REINSURER RISK LOADS FROM MARGINAL SURPLUS REQUIREMENTS

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Abstract

The return on the marginal surplus committed to support the variability of a proposed reinsurance contract is used to derive an appropriate risk load for reinsurers. The risk load is a linear combination of the standard deviation and variance of the return on the contract, and depends upon the covariance of the contract with the existing book, the standard deviation of the contract, the standard deviation of the existing book, the acceptable probability of "ruin" of the company, and the yield required on marginal surplus (the additional surplus required for this contract). A new term is defined, the reluctance to write risk, and relatively simple formulas result for it and the premium, which satisfy intuitive reasonableness criteria. Extensions to include expenses and an existing "bank" are discussed, and application is made to the interesting case of excess layer pricing. Empirical comparison suggests that the market pricing is consistent with this approach.

1. MICROECONOMICS

The underlying economic point of view taken is that of a reinsurer considering a new contract. The reinsurer has committed surplus to support the variability of his existing book; the new contract will require additional surplus to support its variability.[1] The return on this marginal surplus required must be at least as much as is available in the capital markets; otherwise the reinsurer might just as well invest directly. It is assumed, with Brubaker [1], that the company expresses the part of its surplus required to support the variability of a book of business with

[1] The remarks here apply equally well to insurance contracts, but the ratemaking procedures for primary insurers typically do not allow explicit risk loads. An implicit load is present from whatever provisions are present for profit, which is economically the reward for bearing risk.

expected return R and standard deviation S as[2]

$$V = zS - R, \tag{1.1}$$

where z is a distribution percentage point corresponding to the acceptable probability that the actual result will require even more surplus than allocated.[3] For example, if the distribution is Normal, then a z of 3.1 is a 1/1000 probability, and an amount of surplus given as above will cover the actual losses 999 years out of 1000 years, on average. The choice of the appropriate value of z is an upper management decision, reflecting the overall conservatism of the company and explicit and implicit regulatory requirements.

Consider a potential new contract with an expected return (premium less losses and expenses) r and standard deviation σ, and indicate the resulting new book values with a prime ($'$). The new values are given by

$$R' = R + r, \tag{1.2}$$

and

$$V' = zS' - R'. \tag{1.3}$$

It is assumed that the nature of the total book distribution has not changed significantly, so that the same value of z is appropriate. The marginal surplus required by the contract is then given as

$$V' - V = z(S' - S) - r. \tag{1.4}$$

Now, the return from the contract and the amount of the marginal surplus required to support the contract imply a yield rate y on this surplus. The value of y must be (at least) equal to the rate in the capital markets, otherwise management might as well simply invest this surplus.[4] Setting

[2] We take all values as present values. A desirable property possessed by this form of surplus allocation is that it is invariant with respect to change in currency value.

[3] This is very similar in spirit and calculation to the "stability constraint [2]." The total surplus need of a company will consist of this contribution, plus that needed to support expenses and equity in any unearned premium reserve for new writings, plus any other contributions required by regulators and/or management.

[4] There are reasons, such as a desire to maintain market presence, which could allow y to fall below the capital market rate temporarily.

the yield rate equal to the management target gives the required return on the contract[5]

$$r = y(V' - V), \tag{1.5}$$

which leads to

$$r = [yz/(1 + y)](S' - S). \tag{1.6}$$

Denoting by C the correlation of the contract with the existing book,

$$(S')^2 = S^2 + \sigma^2 + 2\sigma SC. \tag{1.7}$$

The value of C will be between -1 and $+1$, and

$$S' - S = \sigma(2SC + \sigma)/(S' + S). \tag{1.8}$$

Finally, combining the above and taking σ as the measure of risk, say that r, the risk load, is equal to reluctance times risk:

$$r = \mathcal{R}\sigma, \tag{1.9}$$

where $\mathcal{R}$, the reinsurer's reluctance to take on risk, is defined by

$$\mathcal{R} = [yz/(1 + y)](2SC + \sigma)/(S' + S). \tag{1.10}$$

2. INSURANCE

If the expected mean losses on the contract are μ and the expenses are E, then the appropriate premium P is given by

$$P = \mu + \mathcal{R}\sigma + E. \tag{2.1}$$

In the overwhelmingly typical case, $\sigma \propto \Sigma$, the reluctance has an excellent approximation as

$$\mathcal{R} = [yz/(1 + y)](C + \sigma/2S). \tag{2.2}$$

[5] This approach is actually an extension of the discussion on page 453 of Patrik and John [3]. We adopt this for its simplicity, while acknowledging that there are interesting questions with respect to the surplus flow needed to support the expected return of the book and of the contract, and the consequent internal rate of return.

