the form of the logistical problem connected with providing a Table L for each state. The number of Table L pages that a home office would be required to maintain would be monstrous if we continued to recognize each state's loss elimination ratio. Perhaps the logistical problem can be minimized by reducing the number of possible loss limitations to a minimum and by grouping states with similar loss distributions by size. Perhaps a formula approach to calculating the incremental charge, which recognizes that the increment must vary with the entry ratio, can be devised. And perhaps this problem is trivial in terms of electronic storage.

It is hoped that the obstacles confining Table L to California can be overcome. It is hard to disagree with the author's contention that from a mathematical point of view, Table L represents an advance over Table M.

AUTHOR'S REVIEW OF DISCUSSIONS

The two reviews suggest alternative approaches to three problems, the incompatibility of California Tables L and M for certain entry values, the multitude of Table L's required for countrywide use, and the difficulty of measuring the incremental charge. Mr. Snader suggests a pragmatic method of graduation to produce a consistent set of tables while maintaining the assumption that the loss elimination ratio is independent of premium size. Mr. Harwayne develops a simple method of estimating the incremental charge for Table M.

This reply includes a previously unpublished method of computing the incremental charge from a risk distribution of losses. The reviews were the stimulus for some further mathematical work, which is also included.

THE "RUINOUS TIDE OF PAPER"

A set of Table L's varying by 52 states, 300 entry ratios, 64 risk sizes, 7 per accident limits, and 4 hazard groups would have 28 million entries filling a hundred thousand pages. To stem this tide, average values are used in place of some of the variables. The California Table L has only 11 size groups and is not subdivided by hazard group. The result is a practical, 66 page table.

The Table L charge $\phi^*(r)$ is the sum of the Table M charge $\phi(r)$ and the incremental charge $\triangle\phi(r)$. Let the per accident charge index $Y(r)$ be the ratio of the incremental charge to the loss elimination ratio. Then

$$\phi^*(r) = \phi(r) + \triangle\phi(r) = \phi(r) + Y(r) \cdot k. \tag{1}$$

A Table L charge is the expected proportion of loss dollars eliminated by excluding the portion of each loss above the accident limit and then excluding the portion of the loss ratio above the entry ratio. These two limiting operations overlap because some loss dollars in excess of the accident limit would have been excluded by the loss ratio limit alone. The greater the overlap, the smaller the per accident charge index.

The amount of overlap depends on the expected loss, the entry ratio, and the accident limit. Harwayne defines the attachment point $r\frac{M}{T}$ by

$$r\frac{M}{T} = (\text{Accident Limit}) \div (\text{Expected Loss}) \tag{2}$$

If the entry ratio is below the attachment point, then for any loss the portion exceeding the accident limit also exceeds the loss ratio limit; the limits entirely overlap, so the per accident charge index is zero. As the entry ratio approaches infinity the overlap disappears and the per accident charge index approaches unity.

The National Council's retrospective rating values come from Table M and from tables of excess loss premium factors. In order to reduce the size of the tables, they averaged over certain variables. Their excess loss premium factors are calculated as if the per accident charge index were always unity; they vary only by state, per accident limit, and hazard group. Their Table M varies only by entry ratio and risk size.

Both reviews point out that it is not feasible to go to a Table L approach and maintain full variation by state and hazard group. A compromise worth considering would be the use of countrywide Table L's varying by hazard group and a choice of about four per accident limits and 20 size groups. This procedure would require the production of 16 separate Table L's, but each would be smaller than the current Table M, and only one would be used to rate a risk. Graduation by size would be easier with fewer size groups. Since the entire Table L charge would vary by hazard group, state, entry ratio, and risk size, this method would be more accurate than the current one. I believe that this increase in accuracy would outweigh the decrease in accuracy from reducing the number of size groups and no longer varying excess loss premium factors by state, particularly because state laws are becoming more uniform as the recommendations of the National Commission on State Workers' Compensation Laws are adopted.

THE HARWAYNE METHOD

Frank Harwayne has discovered a technique by which the National Council can use the Table L method, varying the per accident charge index by entry ratio and premium size, retaining all 73 size groups of the current Table M, and continuing to vary the excess loss premium factor by state and hazard group. The method prevents the paper explosion and uses only currently existing tables, so it can be implemented immediately.

