ACTUARIAL NOTE ON LOSS RATING

RONALD E. FERGUSON

Substantial underwriting losses in the mid-1970's are testimony to the inability of the insurance industry to deal effectively in the pricing process with some of the forces that affect its product. The problems are numerous; however, each of the problems can be subsumed under one of three categories. First of all, our inward looking ratemaking techniques did not equip us to cope with a changing economic environment. Our economic environment includes inflation, recession, a combination of the two, and an economy that is increasingly subject to shocks of various types.[1]. The second major problem is societal changes.[2]. Under this broad heading are changing attitudes about the level of risk one can (or should) bear, changing concepts of entitlement, and the erosion of tort law. The third problem area is unsound or inept ratemaking techniques. In this paper, unsound ratemaking practices are defined to include only the technically unsound aspects of ratemaking.

The objective of this paper is modest in that the focus will be on one relatively small area under the heading of unsound ratemaking practices.

Incredible as it may seem—until the mid 1970's, loss development and trending procedures were not part of most industry loss rating schemes. Although this serious defect has been remedied in the current (ISO) individual risk rating plans, we believe many underwriters continue to use loss rating techniques without paying adequate attention to development and trend. The literature and the day-to-day practices of some segments of the excess loss market suggest that many still ignore the impact of these important forces.

One of the rating concepts developed in many textbooks is the "burning cost."[3] "Burning cost" or pure loss cost[4] is generally defined as the unmodified excess losses divided by the total subject premium. The so-called "burning cost" is then surcharged by the use of a loss conversion factor (e.g. 100/85ths) to provide for the assuming carrier's expenses, risk charge, and profit, and becomes the charged rate. The typical observation period of such a rating scheme is five years. In a static environment (i.e. no inflation), this scheme will produce acceptable results. In fact, it will on average produce a loss ratio equal to the reciprocal of the loss conversion factor × 100. While it is probably obvious that in a changing environment (loss development or inflation) there is a lagging process, such schemes are still in use today. Simple loss rating schemes such as these will produce inadequate premiums.

The loss ratio at any given year under such a scheme can be determined from the formula below. The development of this formula is included in the Appendix.

$$\frac{\left[a(1+i)^5 - R\right] \times 100}{\left\{\frac{a}{5} \cdot \frac{[(1+i)^5 - 1]}{i} - R\right\} \times LCF} \tag{1}$$

Where a = Gross loss
i = Inflation rate
R = Retention
LCF = Loss conversion factor

The inception-to-date loss ratio at any given time (t − 1) will be:

$$\frac{\left[a(1+i)^5 \times \frac{(1+i)^t - 1}{i} - tR\right] \times 100}{\left\{\frac{a}{5}\left[\frac{(1+i)^t - 1}{i} \cdot \frac{(1+i)^5 - 1}{i}\right] - tR\right\} \times LCF} \tag{2}$$

To determine the extent or effect of the lagging process, we sought to find the limiting value of the above expression as t becomes very large. Using L'Hopital's rule[5], it can be demonstrated that:

$$\frac{\left[a(1+1)^5 \times \frac{(1+i)^t - 1}{i} - tR\right] \times 100}{\left\{\frac{a}{5}\left[\frac{(1+i)^t - 1}{i} \times \frac{(1+i)^5 - 1}{i}\right] - tR\right\} \times LCF} \xrightarrow[\text{as } t \to \infty]{} \frac{5i(1+i)^5 \times 100}{[(1+i)^5 - 1]\, LCF} \tag{3}$$

The development of this formula is contained in Part B of the Appendix.

Note that while the original expression was set up to describe an excess of loss situation, the limiting value is independent of R and is therefore applicable to a primary loss rating situation.

Apparently those who use such rating schemes feel that the sequence converges to $(1/LCF) \times 100$ or that the slippage is minor. With the above expression, it can be demonstrated that the sequence does not converge to $(1/LCF) \times 100$. For example, with an overall inflation rate of a modest 3%, the limiting loss ratio becomes 92.80%, and at 6%, it becomes 100.8% even though the conversion factor is 100/85ths.