These two equations form the heart of the paper, and both should and do make sense intuitively. In a competitive market, the yield rate y required will decrease, and so will the reluctance. A more conservative company will have a higher value of z, and hence a higher reluctance. A reinsurer whose book is regional will have a larger reluctance to take on a contract from a national carrier than from a carrier from a different region, and a still higher reluctance for a carrier in his region, because of the increasing values of the covariance.

In the very pessimistic case where $C = 1$, the exact form for $\mathcal{R}$ becomes

$$\mathcal{R} = [yz/(1 + y)], \tag{2.3}$$

which depends only on factors external to the contract. The premium still depends, of course, on μ and σ. Back in the general case, if there is a "bank" B built up,[6] then the marginal surplus required is reduced by B, and the premium becomes

$$P = \mu + \mathcal{R}\sigma + E - yB/(1 + y). \tag{2.4}$$

3. EXCESS LAYERING APPLICATION

In the case of high excess layers, generally speaking the mean loss μ will be a small part of the premium, and the contribution from the risk load will be the most significant. This is intuitive and also mathematically demonstrable.

The layer payout function $P(x;A,L)$ for loss in the layer with attachment point A and limit L from an unlayered loss of x is defined by, as usual,

$$P(x;A,L) = \begin{cases} 0, & x \le A \\ (x - A), & a \le x \le (A + L) \\ \mathrm{L}, & (A + L) \le x. \end{cases} \tag{3.1}$$

[6] That is, on a long-term treaty the premiums have exceeded the losses enough for some years that the reinsurer feels that the reassured has some measure of moral, if not legal, equity. Conversely, the losses may have exceeded the premiums enough that the reinsurer wants to add to the premium "to be made whole," which corresponds here to a negative B.

Denote by $E\{\text{any function of } x\}$ the expected value of that function over the distribution. That is, if $f(x)$ is the probability density function defined on the interval $(0,\infty)$ and $h(x)$ is any function of x, then

$$E\{h\} = \int_0^\infty h(x)f(x)dx. \tag{3.2}$$

Of particular interest are $E\{P\}$ and $E\{P^2\}$. For convenience define $G(x)$ as the probability that a loss is greater than x. That is,

$$G(x) = \int_x^\infty f(x)dx. \tag{3.3}$$

Then, a direct substitution of P in the expectation formula and an integration by parts yields the mean $\mu = E\{P\}$ as

$$\mu = \int_0^L G(A + x)dx; \tag{3.4}$$

and, similarly,

$$E\{P^2\} = \int_0^L 2xG(A + x)dx. \tag{3.5}$$

By definition,

$$\sigma^2 = E\{P^2\} - \mu^2. \tag{3.6}$$

Now keep L fixed and increase A; that is, examine higher and higher layers. Since G goes to zero as its argument becomes large, both μ and σ do also. In the cases of much practical interest (e.g., varieties of Pareto and Burr) where $G(x)$ has a power law behavior for large x,

$$G(x) \sim g/x^\alpha. \tag{3.7}$$

The integrals to lowest order in L/A may be approximated as

$$\mu \sim gL/A^\alpha \text{ and } E\{P^2\} \sim gL^2/A^\alpha. \tag{3.8}$$

Since G is essentially constant across the layer, it should come as no surprise that the result is that of a binomial distribution (with "success" probability μ/L) and that

$$\sigma^2 = \mu(L - \mu). \tag{3.9}$$

Thus, μ goes to zero faster than σ (which goes as $\sqrt{\mu}$); and the risk load dominates the expected loss, as intuition would suggest.

4. A USEFUL LEMMA

Further, one often has many layers stacked to create a program of protection; and a reinsurer may want to be on, for example, the first, fourth, and seventh layers. Clearly, there are correlations between layers, since to reach the fourth layer the loss must have exceeded the limit of the first. Fortunately, it is not necessary to do simulations or calculations for all possible combinations of layers. If one knows L, μ, and σ for each layer, then the appropriate mean and standard deviation can be calculated easily for any contract.