The method is to estimate the Table L charge by interpolating between the Table M charge for the given risk size and the Table M charge for a smaller risk size, which is chosen so that its Table M charge lies above the Table L charge out to a very high entry ratio. The interpolation is performed by filling in a simple worksheet. The resultant approximate incremental charges are reasonably accurate, far surpassing those produced by the current National Council method. (See Exhibit 1.) A disadvantage of the method is that several worksheets have to be filled out each time since a retrospective rating is computed by trial and error. Constructing a set of Table L's would require extra work, but they would be more convenient to use.

For a given risk size the Table L charge $\phi^*(r)$ has certain theoretical properties:

i) The Table L charge equals the Table M charge at entry ratios no greater than the attachment point.

$$\phi^*(r) = \phi(r) \text{ for } r \leq r\frac{M}{T}. \qquad (3)$$

ii) The Table L charge is greater than the Table M charge at entry ratios above the attachment point.

$$\phi^*(r) > \phi(r) \text{ for } r > r\frac{M}{T}. \qquad (4)$$

[1]Proved in a later section.

iii) The Table L charge approaches the loss elimination ratio as the entry ratio becomes large.

$$\lim_{r \to \infty} \phi^*(r) = k. \tag{5}$$

iv) The Table L charge is greater or equal to the loss elimination ratio.

$$\phi^*(r) \geq k. \tag{6}$$

v) The Table L charge is less than or equal to the sum of the Table M charge and the loss elimination ratio.

$$\phi^*(r) \leq \phi(r) + k. \tag{7}$$

vi) The incremental charge is a monotone increasing function of r.[1]

$$\frac{d}{dr} \triangle \phi(r) \geq 0. \tag{8}$$

vii) The Table L charge is a monotone decreasing function of r.

$$\frac{d}{dr} \phi^*(r) \leq 0. \tag{9}$$

viii) The Table L charge is a concave upward function of r.[2]

$$\frac{d^2}{dr^2} \phi^*(r) \geq 0. \tag{10}$$

Exhibit 2 illustrates Harwayne's method. Over most of the range of r the Table L charge $\phi^*(r)$ is close to the Table M charge $\phi(r)$ and far from the reference Table M charge $\phi_1(r)$. This distance is the reason that the curve $\phi_1(r)$ cannot determine $\phi^*(r)$ with perfect accuracy.

The Table L charge produced by Harwayne's formula satisfies the first three theoretical properties and appears to satisfy the fourth, but it need not satisfy the others. The example shown in Exhibit 2 deviates from properties (v) through (viii), although the deviations take place at high entry ratios, which are of little practical importance.

[2]Since $\frac{d}{dr}\left[\int_r^\infty (s\text{-}r)\, f^*(s)\,ds + k\right] = -\int_r^\infty f^*(s)\,ds$ and $\frac{d}{dr}\left[-\int_r^\infty f^*(s)\,ds\right]$

$= f^*(r) \geq 0$

Mr. Harwayne has devised a remarkably simple and effective technique for increasing the accuracy of the National Council's retrospective ratings. His method permits an immediate solution to an important practical problem.

MEASURING THE INCREMENTAL CHARGE

The incremental charge can be computed as the Table L charge minus the Table M charge or estimated by choosing a reasonable curve. It can also be measured directly by means of a method devised by the California Inspection Rating Bureau in 1965.

Given a selection of risks numbered 1, . . ., N for a particular size group, with risk n having expected loss E_n, actual loss A_n, and actual limited loss A_n^*, the per accident charge index at entry ratio r can be estimated as

$$\overline{Y}(r) = \frac{\sum_{n=1}^{N} [(Min\ (r, A_n/E_n) - Min\ (r, A_n^*/E_n)]}{\sum_{n=1}^{N} [A_n/E_n - A_n^*/E_n]}. \tag{11}$$

Let $\cdot\overline{\text{k}}$ be the estimated loss elimination ratio for all premium size groups combined. The incremental charge for the particular size group at entry ratio r is then estimated as

$$\triangle\overline{\phi}(r) = \overline{Y}(r) \cdot \overline{K}. \tag{12}$$

To see why this method works, write k, $\phi(r)$, and $\phi^*(r)$ as

$$k = E\{A/E - A^*/E\} \tag{13}$$

$$\phi(r) = E\{Max\ [(A/E - r), 0]\} \tag{14}$$

$$\phi^*(r) = k + E\{Max\ [(A^*/E - r), 0]\} \tag{15}$$

Then $\triangle\phi(r) = \phi^*(r) - \phi(r)$

$$= E\{A/E - A^*/E + Max\ [A^*/E - r), 0] - Max\ [(A/E - r), 0]\}$$

$$= E\{Min\ [r, A/E] - Min\ [r, A^*/E]\} \tag{16}$$

Equation (13) shows that $\sum_{n=1}^{N} [A_n/E_n - A_n^*/E_n]/N$ is an estimator for k.

