHED EARNING

	Premiums Earned						
Calendar	Policy Year						Cumulative
Year	1	2	3	4	5	Total	Total
1	$1,000					$1,000	$1,000
2	2,000	$1,250				3,250	4,250
3	2,000	2,500	$1,563			6,063	10,313
4	2,000	2,500	3,125	$1,719		9,344	19,657
5	2,000	2,500	3,125	3,438	$1,719	12,782	32,439
6	1,000	2,500	3,125	3,438	3,438	13,501	45,940
7		1,250	3,125	3,438	3,438	11,251	57,191
8			1,562	3,438	3,438	8,438	65,629
9				1,717	3,438	5,155	70,784
10					1,717	1,717	72,501
Total	$10,000	$12,500	$15,625	$17,188	$17,188	$72,501	

NOTE: Dollar amounts are in thousands.

EXHIBIT 1
Part 2

EXAMPLE: LOSS RATIOS UNDER MISMATCHED EARNING

	Losses Incurred						
Calendar Year	Policy Year 1	2	3	4	5	Total	Cumulative Total
1	$375					$375	$375
2	1,500	$469				1,969	2,344
3	3,000	1,875	$586			5,461	7,805
4	4,125	3,750	2,344	$645		10,864	18,669
5	4,125	5,156	4,688	2,578	$645	17,192	35,861
6	1,875	5,156	6,445	5,156	2,578	21,210	57,071
7		2,344	6,445	7,090	5,156	21,035	78,106
8			2,930	7,090	7,090	17,110	95,216
9				3,223	7,090	10,313	105,529
10					3,223	3,223	108,752
Total	$15,000	$18,750	$23,438	$25,782	$25,782	$108,752	

NOTE: Dollar amounts are in thousands.

EXHIBIT 1
Part 3

EXAMPLE: LOSS RATIOS UNDER MISMATCHED EARNING

	Earned Premiums		Incurred Losses		Indicated Loss Ratios		
Calendar Year	Total	Cumulative Total	Total	Cumulative Total	Total	Cumulative Total	Cumulative Profit (Loss)
1	$1,000	$1,000	$375	$375	38%	38%	$625
2	3,250	4,250	1,969	2,344	61	55	1,906
3	6,063	10,313	5,461	7,805	90	76	2,508
4	9,344	19,657	10,864	18,669	116	95	988
5	12,782	32,439	17,192	35,861	135	111	(3,422)
6	13,501	45,940	21,210	57,071	157	124	(11,131)
7	11,251	57,191	21,035	78,106	187	137	(20,915)
8	8,438	65,629	17,110	95,216	203	145	(29,587)
9	5,155	70,784	10,313	105,529	200	149	(34,745)
10	1,717	72,501	3,223	108,752	188	150	(36,251)
Total	$72,501		$108,752		150%		

NOTE: Dollar amounts are in thousands.

EXHIBIT 1
Part 4

EXAMPLE: LOSS RATIOS UNDER MISMATCHED EARNING

Indicated Loss Ratios by Calendar and Policy Year

Calendar Year	Policy Year 1	2	3	4	5	Total	Cumulative Total
1	38%					38%	38%
2	75	38%				61	55
3	150	75	37%			90	76
4	206	150	75	38%		116	95
5	206	206	150	75	38%	135	111
6	188	206	206	150	75	157	124
7		188	206	206	150	187	137
8			188	206	206	203	145
9				188	206	200	149
10					188	188	150
Total	150%	150%	150%	150%	150%	150%	

EXHIBIT 2

SAMPLE FISCAL ACCIDENT YEAR PAID LOSS DEVELOPMENT

(AS OF SEPTEMBER 30, 1992)

Fiscal Accident Year Ending 9/30	Months of Development 24	36	48	60	72	84	Ultimate Forecast	Indicated Loss Reserves
1986	$13	$14	$14	$14	$14	$14	$14	$0
1987	187	188	188	188	188		188	0
1988	1,362	1,383	1,394	1,394			1,394	0
1989	3,622	3,887	3,926				3,926	0
1990	8,616	8,687					8,765	78
1991	15,554						16,098	544
1992							23,677	6,849
							Total	7,471

DEVELOPMENT FACTORS

Fiscal Accident Year Ending 9/30	Months of Development 24/12	36/24	48/36	60/48	72/60	84/72	Ultimate/84
1986	1.625	1.077	1.000	1.000	1.000	1.000	
1987	1.520	1.005	1.000	1.000	1.000		
1988	1.684	1.015	1.008	1.000			
1989	1.945	1.073	1.010				
1990	1.386	1.008					
1991	1.237						
Selected	1.359	1.026	1.009	1.000	1.000	1.000	
Cumulative	1.407	1.035	1.009	1.000	1.000	1.000	1.000

NOTE: Dollar amounts are in thousands.

EXHIBIT 3

SAMPLE FISCAL POLICY YEAR PAID LOSS DEVELOPMENT

(AS OF SEPTEMBER 30, 1992)

Fiscal Policy Year Ending 9/30	Months of Development 12	24	36	48	60	72	84	Ultimate Forecast	Indicated Loss Reserves
1986	$0	$114	$581	$1,294	$2,341	$3,102	$3,176	$3,201	$25
1987	18	406	1,754	4,701	7,198	8,547		8,821	274
1988	16	370	2,974	7,156	10,696			13,477	2,781
1989	26	1,439	7,743	16,588				32,164	15,576
1990	29	1,237	6,516					29,087	22,571
1991	32	1,038						24,688	23,650
1992	29							25,724	25,695
								Total	$90,572

DEVELOPMENT FACTORS

Fiscal Policy Year Ending 9/30	Months of Development 24/12	36/24	48/36	60/48	72/60	84/72	Ultimate/84
1986	--	5.096	2.227	1.809	1.325	1.024	
1987	22.556	4.320	2.680	1.531	1.187		
1988	23.125	8.038	2.406	1.495			
1989	55.346	5.381	2.142				
1990	42.655	5.268					
1991	32.438						
Selected-1	--	--	--	1.539	1.221	1.024	
Cumulative	--	--	--	1.939	1.260	1.032	1.008
Selected-2	37.295	5.328	2.142	1.575	1.250	1.050	
Cumulative	887.024	23.784	4.464	2.084	1.323	1.058	1.008

NOTES: 1. Dollar amounts are in thousands.
2. Selected-1 is used to develop policy years 1989 and prior; Selected-2 is used for policy years 1990 and subsequent.

EXHIBIT 4

SAMPLE FISCAL POLICY YEAR DISTRIBUTION OF PAID LOSSES BY ACCIDENT YEAR

(AS OF SEPTEMBER 30, 1992)

Fiscal Policy Year	Accident Year						
Ending 9/30	PY	PY+1	PY+2	PY+3	PY+4	PY+5	PY+6
1986	$14	$145	$675	$868	$951	$522	$0
1987	43	676	2,112	2,622	2,170	925	
1988	43	814	2,941	4,142	2,756		
1989	132	2,096	6,961	7,399			
1990	77	1,696	4,743				
1991	62	976					
1992	29						

ACCIDENT YEAR DEVELOPMENT FACTORS

	Accident Year Age						
	12	24	36	48	60	72	84
Factor	1.407	1.035	1.009	1.000	1.000	1.000	1.000

ESTIMATED ULTIMATE POLICY YEAR/ACCIDENT YEAR LOSSES

Fiscal Policy	Accident Year						
Year Ending 9/30	PY	PY+1	PY+2	PY+3	PY+4	PY+5	PY+6
1986	$14	$145	$675	$868	$960	$540	$0
1987	43	676	2,112	2,646	2,246	1,301	
1988	43	814	2,967	4,287	3,878		
1989	132	2,115	7,205	10,410			
1990	78	1,755	6,673				
1991	64	1,373					
1992	41						

NOTES: 1. Dollar amounts are in thousands.

2. The Accident Year Development Factors are the cumulative factors from Exhibit 2.

EXHIBIT 5

SAMPLE FISCAL POLICY YEAR BY ACCIDENT YEAR PAID LOSS DEVELOPMENT

(AS OF SEPTEMBER 30, 1992)

Fiscal Policy Year	Number of Accident Years Emerged							Ultimate	Indicated Total
Ending 9/30	1	2	3	4	5	6	7	Forecast	Unpaid
1986	$14	$159	$834	$1,702	$2,662	$3,202	$3,202	$3,202	$26
1987	43	719	2,831	5,477	7,723	9,024		9,024	477
1988	43	857	3,824	8,111	11,989			14,111	3,415
1989	132	2,247	9,452	19,862				34,202	17,614
1990	78	1,833	8,506					32,008	25,492
1991	64	1,437						23,798	22,760
1992	41							15,636	15,607
								Total	$85,392

DEVELOPMENT FACTORS

Fiscal Policy Year	Number of Accident Years Emerged						
Ending 9/30	2/1	3/2	4/3	5/4	6/5	7/6	Ultimate/7
1986	11.357	5.245	2.041	1.564	1.203	1.000	
1987	16.721	3.937	1.935	1.410	1.168		
1988	19.930	4.462	2.121	1.478			
1989	17.023	4.206	2.101				
1990	23.500	4.640					
1991	22.453						
Selected-1	--	--	--	1.463	1.177	1.000	
Cumulative	--	--	--	1.722	1.177	1.000	1.000
Selected-2	23.028	4.401	2.101	1.480	1.210	1.000	
Cumulative	381.367	16.561	3.763	1.791	1.210	1.000	1.000

NOTES: 1. Dollar amounts are in thousands.
2. Selected-1 is used to develop policy years 1989 and prior; Selected-2 is used for policy years 1990 and subsequent.

EXHIBIT 6
Part 1

SUMMARY OF ULTIMATE LOSS FORECASTS
(AS OF SEPTEMBER 30, 1992)

Fiscal Policy Year Ending 9/30	(1) Policy Year Development (Exhibit 3)	(2) Policy-Accident Year Development (Exhibit 5)	(3) Initial Selection	(4) Total Written Contracts	(5) Initial Indicated Pure Premium	(6) Smoothed Pure Premium	(7) Expected Pure Premium Method	(8) Selected Ultimate Losses	(9) Indicated Pure Premium	(10) Losses Paid 9/30/92	(11) Indicated Unpaid Losses
1986	$3,201	$3,202	$3,202	22,399	$143	$143	$3,202	$3,202	$143	$3,176	$26
1987	8,821	9,024	9,024	60,513	149	152	9,024	9,024	149	8,547	477
1988	13,477	14,111	13,953	85,716	163	161	14,065	14,028	164	10,696	3,332
1989	32,164	34,202	33,693	197,116	171	171	33,995	33,894	172	16,588	17,306
1990	29,087	32,008	31,278	186,064	168	163	30,776	30,943	166	6,516	24,427
1991	24,688	23,798	24,243	143,542	169	173	24,770	24,638	172	1,038	23,600
1992	25,724	15,636	---	149,963	---	184	27,562	27,562	184	29	27,533
Total								$143,291		$46,590	$96,701

Indicated Loss Reserves as of 9/30/92 $7,471

Estimated Losses on Future Claims as of 9/30/92 $89,230

NOTES:
1. Amounts in Columns (1), (2), (3), (7), (8), (10) and (11) are in thousands of dollars.
2. The derivation of Column (6) is shown in Column (5) of Part 2.
3. The derivation of Column (7) is shown in Column (9) of Part 2.

EXHIBIT 6
Part 2

DERIVATION OF COLUMNS (6) AND (7) OF EXHIBIT 6, PART 1
(AS OF SEPTEMBER 30, 1992)

Fiscal Policy Year Ending 9/30	(1) Initial Indicated Pure Premium	(2) Estimated New Car Warranty Change Factor	(3) Warranty Adjusted Pure Premium (1)/(2)	(4) Smoothed Warranty Adjusted Pure Premium	(5) Selected Smoothed Pure Premium (2) x (4)	(6) Total Written Contracts	(7) Estimated Ultimate Losses on Emerged Claims	(8) Estimated Percent of Losses Emerged	(9) Expected Pure Premium Method (7)+[1−(8)]×(5)×(6)
1986	$143	1.00	$143	$143	$143	22,399	$3,202	100.00%	$3,202
1987	149	1.00	149	152	152	60,513	9,024	100.00	9,024
1988	163	1.00	163	161	161	85,716	11,989	84.96	14,065
1989	171	1.00	171	171	171	197,116	19,862	58.07	33,995
1990	168	0.90	187	182	163	186,064	8,506	26.57	30,776
1991	169	0.90	188	193	173	143,542	1,437	6.04	24,770
1992	--	0.90	--	205	184	149,963	41	0.26	27,562

NOTES:
1. Column (1) is Column (5) from Part 1.
2. Column (2) is assumed, based on a separate analysis.
3. Column (4) is the result of an exponential fit on Column (3).
4. Column (7) is the last diagonal from Exhibit 5.
5. Column (8) is the reciprocal of the age-to-ultimate factors from Exhibit 5. The cumulative for Selected-1 is used for 1986-1989 and the cumulative for Selected-2 is used for 1990-1992.
6. Amounts in Columns (7) and (9) are in thousands of dollars.

EXHIBIT 7

LOSS EMERGENCE IMPLIED BY POLICY YEAR/ACCIDENT YEAR DISTRIBUTION AND SELECTED ULTIMATE LOSSES BY POLICY YEAR

(AS OF SEPTEMBER 30, 1992)

Fiscal Policy	Accident Year						
Year Ending 9/30	PY	PY+1	PY+2	PY+3	PY+4	PY+5	PY+6
1986	0.4%	5.0%	26.0%	53.2%	83.1%	100.0%	100.0%
1987	0.5	8.0	31.4	60.7	85.6	100.0	
1988	0.3	6.1	27.3	57.8	85.5		
1989	0.4	6.6	27.9	58.6			
1990	0.3	5.9	27.5				
1991	0.3	5.8					
1992	0.1						
Weighted Averages:							
1986-89	0.4%	6.6%	28.2%	58.4%	85.2%	100.0%	100.0%
1990-92	0.2	5.9	27.5	--	--	--	--

EXHIBIT 8

EXAMPLE FLOW OF MONTE CARLO SIMULATION MODEL

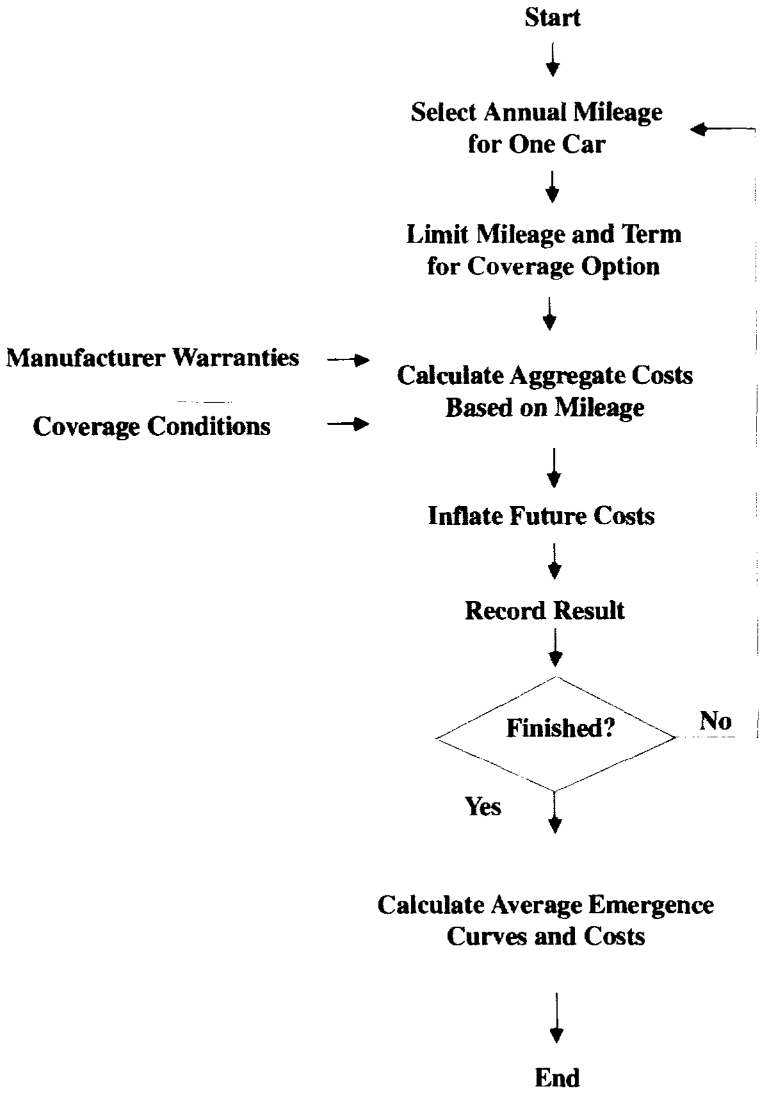

EXHIBIT 9

PERCENTAGE OF VEHICLES BY ANNUAL MILEAGE AND AGE

	Vehicle Age											
Annual Mileage	Under 1	1	2	3	4	5	6	7	8	9	10+	All
0-999	22.0%	4.9%	2.5%	3.6%	4.1%	4.5%	5.2%	5.4%	8.0%	9.0%	19.0%	8.2%
1,000-2,999	13.1	6.1	6.5	7.3	9.6	10.8	11.3	12.9	12.9	16.8	18.9	12.0
3,000-7,999	18.4	22.7	20.9	23.9	26.0	26.9	29.0	32.2	32.8	31.2	31.4	27.7
8,000-12,999	20.0	24.9	30.7	33.2	32.4	31.0	30.1	28.7	30.3	25.6	19.2	27.4
13,000-17,999	9.2	18.8	18.2	15.3	14.6	14.0	13.2	9.4	9.0	9.4	5.6	12.2
18,000-22,999	5.0	8.8	9.8	7.1	6.2	6.7	5.6	6.5	3.9	3.7	2.4	5.7
23,000-27,999	5.6	6.3	4.7	4.4	2.9	2.7	2.7	1.8	1.3	2.0	1.1	2.9
28,000-	6.7	7.5	6.7	5.2	4.2	3.4	2.9	3.1	1.8	2.3	2.4	3.9
Total	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%
Average Annual Mileage	11,268	13,498	13,562	12,261	11,497	10,694	10,624	9,655	8,757	8,714	7,085	10,368

Source: "Household Vehicle Utilization," U.S. Department of Transportation, Federal Highway Administration, Office of Highway Planning, April 1981.

EXHIBIT 10

PERCENTAGE OF AUTOMOBILES PURCHASED NEW AND USED BY AGE OF AUTO AND AUTO OWNERSHIP IN 1977

Age of Autos in Years	Vehicles Purchased New		Vehicles Purchased Used		Total Vehicles	Percentage with Original Owner
	Percentage	Number	Percentage	Number		
Less than 1	2.0%	912	0.0%	0	912	100.0%
1	16.6	7,570	0.9	445	8,015	94.4
2	16.9	7,706	3.4	1,683	9,389	82.1
3	11.2	5,107	5.1	2,525	7,632	66.9
4	11.1	5,062	8.3	4,109	9,171	55.2
5	11.0	5,016	10.1	5,000	10,016	50.1
6	8.5	3,876	10.8	5,346	9,222	42.0
7	5.4	2,462	8.8	4,356	6,818	36.1
8	4.7	2,143	8.8	4,356	6,499	33.0
9	3.4	1,550	9.4	4,653	6,203	25.0
10 or more	9.2	4,196	34.4	17,027	21,223	19.8
Total		45,600		49,500	95,100	47.9%

Source: 1977 values from Table 32 in "Household Vehicle Utilization" published by the U.S. Department of Transportation.

EXHIBIT 11

Estimated Average Costs Per Mile for Unscheduled Repairs and Maintenance

		Vehicle Size				
Year	Annual Miles	Large	Intermediate	Compact	Sub-Compact	Average
1	14,500	$0.0010	$0.0008	$0.0007	$0.0006	$0.0008
2	13,700	0.0045	0.0035	0.0033	0.0029	0.0036
3	12,500	0.0273	0.0291	0.0174	0.0255	0.0248
4	11,400	0.0318	0.0268	0.0198	0.0285	0.0267
5	10,300	0.1203	0.0871	0.0495	0.0664	0.0808
6	9,700	0.0722	0.0756	0.0632	0.1068	0.0795
7	9,200	0.1238	0.1197	0.1588	0.1401	0.1356
8	8,700	0.0751	0.0593	0.0645	0.0596	0.0646
9	8,200	0.0296	0.0292	0.0149	0.0242	0.0245
10	7,800	0.0023	0.0019	0.0013	0.0012	0.0017
11	7,300	0.0019	0.0015	0.0009	0.0008	0.0013
12	6,700	0.0021	0.0017	0.0010	0.0009	0.0014

Source: "Cost of Owning and Operating Automobiles and Vans, 1984," published by the U.S. Department of Transportation.

