THE CALIFORNIA TABLE L

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The retrospective rating plan of the California Inspection Rating Bureau is a tabular plan with a fixed per accident limit. In 1974, in order to bring the rating values up to date, a new table of charges was constructed. In the previous updating the insurance charge had been taken from the countrywide 1965 Table M of the National Council on Compensation Insurance and the charge for the per accident limit had been derived from a study of California claims. The Bureau decided to base the new table of charges wholly upon California experience. Since the per accident limit is fixed in the plan, it was decided to construct a table of charges that would include the cost of the per accident limit in the charge. This table was named "Table L". Its advantage is that it reflects both the charge for limitation of total losses and the charge for limitation of individual accidents, but the overlap between these charges is eliminated.

This article describes the characteristics of Table L and the method by which it was constructed. Section 1 contains a formal definition of the Table L charge and a demonstration of its applicability to retrospective rating. In Section 2 a new formula is derived, which uses Table M to develop retrospective rating plan values for a plan with a per accident limitation. Section 3 contains a description and an explanation of the methodology used by the Bureau to construct Table L. Section 4 describes some of Table L's numerical characteristics.

1. MATHEMATICAL PROPERTIES OF TABLE L

Formal Definition

Assume that a formula for limiting or adjusting individual accidents is given. The Table L charge at entry ratio r, $\phi^*(r)$, is defined as the average difference between a risk's actual unlimited loss and its actual limited loss; plus the risk's limited loss in excess of r times the risk's expected unlimited loss. The Table L savings at entry ratio r, $\psi^*(r)$, is defined as the average amount by which the risk's actual limited loss falls short of r times the expected unlimited loss. The Table L charge and savings are both expressed as ratios to expected unlimited loss.

In general, the "actual limited loss" for a risk may be calculated by adjusting the individual claims according to any pre-set formula, then summing the adjusted claim amounts. The theorems proved in this paper are valid regardless of the type of adjustment formula used, even if the formula prescribes different adjustments for different types of claims. The only requirement is that each adjusted claim amount be completely determined by the unadjusted claim amount and the characteristics of the claim.

For most purposes the adjustment to be used will be the truncation of individual claim amounts at a particular limit. This imposition of a per accident limit will result in a "normal" Table L. In the special case that no adjustment is made to individual claim amounts, the Table L produced is equivalent to a Table M, since the actual limited loss equals the actual unlimited loss.

California law requires that the calculation of a risk's retrospective premium use an average value in place of the actual indemnity loss for any death case. This substitution results in a smoothing of the loss ratios, which was provided for in the California Table L constructed by the Bureau. To accomplish this, an average value of $37,400 was substituted for each actual death indemnity amount before individual losses were truncated at the per accident limit. This use of an average death indemnity value has only a minor effect on the Table L charge, since less than 6% of the loss dollars result from death cases, and most actual death indemnity values are not far from the average value.

In this paper the usual excess pure premium ratio is called a Table M charge and an excess pure premium ratio which includes a provision for a per accident limit (or other adjustment of individual claims) is called a Table L charge.

The definitions will be made precise by utilizing mathematical notation. The annual losses for an insurance risk are a random variable. Let

A = the actual unlimited loss for the risk.

A^* = the actual limited loss for the risk, i.e. the actual loss after adjustment of individual claim amounts.

$E\{\cdot\}$ is the expectation operator: $E\{g(X)\} = \int g(x)\,dF_x(x)$ *for any* random variable X and function g

E = the expected unlimited loss $= E\{A\}$

$F =$ the cumulative distribution function of A/E

$F^* =$ the cumulative distribution function of A^*/E

$k =$ the loss elimination ratio

$$k = (E - E\{A^*\})/E. \tag{1}$$

The Table L charge and savings are defined mathematically for any entry ratio $r \geq 0$ by

$$\phi^*(r) = \int_r^\infty (s-r)dF^*(s) + k \tag{2}$$

$$\psi^*(r) = \int_0^r (r-s)dF^*(s). \tag{3}$$

From these definitions it is possible to prove two results that do not depend upon the application of Table L to any particular retrospective rating plan.