Suppose there is a set of layers P_i, where i runs over the set of values 1 to N; these layers do not overlap; and they are in increasing sequence ($A_i + L_i \leq A_j$ for $i < j$). This is the usual case. The layers need not actually be contiguous, although they generally are. Now, using Σ_i to mean summation over values of the layer index appropriate to the contemplated contract (one, four, and seven in the example above),

$$\mu = E\left\{\sum_i P_i\right\} = \sum_i E\{P_i\} = \sum_i \mu_i. \tag{4.1}$$

This is the obvious, but useful, result; and

$$E\left\{\left(\sum_i P_i\right)^2\right\} = E\left\{\sum_i (P_i)^2\right\} + 2\sum_{i<j} E\{P_iP_j\}, \tag{4.2}$$

where the $\Sigma_{i<j}$ means summation is restricted to values of i and j such that $i < j$. The essential point is that when $i < j$, for values of x where P_j is non-zero, P_i is constant at L_i. Thus,

$$E\{P_iP_j\} = L_iE\{P_j\} = L_i\mu_j \tag{4.3}$$

and

$$E\left\{\left(\sum_i P_i\right)^2\right\} = \sum_i [(\sigma_i)^2 + (\mu_i)^2] + 2\sum_{i<j} L_i\mu_j. \tag{4.4}$$

Hence

$$\sigma^2 = \sum_i (\sigma_i)^2 + 2\sum_{i<j} (L_i - \mu_i)\mu_j. \tag{4.5}$$

The second term represents the covariance between the layers. This formula is a great convenience in actual simulation modeling, since it means one only has to do the layers separately, and then any combination of layers may be easily derived.

5. PRACTICAL CONSIDERATIONS

Where does one obtain μ and σ for the layers? Typically, from doing simulation modeling on the underlying data. There, one has to make explicit the assumptions on trend, development, exposure, curve family, and so on. However, once done, the statistics are obtained fairly easily in these days of powerful personal computers. In principle, σ should contain the uncertainty from the underlying assumptions (parameter variability) as well as the process variance from the distributions.

Expenses of the reinsurer can be modeled as a flat piece, for handling the contract per se, plus a piece proportional to the number of losses, representing the loss handling cost. The expected number of losses is also available from the simulation runs.

One would surmise that the market pricing would be relatively efficient, in the sense of producing rates appropriate to the risk. Reinsurers have, after all, been in the business a long time. Of course, in the golden years of the past, the expectation was that relationships would be long-term, and that rates each year would be adjusted for past results so that in the not too long run reinsurers would make a profit.[7] In such circumstances, precise pricing was not as necessary, nor was competition perhaps as fierce as in today's environment.

Where does "rate on line" pricing fit in? For those not in the reinsurance field, this is the inverse of "payback period:" the number of years that premium would have to be collected to equal one total loss. For example, for a limit of one million dollars, a 10% rate on line gives

[7] Hence the notion of a "bank." See the preceding footnote.

a premium of $100,000, or equivalently a payback period of ten years. Underwriters seem to have definite notions of what a maximum payback period should be, more or less independent of the nature of the cover. Assuming a continuing relationship, what seems to underlie this kind of thinking is the notion that every individual program should make a profit in a time frame during which the reassured is likely to remain solvent. In the present context, this translates into an additional contribution to σ coming from credit risk. This contribution would not go to zero as the layer gets higher.

Returning to the reluctance, it is expected to be relatively constant across layers as long as $\sigma/\Sigma \propto C$, for example when a reinsurer is considering a piece of a layer of a large multi-line primary. Further, as remarked earlier, to the extent that the covariance is large, we would expect the reluctance to be a product of only the reinsurance market conditions and the reinsurer's conservatism measure.

In actual practical use of this work, for any given reinsurance program reluctance has been taken as constant across layers, and σ reflects only the process variability. On a relatively small sample, the reluctance has values varying typically from 30% to 70% or more. Note that with a z of 3.1 and a pessimistic $C = 1$, a 12% return is a reluctance of 33%, and a 20% return is a reluctance of 52%, so this type of range might have been expected.

REFERENCES

[1] R. E. Brubaker, "A Constrained Profit Maximization Model for a Multi-Line Property/Liability Company," 1979 Discussion Paper Program, Casualty Actuarial Society, p. 35.

[2] J. M. Stone, "A Theory of Capacity and the Insurance of Catastrophe Risks," *Journal of Risk and Insurance* Vol. XL, No. 2, 1973, p. 231 and Vol. XL, No. 3, 1973, p. 339.

[3] G. S. Patrik and R. T. John, "Pricing Excess-of-Loss Casualty Working Cover Reinsurance Treaties," 1980 Discussion Paper Program, Casualty Actuarial Society, p. 453.