EXHIBIT 12

ESTIMATED LOSS EMERGENCE FOR ONE 5/100 CONTRACT BASED ON SIMULATION MODEL

Policy Age	Estimated Percentage Emerged
1	3.8%
2	20.2
3	49.3
4	78.3
5	100.0

EXHIBIT 13

ESTIMATED AGGREGATE POLICY YEAR PAYMENT PATTERN FOR 5/100 CONTRACT

			Policy Year Loss Emergence						
Development Year	Accident Year Loss Emergence	Calendar Year	Accident Year 1	Accident Year 2	Accident Year 3	Accident Year 4	Accident Year 5	Accident Year 6	Total
1	71.1%	1	1.4%						1.4%
2	96.6	2	0.5	7.2%					7.7
3	99.1	3	0.0	2.6	16.1%				18.7
4	100.0	4	0.0	0.3	5.8	20.6%			26.7
		5		0.1	0.6	7.4	18.0%		26.1
		6			0.2	0.7	6.5	7.7%	15.1
		7				0.3	0.6	2.8	3.7
		8					0.2	0.3	0.5
		9						0.1	0.1
Policy Year Loss Emergence			1.9%	10.1%	22.7%	29.1%	25.4%	10.8%	100.0%

EXHIBIT 14

ESTIMATED PERCENTAGE OF CONTRACTS HAVING EXPOSURE GREATER THAN INDICATED MILEAGE

	Age of Policy Year					
Mileage	1	2	3	4	5	6
0	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
6,000	37.24	82.17	90.46	92.41	93.13	93.49
12,000	24.79	63.08	79.67	84.23	85.94	86.71
18,000	14.26	45.54	68.09	75.35	78.11	79.47
24,000	7.50	32.07	56.66	66.60	70.56	72.39
30,000	3.97	22.40	45.76	57.96	63.20	65.70
36,000	2.16	15.36	35.95	49.68	56.06	59.19
42,000	1.26	10.49	27.70	41.85	49.24	52.97
48,000	0.83	7.03	20.86	34.70	42.83	47.14
54,000	0.56	4.67	15.55	28.34	36.90	41.70
60,000	0.38	3.14	11.47	22.85	31.42	36.59
66,000	0.26	2.14	8.36	18.14	26.56	31.89
72,000	0.19	1.47	6.04	14.21	22.23	27.66
78,000	0.13	1.02	4.33	11.02	18.40	23.79
84,000	0.09	0.73	3.16	8.45	14.99	20.39
90,000	0.08	0.54	2.29	6.45	12.22	17.30
96,000	0.06	0.40	1.71	4.92	9.75	14.50
100,000	0.05	0.31	1.40	4.14	8.43	12.93

NOTE: Estimates are derived from the Monte Carlo simulation model based on input assumptions from Exhibits 9 and 10.

EXHIBIT 15
Part 1

EXPECTED EXPOSURE BY POLICY AGE EXPECTED TO EXCEED INDICATED MILEAGE

WRITTEN CONTRACTS

Policy Year					
1992	1991	1990	1989	1988	1987
200,000	150,000	100,000	50,000	10,000	1,000

POTENTIAL CONTRACTS EXPOSED BY AGE

Policy	Age of Policy Year					
Year	1	2	3	4	5	6
1987	1,000	1,000	1,000	1,000	1,000	500
1988	10,000	10,000	10,000	10,000	5,000	
1989	50,000	50,000	50,000	25,000		
1990	100,000	100,000	50,000			
1991	150,000	75,000				
1992	100,000					
Total	411,000	236,000	111,000	36,000	6,000	500

EXHIBIT 15
Part 2

ESTIMATED CONTRACTS EXPOSED

	Age of Policy Year						
Mileage	1	2	3	4	5	6	Total
0	411,000	236,000	111,000	36,000	6,000	500	800,500
6,000	153,056	193,921	100,411	33,268	5,588	467	486,711
12,000	101,887	148,869	88,434	30,323	5,156	434	375,103
18,000	58,609	107,474	75,580	27,126	4,687	397	273,873
24,000	30,825	75,685	62,893	23,976	4,234	362	197,975
30,000	16,317	52,864	50,794	20,866	3,792	329	144,962
36,000	8,878	36,250	39,905	17,885	3,364	296	106,578
42,000	5,179	24,756	30,747	15,066	2,954	265	78,967
48,000	3,411	16,591	23,155	12,492	2,570	236	58,455
54,000	2,302	11,021	17,261	10,202	2,214	209	43,209
60,000	1,562	7,410	12,732	8,226	1,885	183	31,998
66,000	1,069	5,050	9,280	6,530	1,594	159	23,682
72,000	781	3,469	6,704	5,116	1,334	138	17,542
78,000	534	2,407	4,806	3,967	1,104	119	12,937
84,000	370	1,723	3,508	3,042	899	102	9,644
90,000	329	1,274	2,542	2,322	733	87	7,287
96,000	247	944	1,898	1,771	585	73	5,518
100,000	206	732	1,554	1,490	506	65	4,553

NOTE: Estimates of contracts exposed are based on the written contracts on the top portion of this exhibit and the percentages in Exhibit 14.

EXHIBIT 16

ESTIMATED COST PER MILE

Mileage at Time of Repair	(1) Total Costs	(2) Estimated Exposed Contracts	(3) Indicated Cost per Mile in Interval (1)/[(2)/5]
$0 < X \leq 6{,}000$	$10,000	800,500	0.0010¢
$6{,}000 < X \leq 12{,}000$	15,000	486,711	0.0026
$12{,}000 < X \leq 18{,}000$	50,000	375,103	0.0111
$18{,}000 < X \leq 24{,}000$	60,000	273,873	0.0183
$24{,}000 < X \leq 30{,}000$	75,000	197,975	0.0316
$30{,}000 < X \leq 36{,}000$	150,000	144,962	0.0862
$36{,}000 < X \leq 42{,}000$	160,000	106,570	0.1251
$42{,}000 < X \leq 48{,}000$	165,000	78,967	0.1741
$48{,}000 < X \leq 54{,}000$	150,000	58,455	0.2138
$54{,}000 < X \leq 60{,}000$	125,000	43,209	0.2411
$60{,}000 < X \leq 66{,}000$	100,000	31,998	0.2604
$66{,}000 < X \leq 72{,}000$	90,000	23,682	0.3167
$72{,}000 < X \leq 78{,}000$	75,000	17,542	0.3563
$78{,}000 < X \leq 84{,}000$	50,000	12,937	0.3221
$84{,}000 < X \leq 90{,}000$	30,000	9,644	0.2592
$90{,}000 < X \leq 96{,}000$	20,000	7,287	0.2287
$96{,}000 < X \leq 100{,}000$	10,000	5,518	0.2265

NOTES:

1. Amounts in Column (2) are from Exhibit 15.
2. The amounts in Column (1) here are hypothetical but could be determined from company loss experience by miles driven at time of repair.

APPENDIX

This appendix contains the proof that the mixing of the inverse Weibull and Gamma distributions produces an inverse Burr distribution. First suppose the variable x has an inverse Weibull distribution with parameters θ and τ with cumulative density function given by:

$$F(x|\theta) = e^{-\theta x^{-\tau}}.$$

This results in a probability density function given by:

$$f(x|\theta) = \theta\tau x^{-(\tau+1)}\, e^{-\theta x^{-\tau}}.$$

Suppose, further, that θ is itself unknown but has a Gamma distribution with the probability density function:

$$g(\theta) = \frac{\lambda^{\alpha}\,\theta^{\alpha-1} e^{-\lambda\theta}}{\Gamma(\alpha)}.$$

In this case the probability density function for the variable x is given by:

$$h(x) = \int_0^\infty f(x|\theta) g(\theta) d\theta$$

$$= \int_0^\infty \frac{\lambda^{\alpha}\,\theta^{\alpha-1}\, e^{-\lambda\theta}}{\Gamma(\alpha)}\, \theta\tau x^{-(\tau+1)} e^{-\theta x^{-\tau}}\, d\theta$$

$$= \frac{\lambda^{\alpha}\,\tau\, x^{-(\tau+1)}}{\Gamma(\alpha)} \int_0^\infty \theta^{\alpha}\, e^{-\lambda\theta - \theta x^{-\tau}}\, d\theta$$

$$= \frac{\lambda^{\alpha}\,\tau\, x^{-(\tau+1)}}{\Gamma(\alpha)} \int_0^\infty \theta^{\alpha}\, e^{-\theta(\lambda+x^{-\tau})}\, d\theta\,.$$

We now make the change of variables with $z = \theta\,(\lambda+x^{-\tau})$, so that

$$d\theta = \frac{1}{\lambda+x^{-\tau}}\, dz\,.$$

The equation now becomes:

$$h(x) = \frac{\tau\lambda^{\alpha}\, x^{-(\tau+1)}}{\Gamma(\alpha)(\lambda+x^{-\tau})} \int_0^\infty \left(\frac{z}{\lambda+x^{-\tau}}\right)^{\alpha} e^{-z} dz$$

$$= \frac{\tau\lambda^{\alpha}\, x^{-(\tau+1)}}{\Gamma(\alpha)(\lambda+x^{-\tau})^{\alpha+1}} \int_0^\infty z^{\alpha}\, e^{-z} dz$$

$$= \frac{\tau\lambda^{\alpha}\, x^{-(\tau+1)}}{\Gamma(\alpha)(\lambda+x^{-\tau})^{\alpha+1}}\, \Gamma(\alpha+1)$$

$$= \frac{\tau\lambda^{\alpha}\, x^{-(\tau+1)}}{\Gamma(\alpha)(\lambda+x^{-\tau})^{\alpha+1}}\, \alpha\Gamma(\alpha)$$

$$= \frac{\alpha\tau\lambda^{\alpha}\, x^{-(\tau+1)}}{(\lambda+x^{-\tau})^{\alpha+1}}$$

$$= \frac{\alpha\tau\lambda^{\alpha+1}\, x^{-(\alpha+1)\tau}\, x^{\alpha\tau-1}}{\lambda(\lambda+x^{-\tau})^{\alpha+1}}$$

$$= \frac{\alpha\tau x^{\alpha\tau-1}(\lambda x^{-\tau})^{\alpha+1}}{\lambda(\lambda+x^{-\tau})^{\alpha+1}}$$

$$= \frac{\alpha\tau x^{\alpha\tau-1}}{\lambda}\left(\frac{\lambda x^{-\tau}}{\lambda+x^{-\tau}}\right)^{\alpha+1}$$

$$= \frac{\alpha\tau x^{\alpha\tau-1}}{\lambda}\left(\frac{1}{\dfrac{\lambda}{\lambda x^{-\tau}}+\dfrac{x^{-\tau}}{\lambda x^{-\tau}}}\right)^{\alpha+1}$$

$$= \frac{\alpha\tau(1/\lambda)x^{\alpha\tau-1}}{((1/\lambda)+x^{\tau})^{\alpha+1}}.$$

This is the probability density function for an inverse Burr distribution with parameters α, $\lambda^{-1/\tau}$, and τ.

UNDERWRITING BETAS—THE SHADOWS OF GHOSTS

THOMAS J. KOZIK

Abstract

This paper critiques the methodologies used in prior studies to estimate underwriting betas for application in the "insurance CAPM." It argues that reliable estimates of underwriting betas do not exist. It also demonstrates the inapplicability of the CAPM to the yield to maturity of a bond or portfolio of bonds. Finally, it demonstrates that the assumption that the yield on a U.S. Treasury bill is risk-free for purposes of applying the CAPM implies that all U.S. Treasury securities, regardless of maturity, have betas of zero.

1. INTRODUCTION

The asset pricing models of modern finance theory are sometimes used in insurance rate hearings to determine the equilibrium rate of return to an insurer, and hence the premium level that is implied by that rate of return. One of those models, the Capital Asset Pricing Model (CAPM) (Sharpe, [21]), asserts that the equilibrium rate of return on any asset is given by

$$r_a = r_f + \beta_a(r_m - r_f) \tag{1.1}$$

where

r_a = expected rate of return on asset a,

r_f = risk-free rate of return,

r_m = expected rate of return on the market portfolio (the market portfolio is the portfolio that includes all risky securities, each in proportion to its market value;

i.e., U.S. stocks, foreign stocks, real estate, precious metals, etc.),

β_a = systematic risk or beta of asset a
$= \mathrm{cov}(r_a, r_m)/\mathrm{var}(r_m)$.

It is possible, under some assumptions, to derive an expression for the equilibrium underwriting rate of return for an insurer's total book of business as a percent of premium by applying the CAPM to an insurer and decomposing the total return into the sum of the underwriting return and the investment return. This results in the model that Cummins [7] has termed the "insurance CAPM." Further, with additional assumptions, estimates of equilibrium underwriting returns for individual lines of insurance can be obtained.

The "insurance CAPM" was first derived by Biger and Kahane [3]. Their version of the model not only assumed a world without taxes, but also that each dollar of premium is invested for exactly one year before being paid out in the form of loss or expense. This latter assumption was relaxed by Kahane in his paper, "The Theory of Insurance Risk Premiums—A Re-examination In The Light of Recent Developments in Capital Market Theory" [15]. Kahane's version of the model is

$$r_u = -kr_f + \beta_u(r_m - r_f) \qquad (1.2)$$

where

r_u = expected underwriting return per dollar of premium,

β_u = systematic risk of underwriting
$= \mathrm{cov}(r_u, r_m)/\mathrm{var}(r_m)$,

k = a measure of the length of time that one dollar of premium is invested before being paid out in the form of loss or expense.

Other authors include taxes in their versions of the model. Both Hill [12] and Fairley [9], for example, include a single average tax rate in their versions of the model. Urrutia [24] recognizes two tax rates—one for underwriting income and another for investment income. Urrutia's model is

$$r_u = -kr_f(1-t_a)\,/\,(1-t_u) + t_a r_f\,/\,(s(1-t_u)) + \beta_u(r_m-r_f) \qquad (1.3)$$

where

$t_a =$ tax rate on investment income,

$t_u =$ tax rate on underwriting income,

$s =$ premium to equity ratio.

Although the "insurance CAPM" purports to give the equilibrium underwriting return, it is increasingly being used in insurance rate hearings to determine the "fair" underwriting return, and thereby, the implied premium level. The magnitude of return that is required in order to be a "fair" return is a constitutional question. An equilibrium return can be a "fair" return only if it meets the standards of "fair" returns that have been enunciated by the United States Supreme Court. It is not universally accepted that equilibrium returns meet the standards of "fair" returns. Whether or not they do, though a significant issue in its own right, is not the focus of this paper. Nor is it the intent of this paper to address the shortcomings of the models in their various forms. Rather, the intent of this paper is to focus solely on the problem of estimating underwriting betas.

It is common for expert witnesses who apply the "insurance CAPM" for the purpose of determining premium levels in insurance rate hearings to assume that underwriting betas are zero or slightly negative. Consider the following comments of two expert witnesses in a recent auto insurance rate hearing:

> Witness 1: "Empirical evidence shows that underwriting has no systematic risk. Remember, only systematic risk is compensable. Some studies have demonstrated very small and even negative underwriting risk, therefore we can assume it is not relevant."
>
> Witness 2: "The underwriting beta (β_u) is assumed to be zero precisely because no published study has shown that underwriting returns are related to market returns. This makes intuitive sense because one does not expect accidents to increase or decrease because stock market returns are increasing or decreasing."

Listening to these witnesses, one might get the impression that the issue is settled—that it is virtually certain that underwriting betas are indistinguishable from zero, and that this conclusion is supported by all of the studies that have addressed the issue. This, however, is not true. Not only do estimates of underwriting betas vary widely from study to study, but also numerous authors note the bias and inaccuracy inherent in the estimates themselves. This paper argues that reliable estimates of underwriting betas do not exist, and hence, the true values of those underwriting betas are unknown.

2. DISCERNING THE GHOST

Underwriting betas are not observable. If the underwriting operations of insurers were publicly traded, then historic underwriting betas might be estimable. All insurers, though, have investment operations as well as underwriting operations. This substantially complicates the estimation of underwriting betas.

A number of authors have tried to estimate underwriting betas using indirect methods (Biger and Kahane; Fairley; Hill; Cummins and Harrington, [6]; Cox and Rudd, [5]). The indirect methods employed are of two general types. One is to regress historic accounting underwriting returns on historic returns for some market index. In this paper, betas estimated using this technique are called "accounting betas." The second method of estimation is based on the notion that the equity beta of an insurer is a linear combination of an investment beta and an underwriting beta (or alternatively, an asset beta and a liability beta). Hence, the underwriting beta (liability beta) can be inferred from estimates of the investment portfolio beta (asset beta) and the equity beta of a publicly traded insurer. In this paper, betas estimated using this technique are called "inferred betas."

The results of these studies vary greatly across lines of insurance, firms, time, choice of the market portfolio, and estimation methodology. In spite of this variation, it is common, as stated earlier, for expert witnesses to assume that underwriting betas are zero or slightly negative. The support for this practice, if and when any is given, is to cite only those studies that affirm the witness's assertion.

Underlying the indirect methods of estimation are numerous unstated assumptions. Moreover, the magnitude of the estimation errors are unknown and potentially enormous. Given that these models and these estimates are increasingly being used to determine premium levels, a critical analysis of the reasonableness of these assumptions is necessary.

3. REVIEW OF THE LITERATURE

Biger and Kahane estimated two sets of accounting betas for each of eighteen lines of insurance. They regressed annual percentage underwriting profits, aggregated for all U.S. non-life stock insurance companies over the period from 1956-1973 (as reported in Best's *Property-Casualty Aggregates and Averages*), against two indices that served as proxies for the market portfolio. The first index is Moody's stock index including dividends. Hence, the market portfolio represented by this index is an all-equity portfolio. The betas corresponding to this portfolio range from -.109 to .199. Further, the betas for fifteen of the eighteen lines of insurance have an absolute value that is less than .100. The second index was constructed from Moody's stock index and the annual holding period returns on U.S. Treasury bonds. The Treasury bonds comprised 70% of the mixed portfolio and Moody's stock index comprised the remaining 30% of the portfolio. The betas that correspond to this mixed stock and bond market portfolio are more variable. They range from -.230 to .371. Biger and Kahane [p. 121] conclude, "systematic risk of underwriting profits approaches zero in most lines. Thus an intuitive solution for underwriting profit rates in these lines equal to minus the riskless interest rate is reasonable."

However, they go on to say [pp. 126-127] that:

> Evaluation of the systematic risk of underwriting, which is not based on market returns but on reported profits, may result in biased estimates of the coefficients . . . Several accounting procedures, unique to the insurance business, make the concept of profit or loss on any particular line of insurance less meaningful than the earnings per share figures for other business firms. In particular, the somewhat arbitrary allocation of overhead to individual lines makes the profit estimates even more questionable, as what is required are specific betas for specific lines. If one adds the empirical inconsistency between accounting betas and market betas for securities, reported in several studies, to these reservations, one must conclude that regulators should be cautious when accounting betas are used for the insurance lines in ratemaking.

Fairley estimates inferred underwriting betas for five lines of insurance. He first estimates beta for all lines combined. This estimate is inferred from the equity betas reported in the *Value Line Investment Survey* [1976] for nine predominantly property-liability stock insurance companies and an investment beta which is estimated using a subsample of the nine insurers. Betas for the various lines are estimated assuming that they are proportional to the ratio of liabilities to premiums. Fairley's estimates of the underwriting betas are as follows: .34 for auto bodily injury; .07 for auto property damage; .07 for homeowners; .34 for workers compensation; and .79 for medical malpractice.

Interestingly, Fairley has little confidence in the accuracy of accounting betas. He states [p. 205] that, "Accounting betas for liabilities or for underwriting determined by regressing annual accounting underwriting returns against annual market index returns are generally near zero in absolute value, though the possible downward bias in these estimates makes them suspect as estimates of true market betas."

Hill presents accounting betas for 14 lines of insurance. They were calculated by regressing underwriting profit rates over the period from 1943 to 1973, as reported in Best's *Aggregates and Averages*, on the logarithm of the return on the market portfolio. Hill does not specify what comprises the market portfolio other than to note [p. 183] that, "The market return is the value weighted index computed by Ibbotsen and Sinquefield [1976]." The underwriting betas for the individual lines vary from -.212 to 1.013. Hill [p. 183] concludes, "Almost all the betas are insignificant. One might draw the weak conclusion that underwriting betas can be positive or negative and that they are generally fairly near zero." Nevertheless, he points out that, "There is a high probability that betas estimated from accounting data are biased towards zero."

Hill also presents inferred all-lines underwriting betas for six publicly traded insurers. They were calculated by regressing the underwriting return on the market return. The underwriting return is calculated by subtracting investment income and capital gains from the change in the market value of the firm plus dividends in each successive one-year period of time. The underwriting betas for the six firms range from -1.03 to .85 and average -.20.

Cummins and Harrington present two sets of all-lines accounting underwriting betas for 14 insurers for two periods of time. The two periods of time are from the first quarter of 1970 to the third quarter of 1975, and from the fourth quarter of 1975 to the second quarter of 1981. One set of betas is based on a regression of quarterly underwriting profits as a percent of earned premium, as reported by the A.M. Best Co., on the market return. The market portfolio is the value weighted index of the New York Stock Exchange and the American Stock Exchange common stocks. The second set of betas is based on a regression of quarterly underwriting profits on five lagged market returns, that is the return on the market portfolio for the current and prior four periods. The underwriting betas from this regression consist of the sum of the coefficients of the five market return variables.

The simple regression estimates are small in absolute value and most are negative. The 14 firm averages are -.05 and -.04 for the two periods of time. Twenty-one of the twenty-eight estimates are negative, and only seven have absolute values greater than .10. The estimates from the second regression are far more variable and average .49 and -1.18 for the two periods of time. Cummins and Harrington state [p. 16], "The results imply that underwriting betas may have been subject to significant instability during the 1970's. This finding suggests extreme caution if underwriting betas are to be used to establish fair profit margins in rate regulations."

They go on to state [p. 38], "Betas have not been stable during the 1970's and may shift again in the early to mid 1980's. Thus regulators should be extremely cautious in using ex post beta estimates to predict ex ante results. Betas also differ across insurers."

Cox and Rudd present two sets of all-lines accounting underwriting betas using the same regression models that Cummins and Harrington used, and one set of inferred underwriting betas for twenty-one insurers for two periods of time. The first period of time is from the first quarter of 1973 to the third quarter of 1977. The second period of time is from the fourth quarter of 1977 to the second quarter of 1982. The accounting betas were calculated using quarterly combined ratios with the Center for Research on Security Prices (CRSP) equally weighted stock index as the proxy for the market portfolio. The accounting betas based on the simple regression model average .068 and -.093 for the two periods of time. Most of the estimates have absolute values less than .100. The accounting betas based on the second regression average .024 and -.027 for the two periods. Most of these estimates also have absolute values less than .100.

The inferred betas were calculated based on data reported in Moody's *Bank and Finance Manual.* The inferred betas are far more variable. They average -.129 and -1.021 for the two periods of time, and range from a low of -2.076 to a high of .164. Cox and Rudd [p. 317] conclude: "Virtually no relationship is observed between the two types of estimates."

There is an extraordinary amount of variation in estimates of underwriting betas across lines of insurance, firms, time, choice of the market portfolio, and methodology. Estimates of underwriting betas are typically measured per dollar of premium. The investor, of course, pays market price when investing in the firm. Accordingly, the relevant risk to the investor is measured per dollar invested. Premium volume generally exceeds the market value of insurers. Hence, betas measured per dollar invested exceed those measured per dollar of premium. For example, if the ratio of premium to market value is two, then beta measured per dollar invested is exactly twice the value measured per dollar of premium. Measuring underwriting betas per dollar of premium thus reduces their apparant variability and contributes to the illusion, in at least some studies, that underwriting betas are near zero.