Lemma 1 Given constants r_1 and r_2 with $0 \leq r_1 \leq r_2$, define the random variable L to be the limited loss restricted to be no more than r_2E and no less than r_1E, i.e.

$$L = \begin{cases} r_1E, \text{ if } A^* \leq r_1E \\ A^*, \text{ if } r_1E < A^* \leq r_2E \\ r_2E, \text{ if } r_2E < A^*. \end{cases} \tag{4}$$

Then $E\{L\}/E = 1 - \phi^*(r_2) + \psi^*(r_1)$.

Proof: The random variable L/E can be represented as $g(A^*/E)$, where

$$g(x) = \begin{cases} r_1, \text{ if } x \leq r_1 \\ x, \text{ if } r_1 < x \leq r_2 \\ r_2, \text{ if } r_2 < x. \end{cases} \tag{5}$$

Then

$$E\{L\}/E = E\{L/E\} = E\{g(A^*/E)\} = \int_0^\infty g(s)dF^*(s)$$

$$= \int_0^{r_1} r_1dF^*(s) + \int_{r_1}^{r_2} sdF^*(s) + \int_{r_2}^\infty r_2\, dF^*(s)$$

$$= \int_0^{r_1} (r_1 - s)dF^*(s) + \int_0^{\infty} s dF^*(s) + \int_{r_2}^{\infty} (r_2 - s)dF^*(s)$$

$$= \int_0^{r_1} (r_1 - s)dF^*(s) + E\{A^*/E\} - \int_{r_2}^{\infty} (s - r_2)dF^*(s)$$

$$= \psi^*(r_1) + 1 - k - \int_{r_2}^{\infty} (s - r_2)dF^*(s)$$

$$= 1 + \psi^*(r_1) - \phi^*(r_2). \quad \text{Q.E.D.}$$

It will now be proved that for Table L, the savings equals the charge plus the entry ratio minus one. This is the same relationship that holds for Table M.

Theorem 1 For any $r \geq 0$, $\psi^*(r) = \phi^*(r) + r - 1$. (6)

Proof: In Lemma 1, take $r_1 = r_2 = r$.

Then, $L \equiv rE$, so that

$$E\{L\}/E = r = 1 + \psi^*(r) - \phi^*(r). \quad \text{Q.E.D.}$$

Application of Table L to Retrospective Rating

In the California Workmen's Compensation Retrospective Rating Plan, the retrospective premium R is given by

$$R = BP + CA^*. \tag{7}$$

subject to a maximum of G and a minimum of H, where

G = the maximum premium

H = the minimum premium

B = the basic premium ratio

P = the standard premium (before any applicable expense gradation.

C = the loss conversion factor (LCF)

Unlike the National Council plans, the California Plan uses only one tax expense ratio, so the tax multiplier is included in the basic premium ratio and the LCF. The formulas derived in this section can also be applied to the National Council plans by adjusting for the different meanings assigned to these two terms.

In order to demonstrate how Table L leads to a balanced plan, it is convenient to introduce the following notation:

$L_G =$ the actual limited losses that will produce the maximum premium

$$L_G = (G - BP)/C \tag{8}$$

$$r_G = L_G/E \tag{9}$$

$L_H =$ the actual limited losses that will produce the minimum premium

$$L_H = (H - BP)/C \tag{10}$$

$$r_H = L_H/E. \tag{11}$$

$L =$ the losses that will produce the retrospective premium in equation (7) without reference to the maximum and the minimum premiums. That is

$$L = \begin{cases} L_H, \text{ if } A^* \leq L_H \\ A^*, \text{ if } L_H \leq A^* \leq L_G \\ L_G, \text{ if } L_G \leq A^*. \end{cases} \tag{12}$$