Equation (16) shows that $\sum_{n=1}^{N} [Min\ (r, A_n/E_n) - Min\ (r, A_n^*/E_n)]/N$ *is* an estimator of $\triangle\phi(r)$. Formula (11), the ratio of these two expressions, is an estimator for $Y(r) = \triangle\phi(r)/k$.

Equation (16) can be used to show that $\triangle\phi(r)$ is a monotone increasing function of r.

Theorem: Assume that the loss limitation procedure never increases a loss, that is, $A \geq A^*$. Let r and s be entry ratios with $0 \leq r < s$. Then $\triangle\phi(r) \leq \triangle\phi(s)$.

Proof: Let $X = Min\ [s, A/E] - Min\ [s, A^*/E] - (Min\ [r, A/E] - Min\ [r, A^*/E])$. Then $E\{X\} = \triangle\phi(s) - \triangle\phi(r)$, from equation (16). The value of the random variable X depends on the relative sizes of r, s, A/E, *and* A^*/E:

Condition	Min [s, A/E] − Min [s, A*/E]	Min [r, A/E] − Min [r, A*/E]	X	Sign of X
$A^*/E \leq A/E \leq r < s$	$A/E - A^*/E$	$A/E - A^*/E$	0	0
$A^*/E \leq r < A/E \leq s$	$A/E - A^*/E$	$r - A^*/E$	$A/E - r$	> 0
$A^*/E \leq r < s < A/E$	$s - A^*/E$	$r - A^*/E$	$s - r$	> 0
$r < A^*/E \leq A/E \leq s$	$A/E - A^*/E$	0	$A/E - A^*/E$	≥ 0
$r < A^*/E \leq s < A/E$	$s - A^*/E$	0	$s - A^*/E$	≥ 0
$r < s < A^*/E \leq A/E$	0	0	0	0

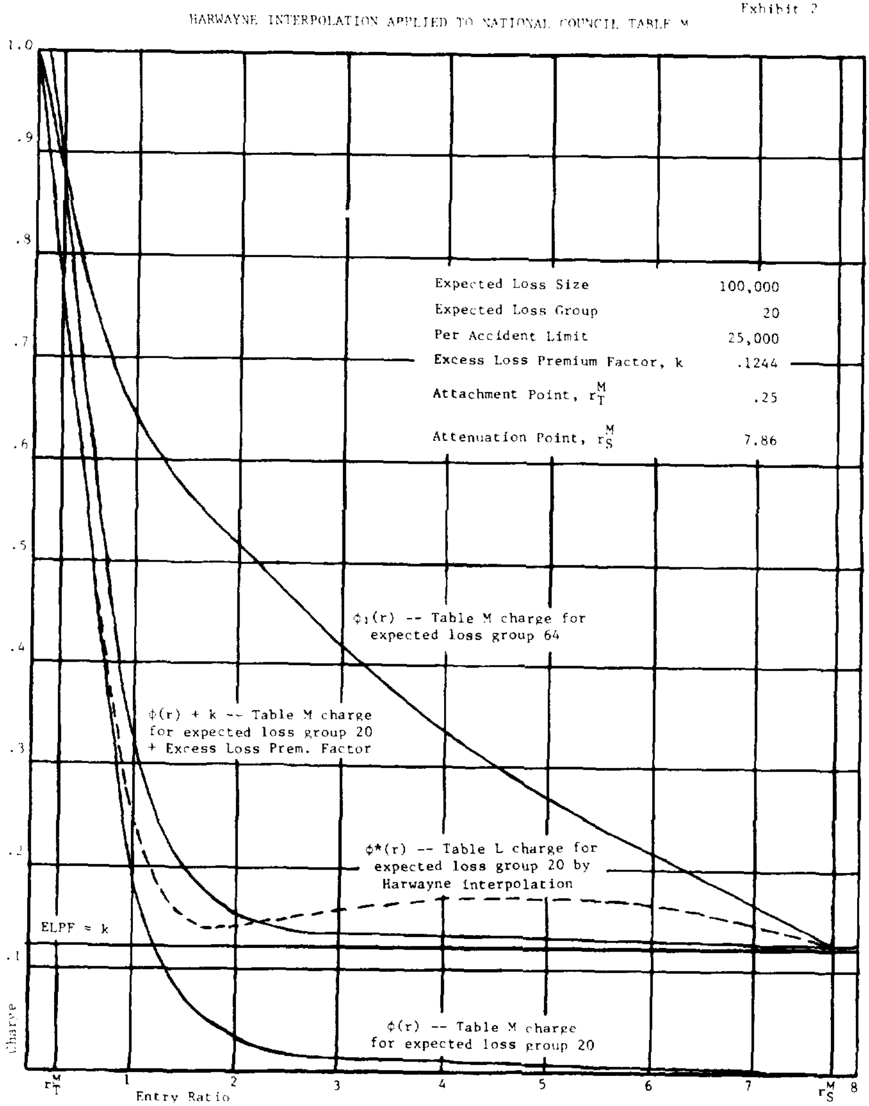
Since $X \geq 0$ in all cases, $E\{X\} \geq 0$.