Is it reasonable to expect true market underwriting betas to vary so greatly? Or is it more reasonable to expect that such variation is caused by faulty estimation methods? If underwriting betas cannot be reliably and accurately estimated, little or no confidence can be placed in the appropriateness of the resulting premiums.

4. UNDERWRITING BETA—WHAT IS IT?

It is important to distinguish future time periods from historic time periods and expectations of future returns from realizations of those expectations. The CAPM is concerned with investors' expectations of future returns. The returns are expected since they are future returns—they have not yet been realized. Moreover, the returns that are realized may not equal those that are expected. Sharpe [21, pp. 85-86] states:

> Capital market theory concerns people's perceptions concerning opportunities. Actual results may (and usually will) diverge from predictions. The values of capital market theory are ex ante (before-the-fact) estimates. Observed values are ex post (after-the-fact) results. The portfolios that do, in fact, turn out to be efficient will lie along some line, but not necessarily the ex ante capital market line. In fact, the market portfolio invariably proves to be inefficient ex post. If the future could be predicted with certainty, investors would shun diversification—the optimal portfolio would contain only the security with the best (actual) performance.
>
> But the future cannot be predicted with certainty. Ex ante estimates must be made. The lack of certainty provides the motivation for both portfolio theory and capital market theory.

According to the CAPM given in Equation 1.1, beta is the covariance of the expected future return on a security with the expected future return on the market portfolio divided by the variance of the expected future return on the market portfolio. The expectation is taken over all investors. The underwriting beta given in Equation 1.2 is the covariance of the investors's expected future underwriting return with the investors's expected future return on the market portfolio divided by the variance of the investors' expected future return on the market portfolio.

Since betas depend on expected future returns, in order to measure betas, those expected future returns must be known. The only way to learn about those expected future returns is to ask investors what they expect. No one has ever done such a study. Rather than measure betas, analysts typically estimate them using the ex post form of the CAPM. That is, betas are estimated using historic realized returns. The ex post form of the model requires additional assumptions that are not required by the ex ante form. According to Copeland and Weston [4, p. 205 and pp. 301-302], the ex post form of the model assumes that the return on any asset is a fair game.[1] Further, when betas that are estimated by using historic realized returns are used to establish premium levels for future periods, it is assumed that these historic estimates of beta apply to future periods.

It is not obvious that these two assumptions are reasonable. In fact, both of them are problematic. The fair game hypothesis, for example, requires that investors have unbiased estimates of expected future returns on each and every asset. In other words, investors must have perfect knowledge of the first moment of the probability distribution of future returns on every asset. This, of course, is highly unlikely. As some of the studies have shown, substantially different estimates of beta result from using different periods of time. Hence, even if investors possess perfect knowledge, the choice of the period of time that is used to estimate the historic beta is critical. Lengthening that period might lessen the chance that returns are not fair games, but it also increases the likelihood that beta has changed over the period.

Empirical applications of the ex post form of the CAPM are subject to unknown and potentially large amounts of estimation error. If any confidence is to be placed in the results, then the model must be validated in some way.

One way of validating the model might be to test how well the model can explain historic returns. The evidence is not reassuring. For example, Fama and French [10] found that historic betas were not able to explain historic returns. They found that size and the book-to-market equity ratio have greater explanatory power than historic betas. Perhaps it is the investors' imperfect knowledge, which prevents returns from being fair games, that limits the ability of historic estimates of beta to explain returns in capital markets. Alternatively,

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[1] A fair game model is one where, on average, across a large number of samples, the predicted future rate of return on an asset, conditioned on current information, is equal to the subsequent realized rate of return.

Bernstein [2, p .1] suggests that, "Despite all the mighty efforts of investment theory, we still do not have a firm handle on a quantitative gauge of risk." Beta may be the proper theoretical measure of risk, but reliable estimates of beta may not yet exist.

Historic returns may have some role in estimating betas, but if historic returns are used, then their use must be validated in some way. Without such validation the estimation error is unknown, and no confidence can be placed in the resulting estimates. For example, it would be enlightening to divide the data into two time periods and test how well estimates derived from the first period explain returns in the second. Rather than cross-validate their results, however, Cox and Rudd present two sets of inferred betas for two periods of time with very different results. Had they cross-validated their results, they may have concluded that estimates from the first period were unable to explain returns in the second period. Either the estimates are devoid of value, or underwriting betas vary enormously over relatively short time periods, and thus historic estimates bear no relation to future periods.

Cummins and Harrington also found that historic betas were not stable over time. If this is indeed true, then unless variation over time can be explained and predicted, historic betas have no relevance for determining premium levels.

There are potentially many ways to estimate betas other than naively using historic returns in a simple-minded way. Jorion [14], for example, uses empirical Bayes estimators. (The actuaries' knowledge of credibility theory uniquely qualifies them to contribute to this line of research.) Rosenberg [18] estimates prospective betas using fundamental factors. He also compares the ability of the predicted betas to explain returns versus that of historic betas. He concludes that the predicted betas are superior to historic betas in explaining subsequent returns. See also Rosenberg and McKibben [19], Rosenberg [17] and Rosenberg and Guy [20]. Whatever methodology is used to estimate underwriting betas, it must be validated in some way.

5. ACCOUNTING BETAS

The use of accounting underwriting betas to estimate true market underwriting betas suffers from a number of flaws. First, accounting underwriting betas are based on historic realized returns rather than on investors' expectations of future returns. As explained earlier, the estimation error is unknown and no confidence can be placed in the resulting estimates.

Second, the historic underwriting returns that are used are not discounted. The market, however, values future cash flows according to their discounted present value. It seems unlikely that undiscounted returns could accurately measure investors' expectations of discounted returns.

Some scholars and analysts have suggested that insurers deliberately smooth underwriting returns by manipulating loss reserves. A more significant source of smoothing of underwriting returns is that reported underwriting returns are undiscounted, and thus do not capture the volatility of interest rates. Another source of smoothing emanates from the way that insurers price their product. When determining premium levels, insurers typically consider investment income by using the portfolio (book) yield which is calculated using the book value of invested assets. Since long term bonds are a large part of most insurers' investment portfolios and are carried on the books at amortized cost rather than at market value, this treatment has the effect of smoothing away short term interest rate volatility and thereby introducing some stability to premium levels. Estimating risk by using a time series of returns where the variability has been smoothed away is obviously going to produce severely biased results.

If accounting betas are to have any value, they must accurately approximate market betas. Unfortunately, this is not the case, as historic returns to the Fortune 500 reveal. In the spring of each year since 1973, *Fortune* magazine reports the median return on equity and the median return to shareholders for the Fortune 500 [11]. Table 1 displays those returns as well as the returns to shareholders in the S&P 500 as reported in *Stocks Bonds Bills and Inflation 1992 Year-*

book [13]. Historic accounting and market betas are calculated for the Fortune 500 using the S&P 500 as a market proxy. The market beta for the Fortune 500 as measured by the median return is 1.00, while the accounting beta is -.02. Could it also be true that an accounting underwriting beta of -.02 corresponds to a market underwriting beta of 1.00? Accounting betas obviously do not provide reliable estimates of market betas.

TABLE 1

ACCOUNTING VS. MARKET BETAS—FORTUNE 500

	Fortune 500		
	Accounting Returns	Market Returns	
Year	Median Return on End of Year Equity	Median Total Return to Shareholders	Return on Market Portfolio S&P 500
1973	12.4%	-25.5%	-14.7%
1974	13.6	-22.4	-26.5
1975	11.6	51.2	37.2
1976	13.3	34.5	23.8
1977	13.5	-3.2	-7.2
1978	14.3	7.2	6.6
1979	15.9	21.3	18.4
1980	14.4	21.1	32.4
1981	13.8	-0.4	-4.9
1982	10.9	21.2	21.4
1983	10.6	30.2	22.5
1984	13.6	-0.8	6.3
1985	11.5	24.1	32.2
1986	11.6	15.5	18.5
1987	13.2	6.8	5.2
1988	16.2	14.1	16.8
1989	15.0	17.5	31.5
1990	13.0	-10.2	-3.2
1991	10.2	29.5	30.6
Standard Deviation	1.7	19.4	17.9
Correlation with market	-.20	.92	1.00
Beta	-.02	1.00	1.00

If the goal is to estimate systematic risk, then accounting returns are the wrong variables to study.

6. INFERRED UNDERWRITING BETAS

Since the total return to an investor consists of an underwriting return and an investment return, it follows that the equity beta of an insurer can be decomposed into a linear combination of an underwriting beta and an investment beta. In its simplest form the decomposition is as follows:

$$\beta_e = (A/E)\beta_a + (P/E)\beta_u \tag{6.1}$$

where

$\beta_e =$ equity beta,

$\beta_a =$ investment beta,

$\beta_u =$ underwriting beta,

$A =$ invested assets,

$E =$ equity,

$P =$ premium.

There are variations to this model. Some authors include a beta for the non-traded assets, and taxes need to be recognized. Nevertheless, for purposes of this discussion, this simple form will suffice.

Historic equity betas for publicly traded insurers can be computed from historic returns. Further, they are available from a number of investment advisory services and brokerage firms. To estimate the underwriting beta, then, it is necessary to estimate the investment beta and the two levers, *(A/E)* and *(P/E)*. At first blush, this method of estimating the underwriting beta seems simple and straightforward. However, it too is fraught with difficulties. This estimation method merely transfers the problems of estimation from underwriting betas to investment and equity betas. Moreover, any error in the estimation of the investment beta is leveraged by the ratio of invested assets to equity. This leveraging of the error can be quite substantial for some insurers, particularly those that write long-tail lines of insurance.

The equity beta applies to the market value of equity. Accordingly, the levers must also be valued at market. The market value of equity, however, is not available for many insurers since they are not publicly traded. Further, the market value of invested assets is not available for any insurer. The market values of some investments are reported. Stocks, for example, are carried on the books at market value. Insurers that are publicly traded report the market value of the bond portfolio in their annual report to shareholders. Some publicly traded insurers also report the market value of mortgage-backed securities in the shareholders' report. Similar information for insurers that are not publicly traded is not available.

For other assets, however, market values are simply unavailable. The market value of mortgage investments, for example, is generally not available regardless of whether the insurer is publicly traded. Real estate investments are carried on the books at cost less depreciation, rather than at market value. The market value of unconsolidated subsidiaries is generally unknown. Market values of other investments such as oil and gas partnerships, limited partnerships, etc. are unavailable. Thus, it is not possible to determine the proper values for the asset lever for any insurer nor the underwriting lever for most insurers.

The Equity Beta

Although historic equity betas can be computed and are available from a number of sources, they are of unknown quality. Many are based on simple regressions of historic returns. All of these estimates depend on the validity of the ex post form of the CAPM. As previously noted, the assumptions that underlie that model are problematic. Further, different analysts and firms calculate historic betas in different ways. For example, different proxies of the market portfolio and different holding periods are used. Theory provides no guidance as to which holding period should be used. Yet changing the holding period can cause significant changes in the estimates of beta. Longstaff [16], for example, states [p. 875]:

> The value of the market beta for firm i is a function of the length of the period over which returns are measured. Thus, betas estimated from daily returns need not equal betas estimated from monthly data, all other estimation problems aside. Perhaps even more important, the relative ranking of firms by betas estimated from daily data need not be the same as the ranking based on betas estimated from monthly returns.

It is well known that the composition of, and returns to, the proper market portfolio are unknown. Typically, a stock market index of some sort, usually a subsample of the entire stock market, such as the S&P 500 or the NYSE, is used as a proxy for the market portfolio. Underlying this practice is the assumption that residential and commercial real estate, farmland, foreign equities, foreign real estate, excluded U.S. equities (such as over the counter stocks and stocks traded on the American or other smaller exchanges), antiques, furs, paintings, precious metals, etc., have no discernable impact on estimates of beta. These excluded assets comprise a much larger part of the market portfolio than the stock indices used as its proxy. Is it reasonable to assume that the tail wags the dog?

Arguably, equity betas estimated by investment advisory services may be more accurate approximations of true equity betas, since investors pay for these services and presumably use them. However, there is great variation in the betas estimated by different firms. Table 2 displays the equity betas estimated by Value Line and Standard & Poors for those property-casualty insurers covered by Value Line which also have a beta published by Standard & Poors. The Value Line betas were published April 10, 1992 [1], and the Standard & Poors betas were current as of March 6, 1992 [23]. The average absolute value of the difference in the two estimates of beta is .26. Both firms use returns over a five year period for calculating betas. Value Line, however, uses a weekly holding period while Standard and Poors uses a monthly holding period.

TABLE 2

ESTIMATES OF BETA

	Betas Published By		Absolute
Insurer	Value Line	Standard & Poors	Difference
Chubb	1.05	.67	.38
Cincinnati Financial	.80	.65	.15
Continental Corp.	1.05	1.02	.03
Frontier Insurance	.90	1.06	.16
Geico	.80	.70	.10
General Re	1.00	.68	.32
Orion Capital	1.10	1.27	.17
Progressive Corp.	.95	.52	.43
Safeco	1.15	.90	.25
St. Paul Cos.	1.05	.73	.32
20th Century	1.00	1.42	.42
USF&G	1.10	.70	.40
Average Absolute Difference			.26

Perhaps the consensus or average estimates of equity betas from all of the investment advisory services would provide truer estimates of investors' expected betas. This hypothesis, however, needs to be tested. In any case, estimating an equity beta is no simple task. The estimation error is unknown and potentially large. Use of the wrong equity beta obviously biases the estimate of the underwriting beta.

The Investment Beta

The investment portfolio beta is the weighted average of the betas of the securities in the portfolio. Bonds are a significant component of most insurers' portfolios. What is the beta of a bond portfolio? How is it estimated?

Historic estimates of bond betas can be computed. However, the estimate of beta varies according to the historic period that is used. For example, the beta of the annual return on long term Treasury bonds from 1926 to 1991, according to data reported in *Stocks Bonds Bills and Inflation 1992 Yearbook*, is .06. However, the estimate of the Treasury bond beta increases almost by a factor of five to .29 if it is based on data from 1970 to 1991. Both of these estimates use the S&P 500 as a proxy for the market. If historic estimates are to be used, then what is the appropriate time period? What assurance is there that the choice of the time period is consistent with the market's expectations?

If using historic estimates of bond betas is problematic, then perhaps the beta can be estimated from the yield to maturity of the bond and the current risk-free rate. Presumably the difference between these two yields is the product of the bond's beta and the market risk premium. One witness, in fact, in a recent auto insurance rate hearing estimated the bond portfolio beta of an insurer in this way. However, as is shown below, CAPM cannot explain the yield to maturity of a bond with a maturity that exceeds the holding period assumed by the CAPM.

If CAPM applies to the yield to maturity of a bond, then it must be able to explain the term structure of interest rates. The theories advanced to explain the term structure of interest rates (expectations theory, liquidity preference theory, and market segmentation theory), however, do not include the CAPM. Further, the implications of the CAPM are inconsistent with these theories. If CAPM applies to the yield to maturity of a bond, then that yield is the sum of a risk-free rate and a risk premium which is proportional to the bond's beta. It follows that risk is the only reason why yields on long term bonds differ from yields on short term instruments.

Consider that the yield on long term Treasury bonds in February, 1989 was approximately 8.8% as reported in the *Wall Street Journal*. The yield on ninety day Treasury bills was also approximately 8.8% and the yield on two year Treasury notes was approximately 9.2%

during that month. If CAPM applies to the yield to maturity of a bond, then it implies that although two year Treasury notes were risky at that point in time, long term Treasury bonds were not. Conversely, in April, 1993, the yield on ninety day Treasury bills was approximately 3.0%, and the yield on long term Treasury bonds was approximately 6.8%. CAPM thus implies that long term Treasury bonds were risky at that time. Hence, if CAPM applies to the yield to maturity of a bond, then bond betas are not stable over time and historic betas have no relevance for determining future premiums.

According to both the expectations theory and the liquidity preference theory, the yield to maturity of a long term bond depends on the market's expectations of future interest rates. Since CAPM, which is a single variable/single period model, does not capture the market's expectations of future interest rates, it cannot explain the yield to maturity of long term bonds with maturities that exceed the holding period assumed by the CAPM.

To demonstrate this, consider a default-free zero coupon bond that pays D dollars at the end of t years. The expected price of the bond at time j is

$$P_j = D/((1+{}_j r_{j+1})(1+{}_{j+1}r_{j+2})....(1+{}_{t-1} r_t)), \tag{6.2}$$

where

$P_j =$ expected price of bond at time j,

$D =$ payment from bond at time t,

${}_{i-1}r_i =$ forward interest rate for a default-free commitment made at time 0 to loan money at beginning of year i, and to be repaid with interest at end of year i.

Hence,

$$P_0 = D/((1+{}_0r_1)(1+{}_1r_2)....(1+{}_{t-1}r_t)) \tag{6.3}$$

and

$$P_1 = D/((1+{}_1r_2)(1+{}_2r_3)....(1+{}_{t-1}r_t)). \tag{6.4}$$

The CAPM is a single period model. In order to apply CAPM, it is necessary to specify the holding period. The holding period in turn determines the risk-free rate, since that rate must prevail over the holding period. The appropriate risk-free rate is thus the interest rate on a risk-free security with a maturity that matches the holding period. Suppose the CAPM with an annual holding period is applied to this bond. The expected return during the first year is

$$(P_1 - P_0)/P_0 = {}_0r_1 \ . \tag{6.5}$$

Thus the return that CAPM would try to explain is ${}_0r_1$. The yield to maturity, however, is given by

$$y_t = ((1+{}_0r_1)(1+{}_1r_2)....(1+{}_{t-1}+r_t))^{1/t} - 1. \tag{6.6}$$

In general, y_t does not equal ${}_0r_1$. Hence, the CAPM cannot explain the yield to maturity of long term bonds.

Note also that ${}_0r_1$ is simply the current interest rate on a default-free security with a maturity equal to the holding period assumed by the CAPM. If yields on default-free securities are assumed to be risk-free, as is commonly done, then ${}_0r_1$ is the appropriate risk-free rate for this application of the CAPM. Hence, the beta for this bond is zero. Further, it follows that by choosing a suitably short holding period, the beta of any default-free bond of any maturity is zero, since the bond is the sum of a portfolio of zero coupon bonds, all of whose betas are zero. This implies that an insurer that invests exclusively in U.S. Treasury securities, regardless of maturity, has an investment beta of zero. Accordingly, the investment portfolio betas of two insurers, one of which invests exclusively in U.S. Treasury bills while the other invests exclusively in thirty-year Treasury bonds, are both equal to zero, even though the latter may have a greater yield to maturity than the former.

Many insurers, of course, invest in municipal bonds, corporate bonds, and mortgages. Estimating the investment portfolio betas in these cases is no simple matter. Historic estimates are problematic since it is unknown which period of history is relevant. Further, CAPM is unable to explain the yield to maturity for these bonds since those yields depend on the market's expectations of future interest rates. Any error that is implicit in the estimate of the investment beta necessarily biases the estimate of the underwriting beta if it is inferred from the former. Moreover, the error in the investment beta is levered up by the ratio of invested assets to equity.

Some bonds that are subject to default risk can be expected to default. Accordingly, the yield to maturity overstates the expected yield on such a bond. The yield to maturity on risky bonds thus includes a default premium which is required to compensate the investor for the expected rate of default.

One way to estimate an upper bound for the beta of a risky bond (or portfolio of bonds) is to compare the yield to maturity (the taxable equivalent yield to maturity in the case of municipal bonds) of the bond with the yield to maturity of a U.S. Treasury bond with the same duration. The yields to maturity of both bonds capture the market's expectations of future interest rates. Hence, the difference in the yields is equal to the sum of the default premium and the risk premium. Since the risk premium is the product of the bond's beta and the market risk premium, it follows that the difference in the yields divided by the market risk premium is an upper bound for the bond's beta.

Contrary to this procedure, a witness in a recent auto insurance rate hearing estimated the beta of a long term high quality bond portfolio to be .24. At the time, Treasury bills and bonds were yielding 6.3% and 8.5% respectively, and long term corporate bonds were yielding 8.9%. Assuming the duration of the corporate bonds matched the duration of the government bonds and assuming a market risk premium of 8.6%, the implied upper bound for the corporate bond beta is .05. Overestimating the bond beta by inferring it from the yield differential over Treasury bills causes an underestimation of the underwriting beta since the levered investment beta is subtracted from the equity beta to get the levered underwriting beta. This further results in the determination of premium levels that are inadequate.

7. INTUITIVE CONSIDERATIONS

On the surface, the notion that underwriting betas for insurance are zero, since the occurrence or non-occurrence of accidents is unrelated to the performance of financial markets, has much intuitive appeal. A closer inspection, however, reveals that this notion is a bit too simple-minded. It confuses accidents with insurance claims, and completely overlooks the severity of those claims. Further, intuition suggests other reasons why underwriting betas might be positive.

There are a number of reasons why underwriting losses increase during times of economic malaise and high unemployment. To the extent that financial market performance is positively related to economic performance, this suggests that underwriting betas may be positive.

The conventional wisdom in the insurance industry is that theft, fire, and arson losses increase during times of high unemployment. Underwriting losses thus increase during such times for auto, homeowners, and commercial theft and fire insurance. When people are out of work, they are more likely to default on their debt. Thus, credit insurance and mortgage insurance losses increase during times of high unemployment. Drivers who are unemployed are more likely to drive without auto insurance, thus increasing the losses under uninsured motorist coverage. It is expected that fraud and misrepresentation increase during times of high unemployment. Misrepresentation such as not disclosing a young driver on an auto insurance policy or lying about the use, annual mileage, or territory of garaging of an insured vehicle deny the insurer the full premium that is required to insure the policy. This increases underwriting losses. It is also expected that when unemployment is high, claimants are more likely to pursue a claim and to exaggerate the value of that claim.

When interest rates are increasing, stock and bond markets tend to perform poorly. Underwriting losses, especially on long tail lines of insurance, also increase as interest rates rise. This suggests that underwriting returns may have positive betas.