The retrospective premium can be written as

$$R = BP + CL \tag{13}$$

$= \text{basic premium} + \text{converted losses}$

$I^* =$ the net Table L insurance charge

$$I^* = [\phi^*(r_G) - \psi^*(r_H)]E. \tag{14}$$

Theorem 2 $\quad E\{L\} = E - I^*. \qquad (15)$

Proof: Apply Lemma 1 taking r_G for r_1 and r_H for r_2. Then

$$E\{L\}/E = 1 - \phi^*(r_G) + \psi^*(r_H)$$

$$E\{L\} = E - [\phi^*(r_G) - \psi^*(r_H)]E. \quad \text{Q.E.D.}$$

A retrospective rating plan is said to be balanced if the expected value of the retrospective premium equals the standard premium adjusted for any expense gradation built into the plan. Let D denote the expense gradation in the plan, expressed as a ratio to P. From equations (13) and (15) balance will require that

$$E\{R\} = BP + CE - CI^* = P(1 - D). \tag{16}$$

It follows that the basic premium ratio must be selected as

$$B = 1 - D - CE/P + CI^*/P. \tag{17}$$

The retrospective premium can be separated into loss and expense components, and it can be shown that the expected value of each of these components equals the value of the corresponding component of standard premium adjusted for expense gradation.

Theorem 3 The use of equations (13) and (17) will produce a plan that is balanced with respect to losses.

Proof: The loss portion of R is $L + I^*$.

$$E\{L + I^*\} = E\{L\} + I^* = E. \quad \text{Q.E.D.}$$

Theorem 4 The use of equations (13) and (17) will produce a plan that is balanced with respect to expenses.

Proof:

$$\text{Expense in basic premium} = P(1 - D) - CE + (C - 1)I^*. \tag{18}$$

$$\text{Expense in converted losses} = (C - 1)L. \tag{19}$$

The expected value of the expense portion of R is

$$\begin{aligned} &E\{P(1 - D) - CE + (C - 1)\ (I^* + L)\} \\ &= P(1 - D) - CE + (C - 1)\ (I^* + E\{L\}) \\ &= P(1 - D) - CE + (C - 1)\ (I^* + E - I^*) \\ &= P(1 - D) - E. \quad \text{Q.E.D.} \end{aligned}$$

Two Useful Formulas

Table M formulas have been derived to express both the entry ratio difference and the charge difference in terms of the minimum premium, the maximum premium and the expense provision. Snader has shown that these formulas must be satisfied in order to have a balanced retrospective rating plan.[1] The formulas are the basis of the National Council's "Method 2" for determining rating values.[2] The use of these formulas facilitates the trial and error search for rating values corresponding to selected maximum and minimum premiums. Comparable formulas also exist for Table L.

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[1] R. H. Snader, "Fundamentals of Individual Risk Rating and Related Topics," CAS Study Note, Part II, p. 3.

[2] National Council on Compensation Insurance, "Rating Supplement for Workmen's Compensation and Employers' Liability Insurance Retrospective Rating Plan D." p. 9.

Theorem 5 $\phi^*(r_H) - \phi^*(r_G) = (P - PD - H)/CE.$ (20)

Proof:

$$H = BP + CEr_H = P(1 - D) - CE + CE[\phi^*(r_G) - \psi^*(r_H)] + CEr_H$$
$$= P(1 - D) + CE[\phi^*(r_G) - \psi^*(r_H) + r_H - 1]$$
$$= P(1 - D) + CE[\phi^*(r_G) - \phi^*(r_H)].$$

Therefore

$$\phi^*(r_H) - \phi^*(r_G) = (P - PD - H)/CE. \quad \text{Q.E.D.}$$

The usual Table M formula for the entry ratio difference also holds for Table L, since the Table L entry ratios are also ratios to expected unlimited loss. This formula is

$$r_G - r_H = (G - H)/CE. \tag{21}$$

Formulas (20) and (21) were used in the construction of the updated California Plan. A selection of rating values will satisfy these two equations if and only if they yield a balanced plan.

2. THE INCREMENTAL CHARGE FOR PER ACCIDENT LIMITATION

In computing rating values for a plan with a per accident limitation, the standard method has been first to use Table M to select a maximum, a minimum, a basic, and an LCF that would provide balance if accidents were not limited; then add an incremental charge to the basic. Dorweiler describes this incremental charge as the increment on the excess pure premium ratio due to superimposing a per case limit on a per loss ratio limit.[3] He points out that the incremental charge will vary between zero and the loss elimination ratio depending upon the per accident limit, the expected loss ratio, the risk premium size and the entry ratio. The variation in incremental charge reflects the varying amount of overlap between the effect of a per accident limit and the effect of an overall loss amount limit. Conceptually, the Table L charge represents the sum of a Table M charge and an incremental charge. Let

[3] P. Dorweiler, "On Graduating Excess Pure Premium Ratios," *PCAS*, XXVIII (1941), p. 140.