If you use a burning cost or simple loss rating scheme such as described above, consider the implications. There is good news and bad news. The bad news is that there is a fundamental lagging process in such a scheme which cannot be overcome even with unlimited time. The good news is that one could very simply work backward from the above formula to determine what *LCF* should be used with a given rate of inflation, *i*, and a target loss ratio after *t* years.

CONCLUSION

Mr. R. E. Stewart, former New York Insurance Department Superintendent, pointed out in a recent essay that it is the business of insurance "to create economic stability for others in the face of certain misfortunes of all kinds—negligent, capricious, malicious, or divine, not to mention social and economic." To fulfill this role, we must overcome what he calls the "fifth legacy of the cartel mind . . ." a feeling "that insurance must have a stable economic and social environment in which to function."[6].

To fulfill its role, the industry must develop ideas and techniques that are suitable for a changing or unstable economic and social environment. In this paper, we have pointed to only one small problem area—industry results suggest there must be many other as yet undiscovered problems.

REFERENCES

[1] The shock theory is put forward by Dr. Otto Eckstein and Sara Johnson in a DRI report dated the summer of 1975 *(The Data Resources—U.S. Long Term Review)*. They noted that we have seen the following major economic shocks in the last 21 years:

1) The end of price controls in 1946;
2) The Korean War;
3) The 116-day steel strike of 1959;
4) The Kennedy-steel industry confrontation of 1962;
5) The Vietnam War (1965-6);
6) The price controls of 1971;
7) The food price explosion of early 1973;
8) The oil embargo of November 1973 and the quadrupling of the price of oil;
9) The second food price explosion and the end of controls in 1974.

[2] "A Culture in Transformation: Toward a Different Societal Ethic?"—Trend Analysis Program Report #12, American Council of Life Insurance.

[3] Munich Re monograph, *Reinsurance and Reassurance*, Volume 2, Page 43 *Property and Liability Reinsurance Management*, Robert C. Reinarz, Mission Publishing Company, 1965, Pages 63, 76.

[4] Sometimes the expressions "pure loss cost," "Carpenter plan," "spread loss plan," are used to describe concepts similar or identical to the burning cost idea.

[5] If $F(t) \longrightarrow \infty$ and $q(t) \longrightarrow \infty$ as $t \longrightarrow a$ and if the limit of the ratio $F'(t)/q'(t)$ as t approaches a exists, then:

$$\lim_{t \to a} \frac{F(t)}{q(t)} = \lim_{t \to a} \frac{F'(t)}{q'(t)}$$

In the notation $t \to a$, a may either be finite or infinite.

[6] Richard E. Stewart, "On the 'Commodity' of Insurance," *The National Underwriter*, December 16, 1977.

APPENDIX

A. Background for formula (1)

$$\begin{aligned}
\text{Gross loss in year } -5 &= a \\
-4 &= a\,(1+i) \\
-3 &= a\,(1+i)^2 \\
-2 &= a\,(1+i)^3 \\
-1 &= a\,(1+i)^4 \\
0 &= a\,(1+i)^5
\end{aligned}$$

Premium for year 0:

$$\frac{(a + a(1+i) + a(1+i)^2 + a(1+i)^3 + a(1+i)^4 - 5R)}{5} \times LCF$$

Loss ratio in year 0:

$$\frac{[a\,(1+i)^5 - R] \times 100}{\frac{1}{5}[a + a(1+i) + a(1+i)^2 + a(1+i)^3 + a(1+i)^4 - 5R] \quad \times LCF}$$

or

$$\frac{a\,(1+i)^5 - R \times 100}{\left\{\frac{a}{5}\ \frac{(1+i)^5 - 1}{i} \quad - R\right\} \quad \times LCF}$$