Finally, catastrophes destroy business property and may depress economic activity from the resulting unemployment and business interruption.

Thus, the intuitive considerations are ambiguous. Intuition is insufficient to determine the value of underwriting betas.

8. CONCLUSIONS

The underwriting beta is a useful theoretical concept. However, it is not possible to measure it directly. The indirect methods that have been used to estimate underwriting betas are flawed and result in estimates that vary greatly across lines of insurance, firms, time, choice of the market portfolio, and estimation technique. Thus, reliable estimates of underwriting betas do not exist. Perhaps better methods of estimation may some day be developed. Until that time, however, underwriting betas will remain as visible as the shadows of ghosts.

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ADDRESS TO NEW MEMBERS—NOVEMBER 13, 1994

W. JAMES MACGINNITIE

In preparation for this occasion, I looked back thirty-one years to 1963, when I was admitted as a new Fellow. Perhaps in 2025 one of you will be at this podium. We had fewer new Fellows and Associates then, but of course the exams were tougher. The meeting drew about 175 attendees, versus 1,100 at this meeting. The venues in 1963 were the Catskills and Atlantic City, two resort areas in decline, whereas this year the CAS meets in Boston and Orlando. In 1963, the premium volume for the property-casualty insurance industry was less than $20 billion; today that's just a good-sized catastrophe loss.

In the ensuing thirty-one years, the CAS has grown from fewer than 400 members to 2,300. Premium volume for the property-casualty insurance industry has grown at a compound rate in excess of 10% per year, and the growth of captive, self-insured, and off-shore premium is substantially greater.

That growth is a function of many forces. One of those forces is the litigiousness of our society. I noted in my review of 1963 that the Secretary of the CAS was authorized to take out the Society's first public liability policy. Another force is the growth of both population and the economy, with a great deal of the latter being inflationary in nature. The breakdown of the system of making rates in concert has also contributed to the growth of the CAS. The largest employers of CAS members are no longer rating bureaus or companies but now are consulting firms, reflecting the rapid growth in that area of practice. The availability of data and computer power with which to manipulate it have also helped the growth. You can get more power in a few pounds of a notebook today than in a roomful of tube machines thirty years ago. But perhaps the largest contribution to the growth and membership of the CAS has been the perceived value of actuarial training, and the ability of those so trained to provide useful solutions to the many problems that face the risk and insurance business.

Some things have not changed much in thirty-one years. In 1963, the President, Laurence Longley-Cook, reported that "...greatly increased competition in the industry has forced rates for certain lines too low...other causes include inflation and greater claims consciousness." He went on to touch on "astronomical legal fees" and he observed that "ignorance and fear of loss of business sometimes lead to inadequate rate filings."

The challenges and opportunities that we faced thirty-one years ago were many and varied. Those that you face today are even more numerous and more varied. I recently had the opportunity to facilitate a gathering of actuaries chosen from the world-wide staff of a multinational insurance and financial services company, focusing on the role of the actuary in the 21st Century. The actuarial needs of that organization, and many like it, are truly exciting. They go well beyond traditional actuarial roles, into risk control, capital management, management of the claim process, sophisticated marketing, and many others. All of this will be on a multinational basis, requiring that many of the actuaries develop familiarity with the culture of other societies. Also, the growing interconnections between the insurance and alternative risk handling mechanisms with which we are so familiar, on the one hand, and the banking, securities, and other financial services industries, on the other hand, will provide challenge and opportunity for many of you.

None of this will be easy. It will require continual re-education on your part—perhaps not at the level of intensity of the exams that you have just completed, but still at a very rigorous level. Because in order to be of value to your clients and employers in these new areas, you must have the same in-depth knowledge and understanding that you have so recently demonstrated for the traditional property-casualty risks.

None of this growth and none of these opportunities would have been possible without the selfless work of generations of actuaries that have gone before you. You are the beneficiaries of that work, and so as you now receive your Fellowship or Associateship, you must also assume the many responsibilities that go with the designation:

- Responsibility to continue your education through study, discussion, and participation in continuing education opportunities.
- Responsibility to extend the expertise of the profession through your research, both theoretical and applied, and through the sharing of your results at meetings such at this.
- Responsibility to recruit and train the next generation, which means for many of you work on the education and examination committees.
- Responsibility to contribute to the continued growth and development of the profession.
- Responsibility to conduct your affairs in a professional and ethical manner, and especially to recognize your fiduciary obligation for the financial soundness of the organizations you serve, with their promises to provide protection and payment far into the future.

The accomplishment for which we recognize you today is a substantial one. You and your families and colleagues can be justly proud. Decades from now, when you look back to today, much will have changed and much will remain unchanged. We trust that as you look back from that future perspective, your discharge of your responsibilities will give you as much pride then as your Fellowship or Associateship does now.

PRESIDENTIAL ADDRESS—NOVEMBER 14, 1994

THE CAS: A HOLOGRAM OF VISIONS

IRENE K. BASS

It's that time of the CAS year when it's expected that I give an address. A couple of years ago, when I received the news that I had been nominated to this office, the first thing that shot through my mind was the terror of knowing I would have to give such an address before you, my colleagues. I would have to say something that would be profound and memorable. It nearly caused me to turn down the nomination. But then I realized that this address comes at the *end* of the year, when you can no longer impeach me.

Before I get to the address, there are two things that I would like to say. The first one is: *Thanks!*

The end of the CAS year is always a good time to reflect on our collective accomplishments, and to recognize those who have made it happen.

To begin the thanks: What would we be without the support of the CAS Office staff under the leadership of Tim Tinsley? Many of us in the room remember the days when the office staff of the CAS could have been described as one woman and a parrot. Now we have a staff of top-drawer professionals who reflect well on our profession. I know of no better executive director than Tim Tinsley. Many of you may not know this but, in addition to Tim's successful military career, he received a college degree in operations research, and in many ways is just as "numerate" as we actuaries. I wonder what Tim's strange karma must be that he is so skilled in quantitative matters but would choose to work for an organization where *everyone* thinks he or she knows more about numbers than the entire rest of the world. But Tim does it with style and grace.

And, of course, many of you know the terrific meeting planning done by Kathy Spicer and Gwynne Hill. That is obvious to us every time we have a meeting or seminar, without exception.

However, a lot of support to our membership and our future membership comes from the behind the scenes Office staff. For example, Michele Lombardo stands strong and deals with all the issues surrounding the exam process, and she is now venturing into the MIS arena. Paula Miller produces the *Actuarial Review* and other CAS publications. As for the other CAS staff: Each contributes and we thank you all.

Another group of people that I would like to thank is you, the leaders of the CAS! In a column that I wrote for the *Actuarial Review* this year, I quoted Mohandas Gandhi who said, "I must hurry and catch up with my people, for I am their leader." That statement says quite a lot about leadership. There are so many members of the CAS who actively participate in the work that we must be the envy of many professional societies. I can't read the names of everyone who participated in the leadership of the CAS, but it would be interesting for everyone in the room to *see* just how many people that really is. Let me ask you for a favor. Let me ask you to rise in place and stay standing until I have completed the list of those who have participated.

- All those who served last year on the Board of Directors or the Executive Council.
- All those who chaired committees.
- All those who served on CAS committees.
- All those who wrote papers or reviewed papers.
- All those who served as official liaisons for the CAS.
- All those who served as officers of regional affiliates and special interest groups.
- All those who served on programs and seminars of the CAS.
- All those who served as facilitators at the Course on Professionalism.
- All those who wrote articles for the *Actuarial Review* or the *Forum*.

My thanks to you, the leaders of this profession.

And it wouldn't be right if I didn't thank those who helped me personally to get through the year. Thanks to my business partners for being understanding of the fact that there are some things more important than the billable hour—at least temporarily. And thanks to my husband, Stan Khury, an actuary, a past president of this society. He is the wind beneath my wings. (Of course, on occasion, some have confused this for hot air.)

Now the second thing that I'd like to do before I get to that address that I'm supposed to deliver, is to ask you to stroll down actuarial memory lane with me for a moment and reflect on your lives as actuaries.

- Think back to the first time you ever heard of an actuary. When was it and where were you?
- Think about the *first* actuarial exam that you took.
- Then think about the relief of that *last* actuarial exam, or if you have not finished them yet, think of what it will be like to finish the last one.
- Think of those job interviews.
- And think of your first days on the job.
- Think of the job you had then, the job you have now, and all you learned in between.
- Think how relatively simple things were then; how complex they have become.
- Think of the first CAS meeting you attended and all the people you came to know by going to meetings.
- Think of grading exams after having finished taking them yourself.
- Think of the committees on which you served and all they accomplished.

- Think of what the CAS looked like when you started in this profession and think of what it looks like today.

Now as we finish our stroll down memory lane, I'd like to ask you to join with me in a moment of quiet reflection. I'm going to ask you to close your eyes. Close them for just a minute and let the thoughts go from your mind. Relax.

I ask you to envision the CAS today. Keeping your eyes closed, think of what the CAS looks like today.

- What does it mean for you to be a member?
- What is the community that it represents?

Keeping your eyes closed for just a moment longer, I ask you to envision the CAS five or ten years from now.

Just contemplate that future vision for a moment and hold it in your mind.

Now you can open your eyes again, and I'll finally get to that address. So, here it is:

"Do whatever is in your power to make your vision of the CAS come true."

MINUTES OF THE 1994 ANNUAL MEETING

November 13 - 16, 1994

THE HILTON AT WALT DISNEY WORLD VILLAGE

ORLANDO, FLORIDA

Sunday, November 13, 1994

The Board of Directors held their regular quarterly meeting from noon to 5:00 p.m.

Registration for the Annual Meeting occurred from 4:00 p.m. to 6:00 p.m.

From 5:30 p.m. to 6:30 p.m., new Associates and their guests attended a reception that featured an introduction of the CAS Executive Council.

A welcome reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Monday, November 14, 1994

Registration continued from 7:30 a.m. to 8:30 a.m.

CAS President Irene K. Bass opened the business session at 8:30 a.m. and recognized past CAS presidents in the audience, as well as special guests: Charles A. Bryan, President, American Academy of Actuaries; José Luis Salas, General Coordinator, Mexican Committee for the International Actuarial Practice; Jack M. Turnquist, President-Elect, American Academy of Actuaries; Sam Gutterman, President-Elect, Society of Actuaries; Roberto Westenberger, Director, Institute of Actuaries of Brazil; U. Richard Neugebauer, Executive Director, Canadian Institute of Actuaries.

Ms. Bass announced the results of the election of CAS officers. The members of the 1995 Executive Council will be Vice President-Administration, Paul Braithwaite; Vice President-Admissions, John J. Kollar; Vice President-Continuing Education, David N. Hafling; Vice

President-Programs and Communications, Alice H. Gannon; Vice President-Research and Development, Michael J. Miller. President-Elect will be Albert J. Beer, and President will be Allan M. Kaufman. New Board members will be Robert F. Conger, John M. Purple, Richard H. Snader, and Kevin B. Thompson.

Ms. Bass thanked outgoing Executive Council and Board members for their service to the CAS.

Allan Kaufman, announced the 86 new Fellows. The names of these individuals follow:

NEW FELLOWS

Todd R. Bault
John A. Beckman
Jennifer L. Biggs
Betsy L. Blue
Mark L. Brannon
Anthony J. Burke
Janet L. Chaffee
Jessalyn Chang
Scott K. Charbonneau
Michael A. Coca
Gregory L. Cote
Michael T. Curtis
Edgar W. Davenport
Michael L. DeMattei
Jeffrey L. Dollinger
Maribeth Ebert
Matthew G. Fay
Daniel B. Finn
Yves Francoeur
Russell Frank
Kim B. Garland
Donna L. Glenn
Linda M. Goss
Farrokh Guiahi
Jonathan M. Harbus
Lisa A. Hays
Deborah G. Horovitz
Nancy E. Kot
John M. Kulik
James W. Larkin
Michael D. Larson
Christopher Lattin
Michel Laurin
France LeBlanc
Elise C. Liebers
William G. Main
Daniel J. Mainka
Donald F. Mango
Blair E. Manktelow
Katherine A. Mann
James B. McCreesh
John W. McCutcheon, Jr.
M. Sean McPadden
John P. Mentz
Paul Allen Mestelle
Robert J. Meyer
Stephen J. Meyer
Stacy L. Mina
Kelly L. Moore
Michelle M. Morrow
David A. Murray
Robin N. Murray
Stephen R. Noonan
Laura A. Olszewski
William L. Oostendorp
Timothy A. Paddock
Rudy A. Palenik
Jennifer J. Palo
Chandrakant C. Patel
Charles C. Pearl, Jr.
Andre Perez
Marvin Pestcoe
Daniel C. Pickens
Marian R. Piet
Brian D. Poole
Donna J. Reed
Elizabeth M. Riczko
James Joseph Romanowski
Kevin D. Rosenstein
Gregory R. Scruton
Derrick D. Shannon
David M. Shepherd
Barbara A. Stahley
Thomas N. Stanford
Paul J. Struzzieri
Richard D. Thomas
Barbara H. Thurston
Michael Toledano
Charles F. Toney, II

Dale G. Vincent, Jr.
Scott P. Weinstein
L. Nicholas Weltmann, Jr.
Debra L. Werland
Marcia C. Williams
William M. Wilt
Ralph T. Zimmer

John J. Kollar, Michael J. Miller and David N. Hafling announced the 76 new Associates. The names of these individuals follow.

NEW ASSOCIATES

Shawna S. Ackerman
Larry D. Anderson
Barry Luke Bablin
James M. Bartie
Andrea C. Bautista
Lori Michelle Bradley
Kevin Joseph Brazee
Russell J. Buckley
Kristi Irene Carpine-Taber
Brian A. Clancy
Kirsten J. Costello
Wayde Alfred Daigneault
Thomas V. Daley
Smitesh Dave
Laura B. Deterding
Gregg Evans
Charles V. Faerber
Bruce Daniel Fell
Ginda Kaplan Fisher
Robert F. Flannery
Margaret Wendy Germani
Julie Terese Gilbert
Nicholas P. Giuntini
William Alan Guffey
Marc S. Hall
Elizabeth E. L. Hansen
Jonathan B. Hayes
David B. Hostetter
Brian Danforth Kemp
Rebecca A. Kennedy
Bradley J. Kiscaden
Paul Henry Klauke
Joan M. Klucarich
Eleni Kourou
Kenneth Allen Kurtzman
Edward M. Kuss
Matthew G. Lange
John P. Lebens
P. Claude Lefebvre
Gary P. Maile
Janice L. Marks
Anthony G. Martella, Jr.
Peter R. Martin
Michael Boyd Masters
Brian James Melas
Anne C. Meysenburg
Camille Diane Minogue
James Edward Monaghan
Matthew C. Mosher
Turhan E. Murguz
Aaron West Newhoff
Marc Freeman Oberholtzer
John R. Pedrick
Anne Marlene Petrides
Michael David Price
Karen L. Queen
Kathleen Mary Quinn
Yves Raymond
Victor Unson Revilla
Brad Michael Ritter
Jay Andrew Rosen
Christine R. Ross
Matt J. Schmitt
Jeffrey Parviz Shirazi
Nathan Ira Shpritz
Kerry S. Shubat
Charles Leo Sizer
Carl J. Sornson
Klayton N. Southwood
Angela Kaye Sparks
Linda F. Ward
James C. Whisenant
Wyndel S. White
William Robert Wilkins
Jeanne Lee Ying
Doug Alan Zearfoss

Ms. Bass introduced W. James MacGinnitie who gave the Address to New Members.

Ms. Bass then presented the 1994 Matthew S. Rodermund Service Award to Robert A. Miller, III.

Ms. Bass requested a moment of silence to mark the passing of five members of the CAS during the past year.

John M. Purple read the Vice President-Administration's Report.

Alice Gannon presented the highlights of the program. David L. Miller, Chairperson of the Committee on Review of Papers, summarized the two new *Proceedings* papers being presented.

David N. Hafling presented the Woodward-Fondiller Prize to Daniel M. Murphy for his paper, "Unbiased Loss Development Factors," and the Dorweiler Prize to Daniel F. Gogol for his paper, "An Actuarial Approach to Property-Catastrophe Cover Rating."

A general session panel on "The Property/Casualty Industry: 2000 and Beyond" took place from 10:30 a.m. to noon. Albert J. Beer, CAS President-Elect and a Senior Vice President with American Re-Insurance Company, moderated the panel discussion that included Steven M. Gluckstern, Chairman, President and Chief Executive Officer with Zurich Reinsurance Centre, Inc.; Jeffrey W. Greenberg, Executive Vice President of American International Companies; and Michael A. Smith, Senior Vice President with Lehman Brothers.

Following the panel, there was a luncheon from noon to 1:30 p.m., highlighted by the Presidential Address from Irene K. Bass.

The afternoon's concurrent sessions ran from 1:30 p.m. to 5:00 p.m. and consisted of various panels and presentations of papers.

The panel presentations covered the following topics:

1. "Health Care Reform Assessments"

Moderator: Paul G. O'Connell
Vice President and Actuary
Continental Insurance

Panelists: William E. Burns
Director of Actuarial Services
Medical Inter-Insurance Exchange of New Jersey

Gary D. Hendricks
Director of Government Information
and Chief Economist
American Academy of Actuaries

Edward M. Wrobel, Jr.
Consulting Actuary
Tillinghast/Towers Perrin

2. "No Fault Automobile Insurance"

Moderator: Michael A. LaMonica
Vice President and Actuary
Allstate Insurance Company

Panelists: Professor Jeffrey O'Connell
The McCoy Professor of Law
University of Virginia

Richard Lynde
Supervising Insurance Examiner
New York State Insurance Department

Chester Szczepanski
Chief Actuary
Pennsylvania Insurance Department

3. "Environmental Liability Exposure"

Moderator: Charles W. McConnell, II
Senior Vice President and Chief Actuary
The Home Insurance Company

Panelists: Raja R. Bhagavatula
Consulting Actuary
Milliman & Robertson, Inc.

John Butler
Principal
Putnam, Hayes, Bartlett, Inc.

Susan K. Woerner
Corporate Actuary
Nationwide Insurance Company

4. "Derivatives"

Panelist: Dr. Joseph B. Cole
Managing Director
Centre Financial Products

5. "Disaster Recovery and You"

Moderator: Robert B. Downer
Vice President and Chief Actuary
Farmers Insurance Group

Panelists: Marcia A. Carpenter
World Wide Practice Leader
Business Recovery Consultation
IBM Consulting Group

Karen Whitlatch
Specialist, Contingency Planning
AT&T Financial Services Organization

6. "The Human Genome Project"

Moderator: David N. Hafling
Senior Vice President and Actuary
American States Insurance Companies

Panelist: Ray Mosely, Ph.D.
Director of the Medical Humanities Program
University of Florida College of Medicine

7. "Mini-Course on Professionalism"

Panelists: Members of the CAS Committee
on the Course on Professionalism

8. "Call Papers on Environmental Issues"

Panel: "Measurement of Asbestos Bodily
Injury Liabilities"

Authors: Susan L. Cross
Consulting Actuary
Tillinghast/Towers Perrin

John P. Doucette
Consulting Actuary
Tillinghast/Towers Perrin

Panel: "Measurement of Pollution Liabilities"

Authors: Amy S. Bouska
Consulting Actuary
Tillinghast/Towers Perrin

Thomas S. McIntyre
Consulting Actuary
Tillinghast/Towers Perrin

9. "The Actuary in Mexico"

Moderator: Thomas R. Bayley
Deputy Managing Director
Seguros Monterrey/Aetna

Panelists: Ignacio Gurza
Consultant
Tillinghast/Towers Perrin

Gloria Leal
International Regulatory Council
Texas Department of Insurance

10. "Insurance Fraud"

Moderator: Ronald C. Retterath
Senior Vice President and Actuary
National Council on Compensation Insurance

Panelist: Frank E. Doolittle
Director, Division of Insurance Fraud
Florida Department of Insurance

Dennis Jay
Executive Director
Coalition Against Insurance Fraud

Bill Kizorek
President
InPhoto Surveillance

The officers held a reception for new Fellows and their guests from 5:30 p.m. to 6:30 p.m. A children's reception was held from 6:00 p.m. to 7:30 p.m., and a general reception for all members and guests was held from 6:30 p.m. to 7:30 p.m.

Tuesday, November 15, 1994

From 8:00 a.m. to 9:30 a.m., simultaneous general sessions were offered.

One general session, "Economics/Finance," was led by Jeanne M. Hollister, Vice President, Aetna Life & Casualty. Panelists included Celeste A. Guth, Vice President of Goldman Sachs; Vincent J. Dowling, Jr., Principal with Paulsen, Dowling Securities, Inc.; Gary R. Ransom, Senior Vice President for Conning & Company; and John H. Snyder, Senior Vice President for A.M. Best Company.

The other general session, "Re-Engineering," was moderated by Alan E. Kaliski, Vice President and Actuary, Royal Insurance Company. Panelists were Gregory L. Gleason, Principal with CSC Index; Tom Valerio, Senior Vice President, Reengineering, CIGNA Corporation; and Peter T. Bothwell, Vice President and Actuary with United States Fidelity and Guaranty Company.

From 10:00 a.m. to 11:30 a.m., several concurrent sessions were conducted. The panel presentations, in addition to repeats of some of the subjects covered on Monday, were:

1. "Workers' Compensation State Reforms"

Moderator: Barry Llewellyn
Senior Vice President and Actuary
National Council on Compensation Insurance

Panelists: Joseph Edwards
Independent Consultant

David Durbin
Vice President, Claims Research
National Council on Compensation Insurance

Dale Peterson
Manager of Employee Health Benefits
General Mills Restaurants

2. "A Case Study on Treaty Reinsurance: I & II"

Moderator: Russell S. Fisher
Vice President
General Reinsurance Corporation

Panelists: James M. Dekle
Vice President
North American Reinsurance

John W. Buchanan
Consulting Actuary
Tillinghast/Towers Perrin

3. "State and Federal Assistance for Insured Catastrophic Loss"

Moderator: Karen F. Terry
Principal and Consultant
Miller, Rapp, Herbers, Brubaker & Terry, Inc.