$\phi(r)$ = the Table M charge at entry ratio $r = \int_r^\infty (s-r)dF(s)$

$\psi(r)$ = the Table M savings at entry ratio $r = \int_0^r (r-s)dF(s)$

$\triangle\phi(r)$ = the increment on the Table M charge due to superimposing a per accident limit (or otherwise adjusting individual claims)

$$\triangle\phi(r) = \phi^*(r) - \phi(r). \tag{22}$$

Uhthoff describes a convenient method of using excess loss premium factors to calculate approximate incremental charges, which do not vary by entry ratio, but which do vary by state.[4] This method is currently in use in most jurisdictions. In the 1966 updating of the California Retrospective Rating Plan the C.I.R.B. computed incremental charges that did vary by entry ratio. Although the incremental charges actually used may be approximate, the formula by which they modify the rating values can be precise.

Under the usual methodology the incremental charge $\triangle\phi(r)$ is estimated, and then the basic (including excess loss) premium ratio is taken as

$$1 - D - CE/P + C[\phi(r_G) - \psi(r_H) + \triangle\phi(r_G)]E/P. \tag{23}$$

Formula (23) is evidently not exact since it is unequal to the basic premium ratio as defined in equation (17). While formula (23) takes into account the incremental effect of a per accident limit on the Table M charge, it fails to include the incremental effect of a per accident limit on the Table M savings. It will be shown that the incremental savings, $\psi^*(r) - \psi(r)$, equals the incremental charge, so that formula (23) can be corrected by subtracting the incremental charge at the entry ratio producing the minimum premium from the incremental charge at the entry ratio producing the maximum premium.

Theorem 6

$$\phi^*(r) - \phi(r) = \psi^*(r) - \psi(r). \tag{24}$$

Proof: Theorem 1 and its Table M analogue show that

$$\phi^*(r) - \psi^*(r) = 1 - r = \phi(r) - \psi(r). \quad \text{Q.E.D.} \tag{25}$$

[4] D. R. Uhthoff, "Excess Loss Ratios Via Loss Distributions," *PCAS*, XXXVII (1950), p. 82.

Theorem 7

Use of the basic (including excess loss) premium ratio

$$1 - D - CE/P + C[\phi(r_G) + \triangle\phi(r_G) - \psi(r_H) - \triangle\phi(r_H)]E/P \tag{26}$$

will produce a plan that is balanced with respect to losses and expenses.
Proof: From Theorems 3 and 4 it is sufficient to show that

$$I^* = [\phi(r_G) + \triangle\phi(r_G) - \psi(r_H) - \triangle\phi(r_H)]E. \tag{27}$$

Indeed, equations (14) and (24) imply that

$$I^*/E = \phi^*(r_G) - \psi^*(r_H) = \phi(r_G) + \triangle\phi(r_G) - \psi(r_H) - \triangle\phi(r_H). \quad \text{Q.E.D.}$$

In actual practice $\triangle\phi(r_H)$ is small for most retrospective rating plans, so Formula (23) generally provides a good approximation.

3. CONSTRUCTION OF TABLE L

Adjustments to Current Level

The Bureau constructed Table L's for eleven premium size intervals separately for six different per accident limits. All the tables reflect California workmen's compensation experience from policy year 1969 second reports, adjusted to April 1, 1974 rate and benefit levels. A Table M was also constructed from the same data.

Premiums and losses at April 1, 1974 rate and benefit levels were used throughout the construction of the tables. Since all the data came from a single state, it was possible to bring losses to an April 1, 1974 benefit level using separate California benefit increase factors for Death, Permanent Total, Major, Minor, and Temporary claims. Premium was brought to an April 1, 1974 rate level by using a factor reflecting only the portion of the rate level change due to benefit increases and experience. A California permissible loss ratio of .635 was used to estimate the expected loss.

