DISCUSSION BY RICHARD H. SNADER

Mr. Skurnick's thoughtful and thought provoking paper is destined to become required reading for actuarial students and practicing actuaries alike. Growing out of the need to solve a specific, local problem, this fine article accomplishes much more. Clear and compact, it can serve by example as a miniature manual of style for those of us who may feel inclined, in the future, to submit our own ideas for publication.

Apart from the introduction and conclusion, the paper is divided into four distinct sections, each of which serves a specific purpose. In the first two sections we are treated to an elaboration of the purely mathematical qualities of Table L. Section 1, in fact, provides us with an excellent review of the properties of Table M. By making the simple adjustment of mentally dispensing with the asterisks and k's, we have at our disposal a concise and truly rigorous mathematical development of the Table M concept. By the simple readjustment of mentally replacing the asterisks and k's, the transition to Table L is easily made.

In the third section theory is applied, and the continuous form is neatly converted to the discrete situation. This section is a boon to anyone who has ever been perplexed by that mysterious entity known as "Sum 2."

In the fourth section, the numerical properties of Table L are discussed, and the thorough reader is compelled to acquire his own copy of the complete table to supplement the discussion.

It is somewhat disturbing to note that the Table L charge is so close to the corresponding Table M charge over much of the table. It is also mildly disturbing to note the many instances where the Table M charge actually exceeds the Table L charge. There are also numerous instances where the Table L charge for a particular limitation at a given entry ratio exceeds the Table L charge for the next lower limitation.

The author is well aware of these anomalies and discusses them at some length. He correctly reasons that the column of Table M charges is less

accurate than the corresponding Table L charges because the Table M data are distorted by large losses. In fact, as the accident limitation increases, each successive column of Table L charges becomes more vulnerable to the distorting effect of large losses. These inconsistencies can be eliminated, suggests the author, by allowing the charges for each premium size group to be developed from the loss elimination ratio pertaining to that group. This procedure will most probably succeed, but the propriety of the measure is somewhat questionable in view of the extreme magnitude of the fluctuations in the k values between premium size groups.

It is possible, of course, that the inconsistencies result from the size of the sample. Perhaps if more data were available, the fluctuations in loss elimination ratios would be less pronounced. In that event, the inconsistencies would tend to eliminate themselves. Perhaps a definite trend in the loss elimination ratios exists but is masked by sparse data. If more data were available, such a trend would become apparent. In that event, it would be necessary for each premium size group to reflect its own loss elimination ratio in the manner suggested by the author.

An alternate approach might also be considered. The problem might be solved by simply requiring that $\phi^*(r_i^L) \geq \phi(r_i^M)$ within each premium size group. Or, if we may allow the superscript L to become specific by substituting a number for a particular loss limitation (for example, let $\phi^*(r_i^{25})$ denote the Table L charge for the $25,000 limitation), we can require that $\phi^*(r_i^{25}) \geq \phi^*(r_i^{30}) \geq \cdots\cdots \geq \phi^*(r_i^{100}) \geq \phi(r_i^M)$.

The enforcement of this constraint must be embodied in an appropriate graduation procedure. The problem is one of obtaining a smooth surface of Table L charges consistent with the array of tabulated values. The problem is quite similar to the one faced by our life insurance counterparts when graduating a select and ultimate mortality table.

I have chosen to dwell on this particular aspect of the paper because I feel it is an important one with respect to the possible extension of Table L to other states. It is clear that these inconsistencies must be dealt with before the Table L concept can gain acceptance elsewhere.

The consistency problem is by no means the major impediment to universal acceptance. A much more formidable obstacle must be faced in

the form of the logistical problem connected with providing a Table L for each state. The number of Table L pages that a home office would be required to maintain would be monstrous if we continued to recognize each state's loss elimination ratio. Perhaps the logistical problem can be minimized by reducing the number of possible loss limitations to a minimum and by grouping states with similar loss distributions by size. Perhaps a formula approach to calculating the incremental charge, which recognizes that the increment must vary with the entry ratio, can be devised. And perhaps this problem is trivial in terms of electronic storage.

It is hoped that the obstacles confining Table L to California can be overcome. It is hard to disagree with the author's contention that from a mathematical point of view, Table L represents an advance over Table M.

AUTHOR'S REVIEW OF DISCUSSIONS

The two reviews suggest alternative approaches to three problems, the incompatibility of California Tables L and M for certain entry values, the multitude of Table L's required for countrywide use, and the difficulty of measuring the incremental charge. Mr. Snader suggests a pragmatic method of graduation to produce a consistent set of tables while maintaining the assumption that the loss elimination ratio is independent of premium size. Mr. Harwayne develops a simple method of estimating the incremental charge for Table M.

This reply includes a previously unpublished method of computing the incremental charge from a risk distribution of losses. The reviews were the stimulus for some further mathematical work, which is also included.

THE "RUINOUS TIDE OF PAPER"

A set of Table L's varying by 52 states, 300 entry ratios, 64 risk sizes, 7 per accident limits, and 4 hazard groups would have 28 million entries filling a hundred thousand pages. To stem this tide, average values are used in place of some of the variables. The California Table L has only 11 size groups and is not subdivided by hazard group. The result is a practical, 66 page table.