Panelists: David R. Chernick
Senior Actuary
Allstate Insurance Company

Myron L. Dye
Vice President, Property and Financial Actuary
United Services Automobile Association

4. "Financial Risk Transfer Markets"

Moderator: J. Scott Bradley
Senior Vice President and Actuary
Richmond Insurance Company

Panelists: Michael J. Cascio
Vice President and Chief Underwriting Officer
CTC Ltd.

Brian E. MacMahon
Vice President and Actuary
Centre Reinsurance Ltd.

W. Allen Taft
Director of Alternative Risk Marketing
American International Group

5. "Questions and Answers with the CAS Board of Directors"

Moderator: Allan M. Kaufman
Principal
Milliman & Robertson, Inc.

Panelists: Kevin B. Thompson
Assistant Vice President and Actuary
Insurance Services Office, Inc.

Anne E. Kelly
Chief Casualty Actuary
New York Sate Insurance Department

Robert S. Miccolis
Senior Vice President and Actuary
Reliance Reinsurance Corporation

6. "Actuarial Research Corner"

Moderator: Gary G. Venter
President
Workers Compensation Reinsurance Bureau

The afternoon was reserved for committee meetings and regional affiliate meetings.

A Beach Party was held at Disney's River Country from 6:00 p.m. to 10:00 p.m.

Wednesday, November 16, 1994

From 8:00 a.m. to 9:15 a.m., concurrent sessions were held. Sessions that were not offered on Monday or Tuesday included:

1. "Database Administration and Development"

   Panelists: Arthur R. Cadorine
   Assistant Vice President
   Insurance Services Office, Inc.

   Patrick A. Belle
   Customer Advisory Systems Support Representative
   AT&T Global Information Solutions

   Richard W. Nichols
   Associate Actuary
   Aetna Life & Casualty

The following *Proceedings* papers were presented:

1. "Underwriting Betas—The Shadows of Ghosts"

   Author: Thomas J. Kozik
   Senior Actuary
   Allstate Insurance Company

2. "Extended Service Contracts"

   Author: Roger M. Hayne
   Consulting Actuary
   Milliman & Robertson, Inc.

From 9:45 a.m. to 11:15 a.m., a general session was held on "The Future Evolution of Insurance Regulation." Kevin M. Ryan, President of Wexford Actuarial and Consulting Services, moderated the panel.

Panelists included William H. McCartney, Counsel for Kutack Rock Company; Mavis A. Walters, Executive Vice President for Insurance Services Office, Inc.; Craig Berrington, Senior Vice President and General Counsel for American Insurance Association; and Larry Forrester, President of NAMIC.

After the general session, Irene K. Bass introduced the keynote speaker, Giandomenico Picco.

Ms. Bass announced future CAS meetings in 1995 and thanked the Program Planning Committee for coordinating the meeting. David P. Flynn presented Ms. Bass with a CAS plaque, and Ms. Bass officially passed the gavel to new CAS President Allan M. Kaufman, who adjourned the meeting at 12:30 p.m.

November 1994 Attendees

In attendance, as indicated by the registration records, were 443 Fellows, and 222 Associates. The list of members' names follows.

FELLOWS

Ralph L. Abell
Barbara J. Addie
Gregory N. Alff
Terry J. Alfuth
Rebecca C. Amoroso
Karen E. Amundsen
Charles M. Angell
Kenneth Apfel
Nolan E. Asch
Irene K. Bass
Todd R. Bault
Thomas R. Bayley
Allan R. Becker
John A. Beckman
Albert J. Beer
Linda L. Bell
David M. Bellusci
William H. Belvin
Phillip N. Ben-Zvi
Robert S. Bennett
Regina M. Berens
Lisa M. Besman
Raja R. Bhagavatula
David R. Bickerstaff
William P. Biegaj
Jennifer L. Biggs
Richard A. Bill
Holly L. Billings
Richard S. Biondi
Robert G. Blanco
Cara M. Blank
Betsy L. Blue
Joseph A. Boor
Ronald L. Bornhuetter
Peter T. Bothwell
Charles H. Boucek
Amy S. Bouska
David S. Bowen
J. Scott Bradley
Paul Braithwaite
Mark L. Brannon
Malcolm E. Brathwaite
Yaakov B. Brauner
Paul J. Brehm
Dale L. Brooks
J. Eric Brosius
Charles A. Bryan
John W. Buchanan
James E. Buck
George Burger
Anthony J. Burke
Patrick J. Burns
George R. Busche
John E. Captain
Christopher S. Carlson
Lynn R. Carroll
Edward J. Carter
Andrew R. Cartmell
Michael J. Cascio
Sanders B. Cathcart
Michael J. Caulfield
Janet L. Chaffee
Jessalyn Chang
Scott K. Charbonneau
Joseph S. Cheng
David R. Chernick

James K. Christie
Walter P. Cieslak
R. Kevin Clinton
Michael A. Coca
Jeffrey R. Cole
Robert F. Conger
Francis X. Corr
Gregory L. Cote
Michael Dennis Covney
Mark Crawshaw
Frederick F. Cripe
Susan L. Cross
Alan M. Crowe
Richard M. Cundy
Diana M. Currie
Ross A. Currie
Alan C. Curry
Robert J. Curry
Michael T. Curtis
Daniel J. Czabaj
Robert A. Daino
Robert N. Darby, Jr.
Edgar W. Davenport
Curtis Gary Dean
Thomas J. DeFalco
James M. Dekle
Michael L. DeMattei
Carol Desbiens
Robert V. Deutsch
Edward D. Dew
Kevin G. Dickson
George T. Dodd
John L. Doellman
Jeffrey L. Dollinger
James L. Dornfeld
Robert B. Downer
Karl H. Driedger
Michael C. Dubin
Diane Symnoski Duda
Brian Duffy
N. Paul Dyck
Myron L. Dye
Richard D. Easton
Bradley C. Eastwood
Maribeth Ebert
Dale R. Edlefson
Bob D. Effinger, Jr.
Gary J. Egnasko
Valere M. Egnasko
Douglas D. Eland
Donald J. Eldridge
Edward B. Eliason
John W. Ellingrod
Charles C. Emma
Jeffrey A. Englander
David Engles
Philip A. Evensen
Dennis D. Fasking
Matthew G. Fay
Frank Fedele
Richard I. Fein
Mark E. Fiebrink
Robert J. Finger
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Russell S. Fisher
William G. Fitzpatrick
David P. Flynn
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John R. Forney, Jr.
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Michael Fusco
Scott F. Galiardo
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Farrokh Guiahi
Sam Gutterman
David N. Hafling
Kyleen Knilans Hale
James A. Hall, III
George M. Hansen
Jonathan M. Harbus
David G. Hartman

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Lisa A. Hays
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Kathleen A. Hinds
Jeanne M. Hollister
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Nancy E. Kot
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Rodney E. Kreps
David J. Kretsch
Jane Jasper Krumrie
John R. Kryczka
Andrew E. Kudera
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John M. Kulik
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Patricia Laracuente
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Phillip C. Vigliaturo	William Robert Wilkins	Ronald J. Zaleski
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REPORT OF THE VICE PRESIDENT-ADMINISTRATION

The objective of this report is to provide a brief summary of Casualty Actuarial Society (CAS) activities since the last annual meeting.

I will first comment on these activities as they relate to the purposes of the CAS which are stated in our Constitution as follows:

1. Advance the body of knowledge of actuarial science in applications other than life insurance;
2. Establish and maintain standards of qualification for membership;
3. Promote and maintain high standards of conduct and competence for the members; and
4. Increase the awareness of actuarial science.

I will then provide a summary of other activities that may not relate to a specific purpose but yet are critical to the ongoing vitality of the CAS. Finally, I will update you on the current status of our finances and key membership statistics.

Undoubtedly the major activity during the past year has been the CAS's efforts with regard to the Appointed Actuary concept, or more descriptively, Dynamic Financial Analysis (DFA). This major initiative, under the direction of the Appointed Actuary Advisory Committee and President-Elect Allan M. Kaufman, impacted each of the four purposes listed above. Virtually all the CAS Vice Presidents had specific 1994 goals in support of DFA, and the CAS Board was apprised of progress through status reports at each of the four Board meetings.

Related to purpose one, there are a number of DFA related research initiatives underway. The highest priority areas of research during the year included the following projects as assigned to the appropriate committee:

- A survey of financial models in use (Committee on Valuation and Financial Analysis);
- An outline of financial modeling needs (Committee on Valuation and Financial Analysis);
- Development of a model Appointed Actuary report (Committee on Valuation and Financial Analysis);
- Identification of key reinsurance issues (Committee on Reinsurance Research); and
- Analysis of variability in loss ratios and reserves (Committee on Theory of Risk).

It is expected that all of these, except the last, will be completed by year-end 1994.

In addition, there are a number of other research projects underway that will produce results at various points in time in the coming years.

Continuing education opportunities help fulfill purpose three, and a significant amount of DFA material was offered in this year's programs. Specific seminars relating to DFA topics included the limited attendance seminar on "Principles of Finance in Pricing Property and Casualty Insurance," which was held twice; a limited attendance seminar on "Financial Models with Practical Insurance and Reinsurance Application"; the AFIR Colloquium in April 1994; and the CIA/CAS Seminar for the Appointed Actuary held in September 1994. In addition, our May and November meetings, as well as other CAS seminars including the Casualty Loss Reserve Seminar, the June Reinsurance Seminar, and the Ratemaking Seminar, all contained sessions on DFA subjects.

An overall continuing education plan to support DFA was developed, and opportunities to be offered next year have already been identified. The 1995 Call Paper Program topic will be "Incorporating Risk Factors in Dynamic Financial Analysis." A limited attendance seminar on "Managing Asset and Investment Risk" will be offered in early 1995. And the "Principles of Finance" seminar will again be offered twice.

The education and examination process supports both purposes two and three. DFA-related activities during the year included revisions to the Part 10 syllabus to add asset material and establishment of a task force to review the feasibility of establishing post-Fellowship education for actuaries involved in providing Appointed Actuary reports. Further review of syllabus material relating to DFA topics is ongoing.

The role of increasing the awareness of actuarial science (purpose four) as it relates to DFA falls to the Appointed Actuary Advisory Committee and the President-Elect. A key component of the CAS's activity with DFA is to help define the role that actuaries will play. This will involve coordination and feedback with regulators, industry leaders, and the American Academy of Actuaries. These activities have been ongoing and will continue as the scope of the actuary's involvement in DFA evolves.

In addition to the progress made on the DFA initiative, there were other activities supporting our four purposes during the past 12 months.

New papers published in the *Proceedings,* the *Forum,* and the other CAS publications all increase the body of knowledge available to our profession. The summer *Forum* included the Call Papers on Environmental Liabilities, while the spring *Forum* published selected papers from the 1994 Variability in Reserves Prize Program and the paper on "Accounting for Risk Margins." The 1994 Ratemaking Call Papers and other non-call papers on ratemaking topics were released in the winter issue of the *Forum.* The 1993 *Proceedings* included eight new papers on a variety of topics.

In addition to the publication of papers, other research continued within various committees on topics that included catastrophe modeling, reinsurance risk transfer, and database development.

The Admissions Committees provide the major support for purpose two. They make continuous improvements to the exam preparation and grading process while overseeing the administration of the testing of approximately 6,500 candidates.

A major initiative during 1994 was the work of the Travel Time Working Group, which was charged with developing the necessary information to monitor travel time through the CAS exam process. The task force members, with significant staff support from the CAS Office, have developed the necessary historical data and a proposed approach for accomplishing the goal that was presented in a draft report to the CAS Board in September. A final report incorporating changes recommended by the Board will be presented and discussed at the February Board meeting.

High standards of qualifications and conduct are essential components of our profession and are embodied in CAS purposes two and three. Enforcement of compliance with professional standards was strengthened as the Board adopted the CAS Rules of Procedure for Disciplinary Action. The necessary Bylaws revisions to incorporate these changes were subsequently approved by the membership.

Maintaining our high standards is also accomplished through a quality program of continuing education. The CAS provides these opportunities through the publication of actuarial materials and the sponsorship of a number of meetings and seminars. This year's sessions included:

- the spring and fall meetings in Boston and Orlando;
- the Ratemaking Seminar, held in Atlanta, Georgia, which had 648 registrants;
- the Casualty Loss Reserve Seminar in Boston, Massachusetts, of which the CAS is a co-sponsor, attended by 815;
- the special interest seminar "AFIR Colloquium" in April, attended by 69 CAS members;
- the special interest seminar on "Medical Cost Containment and Health Issues" held last month in Minneapolis, Minnesota, attended by 75;

- the Reinsurance Seminar in June, which attracted 246 attendees to Washington, D.C.;
- the CIA/CAS Seminar for the Appointed Actuary in Toronto sponsored by the Canadian Institute of Actuaries and the CAS, attended by 300; and
- the previously mentioned limited attendance seminars on DFA topics.

The Continuing Education Committees continue to explore ways to provide additional opportunities to our membership. This year, we saw the use of a relatively new forum: limited attendance seminars with academic instructors. These have been well received and a task force has been established to investigate the use of limited attendance focus group seminars.

The CAS regional affiliates also provide valuable opportunities for the members to participate in educational forums. In addition, the regional affiliates are a resource to help increase the awareness of the profession (purpose four) at the local level. Discussions are underway with the leadership of the regional affiliates to encourage more communication at the high school level.

There were other initiatives during the year in support of our goal to enhance the actuarial profession. A career video tape and accompanying brochure were developed in conjunction with the SOA for use in recruiting in high schools and colleges. Also, the CAS has reached agreement with the American Academy's Casualty Practice Council to better coordinate planning and goal setting processes to maximize the benefits we derive from the Academy's public interface role.

More related to the fourth purpose, but generally impacting all purposes, are the CAS's international activities. In addition to the ongoing attendance at various international actuarial society meetings by the CAS leadership, the following items highlight a few of the many international activities that took place during 1994:

- The CAS was a co-sponsor of the AFIR colloquium in April.

- An exam waiver policy with the Institute/Faculty of Actuaries was completed.
- The Working Agreement Task Force was expanded to include Mexican actuaries.
- A new regional affiliate from Asia, "Casualty Actuaries of the Far East," was formed.
- The CAS participated in discussions for a proposed International Federation of Actuarial Associations (IFAA).
- Liaisons were formed between the CAS and the General Insurance Study Group (GISG) and Joint Education Committee of the Institute/Faculty of Actuaries.

The CAS Office continues to provide excellent support and expand its services and capabilities. Significant productivity gains have been realized with their enhanced MIS capabilities, while support for exam administration and the annual budget process have been greatly enhanced. New member services introduced this year include a yearbook listing of members on diskette and the CAS Bulletin Board System (BBS), which will allow for electronic exchange of information and ideas among the membership.

Another resource of the CAS is that an integral part of its fabric and success is its committees and many volunteers. Member participation on our committees remains high. The annual committee chairpersons' meeting in April was highlighted by breakout group discussions of key CAS issues.

In closing, I will provide a brief status of our membership and financial condition. Our size continued its rapid increase as we added 225 new Associates and 103 new Fellows. Our membership now stands at 2,299.

New members elected to the Board of Directors for next year include Robert F. Conger, John M. Purple, Richard H. Snader, and Kevin B. Thompson. The membership elected Albert J. Beer to the position of President-Elect, while Allan M. Kaufman will assume the presidency.

The Executive Council, with primary responsibility for day-to-day operations, met either by teleconference or in person at least once a month during the year. The Board of Directors elected the following Vice Presidents for the coming year:

Vice President-Administration, Paul Braithwaite

Vice President-Admissions, John J. Kollar

Vice President-Continuing Education, David N. Hafling

Vice President-Programs and Communications, Alice H. Gannon

Vice President-Research and Development, Michael J. Miller

The CPA firm of Feddeman & Company has been engaged to examine the CAS books for fiscal year 1994. Its findings will be reported by the Audit Committee to the Board of Directors in February 1995. The fiscal year ended with an unaudited net income of $118,325 which compares favorably to a budgeted amount of approximately $35,000. Members' equity now stands at $1,252,389, subdivided as follows:

Michelbacher Fund	$87,896
Dorweiler Fund	5,823
CAS Trust	3,305
Scholarship Fund	7,447
Rodermund Fund	14,222
CLRS Fund	5,000
ASTIN Fund	2,000
Research Fund	178,165
CAS Surplus	948,532
TOTAL MEMBERS' EQUITY	$1,252,389

This represents an increase in equity of $201,739 over the amount reported last year.

For 1994-95, the Board of Directors has approved a budget of approximately $2.6 million. Members' dues for next year will be $250, an increase of $10, while fees for the Invitational Program will increase by $15 to $305.

Respectfully submitted,

John M. Purple

John M. Purple
Vice President-Administration
November 14, 1994

FINANCIAL REPORT
FISCAL YEAR ENDED 9/30/94

OPERATING RESULTS BY FUNCTION

FUNCTION	*INCOME*	*EXPENSE*	*DIFFERENCE*
Membership Services	$ 579,966	$ 760,351	($ 180,385)
Seminars	466,881	307,060	159,821
Meetings	533,827	422,901	110,926
Exams	755,352	651,034	104,318
Publications	69,255	55,332	13,923
TOTAL	$ 2,405,281	$ 2,196,678	$ 208,603*

*NOTE: Change in surplus before interfund transfers of $102,000 for research and ASTIN funds.

BALANCE SHEET

ASSETS	*9/30/93*	*9/30/94*	*DIFFERENCE*
Checking Account	$ 165,981	$ 366,425	$ 200,444
T-Bills/Notes and Accrued Interest	1,100,627	1,216,193	115,566
Prepaid Expenses	22,383	63,322	40,939
Account Receivable	50,000	45,000	(5,000)
Property and Equipment	223,533	233,279	9,746
Less: Accumulated Depreciation	(84,770)	(149,899)	(65,129)
TOTAL ASSETS	$ 1,477,754	$ 1,774,320	$ 296,566

LIABILITIES	*9/30/93*	*9/30/94*	*DIFFERENCE*
Exam Fees Deferred	$ 236,765	$ 296,989	$ 60,224
Meeting, Seminar Fees Deferred	103,663	109,594	5,931
Subscriber Fees Deferred	624	0	(624)
Accounts Payable	20,967	58,335	37,368
Deferred Rent	53,145	45,074	(8,071)
Accrued Pension	11,940	23,661	11,721
TOTAL LIABILITIES	$ 427,104	$ 533,653	$ 106,549

MEMBERS' EQUITY	*9/30/93*	*9/30/94*	*DIFFERENCE*
Michelbacher Fund	$ 85,336	$ 87,896	$ 2,560
Dorweiler Fund	6,624	5,823	(801)
CAS Trust	3,208	3,305	97
Scholarship Fund	7,715	7,446	(269)
Rodermund Fund	14,895	14,222	(673)
CLRS Fund	5,000	5,000	0
Research Fund	97,665	178,165	80,500
ASTIN Fund	0	2,000	2,000
CAS Surplus	830,207	936,810	106,603
TOTAL EQUITY	$ 1,050,650	$ 1,240,667	$ 190,017

John M. Purple, Vice President-Administration

This is to certify that the assets and accounts shown in the above financial statement have been audited and found to be correct.

CAS Audit Committee: Sheldon Rosenberg, Chairperson; Steven F. Goldberg, Anthony J. Grippa, and William M. Rowland.

1994 EXAMINATIONS—SUCCESSFUL CANDIDATES

Examinations for Parts 3B, 4A, 4B, 5A, 5B, 6, 8, 8C (Canadian), and 10 of the Casualty Actuarial Society were held on May 2, 3, 4, 5, and 6, 1994. Examinations for Parts 3B, 4A, 4B, 5A, 5B, 7, and 9 of the Casualty Actuarial Society were held on November 1, 2, 3, and 4, 1994.

Examinations for Parts 1, 2, 3A and 3C (SOA courses 100, 110, 120, and 135) are jointly sponsored by the Casualty Actuarial Society and the Society of Actuaries. Parts 1 and 2 were given in February, May, and November of 1994, and Parts 3A and 3C were given in May and November of 1994. Candidates who were successful on these examinations were listed in joint releases of the two societies.

The Casualty Actuarial Society and the Society of Actuaries jointly awarded prizes to the undergraduates ranking the highest on the Part 1 examination.

For the February 1994 examination, the $200 first prize was awarded to Marie-Eve Lachance. The $100 second prize winners were David W. Jelinek, Samuel Johnson, Aleksandr Khazanov, and Maria Zaretsky.

For the May 1994 examination, the $200 first prize was awarded to Darryl H. Yong. The $100 second prize winners were Kevin P. Brennan, Douglas S. Freedman, Jui-Ruei Hung, Brent R. Logan, Bryant A. Swanson, and Kristopher A. Swayze.

For the November 1994 examination, the $200 first prize was awarded to Thomas G. Draper. The $100 second prize winners were Hing S. Lau, Kok B. Liew, Ching Ng, and Julia Stetter.

The following candidates were admitted as Fellows and Associates at the CAS Spring Meeting in May 1994 as a result of their successful completion of the Society requirements in the November 1993 examinations.