As shown in Exhibit 1, the benefit increase factors were particularly high for Deaths and Permanent Totals, the categories with the largest claims. Consequently, inadequate charges would have resulted if an average benefit increase factor had been used for all types of claims, as was done in the construction of the 1965 Table M. From the standpoint of use in California, another advantage of the 1974 California Table L over the

1965 Table M is that a California permissible loss ratio was used, rather than a countryside average.

The Table L Tabulation

A California Table M was constructed by means of Simon's procedure.[5] The risks were sorted by premium size group. Working with one group at a time, the standard premium P for each risk was multiplied by the permissible loss ratio to obtain the estimated expected loss E. The ratio of the actual unlimited loss to the estimated expected loss was designated R^M. The risks were then sorted on R^M and each premium size group was tabulated as in Exhibit 2. The smallest value of R^M was zero.

In the construction of Table L, losses were limited by substituting the average death indemnity value for the actual indemnity in each death case and truncating at the per accident limit. The same premium size groups, permissible loss ratio and estimated expected loss were used as for the California Table M. Within each premium size group, the ratio of the actual limited loss to estimated expected loss wes denoted E^L. The risks were then sorted on R^L and each premium size group was tabulated as in Exhibit 3. The value of the loss elimination ratio k was based upon all premium size groups combined.

The tabulations for Table M and Table L will now be compared column by column. Note that superscripts ^{L} and ^{M} are used to denote values in, or corresponding to, the tabulations. A subscript denotes the row of the table. The absence of a subscript in a symbol indicates that it represents a theoretical value for an individual task.

[5] L. J. Simon, "The 1965 Table M," *PCAS*, LII (1965), p. 1 ff.

Table M		Table L	
Standard Premium (P_i^M)		Standard Premium (P_i^L)	
Actual Unlimited Loss (A_i^M)		Actual Limited Loss (A_i^L)	
Ratio (R_i^M) $= A_i^M/.635\ P_i^M$	(28)	Ratio (R_i^L) $= A_i^L/.635\ P_i^L$	(33)
Number of Risks (N_i^M) $=$ number with a ratio of R_i^M		Number of Risks (N_i^L) $=$ number with a ratio of R_i^L	
Sum 1 ($S_{1,i}^M$) $= \sum_{j \geq i} N_j^M$	(29)	Sum 1 ($S_{1,i}^L$) $= \sum_{j \geq i} N_j^L$	(34)
Sum 2 ($S_{2,i}^M$) $= S_{2,i+1}^M + (R_{i+1}^M - R_i^M)\ S_{1,i+1}^M$	(30)	Sum 2 ($S_{2,1}^L$) $= S_{2,i+1}^L + (R_{i+1}^L - R_i^L)\ S_{1,i+1}^L$	(35)
Adjusted Ratio (r_i^M) $= R_i^M\ S_{1,0}^M/S_{2,0}^M$	(31)	Adjusted Ratio (r_i^L) $= R_i^L\ (1 - k)\ S_{1,0}^L/S_{2,0}^L$	(36)
Charge (ϕ_i^M) $= S_{2,i}^M/S_{2,0}^M$	(32)	Charge (ϕ_i^L) $= k + (1 - \mathrm{k})\ S_{2,i}^L/S_{2,0}^L$	(37)

Notes: These formulas correspond to the tabulation shown in Exhibit 2 and Exhibit 3. The index i descends in magnitude, going from top to bottom on the tabulations.

Explanation of the Tabulation

Here is an intuitive explanation of why the charge produced by the Table L tabulation is an estimate of the Table L charge as defined by equation (2). The first six columns are the beginning of a Table M calculation based on limited loss. The Table L adjustment factor would be $S_{1,0}^L/S_{2,0}^L$ if one wanted the Table L entry ratio to be a ratio of expected limited losses. Since it is desired that the Table L entry ratio be a ratio to expected unlimited losses, an adjustment factor of $(1 - k)\ S_{1,0}^L/S_{2,0}^L$ is used instead. (Recall $1 - k =$ expected limited losses $\div$ expected unlimited losses.)

The term $S_{2,i}^L/S_{2,0}^L$ would be an appropriate charge for only the limited losses, with this charge represented as a ratio to the expected limited loss. The expression $(1 - k)\ S_{2,i}^L/S_{2,0}^L$ is the charge for limited losses, with the charge now represented as a ratio to expected unlimited losses. Finally, adding k to the $(1 - k)\ S_{2,i}^L/S_{2,0}^L$ includes in the charge a provision for the per accident limitation, also expressed as a ratio to the expected unlimited loss.