Exhibit 1

TEST OF HARWAYNE INTERPOLATION APPLIED TO CALIFORNIA DATA AT SELECTED ENTRY RATIOS ABOVE THE ATTACHMENT POINT

Expected Loss Size	Per Accident Limit	Entry Ratio	(1) Calif. Table L Charge	(2) Calif. Table M Charge	(3) Calif. Incremental Charge	(4) Incremental Charge by Current National Council Method	(5) Error	(6) Incremental Charge by Harwayne Interpolation	(7) Error
					(1)–(2)		(4)–(3)		(6)–(3)
22,014	25,000	2.0	.193	.188	.005	.124	.119	.037	.032
38,819	25,000	1.5	.207	.194	.013	.124	.111	.044	.031
		2.0	.157	.126	.031	.124	.093	.067	.036
54,781	25,000	1.0	.300	.287	.013	.124	.111	.027	.014
		1.5	.179	.130	.049	.124	.075	.062	.013
		2.0	.139	.061	.078	.124	.046	.090	.012
	50,000	1.0	.290	.287	.003	.050	.047	.003	—
		1.5	.140	.130	.010	.050	.040	.020	.010
		2.0	.082	.061	.021	.050	.029	.034	.013
	100,000	2.0	.070	.061	.009	.027	.018	.004	−.005
161,892	25,000	.5	.533	.529	.004	.124	.120	.011	.007
		1.0	.243	.219	.024	.124	.100	.050	.026
		1.5	.151	.084	.067	.124	.057	.087	.020
		2.0	.130	.039	.091	.124	.033	.109	.018
	50,000	.5	.529	.529	—			.004	.004
		1.0	.222	.219	.003	.050	.047	.027	.024
		1.5	.098	.084	.014	.050	.036	.047	.033
		2.0	.063	.039	.024	.050	.026	.053	.029
	100,000	1.0	.220	.219	.001	.027	.026	.011	.010
		1.5	.088	.084	.004	.027	.023	.024	.020
		2.0	.046	.039	.007	.027	.020	.030	.023

Exhibit 2


HARWAYNE INTERPOLATION APPLIED TO NATIONAL COUNCIL TABLE M
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
Charge
Expected Loss Size 100,000
Expected Loss Group 20
Per Accident Limit 25,000
Excess Loss Premium Factor, k .1244
Attachment Point, r_T^M .25
Attenuation Point, r_S^M 7.86
$\phi_1(r)$ -- Table M charge for expected loss group 64
$\phi(r) + k$ -- Table M charge for expected loss group 20 + Excess Loss Prem. Factor
$\phi^*(r)$ -- Table L charge for expected loss group 20 by Harwayne interpolation
ELPF = k
$\phi(r)$ -- Table M charge for expected loss group 20
r_T^M 1 2 3 4 5 6 7 r_S^M 8
Entry Ratio