FELLOWS

Richard R. Anderson	Warren A. Klawitter	Robert L. Miller
Benoit Carrier	Gilbert M. Korthals	Donald D. Palmer
Stephen R. DiCenso	Paul W. Lavrey	Karen L. Pehrson
Shawn F. Doherty	John J. Limpert	Tom A. Smolen
George Fescos	Paul R. Livingstone	Beth M. Wolfe
Allan A. Kerin	Cassandra M. McGill	

ASSOCIATES

Mark A. Addiego	John S. Chittenden	Lise A. Hasegawa
Elise M. Ahearn	Kuei-Hsia R. Chu	Amy J. Himmelberger
Timothy P. Aman	Rita E. Ciccariello	Thomas A. Huberty
Michael J. Andring	Laura R. Claude	Sandra L. Hunt
William M. Atkinson	J. Paul Cochran	Fong-Yee J. Jao
Lewis V. Augustine	Frank S. Conde	June V. Jarvis
Robert S. Ballmer, II	Pamela A. Conlin	Charles N. Kasmer
Jack Barnett	Francis L. Decker, IV	Mark J. Kaufman
Rose D. Barrett	Kurt S. Dickmann	Louis K. Korth
Martin J. Beaulieu	Andrew J. Doll	Mary D. Kroggel
Brian P. Beckman	John P. Doucette	Cheung S. Kwan
Richard Belleau	Robert G. Downs	Mylene J. Labelle
Cynthia A. Bentley	Bernard Dupont	Bertrand J. LaChance
LaVerne J. Biskner, III	David M. Elkins	Blair W. Laddusaw
Suzanne E. Black	Martin A. Epstein	Elaine Lajeunesse
Michael G. Blake	Dianne L. Estrada	Lewis Y. Lee
Erik R. Bouvin	Michael A. Falcone	Julie Lemieux-Roy
Robert E. Brancel	Karen M. Fenrich	Paul B. LeStourgeon
Christopher G. Brunetti	Stephen A. Finch	Aaron S. Levine
Mark E. Burgess	Daniel B. Finn	Kenneth A. Levine
Mark W. Callahan	Brian C. Fischer	Frank K. Ling
Robert N. Campbell	Douglas E. Franklin	Andrew M. Lloyd
Daniel G. Carr	Kirsten A. Frantom	Ronald P. Lowe, Jr.
Julia C. Causbie	Nathalie Gamache	Robert G. Lowery
Maureen A. Cavanaugh	Christine A. Gennett	Christopher J. Luker
Francis D. Cerasoli	Joyce G. Hallaway	Barbara S. Mahoney
Julie S. Chadowski	William D. Hansen	Robert G. Mallison, Jr.
Daoguang E. Chen	Steven T. Harr	Gabriel O. Maravankin

Robert F. Maton
Emma Macasieb McCaffrey
Charles L. McGuire, III
David W. McLaughry
Kathleen A. McMonigle
Robert F. Megens
Daniel J. Merk
Timothy Messier
Stephen J. Mildenhall
Scott M. Miller
Gregory A. Moore
Robert J. Moser
Mark A. O'Brien
Denise R. Olson
John E. Pannell
Wende A. Pemrick
Robert L. Penick
Beverly L. Phillips
Mark A. Piske
Gregory J. Poirier
Mark Priven
Arlie J. Proctor
Donald A. Riggins
Tracey S. Ritter
Douglas S. Rivenburgh
Paul J. Rogness
David A. Russell
Sean W. Russell
Stephen P. Russell
Linda M. K. Saunders
Gerson Smith
Gina B. Smith
Louis B. Spore
Douglas W. Stang
Laurence H. Stauffer
Judith L. Stolle
Ilene G. Stone
Collin J. Suttie
Jeanne E. Swanson
John P. Thorrick
Tony King Gwan Tio
Dom M. Tobey
Glenn A. Tobleman
Cynthia J. Traczyk
Theresa A. Turnacioglu
Robert C. Turner, Jr.
Ching-Hom Rick Tzeng
Robert W. Van Epps
Jeffrey A. Van Kley
Mark D. van Zanden
Trent R. Vaughn
Robert J. Vogel
W. Olivia Wacker
Joseph W. Wallen
Lisa Marie Walsh
Alice M. Wang
Gregory S. Wanner
Michelle M. Wass
Geoffrey T. Werner
Tad E. Womack
Robert S. Yenke
Benny S. Yuen
George H. Zanjani
Joshua A. Zirin
Rita M. Zona

The following is a list of successful candidates in examinations held in May 1994.

Part 3B

Sarah Albro
Fred S. Allsbrook
Nabila Audi
Craig V. Avitabile
David M. Baxter
Robert S. Beatman
Anna Marie Beaton
Nathalie Belanger
Michael J. Belfatti
Jeffrey D. Benelli
Terry R. Benz
Rodney L. Blacklock
Winfred N. Botchway
Edmund L. Bouchie
Kimberly Bowen
Thomas G. Bowyer
Lisa A. Cabral
Amy M. Campbell
Rutledge M. Capers
Stephen P. Carlson
Milissa D. Carter
Daero Choi
Philip A. Clancey, Jr.
David A. Clute
Christopher P. Coelho
Michele Cohen
Francis Kevin Connors
David G. Cook
David E. Corsi
Jonathan S. Curlee
John E. Daniel
Vickie L. Davis
Michael T. Decker

Sharon D. Devanna
Stefvan S. Drezek
Ross Dunlop
Cynthia Durbin
Melissa M. Emmendorfer
Kristine M. Esposito
Jonathon E. Fassett
Karen L. Field
Ginda Kaplan Fisher
Ronnie S. Fowler
Mark A. Fretwurst
Rosemary D. Gabriel
Serge Gagne
Michael A. Garcia
Marilyn M. Giannos
Michael P. Gibson
Olga Golod
Jay C. Gotelaere
Laurie L. Griffin
Lora L. Gruesbeck
Brian T. Hanrahan
Eric C. Hassel
Lisa M. Hawrylak
Jodi J. Healy
Kevin B. Held
Deborah L. Herman
Twiggy Hernandez
Lisa K. Hiatt
Dave R. Holmes
Eric A. Hoppe
Brett Horoff
Gail Hossin
Candace Yolande Howell
Heidi L. Hower
Rebecca R. Hunt
Scott R. Jean
Jeffery F. Johnson
William R. Johnson
Burt D. Jones
William Rosco Jones
Michael S. Kahlowsky
Rebecca T. Katz
Hsien-Ming K. Keh
Mary C. Kellstrom
William J. Keros
David N. Kightlinger
He-Jin Kim
Patricia Kinghorn
Jill E. Kirby
Kristie L. Klekotka
Robert A. Kranz
John J. Kraska, III
Scott C. Kurban
Kenneth Allen Kurtzman
Laura S. Larson
Douglas W. Latimer
Rocky S. Latronica
Michelle J. Leeper
Todd W. Lehmann
Bradley H. Lemons
Steven J. Lesser
Charles Letourneau
Kuen-Shan Ling
Yih-Jiuan B. Lu
James W. Luedtke
Sak-Man Luk
Victoria S. Lusk
Jacob Margulis
Julie Martineau
Victor Mata
Stephen J. McAnena
Richard M. McGowan
Shawn Allan McKenzie
Rae F. McPhail
Jill M. Merchant
Paul D. Miotke
Matthew K. Moran
Roosevelt C. Mosley
Robert J. Moss
Michael D. Neubauer
Gary R. Nidds
Liam F. O'Connor
Randall W. Oja
Kevin J. Olsen
Michael A. Onofrietti
Grace A. Orsolino
Kelly A. Paluzzi
Mark Paykin
Harry T. Pearce
Portia E. Pelt
Robert B. Penwick
Priyantha L. Perera
Christopher K. Perry
Daniel B. Perry
John S. Peters
Kraig P. Peterson
Judy L. Pool
Edward L. Pyle
Penelope A. Quiram
Jill A. Raike
Jeffrey T. Rasmussen
Janice L. Rexroth
Melissa K. Ripper
Karen L. Rivara
Anthony V. Rizzuto
Christopher D. Ruckman
Anthony S. Ruscitto
Joseph J. Sacala
Kashyap C. Saraiya
Anne T. Schalda
Ryan D. Schave
Christy B. Schreck
Bradley J. Schroer

Ernest C. Segal
Michele Segreti
Tina Shaw
Kelli D. Shepard-El
Meyer Shields
Bret C. Shroyer
Richard Sieger
David C. Sky
Ronald L. Smith
Kendra Barnes South
Caroline B. Spain
Lloyd M. Spencer
Daniel J. Spillane
Carol A. Stevenson
Therese M. Stom
Thomas Struppeck
Mark Sturm
Sherri C. Sturm
Edward T. Sweeney
Christopher C. Swetonic
Charles A. Thayer
Amy L. Tucker
Jeffrey E. Tucker
Jordan N. Uditsky
Joel A. Vaag
Martin Vezina
Melodee A. Wallace
Norman E. Watkins
Joseph C. Wenc
James C. Whisenant
Patricia C. White
Vanessa C. Whitlam-Jones
Brenda K. Wilson
Tamara M. Winton
Wendy L. Witmer
Karin H. Wohlgemuth
Simon Wong
Jun Yan
Michael J. Yates
David P. Zanutto

Part 4A

Sarah Albro
Madhu G. Amar
Paul D. Anderson
Carl X. Ashenbrenner
Barry Luke Bablin
James V. Barilaro
Sabine C. Barksdale
Cheryl L. Barnett
Karen E. Bashe
Elizabeth F. Bassett
David W. Batten
Michael J. Bednarick
Saeeda Behbahany
Julie Bennett
Wayne F. Berner
Frank J. Bilotti
Kevin M. Bingham
Jonathan E. Blake
Mariano R. Blanco
Luc Boissiere
Thomas L. Boyer, II
Rebecca S. Bredehoeft
Cary J. Breese
Steven A. Briggs
Karen A. Brostrom
Lori L. Burton
John J. Carroll
Peter T. Chang
Yu L. Chen
Peggy Cheng
Richard M. Chiarini
Jamie Chow
Michael J. Christian
Charles A. Cicci
Pamela A. Connors
Sean O. Cooper
Brenda K. Cox
Spencer L. Coyle
Richard S. Crandall
Michael B. Cray
Mary Katherine T. Dardis
Sheri L. Daubenmier
Dawne L. Davenport
Willie L. Davis
Harin A. De Silva
Nancy K DeGelleke
Emily Y. Deng
Alain P. DesChatelets
John T. Devereux
Nelson T. Dismukes
Dean P. Dorman
John C. Dougherty
Cindy L. Dube
Rachel Dutil
Mark Kelly Edmunds
Jane Eichmann
Tanya E. Eng
Kristine M. Esposito
Jonathan Palmer Evans
Jui-Chuan Fan
Yen Fang
Junko K. Ferguson
Chantal N. Fitzgerald
Mary E. Fleischli
Hugo Fortin

Nathalie Fortin
Christian Fournier
Noelle C. Fries
Richard A. Fuller
Serge Gagne
David E. Gansberg
Michael H. Gay
Margaret Wendy Germani
Isabelle Gingras
Theresa Giunta
Moshe D. Goldberg
Carol A. Goodrich
Lori A. Gordon
Jay C. Gotelaere
John P. Gots
Steven C. Gross
William Alan Guffey
Greg M. Haft
Marc S. Hall
Faisal O. Hamid
Gregory Hansen
Jean-Francois Hebert
David E. Heppen
Timothy E. Hill
Daniel L. Hogan
Eric J. Hornick
Geoffrey W. Horton
Jeff S. Howatt
Marguerite M. Hunt
Jamison J. Ihrke
Cindy Jacobowitz
Gregory O. Jaynes
Walter L. Jedziniak
Neal O. Jettpace
Edward Jhu
Daniel K. Johnson
Michael S. Johnson
Paul J. Johnson
Derek A. Jones
Theodore A. Jones
Jeremy M. Jump
Jong-Ming Kan
Philip A. Kane, IV
Chad C. Karls
Anthony N. Katz
Kathryn E. Keehn
Mary C. Kellstrom
David N. Kightlinger
Cameron D. Kimbrough
Debra L. Kocour
James J. Konstanty
Eleni Kourou
John J. Kraska, III
Regina Krasnovsky
Richard S. Krivo
Stephane Lalancette
Rita Ann B. Lamb
Debra K. Larcher
Gregory D. Larcher
Valerie Lavoie
Yin Lawn
Henry T. Lee
Christian Lemay
Xiaoyin Li
Christina Link
Serge M. Lobanov
Yih-Jiuan B. Lu
Vahan A. Mahdasian
Laura S. Marin
Robert H. Marks
Kirk E. Marnin
Richard E. Marrs
Jason A. Martin
Stanislav D. Maydan
Claudia A. McCarthy
Patrice McCaulley
Mark Z. McGill, III
Michele L. McKay
Brian James Melas
Alan E. Morris
John V. Mulhall
Jarow G. Myers
Yinay Nadkarni
Jennifer A. Nelson
Thomas E. Newgarden
Marc Freeman Oberholtzer
Avital Ohayon
Helen S. Oliveto
Apryle L. Oswald
Andrew J. Owen
Carole K. Payne
Tracie L. Pencak
Cynthia Perrault
Julie Perron
Anthony G. Phillips
Mitchell S. Pollack
Brentley J. Radeloff
Kimberly E. Ragland
Sundar Ramaswami
Ricardo A. Ramotar
John F. Rathgeber
Dean R. Reigner
Teresa M. Reis
Rebecca J. Richard
Melissa K. Ripper
David Roberge
Linda L. Roberts
Michelle N. Rodriguez
Jay Andrew Rosen
Richard A. Rosengarten
Christine R. Ross
Robert R. Ross
Michael M. Rubin
Brian P. Rucci
Benoit St-Aubin

Jeffrey T. Sallee
Anthony N. Sammur
Glenn R. Scharf
Christy B. Schreck
Peter A. Scourtis
Robin M. Seifert
Stacy L. Shimizu-Hall
Nathan Ira Shpritz
Donna K. Siblik
Steven A. Smith, II
Carl J. Sornson
Jennifer A. Sovell
Caroline B. Spain
T. Matthew Steve
Elizabeth A. Sullivan
Roman Svirsky
Jo D. Thiel
Tammy M. Titus
Stephanie J. Traskos
Beth S. Tropp
Scott M. Tulloch
David S. Udall
Richard A. Van Dyke
Matthew J. Wasta
William Robert Wilkins
Kendall P. Williams
L. Alicia Williams
Laura M. Williams
Frances E. Wilson
Rick A. Workman
Eric E. Zlochevsky

Part 4B

Lynn A. Allen
Michael J. Anstead
Stephen M. Arnhold
Timothy W. Atwill
Nathan J. Babcock
Calvin L. Baker
Bassam B. Barazi
J. Bradford Barlow
Kimberly M. Barnett
Elizabeth F. Bassett
Ghislaim Beattie
Nicolas Beaupre
Saeeda Behbahany
David J. Belany
Robert W. Bell
Darryl R. Benjamin
Suzanne Berendsen
Bruce J. Bergeron
Martin Bernard
Robert C. Birmingham
Jennifer L. Blank
Daniel R. Boerboom
Michele Boivin
Jean-Pierre Bolduc
Joseph V. Bonanno, Jr.
Caleb M. Bonds
Francois Bourdon
Andree-Anne Bourgeois
Danny Boutin
Kimberly Bowen
Patrice Brassard
David L. Braun
Tommie D. Brooks
James D. Buntine
Kevin D. Burns
Donia N. Burris
Robert Buzecan
Donna L. Callison
Thomas K. Calvert
Rong Rose Cao
Ann Marie L. Cariglia
Thomas P. Carlson
Peter Chae
Thierry Chamberland
Kelly C. Chang
Daniel G. Charbonneau
Nathalie Charbonneau
Todd D. Cheema
Cho-Jieh Chen
Yvonne W. Y. Cheng
Theresa A. Christian
Louise Chung-Chum-Lam
Bernadette M. Chvoy
Lori Anne Cieri
Stephen D. Clapp
Susan M. Cleaver
Bruce Jay Collings
Margaret E. Conroy
Sharon R. Corrigan
Edgar B. Cruz
Michael J. Curcio
Sheri L. Daubenmier
John D. Deacon
Brian H. Deephouse
Romulo N. Deo-Campo Vuong
Krikor Derderian
Giuseppe C. Di Tullio
Anthony M. DiLapi
David A. Dolly
Christopher S. Downey
James A. Doyle
Chris L. Draper
Stephen C. Dugan
Rachel Dutil
Jennifer S. Ebert

Ian H. Edelist
Jane Eichmann
Jason L. Ellement
Kristen E. Erickson
Juan Espadas
Edward H. Eun
Brian A. Evans
Rebecca F. Evans
Robin S. Fader
Charles V. Faerber
Karl F. Farmer
Sylvain Fauchon
Michael N. Ferik
Elizabeth A. Fish
Chauncey E. Fleetwood
Michael A. Fradkin
Daniel Gagne
Yannick Gagne
John E. Gaines
Michelle R. Garnock
John J. Garrett
Kathy H. Garrigan
Abbe B. Gasparro
Margaret Wendy Germani
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Gary J. Goldsmith
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Brian R. Knox
Jeffrey J. Krygiel
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Rita Ann B. Lamb
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Kevin A. Lee
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James P. Leise
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Sylvain Leonard
Steven J. Lesser
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Man Yan Leung
Emmanuel Serge Levi
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Janet G. Lindstrom
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Mitchel Merberg
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Eric Millaire-Morin
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Isabelle Morin
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Tieyan Tina Ni
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Brad Michael Ritter
Carmilla T. Rivera
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Lynn A. Ruezinsky
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Giuseppe Russo
Brian C. Ryder
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Jennifer N. Williams
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Anne M. DelMastro
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Dawn E. Elzinga
Benedick Fidlow
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Eric J. Hornick
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Michael K. Curry
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Arthur J. Schwartz
Gregory R. Scruton
Peter Senak
Robert D. Share
Michelle G. Sheng
Gary E. Shook
Patricia E. Smolen
Douglas W. Stang
Richard A. Stock
Ilene G. Stone
Paul J. Struzzieri
James F. Tygh
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Jeffrey L. Dollinger
John P. Doucette
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Russell Frank
Kim B. Garland
Donna L. Glenn
Linda M. Goss
Bradley A. Granger
Russell H. Greig, Jr.
Leigh Joseph Halliwell
Jonathan M. Harbus
Lisa A. Hays
Suzanne E. Henderson
Wayne Hommes
Craig W. Kliethermes
Nancy E. Kot
Cheung S. Kwan
Mathieu Lamy
James W. Larkin
Michael D. Larson
Michel Laurin
France LeBlanc
Thomas L. Lee
Siu K. Li
Richard S. Light
Maria Mahon
William G. Main
Lawrence F. Marcus
John W. McCutcheon, Jr.
Richard T. McDonald
M. Sean McPadden
Christopher J. McShea
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Stephen J. Meyer
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Kelly L. Moore
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W. Randall Naylor
John Nissenbaum
Laura A. Olszewski
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Andre Perez
Marian R. Piet
Brian D. Poole
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Eduard J. Pulkstenis
Donna J. Reed
Elizabeth M. Riczko
James Joseph Romanowski
Kevin D. Rosenstein
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Derrick D. Shannon
David M. Shepherd
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Barbara H. Thurston
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Charles F. Toney, II
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Scott P. Weinstein
L. Nicholas Weltmann, Jr.
Debra L. Werland
Steven B. White
Marcia C. Williams
Tad E. Womack
Floyd M. Yager
Edward J. Yorty
Ralph T. Zimmer

The following candidates were admitted as Fellows and Associates at the CAS Annual Meeting in November 1994 as a result of their successful completion of the Society requirements in the May 1994 examinations.

FELLOWS

Todd R. Bault
John A. Beckman
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Michael A. Coca
Gregory L. Cote
Michael T. Curtis
Edgar W. Davenport
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Daniel B. Finn
Yves Francoeur
Russell Frank
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Linda M. Goss
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Deborah G. Horovitz
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Michel Laurin
France LeBlanc
Elise C. Liebers
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Camille Diane Minogue
James Edward Monaghan
Matthew C. Mosher
Turhan E. Murguz
Aaron West Newhoff
Marc Freeman Oberholtzer
John R. Pedrick
Anne Marlene Petrides
Michael David Price
Karen L. Queen
Kathleen Mary Quinn
Yves Raymond
Victor Unson Revilla
Brad Michael Ritter
Jay Andrew Rosen
Christine R. Ross
Matt J. Schmitt
Jeffrey Parviz Shirazi
Nathan Ira Shpritz
Kerry S. Shubat
Charles Leo Sizer
Carl J. Sornson
Klayton N. Southwood
Angela Kaye Sparks
Linda F. Ward
James C. Whisenant
Wyndel S. White
William Robert Wilkins
Jeanne Lee Ying
Doug Alan Zearfoss

The following is the list of successful candidates in examinations held in November 1994.