Here is a formal explanation of why the charge produced by the Table L tabulation is an estimate of the Table L charge as defined by equation (2). The entries in the tabulation are indexed by i, where i goes up the table from the zero entry. $R^L_{i+1} > R^L_i$ and $R^L_0 = 0$. P^L_i is the amount of standard premium for the risks with Ratio R^L_i. A^L_i is the limited loss for these risks. $\hat{E}^L_i$ the estimated expected unlimited losses for these risks, is taken as $.635P^L_i$, under the assumption that the expected loss is the same for all risks in the premium size group. Other columns are as defined. From the recursive definition of $S^L_{2,i}$ it can be shown by downward induction that

$$S^L_{2,i} = \sum_{j \geq i+1} (R^L_j - R^L_i)\ N^L_j. \tag{38}$$

It is assumed that the mean of the limited loss ratios over a premium size group equals the expected value of the limited loss ratio for any risk in the group, where all these ratios are to the estimated expected loss. That is,

$$\sum_{j \geq 0} (A^L_j/\hat{E}^L_j)\, N^L_j \ / \ \sum_{i \geq 0} N^L_j = E\{A^*\}/\hat{E}. \tag{39}$$

An analagous assumption was made for unlimited losses in the construction of the 1965 Table M. From equation (38) and the fact that $A^L_j/\hat{E}^L_j$ equals R^L_j, it follows that the left-hand side of equation (39) equals $S^L_{2,0}/S^L_{1,0}$.

It is also assumed that the actual loss elimination ratio for all premium size groups combined equals the expected loss elimination ratio for a risk in the group, k. That is, for any risk,

$$k = 1 - \left[\left(\sum_{\substack{\text{all premium} \\ \text{size groups}}} \sum_{i \geq 0} A^L_i \right) \div \left(\sum_{\substack{\text{all premium} \\ \text{size groups}}} \sum_{i \geq 0} A^M_i \right) \right]. \tag{40}$$

This assumption is supported by Exhibit 4, which shows that the percentage of losses eliminated by per accident limitation does not vary by premium size in any meaningful manner.

From equation (39) an expression for the estimated expected unlimited loss of a risk can be obtained by substituting from equations (1), (34), (33) and (38):

$$E = (1-k)E\, S^L_{1,0}/S^L_{2,0} \tag{41}$$

Letting E_i^L denote the expected unlimited loss for the risks in row i, it follows that

$$\hat{E}_i^L/E_i^L = (1-k)S_{1,0}^L/S_{2,0}^L. \tag{42}$$

It is desired that r_i^L be the ratio of actual limited loss to expected unlimited loss. Then

$$\begin{aligned} r_i^L &= A_i^L/E_i^L \\ &= \frac{A_i^L}{\hat{E}_i^L} \times \frac{\hat{E}_i^L}{E_i^L} \\ &= R_i^L(1-k)/S_{1,0}^L S_{2,0}^L \end{aligned}$$

Thus equation (36) is justified.

In order to justify equation (37), the definition of the Table L charge, equation (2), is applied to a particular entry ratio r_i^L.

$$\phi^*(r_i^L) = k + \int_{r_i^L}^{\infty} (s - r^L)dF^*(s).$$

From the two prior assumptions and from the assumption that the actual distribution of limited loss ratios is the same as the theoretical distribution of limited loss ratios, it follows that

$$\begin{aligned} \phi^*(r_i^L) &= k + \sum_{j \geq i+1} (r_j^L - r_i^L) \text{ Prob } \{A^*/E = r_j^L\} \\ &= k + \sum_{j \geq i+1} (r_j^L - r_i^L) \; N_j^L/S_{1,0}^L \\ &= k + (1-k) \sum_{j \geq i+1} R_j^L - R_i^L) \; N_j^L/S_{2,0}^L \\ &= k + (1-k)\; S_{2,1}^L/S_{2,0} = \phi_i^L. \end{aligned}$$

This justifies equation (37).