B. Background for development of the limit:

Show that:

$$\lim_{t \to \infty} \frac{\left[a(1+i)^5 \cdot \frac{(1+i)^t - 1}{i} - tR\right] \times 100}{\left[\frac{a}{5} \cdot \frac{(1+i)^t - 1}{i} \cdot \frac{(1+i)^5 - 1}{i} - tR\right] \times LCF} = \frac{5\, i\, (1+i)^5 \times 100}{[(1+i)^5 - 1] \times LCF}$$

Proof: Let f and g be functions, such that:

$$f(t) = \left[a\,(1+i)^5 \cdot \frac{(1+i)^t - 1}{i} - tR\right] \times 100, \text{ and}$$

$$g(t) = \left[\frac{a}{5} \cdot \frac{(1+i)^t - 1}{i} \cdot \frac{(1+i)^5 - 1}{i} - tR\right] \times LCF$$

on the interval $(0, \infty)$

By simple algebraic manipulation, we have:

$$f(t) = t\left[\frac{a\,(1+i)^5}{i} \cdot \frac{(1+i)^t - 1}{t} - R\right]$$

Since $(1+i)^t - 1 \to \infty$ as $t \to \infty$,

$\frac{(1+i)^t - 1}{t}$ is an indeterminate form of type ∞/∞.

Apply L'Hôpital's rule:

$$\lim_{t \to \infty} \frac{(1+i)^t - 1}{t} = \lim_{t \to \infty} \frac{(1+i)^t \log(1+i)}{1} = \infty$$

It follows that $\lim_{t \to \infty} f(t) = \infty$.

Similarly, $\lim_{t \to \infty} g(t) = \infty$.

Hence, $\frac{f(t)}{g(t)}$ is an indeterminate form of type ∞/∞.

Since,

$$f'(t) = \frac{d}{dt} f(t) = \left[\frac{a(1+i)^5}{i} \cdot (1+i)^t \log(1+i) - R \right] \times 100,$$

and

$$g'(t) = \frac{d}{dt} g(t) = \left[\frac{a}{5} \cdot \frac{(1+i)^5 - 1}{i^2} \cdot (1+i)^t \log(1+i) - R \right] \times LCF,$$

it is evident that

$\lim_{t \to \infty} f'(t) = \infty$ and $\lim_{t \to \infty} g'(t) = \infty$, therefore

$\dfrac{f'(t)}{g'(t)}$ is also an indeterminate form of type ∞/∞.

Differentiate $f'(t)$ and $g'(t)$ with respect to t:

$$f''(t) = \frac{d}{dt} f'(t) = a \cdot \frac{(1+i)^5}{i} \cdot (1+i)^t \cdot [\log(1+i)]^2 \times 100$$

$$g''(t) = \frac{d}{dt} g'(t) = \frac{a}{5} \cdot \frac{[(1+i)^5 - 1]}{i^2} \cdot (1+i)^t \cdot [\log(1+i)]^2 \times LCF$$

It is easy to prove that:

$$\frac{f''(t)}{g''(t)} = \frac{5\, i\,(1+i)^5 \times 100}{[(1+i)^5 - 1] \times LCF,}$$

which is independent of t. Applying L'Hôpital's rule twice, we should have

$$\lim_{t \to \infty} \frac{f(t)}{g(t)} = \lim_{t \to \infty} \frac{f'(t)}{g'(t)} = \lim_{t \to \infty} \frac{f''(t)}{g''(t)} = \frac{5\, i\,(1+i)^5 \times 100}{[(1+i)^5 - 1] \times LCF}$$

C. For example, using formula (2), we can find the appropriate *LCF* given inflation of 7%, a planning horizon of 10 years, and a target loss ratio of 90%. Assuming a is 100,000 and R is \$50,000—it appears that an *LCF* of $\dfrac{100}{68.18}$ would satisfy all requirements.