Part 3B

Jeffrey R. Adcock
Ethan D. Allen
John P. Alltop
Silvia J. Alvarez
Mary K. Anderson
Amy P. Angell
Melissa J. Appenzeller
Anju Arora
Carl X. Ashenbrenner

Afrouz Assadian
Robert D. Bachler
Dee A. Bailey
Brent W. Barney
Justin R. Barrow
Cortney A. Bass
Julie-Ann Basso
James H. Bennett
Peter A. Bennett
Sheri L. Bieske
Brian A. Bingham
Tony F. Bloemer
Christopher D. Bohn
David R. Border
Christopher L. Bowen
Jennifer L. Bramschreiber
Brenda A. Brazil
Stephane Brisson
Angela D. Burgess
Christopher J. Burkhalter
Judith E. Callahan
Allison F. Carp
William Brent Carr
Alison S. Carter
Patrick J. Causgrove
Raji H. Chadarevian
Chien-yu Chan
Suhui Chen
Brian K. Ciferri
Jason T. Clarke
James P. Cleary
Kevin M. Cleary
Sean O. Cooper
Michelle A. Corso
William F. Costa
Michael J. Curcio
Mary Katherine T. Dardis
John D. Deacon
Brian H. Deephouse
Wesley J. DeNering
D. Vance C. DeWitt
Thomas R. Dlouhy
David L. Drury
Tammi B. Dulberger
Marcus W. Dummer
Rachel Dutil
Wayne W. Edwards
James R. Elicker
Sylvain Fauchon
Julia M. Ford
Hugo Fortin
Martine Gagnon
David M. Galko
David E. Gansberg
Michael K. Gastineau
Arthur L. Georges
Klete D. Geren
Matthew J. Gillette
Andrew S. Golfin, Jr.
Amy L. Grbcich
Robert A. Grocock
Greg M. Haft
Julie K. Halper
Marcus R. Hamacher
Craig E. Hanford
Scott W. Hanson
Esther Harrison
Jean-Francois Hebert
James A. Heer
James D. Heidt
Christopher R. Heim
William N. Herr, Jr.
Cynthia J. Heyer
Peter B. Hindman
Bradford K. Hoagland
Jody M. Hoffman
Joseph H. Hohman
Allen J. Hope
Mary E. Hromco
Catherine L. Hudson
Diane L. Hudson
Kristina M. Hummel
Philip M. Imm
Neal O. Jettpace
Tricia L. Johnson
Bryon R. Jones
Jeremy M. Jump
Brian A. Junod
Tamora A. Kapeller
Richard T. Kelly
Lauren A. Kerr
He-Jung Kim
Jean Y. Kim
James F. King
Susan L. Klein
Bradley S. Kove
Ignace Y. Kuchazik
Matthew R. Kuczwaj
James D. Kunce
William J. Lakins
Khanh M. Le
David Leblanc-Simard
Karen J. Lee
Christian Lemay
Daniel E. Lents
Marc E. Levine
Richard P. Lonardo
Michelle Luneau
Cynthia K. Lysne
Craig MacIntyre
Daniel Patrick Maguire

Michael C. Malone
David K. Manski
Joanne E. Marshall
Jason A. Martin
George J. McCloskey
Martin Menard
Ross H. Michehl
David P. Moore
Kenneth B. Morgan, Jr.
Ethan Mowry
Matthew D. Myshrall
Kari A. Nicholson
Gregory P. Nini
Michael P. O'Connor
Marsi A. O'Malley-Riley
Oscar J. Orban
Matthew R. Ostiguy
Kevin T. Peterson
Terry C. Pfeifer
Jeffrey J. Pfluger
Igor Pogrebinsky
Scott W. Pollard
Jennifer K. Price
Anthony E. Ptasznik
Beth A. Pyle
Kara L. Raiguel
Ricardo A. Ramotar
Peter S. Rauner
William J. Raymond
Delia E. Roberts
Efrain Rodriguez
Nathan W. Root
Piya Roy
Jennifer L. Rupprecht
Bryant E. Russell
Frederick D. Ryan
Matthew L. Sather
Suzanne K. Sauers
Gary F. Scherer
Gena A. Shangold
Bintao Shi
Rebecca L. Simons
Jason R. Smith
Stephen M. Smith
Thomas M. Smith
Monika Soja
Benoit St-Aubin
Patrick C. Steuber
Avivya S. Stohl
Roman Svirsky
C. Steven Swalley
Karrie L. Swanson
Edward Sypher
Stephen J. Talley
Craig D. Thomas
Craig Tien
James H. Tran
Michael C. Tranfaglia
Nathalie Tremblay
Karen J. Triebe
Lisa E. Tripp
Brian K. Turner
Sharon E. Tuttle
David S. Udall
David Uhland
Kevin E. Weathers
William J. Webb
Dana S. Weisbrot
Bruce A. Werner
Hau L. Ying
Anthony C. Yoder
Ruth Zea

Part 4A

Jeffrey R. Adcock
Sajjad Ahmad
Josee Allard
Gwendolyn R. Anderson
Kevin L. Anderson
Mark B. Anderson
Sheila M. Aranyos
Wendy L. Artecona
Craig V. Avitabile
Bassam B. Barazi
Thomas C. Bates
Mary P. Bayer
Michael J. Belfatti
Dwight D. Bell
Jennifer L. Blackmore
Daniel R. Boerboom
Joseph V. Bonanno, Jr.
James D. Buntine
Hugh E. Burgess
Kevin C. Burke
Donia N. Burris
Pamela A. Burt
Aleksandr A. Bushel
Daniel G. Charbonneau
Nathalie Charbonneau
Hongyan Chen
Christopher P. Coelho
Anna V. Colelli
Robert B. Collins
Margaret E. Conroy
David F. Dahl
Kristin J. Dale
Mark A. Davenport
Douglas L. Dee
Brian H. Deephouse

Anthony M. DiLapi
Nancy Ding
Francis J. Dooley
Christopher S. Downey
Tammy L. Dye
Keith A. Engelbrecht
Richard A. Farrow
Janine A. Finan
Chauncey E. Fleetwood
Sean P. Forbes
Sarah J. Fore
Martin Fortin
Ronnie S. Fowler
Mark R. Frank
Timothy J. Friers
Amy A. Gadsden
Gina L. Gagliardi
John E. Gaines
Michael A. Garcia
Ellen M. Gavin
Siddhartha Ghosh
James W. Gillette
Sanjay Godhwani
Olga Golod
Melanie T. Green
Jacqueline L. Gronski
Curtis A. Grosse
Lora L. Gruesbeck
David B. Hackworth
Lisa M. Hawrylak
Kimberly A. Heiligenberg
Kevin B. Held
Sally Dunlap Hendrick
Tina M. Henninger
Twiggy Hernandez
Brook A. Hoffman
Christopher R. Jarvis
Stephen L. Jauss
Michael J. Kallan
Robert C. Kane
Glenda J. Kettelson
Linda I. Kierenia
Patricia Kinghorn
Joseph P. Kirley
Kwabena A. Koranteng
Tanya M. Kovacevich
Scott C. Kurban
Timothy J. Landick
Laura S. Larson
Peter Latshaw
Dennis H. Lawton
Dzung Le
Bradley R. Leblond
Todd W. Lehmann
Steven E. Levitt
Craig A. Levitz
Jamison W. Lindsey
Rebecca M. Locks
Wayne L. Lowe
Mark S. Lu
John Lum
Jason K. Machtinger
Daniel Patrick Maguire
Stephen P. Marsden
William J. Mazurek
William R. McClintock
Peter B. McCloud
Shawn Allan McKenzie
Sarah K. McNair-Grove
Kirk F. Menanson
Michelle L. Merkel
Eric Millaire-Morin
Kathleen C. Miller
Paul W. Mills
Paul D. Miotke
Roosevelt C. Mosley
Gwendolyn D. Moyer
Michael D. Nielsen
Eng Loke Ong
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Jennifer L. Paris
Michael C. Parsons
Nilesh T. Patel
Harry T. Pearce
Wendy W. Peng
John S. Peters
Charles V. Petrizzi
Jeffrey J. Pfluger
Deborah J. Pomerantz
Dale S. Porfilio
Edward L. Pyle
Kiran Rasaretnam
Nathan W. Root
Jaime J. Rosario
Denise F. Rosen
Brian C. Ryder
Julie A. Schneider
Timothy D. Schutz
Lisa M. Scorzetti
Michele Segreti
Tina Shaw
Scott A. Sheldon
Meyer Shields
Bret C. Shroyer
Allison M. Skolnick
Robert K. Smith
Thomas M. Smith
William L. Smith
George Dennis Sparks
William A. Spoerner
Carol A. Stevenson
Steven J. Symon
Ming Tang
Varsha A. Tantri
Daniel A. Tess
Hugh T. Thai

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Milton F. Yee
Kathryn L. Zaelit
Grace Zakaria

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Perri Ann Allen
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Gilla A. Amar
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Julie A. Anderson
Paul D. Anderson
Todd C. Anderson
Cheng-Hong Ang
Frank A. Aritz
Wendy L. Artecona
Julie R. Augustine
Brandon E. Auster
Rona A. Axelrod
Robert D. Bachler
Lisa Buchman Barshay
Thomas C. Bates
Mourad Bentoumi
Mario Binetti
Kevin M. Bingham
Linda J. Bjork
Jonathan E. Blake
Joseph Bojman
Andrea Bolliger
Edith Boucher
John B. Brady
Jennifer L. Bramschreiber
Richard A. Brassington
Cary J. Breese
Andrew J. Bren
Willard E. Brown
Bruce D. Browning
Julie Burdick
Alan Burns
Elise S. Burns
Michael L. Burruss
Pamela A. Burt
Jason B. Bushey
Matthew R. Carrier
John J. Carroll
Milissa D. Carter
Anne-Marie Castilloux
Harvey C. C. Chan
Simon Hei-Yin Chan
Valerie C. Chan
Sabine Chapus
Chun-Nan Chen
Cindy X. Chen
Ja-Lin Chen
Lisa C. Chen
Peggy Cheng
Richard M. Chiarini
Li-Chen Chou
Chin-mei Y. Chueh
Jeffrey A. Clements
Lynn D. Coleman
Thomas P. Collins
Greg E. Conklin
Christopher W. Cooney
David E. Corsi
Renee Couture
Catherine Cresswell
Bryan J. Curley
Stephen T. Custis
Vick Dannon
Mary Katherine T. Dardis
Willie L. Davis
Nicholas J. De Palma
Harin A. De Silva
Nancy K. DeGelleke
Michael B. Delvaux
Emily Y. Deng
Alain P. DesChatelets
Jonathan M. Deutsch
Mary Jane B. Donnelly
Kevin F. Downs
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Cindy L. Dube
Martin Dubeau
Patrice Duchaine
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Ruchira Dutta
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Brian M. Fernandes
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James M. Filmore
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Keith E. Floman
William J. Fogarty
Dennis Anthony Fong
Sean P. Forbes
Marie-Josee Forcier
Sarah J. Fore
Michelle M. Forst
Martin Fortier
Nathalie Fortin
Patrice Fortin
Robert C. Fox
Joseph K. Fung
James M. Gabriel
Nathalie Gagnon
Sherri L. Galles
Dennis Gan
Marie-Elaine Gaudreault
Ellen M. Gavin
Glenn J. Gazdik
Michael L. George
John J. Gericke, III
Wendi J. Giachino
Dawn M. Giglio
James B. Gilbert
James W. Gillette
Theresa Giunta
Nik Godon
Eric P. Goetsch
Moshe D. Goldberg
Olga Golod
Christian N. Goodman
John P. Gots
Leslie B. Graham
Daniel C. Greer
Michael L. Greer
Catherine E. Griffin
Jennifer T. Grimes
Leslie E. Gross
Josephine A. Gurreri
Greg M. Haft
William Woojae Hahn
Barry R. Haines
Constance B. Hall
Thomas Hamm
Alex A. Hammett
Gregory Hansen
Kurt D. Hanson
Kevin A. Harris
Jodi J. Healy
Dale A. Helzerman
David E. Heppen
Linda M. Hewitt
Cynthia J. Heyer
Jay T. Hieb
Luke D. Hodge
Shawn C. Howell
Gordon R. Hugh
Daniel C. F. Hui
Jamison J. Ihrke
Mario A. Imbarrato
Susan E. Innes
Bryan B. Jaicks
Eric Janecek
Gregory O. Jaynes
Neal O. Jettpace
Donna G. Jockers
Kathleen M. Johnson
Paul J. Johnson
William Rosco Jones
Eunsook Joo
Anthony Ernest Jung
Daniel R. Kamen
Jong-Ming Kan
Robert C. Kane
Joseph M. Kaner
Rishi Kapur
Chad C. Karls
Kimberly S. H. Kaune
Kathryn E. Keehn
Shannan R. Keet Corey
Mary C. Kellstrom
Linda M. Kiene
David N. Kightlinger
Kari L. Killing
Linda Kong
Kimberly A. Kracht
John J. Kraska, III
Richard S. Krivo
Sarah Krutov
Robert W. Kuchler
Micheline M. Lafond
Mai B. Lam
David Eric Lamoureux
Robert G. Landau
Timothy J. Landick
Debra K. Larcher
Isam Laroui
Valerie Lavoie
Yin Lawn
Donald G. Lawrence
Eric T. Le
Bradley R. Leblond
Borwen Lee
Henry T. Lee
Joan K. Lee
Todd W. Lehmann
Neal M. Leibowitz
Gerald E. Lenis
W. Scott Lennox
Brendan M. Leonard
Jing Li
Lei Li

Xiaoyin Li
Yu-Ching Lin
Jamison W. Lindsey
Hwa-Lin Liu
Xisuo Liu
Deborah Livingston
Brian D. Loewen
Peter R. Lopatka
Paul R. Lorentz
Yongchil Ly
Jaclyn B. Maher
Alexander P. Maizys
James W. Malin
Lisa K. Manderson
William J. Manternach
Michelle D. Marks
Jason A. Martin
Luc Martin
Annie Massicotte
Stanislav D. Maydan
Arkadiy B. Maydanchik
Randall Mays
Terry J. McFadden
Patricia McGahan
Mark E. McGuire
Michele L. McKay
Amy S. McLaughlin
Jennifer A. Medvec
John D. Meerschaert
Raveendran Menon
Etienne Mercier
Richard E. Meuret
Cory S. Michel
Paul W. Mills
Dmitriy Mindlin
Paul D. Miotke
Michael J. Miraglia
Ajit D. Mistry
David Molyneux
Celso M. Moreira
Alan E. Morris
Jennifer A. Moseley
Janice C. Moskowitz
Roosevelt C. Mosley
Robert J. Moss
Brian J. Mullen
Syed A. Murtaza
Tawnia L. Newton
Khanh K. Nguyen
Tu T. Nguyen
Wendy A. Nichols
Serge Yanic Nana Njike
Douglas K. Noble
Chad W. Noehren
Miodrag Novakovic
Martin J. O'Connell
Karl K. Oman
Leo M. Orth, Jr.
Maria T. Palandra
James A. Partridge
Kamlesh M. Patel
Prashant Patel
Carole K. Payne
Harry T. Pearce
James A. Pederson
Sylvain Perrier
John S. Peters
Wesley R. Peterson
Thomas L. Poklen, Jr.
Deborah J. Pomerantz
Josee Pomerleau
Stephen L. Pontecorvo
Kathy A. Poppe
Dale S. Porfilio
Scott F. Porter
Scott M. Priebe
Gariguin E. Prilepski
Warren T. Printz
Troy J. Pritchett
Lisa Procaccitto
Anthony E. Ptasznik
Penelope A. Quiram
Brentley J. Radeloff
Ricardo A. Ramotar
Amir Rasheed
Beth A. Rasmussen
John F. Rathgeber
Teresa M. Reis
Jennifer L. Reisig
Natalie J. Rekittke
Janice L. Rexroth
Andrew S. Ribaudo
Jason H. Rickard
David C. Riek
Jean-Yves Rioux
Melissa K. Ripper
Karen L. Rivara
David Roberge
Scott A. Robinson
Mark J. Rodts
Sharon G. Rothwachs
Brian P. Rucci
Joanne E. Russell
Anna May P. Sadler
Sharon R. Saleh
Juliet R. Sandrowicz
Jason R. Santos
Frances G. Sarrel
Barbara A. Satsky
David M. Savage
Daniel V. Scala
Steven M. Schatt
Thomas C. Schultz
Terri L. Schwomeyer
Glenn C. Scott
Peter A. Scourtis
Michele Segreti

Ronald G. Sevold
Amresh Mansukhlal Shah
Linda R. Shahmoon
Mohammad A. Sharif
Jonathan A. Shelon
Meyer Shields
Glenn D. Shippey
Maria Shlyankevich
Bret C. Shroyer
Kanhiya Lal Shukla
Brent A. Simmons
Gregory A. Simmons
Brian A. Simpson
Donna M. Sivigny
Allison M. Skolnick
Gregory M. Smith
Scott G. Sobel
Charles J. Song
Caroline B. Spain
Kristen L. Sparks
Daniel J. Spillane
Dawn L. Stamets
Ana M. Stangl
Barry P. Steinberg
Carol A. Stevenson
Elizabeth A. Sullivan
Randall A. Swanson
David J. Tenembaum
Steve D. Tews
Harlan H. Thacker
Laura L. Thorne
Jennifer L. Throm
David A. Tobin
Michael G. Townsend
Quynh-Le Tran
Stephanie J. Traskos
Jeffrey S. Trichon
Andrea E. Trimble
Herman T. Tse
Brian D. Ulery
Dennis R. Unver
Joel A. Vaag
Steven J. Vercellini
Vratislav Vodrazka
Josephine M. Waldman
Donald M. Walker
Jon M. Wander
Helen R. Wargel
Chang-Hsien Wei
Mary A. Weiler
Min-Ming Wen
Shari A. Westerfield
Dean A. Westpfahl
Joel D. Whitcraft
Vanessa C. Whitlam-Jones
Joseph R. Wille
Kendall P. Williams
Victor S. F. Wong
Ruth Ann Woodley
Haichuan Wu
Pearson K. Wu
Walter R. Wulliger
Armand M. Yambao
Hailiang Yang
Michael Yarmish
Michael G. Young
In Sung Yuh
Paula S. Ziegelbein

Part 5A

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Michael W. Barlow
Paul C. Barone
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Anna Marie Beaton
Jennifer L. Blackmore
Stephen D. Blaesing
Daniel R. Boerboom
Kimberly Bowen
Kevin M. Brady
Cary J. Breese
Karen A. Brostrom
Robert Lindsay Brown
James D. Buntine
Kevin D. Burns
Sandra J. Callanan
Sharon A. Carroll
Todd D. Cheema
Gary C. K. Cheung
Stephen D. Clapp
Jeffrey J. Clinch
Peter J. Cooper
Kevin A. Cormier
Hall D. Crowder
Michael J. Curcio
Charles A. Dal Corobbo
Sheri L. Daubenmier
Raymond Demers
David A. DeNicola
Thomas J. Dwyer
Brian A. Evans
Sylvain Fauchon
Brian M. Fernandes
Tracy M. Fleck
Mary E. Fleischli
Shina N. Fritz
Serge Gagne

Kathy H. Garrigan
Christopher H. Geering
Barry A. Gertschen
Jie Gong
Allen J. Gould
Mari L. Gray
Karen L. Greene
David T. Groff
Alex A. Hammett
Joel D. Hanson
Michelle L. Harnick
Bryan Hartigan
Lisa M. Hawrylak
Daniel L. Hogan
Susan E. Innes
David R. James
Christopher R. Jarvis
Christian Jobidon
Burt D. Jones
Alexander Kastan
Robert B. Katzman
Claudine H. Kazanecki
Timothy P. Kenefick
Michael B. Kessler
Ruta V. Kher
Deborah M. King
James F. King
Omar A. Kitchlew
David Kodama
Kimberly A. Kracht
Gary R. Kratzer
Jean-Francois Larochelle
Yin Lawn
Emily C. Lawrance
Guy Lecours
Kevin A. Lee
Bradley H. Lemons
Steven J. Lesser
John N. Levy
Sally M. Levy
Michael Leybov
Janet G. Lindstrom
William F. Loyd
Vahan A. Mahdasian
Alexander P. Maizys
Timothy C. McAuliffe
Claudia A. McCarthy
Patrice McCaulley
Douglas W. McKenzie
Shawn Allan McKenzie
Scott A. McPhee
Michelle L. Merkel
Claus S. Metzner
Randy J. Murray
Vinay Nadkarni
Ronald T. Nelson
William F. Nicodemus
Michael D. Nielsen
Kathleen C. Odomirok
Christopher E. Olson
Charles Pare
Bhikhabhai C. Patel
Julie Perron
Andrea L. Phillips
Genevieve Pineau
David J. Pochettino
Mitchell S. Pollack
John L. Quigley
William D. Rader, Jr.
Kimberly E. Ragland
James J. Rehbit
Ellen K. Rein
James C. Sandor
Annmarie Schuster
Andrea W. Sherry
Theodore J. Shively
Laura E. Siegel
Jason R. Smith
Robert K. Smith
Daniel J. Spillane
Beth A. Stahelin
Christine L. Steele-Koffke
C. Steven Swalley
Michael J. Tempesta
Glenda O. Tennis
Mark L. Thompson
Philippe Trahan
Huguette Tran
Stephanie J. Traskos
Janet K. Vollmert
Benjamin A. Walden
Christopher B. Wei
Miroslaw Wieczorek
Joel F. Witt
Simon Wong
Virginia R. Young
Fengming Zhang

Part 5B

Rachelle R. Ambrose
Timothy W. Atwill
Bassam B. Barazi
Emmanuil Bardis
Nancy Barry
Elizabeth F. Bassett
David M. Baxter
Michael J. Bednarick
Sheila J. Bertelsen
Frank J. Bilotti
Kevin M. Bingham
Kimberly Bowen

Rebecca S. Bredehoeft
Jeffrey H. Brooks
Elise S. Burns
Aleksandr A. Bushel
John G. Butler
Donna L. Callison
Tamela Canora
Sharon L. Chapman
Gary C. K. Cheung
Heng Seong Cho
Stephen D. Clapp
Susan M. Cleaver
Jeffrey J. Clinch
Margaret E. Conroy
Kenneth S. Dailey
John E. Daniel
John D. Deacon
Donna K. DiBiaso
Christopher S. Downey
Wayne W. Edwards
Dawn E. Elzinga
Vicki A. Fendley
Junko K. Ferguson
Benedick Fidlow
Tracy M. Fleck
Christian Fournier
Walter H. Fransen
Shina N. Fritz
James M. Gallagher
Isabelle Gaumond
Michael H. Gay
Moshe D. Goldberg
Jay C. Gotelaere
Mari L. Gray
Paul E. Green
David J. Gronski
John A. Hagglund
Jodi J. Healy
Rhonda R. Hellman
Jay T. Hieb
Christopher T. Hochhausler
Daniel L. Hogan
Todd H. Hoivik
Dave R. Holmes
Tina T. Huynh
Jean-Claude J. Jacob
Walter L. Jedziniak
Christian Jobidon
Brian E. Johnson
William Rosco Jones
Philip A. Kane, IV
Chad C. Karls
Mary C. Kellstrom
Michael B. Kessler
David N. Kightlinger
Deborah M. King
Brant L. Kizer
Robert A. Kranz
John J. Kraska, III
Richard S. Krivo
Dar-Jen D. Kuo
Douglas H. Lacoss
Salvatore T. LaDuca
Jean-Francois Larochelle
Michael L. Laufer
Khanh M. Le
Guy Lecours
Bradley H. Lemons
Steven J. Lesser
Michael Leybov
Janet G. Lindstrom
Christina Link
Yih-Jiuan B. Lu
Barbara D. Majcherek
Dina M. Maloney
Victor Mata
James P. Mathews
Patrice McCaulley
Smith W. McKee
Douglas W. McKenzie
Shawn Allan McKenzie
Raveendran Menon
Richard E. Meuret
Susan A. Minnich
Janice C. Moskowitz
John V. Mulhall
Ronald T. Nelson
John E. Noble
Michael A. Nori
Corine Nutting
Steven B. Oakley
Lowell D. Olson
Milary N. Olson
Alan M. Pakula
Julie Perron
Jeffrey J. Pfluger
Frank P. Pittner
David J. Pochettino
Mitchell S. Pollack
Kathy A. Poppe
Ellen K. Rein
Brad E. Rigotty
Denise F. Rosen
Janelle P. Rotondi
Joanne E. Russell
Charles J. Ryherd
Glenn R. Scharf
Christine E. Schindler
Bradley J. Schroer
Timothy D. Schutz
Kelli D. Shepard-El
James S. Shoenfelt
Bret C. Shroyer
Jeffrey T. Snook
Curt A. Stewart

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Ming Tang
Varsha A. Tantri
David J. Tenembaum
Jo D. Thiel
Mark L. Thompson
Laura L. Thorne
W. Mont Timmins
Philippe Trahan
Jeffrey S. Trichon
Beth S. Tropp
Bonnie J. Trueman
Timothy J. Ungashick
Mary H. Vale
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Lynne K. Wehmueller
Christopher B. Wei
Dean A. Westpfahl
Trevar K. Withers
Joel F. Witt
Amy M. Wixon
Brandon L. Wolf

Part 7

Rimma Abian
K. Athula P. Alwis
Steven D. Armstrong
Martin S. Arnold
Richard J. Babel
Karen L. Barrett
Claudia M. Barry
David B. Bassi
Brian K. Bell
Bruce J. Bergeron
Steven L. Berman
Corey J. Bilot
Lisa A. Bjorkman
Carol A. Blomstrom
Raju Bohra
John T. Bonsignore
Lee M. Bowron
Douglas J. Bradac
Betsy A. Branagan
Michael D. Brannon
David J. Braza
James L. Bresnahan
Margaret A. Brinkmann
Lisa J. Brubaker
Marian M. Burkart
Elliot R. Burn
Michelle L. Busch
Tara E. Bush
J'ne E. Byckovski
Sandra L. Cagley
Pamela J. Cagney
Jeanne L. Carey
Douglas A. Carlone
Jill C. Cecchini
Heather L. Chalfant
Jean-Francois Chalifoux
Hong Chen
Christopher J. Claus
Brian C. Cornelison
Christopher G. Cunniff
M. Elizabeth Cunningham
Angela M. Cuonzo
Malcolm H. Curry
John T. Devereux
Sean R. Devlin
Behram M. Dinshaw
Patricia J. Donnelly
William A. Dowell
Kimberly J. Drennan
Barry P. Drobes
Pierre Drolet
Stephen C. Dugan
Kevin M. Dyke
Jeffrey Eddinger
Anthony D. Edwards
S. Anders Ericson
James G. Evans
Joseph G. Evleth
Alexander Fernandez, Jr.
Steven J. Finkelstein
William P. Fisanick
Daniel J. Flick
Andre F. Fontaine
Kevin J. Fried
Richard A. Fuller
Gary J. Ganci
Susan T. Garnier
Abbe B. Gasparro
Micah R. Gentile
Eric J. Gesick
Thomas P. Gibbons
Stewart H. Gleason
John T. Gleba
Annette J. Goodreau
Chris D. Goodwin
Mark A. Gorham
John E. Green
Steven A. Green
Daniel E. Greer
Charles R. Grilliot
Brian D. Haney
David S. Harris
Adam D. Hartman
Scott J. Hartzler

Fritz J. Heirich
Daniel F. Henke
Betty-Jo Hill
John V. Hinton
Jason N. Hoffman
Eric J. Hornick
Brett Horoff
Marie-Josee Huard
John F. Huddleston
David D. Hudson
Li Hwan Hwang
Brian L. Ingle
C. M. Ali Ishaq
Paul Ivanovskis
Randall A. Jacobson
Suzanne G. James
Brian J. Janitschke
Joseph W. Janzen
Patrice Jean
Daniel K. Johnson
Michael S. Johnson
Philip A. Kane, IV
Mary Jo Kannon
Ira M. Kaplan
Gail E. Kappeler
Hsien-Ming K. Keh
Lowell J. Keith
Thomas P. Kenia
Jean-Raymond Kingsley
Therese A. Klodnicki
Brian S. Krick
Salvatore T. LaDuca
Jocelyn Laflamme
Marc LaPalme
Gregory D. Larcher
Ramona C. Lee
Thomas C. Lee
Isabelle Lemay
Charles R. Lenz
Brian P. LePage
Jennifer M. Levine
Edward A. Lindsay
Richard B. Lord
Laura J. Lothschutz
Robert A. MacKenzie
James M. MacPhee
Cornwell H. Mah
Anthony L. Manzitto
Leslie A. Martin
Scott A. Martin
Tracey L. Matthew
Camley A. Mazloom
Deborah L. McCrary
Michael K. McCutchan
Kelly S. McKeethan
Lynne S. McWithey
Jeffrey A. Mehalic
James R. Merz
Stephanie J. Michalik
Anne Hoban Moore
Kevin T. Murphy
Kari S. Nelson
Hiep T. Nguyen
Mindy Y. Nguyen
James L. Nutting
Mihaela L. O'Leary
Richard D. Olsen
Michael G. Owen
Dmitry Papush
James A. Partridge
Thomas Passante
Nicholas H. Pastor
Abha B. Patel
Claude Penland
William Peter
Robert E. Quane, III
Patrice Raby
Brenda L. Reddick
Scott Reynolds
Meredith G. Richardson
Dennis L. Rivenburgh, Jr.
Jeremy Roberts
John W. Rollins
Peter A. Royek
Jason L. Russ
Thomas A. Ryan
Rajesh V. Sahasrabuddhe
Elizabeth A. Sander
Manalur S. Sandilya
Christina L. Scannell
Marilyn E. Schafer
Christine E. Schindler
Michael C. Schmitz
Michael J. Scholl
Craig J. Scukas
Terry M. Seckel
Raleigh R. Skaggs, Jr.
L. Kevin Smith
M. Kate Smith
Lori A. Snyder
John B. Sopkowicz
Jay M. South
Linda M. Sowter
Michael J. Sperduto
Scott D. Spurgat
Nathan R. Stein
Scott T. Stelljes
Kevin D. Strous
Thomas Struppeck
Joy Y. Takahashi
Yuan Yew Tan
David M. Teme
Diane R. Thurston
Son T. Tu

Marie-Claire Turcotte
Eric Vaith
Cynthia L. Vidal
Jennifer S. Vincent
Edward H. Wagner
Robert J. Wallace
Robert J. Walling, III
Isabelle T. Wang
Jeffrey D. White
Steven B. White
Thomas J. White
Elizabeth R. Wiesner
Michael J. Williams
Kirby W. Wisian
David S. Wolfe
Floyd M. Yager
Richard L. Zarnik

Part 9

Shawna S. Ackerman
Mark A. Addiego
Elise M. Ahearn
Craig A. Allen
Timothy P. Aman
Larry D. Anderson
Michael E. Angelina
Mohammed Q. Ashab
William M. Atkinson
Lewis V. Augustine
Nathan J. Babcock
Philip A. Baum
Daniel D. Blau
Ann M. Bok
Tobias E. Bradley
Dominique E. Brassier
Tracy L. Brooks-Szegda
Peter V. Burchett
Mark E. Burgess
Martin Carrier
Maureen A. Cavanaugh
Francis D. Cerasoli
Dennis K. Chan
Bryan C. Christman
Darrel W. Chvoy
Gary T. Ciardiello
Laura R. Claude
Frank S. Conde
Kirsten J. Costello
Brian K. Cox
Timothy J. Cremin
Catherine Cresswell
Joyce A. Dallessio
David J. Darby
Smitesh Dave
Karen L. Davies
Marie-Julie Demers
Kurt S. Dickmann
John P. Doucette
Robert G. Downs
William E. Emmons
Martin A. Epstein
Dianne L. Estrada
Michael A. Falcone
David I. Frank
Kirsten A. Frantom
James E. Gant
Julie Terese Gilbert
Nicholas P. Giuntini
Marc C. Grandisson
Bradley A. Granger
Terry D. Gusler
Paul James Hancock
William D. Hansen
Bradley A. Hanson
David L. Homer
Wayne Hommes
Robert J. Hopper
Sandra L. Hunt
Paul R. Hussian
Fong-Yee J. Jao
Hou-wen Jeng
Mark R. Johnson
Edwin G. Jordan
Janet S. Katz
Mark J. Kaufman
Brian Danforth Kemp
Craig W. Kliethermes
Terry A. Knull
Louis K. Korth
Jason A. Kundrot
Howard A. Kunst
Kenneth Allen Kurtzman
Bertrand J. LaChance
Blair W. Laddusaw
Mathieu Lamy
John P. Lebens
Lewis Y. Lee
Thomas L. Lee
Scott J. Lefkowitz
Steve E. Lehecka
Paul B. LeStourgeon
Kenneth A. Levine
Maria Mahon
Barbara S. Mahoney
Stephen N. Maratea
Richard J. Marcks
Peter R. Martin
Suzanne Martin
Michael Boyd Masters
Robert F. Maton
Richard T. McDonald

Stephen J. McGee
Kathleen A. McMonigle
Robert F. Megens
Daniel J. Merk
Stephen V. Merkey
Stephen J. Mildenhall
Brett E. Miller
Camille Diane Minogue
Madan L. Mittal
Robert J. Moser
Kimberly J. Mullins
David Y. Na
Victor A. Njakou
Peter M. Nonken
Melinda H. Oosten
Todd F. Orrett
Joseph M. Palmer
Edward F. Peck
Wende A. Pemrick
Anne Marlene Petrides
Mark W. Phillips
Joseph W. Pitts
Daniel A. Powell
Mark Priven
Arlie J. Proctor
Eduard J. Pulkstenis
Mark S. Quigley
Donald A. Riggins
Andrew T. Rippert
Brad Michael Ritter
Tracey S. Ritter
Douglas S. Rivenburgh
Jay Andrew Rosen
James B. Rowland
David A. Russell
Kevin L. Russell
Sean W. Russell
Melodee J. Saunders
Letitia M. Saylor
Matt J. Schmitt
Jeffery J. Scott
Michelle G. Sheng
Jeffrey Parviz Shirazi
Douglas W. Stang
Russell Steingiser
John A. Stenmark
Brian M. Stoll
Ilene G. Stone
Collin J. Suttie
Eileen M. Sweeney
Christopher Tait
Yuan-Yuan Tang
Georgia A. Theocharides
Glenn A. Tobleman
Thomas C. Toce
Cynthia J. Traczyk
Patrick N. Tures
Robert C. Turner, Jr.
John V. Van de Water
Jeffrey A. Van Kley
Mark D. van Zanden
Trent R. Vaughn
W. Olivia Wacker
Lisa Marie Walsh
Geoffrey T. Werner
Wyndel S. White
Peter G. Wick
Gayle L. Wiener
Tad E. Womack
Claude D. Yoder
Edward J. Yorty
Benny S. Yuen
Doug Alan Zearfoss
Guangjian Zhu
Joshua A. Zirin

NEW FELLOWS ADMITTED MAY, 1994: Fifteen of the seventeen new Fellows admitted in Boston are shown with CAS President Irene K. Bass.

NEW ASSOCIATES ADMITTED MAY, 1994: Thirty-eight of the 149 new Associates admitted in Boston are shown with CAS President Irene K. Bass.

NEW ASSOCIATES ADMITTED MAY, 1994: Forty-seven of the 149 new Associates admitted in Boston are shown with CAS President Irene K. Bass.

NEW ASSOCIATES ADMITTED MAY, 1994: Thirty-seven of the 149 new Associates admitted in Boston are shown with CAS President Irene K. Bass.

NEW FELLOWS ADMITTED NOVEMBER, 1994: (Front row, from left) Jessalyn Chang, Scott K. Charbonneau, M. Sean McPadden, Deborah G. Horovitz, Mark L. Brannon, CAS President Irene K. Bass, Michelle M. Morrow, John P. Mentz, Katherine A. Mann, Betsy L. Blue. Second row: Michel Laurin, William G. Main, Michael D. Larson, Edgar W. Davenport, Jonathan M. Harbus, Yves Francouer, Kelly L. Moore, Maribeth Ebert, Robin N. Murray, David A. Murray. Third row: John W. McCutcheon Jr., Michael A. Coca, Stacy L. Mina, Donna L. Glenn, Gregory L. Cote, Nancy E. Kot, Janet L. Chaffee, Lisa A. Hays. Fourth row: Anthony J. Burke, Christopher Lattin, John M. Kulik, Farrokh Guiahi, Daniel B. Finn, John A. Beckman, Russell Frank, Jennifer L. Biggs. Last row: Jeffrey L. Dollinger, Stephen J. Meyer, Todd R. Bault, Blair E. Manktelow, France LeBlanc, Michael T. Curtis, Robert J. Meyer, Michael L. DeMattei, Paul Allen Mestelle, Kim B. Garland, James B. McCreesh.

NEW FELLOWS ADMITTED NOVEMBER, 1994: (Front row, from left) Ralph T. Zimmer, Gregory R. Scruton, Andre Perez, Donna J. Reed, CAS President Irene K. Bass, Scott P. Weinstein, Chandrakant C. Patel, Marian R.Piet, Laura A. Olszewski. Second row: Daniel C. Pickens, Elizabeth M. Riczko, Rudy A. Palenik, Barbara A. Stahley, Brian D. Poole, Barbara H. Thurston, Jennifer J. Palo, Debra L. Werland. Third row: James Roseph Romanowski, L. Nicholas Welstman Jr., Paul J. Struzzieri, Elise C. Liebers, Timothy A. Paddock, Charles C. Pearl Jr. Fourth row: William L. Oostendorp, Richard D. Thomas, Kevin D. Rosenstein, Michael Toledano, Dale G. Vincent Jr., Stephen R. Noonan. Last row: Thomas N. Stanford, David M. Shepherd, Derrick D. Shannon.

NEW FELLOWS ADMITTED IN NOVEMBER, 1994, WHO ARE NOT PICTURED: Matthew G. Fay, Linda M. Goss, James W. Larkin, Daniel J. Minka, Donald F. Mango, Marvin Pestcoe, Charles F. Toney II, Marcia C. Williams, William M. Wilt.

NEW ASSOCIATES ADMITTED NOVEMBER, 1994: (Front row, from left) Jeanne Lee Ying, Anne C. Meysenburg, Anne Marlene Petrides, Angela Kaye Sparks, Andrea C. Bautista, CAS President Irene K. Bass, Camille Diane Minogue, Rebecca A. Kennedy, Kirsten J. Costello, Lori Michelle Bradley. Second row: Smitesh Dave, Kevin J. Brazee, Jeffrey Parvis Shirazi, Klayton N. Southwood, Turhan E. Murguz, Yves Raymond, Kathleen Mary Quinn, Kristie Irene Carpine-Taber, Janice L. Marks, Linda F. Ward, Wyndel S. White, Ginda Kaplan Fisher, Victor Unson Revilla, John P. Lebens. Third row: Matt J. Schmitt, Joan M. Klucarich, Elizabeth E.L. Hansen, Michael David Price, Christine R. Ross, Karen L. Queen, Gregg Evans, Charles Leo Sizer, Kerry S. Shubat, Julie Terese Gilbert, Brian A. Clancy. Fourth row: Nicholas P. Giuntini, Brian Danforth Kemp, Marc S. Hall, Brian James Melas, Charles V. Faerber, Laura B. Deterding, Matthew G. Lange, Brad Michael Ritter, Anthony G. Martella Jr., Margaret Wendy Germani, James M. Bartie. Fifth row: Kenneth Allen Kurtzman, Larry D. Anderson, Bruce Daniel Fell, Nathan Ira Shpritz, David B. Hostetter, Paul Henry Klauke. Sixth row: John R. Pedrick, Marc Freeman Oberholtzer, Carl J. Sornson, Aaron West Newhoff, Peter R. Martin. Seventh row: Edward M. Kuss, P. Claude Lefebvre, William A. Guffey, Bradley J. Kiscaden, Barry Luke Bablin, Jay A. Rosen, Russell J. Buckley, Matthew C. Mosher, Michael Boyd Masters. Last row: Gary P. Maile, Doug Alan Zearfoss, James Edward Monaghan, Jonathan B. Hayes. **NEW ASSOCIATES ADMITTED IN NOVEMBER, 1994, WHO ARE NOT PICTURED:** Shawna S. Ackerman, Wayde Alfred Daigneault, Thomas V. Daley, Robert F. Flannery, Eleni Kourou, James C. Whisenant, William Robert Wilkins.

OBITUARIES

Clarence Ray Atwood
Elgin R. Batho
Harold J. Ginsburgh
Raymond S. Lis, Jr.
Henry F. Rood

CLARENCE RAY ATWOOD

1937-1994

Clarence Ray Atwood died December 19, 1994, from injuries he suffered in a traffic accident which occurred two months prior.

Mr. Atwood became an Associate of the Casualty Actuarial Society in 1968 and a Fellow in 1971. He attended the University of Nebraska, then later obtained a degree from a university in Denver, Colorado.

Though Atwood was born in Nebraska on August 12, 1937, he considered Los Angeles and San Diego, California to be his hometowns. He began an actuarial consulting firm, Atwood & Co., Consulting Actuaries, in Rancho Bernardo, California, during the late 1970's. Atwood served as Founder, Chairman of the Board, and President of the consulting firm until his death.

Atwood served two years on the CAS Education and Examination Committee, and two years on the CAS Committee on Financial Reporting.

ELGIN R. BATHO

1904-1994

Elgin R. Batho died July 24, 1994 from a heart attack at the age of 89 in Cape Coral, Florida.

Mr. Batho was a resident of Cape Coral since retiring from Berkshire Life Insurance Company in Pittsfield, Massachusetts, where he served as Vice President and Actuary until October 1969.

He became a Fellow of the Casualty Actuarial Society in 1931, and served as chairman of the Boston Actuaries Club and the Actuaries Club of Hartford.

He was born in Winnipeg, Manitoba, and earned bachelor's and master's degrees from the University of Manitoba, specializing in actuarial science. After graduation, he worked in the actuarial department of the Great West Life Assurance Company in Winnipeg. He moved to the Bankers Life Company in Des Moines, Iowa; then back to Canada as Assistant Actuary for the Equitable Life Insurance Company of Canada in Waterloo, Ontario. In 1946, he left Canada to join Berkshire Life.

Batho joined Berkshire Life in 1946 as Assistant Actuary. He was promoted to Associate Actuary the next year. In 1956, Batho was promoted to Assistant Vice President and Actuary. In 1958, he was named Vice President and Actuary, serving in that position until retiring in 1969.

He is survived by a son, Lester Batho of Richmond, Massachusetts; two daughters, Mrs. Charles Thompson (Marion) of Lenox, Massachusetts, and Mrs. John E. Pryzby (Phyllis) of Pittsfield, Massachusetts; six grandchildren; and four great-grandchildren. Batho was predeceased by his first wife, the former Olive H. Rees, and his second wife, the former Monna S. Asby.

HAROLD J. GINSBURGH

1899-1994

Former CAS President Harold J. Ginsburgh died February 2, 1994, in Laguna Hills, California after a long illness.

Mr. Ginsburgh was born September 21, 1899, in Rochester, New York, and graduated from Harvard University in 1920. Ginsburgh, who served in the United States Army, became an Associate of the Casualty Actuarial Society in 1922 and a Fellow in 1924.

Ginsburgh was well-known as a pioneer in the development of the automobile insurance policy. In the early 1920's, when the first automobile policies were being developed, Ginsburgh's work strongly influenced the shape of those policies. By the early 1940's, Ginsburgh's knowledge and expertise had established him as a highly-respected individual in actuarial circles.

In 1922, Ginsburgh was employed with Aetna Life Insurance Company in Hartford, Connecticut. He was named Vice President at Aetna in 1943. In 1954, he became a Senior Vice President of American Mutual Liability Insurance Company in Boston, Massachusetts, where he remained until retiring in 1961.

Of his many contributions to actuarial science, what Ginsburgh enjoyed the most about his career was developing and training young professional actuaries.

Ginsburgh held many leadership roles for the Society from 1925 to 1956. He was elected to the CAS Council three times, served as President of the Society from 1943 to 1944, Vice President from 1940 to 1941, and participated in panels and workshops for many CAS meetings and seminars. Among his many contributions to CAS literature were "Rate Regulation and the Casualty Actuary," which appeared in the *PCAS*, Volume XXXVIII (1952), and "The Retrospective Rating Plan for Workmen's Compensation Risks," which appeared in the *PCAS*, Volume XV (1939).

After retiring from American Mutual Liability, Ginsburgh served as a consultant for the Commonwealth of Massachussetts, appearing as an expert witness and advisor.

Ginsburgh is survived by wife, Betty; a son, Allen Ginsburgh, of Illinois; a daughter, Elgie Ginsburgh, of Massachusetts; five grandchildren; and five great-grandchildren.

RAYMOND S. LIS, JR.

1948-1994

Raymond S. Lis, Jr. died on March 28, 1994, after a long illness.

Mr. Lis was born November 24, 1948, in Willimantic, Connecticut, and graduated in 1970 from the University of Connecticut with a degree in mathematics.

Lis became an Associate of the Society in 1973, and a Member of the American Academy of Actuaries in 1977. He began his career in 1971 at The Travelers Insurance Company in Hartford, Connecticut, where he was promoted to Assistant Director in the casualty division. He remained there until retiring in 1993. Lis found the actuarial profession rewarding and intellectually challenging; he enjoyed being in management at Travelers and working with people.

At the time of his death, Lis was working toward completing Parts 8 and 9 of the CAS examinations.

Lis is survived by his mother, Florence Lis, of Willimantic, Connecticut; a son, Randall Lis, of Boulder, Colorado; and a daughter, Jennifer Lis, of Brooklyn, New York.

HENRY F. ROOD

1907-1994

Henry F. Rood, age 87, died June 11, 1994, in Fort Wayne, Indiana.

Mr. Rood became an Associate of the Casualty Actuarial Society in 1962. He was a member of the Society of Actuaries and served as President of that society.

Rood retired in 1971 as Chairman of the Board and President of Lincoln National Corporation after 40 years of service to that company.

He was a native of Port Chester, New York, and joined Lincoln National as an actuary in 1931. He was also a director of Lincoln National Bank and Trust Company and Lincoln Financial Corporation. He was an advisor for life insurance taxes to the United States Department of Treasury from 1957-1959, and a member of the board of advisers for Purdue University from 1975-1994.

Rood is survived by his wife, Ruth; two sons, Win and Douglas; and one grandchild.

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