4. NUMERICAL PROPERTIES OF TABLE L

The Table L charge is a function of entry ratio and premium size. The asymptotic properties of this charge are important for extrapolating it to those risks of premium size above the average of the largest size group or below the average of the smallest size group. These properties can be inferred from the properties of the Table M charge and of the incremental charge, described by Dorweiler.[6]

For a given premium size, the Table L charge approaches the loss elimination ratio as the entry ratio goes to infinity. The asymptotic behavior of the charge for a fixed entry ratio depends upon whether the entry ratio is smaller or larger than the complement of the loss elimination ratio. For a fixed entry ratio r, as the premium size approaches infinity, the charge approaches

$$\begin{cases} k, & \text{if } r \geq 1 - k \\ 1 - r, & \text{if } r < 1 - k. \end{cases}$$

For a fixed premium size, the charge approaches unity as the entry ratio approaches zero. For a fixed entry ratio, the charge also approaches unity as the premium size approaches zero.

Exhibit 6 is a graph of California Table L charges for a per accident limit of $25,000. It can be seen that the charge is a monotone decreasing, concave function of the entry ratio and a monotone decreasing function of premium size.

Exhibit 5 lists comparative insurance charges from the 1974 California Tables L and M and the 1965 and 1972 countrywide Table M's of the National Council. California charges from each size group were applied only to the average premium size for the group. Charges for other premium sizes were interpolated from these average values.

The charges in the 1974 California Table M are much higher than the charges in either National Council Table M. The higher California charges reflect a higher variation in loss ratio for risks of a given premium size, which may be the result of higher benefits. The differences between the California Table L and Table M charges are much smaller than the loss elimination ratios shown in Exhibit 4, due to the overlap, discussed in §2. It is apparent that the use of an incremental charge that does not vary by premium size results in the overcharging of small risks and the undercharging of very large risks.

In some instances the California Table M charge is a little higher than the corresponding California Table L charge. The cause of this

[6] P. Dorweiler, op. cit., p. 133 ff.

slight incompatibility is that each table was compiled on an individualized basis. In the construction of Table M an estimated expected loss ratio of .635 was initially used, but each premium size group had its figures adjusted to reflect the actual unlimited loss ratio of the group. This is the procedure used by Simon.[7] Similarly, each premium size group of each Table L had its figures adjusted to reflect the actual limited loss ratio of the group and the loss elimination ratio for all size groups combined. This was regarded as the most accurate method for constructing Table L, although the anomaly suggests that a more accurate Table M could be constructed if a special adjustment were made to correct for any irregularity in the distribution of large losses from one size group to another.

If each size group had been allowed to determine its own loss elimination ratio by using the formula $1 - S^L_{2,0}/S^M_{2,0}$, then formula (36) would have been replaced by

$$r^L_i = R^L_i S^L_{1,0}/S^M_{2,0} \tag{44}$$

and formula (37) would have been replaced by

$$\phi^L_i = 1 - (S^L_{2,0}/S^M_{2,0}) + (S^L_{2,0}/S^M_{2,0}). \tag{45}$$

Table L's for various per accident limits produced using these formulas would be consistent with each other and with the Table M actually produced.

5. CONCLUSION

From a mathematical point of view, Table L represents an advance over Table M. Every important Table M formula has an appropriate Table L generalization. The Table L versions are stronger than the Table M versions, since Table M is a special case of Table L.

From a practical point of view a Table L should produce more accurate rating values than a Table M. An incremental charge that does not vary by entry ratio and risk size does not take into account variation in the overlap between per accident limitation and overall loss amount limitation. Table L takes this variation into account. A retrospective rating plan constructed from Table L automatically includes the effect of the incremental savings. A Table L can be adapted to a retrospective plan that requires special adjustments of individual cases. Table L is no more difficult to con-

[7] L. J. Simon, op. cit., p. 4.

struct than Table M, if the data base includes individual large losses. Retrospective rating plan values can be found as easily from a Table L as from a Table M, using equations (20) and (21). The use of a Table L also helps by making excess loss premium factors unnecessary. It follows that Table L is preferable to Table M for any retrospective rating plan with a fixed per accident limit.

Even for a plan with a choice of per accident limits, it may be desirable to develop a set of Table L's corresponding to the various per accident limits in order to obtain more accurate insurance charges. Formulas (44) and (45) can be used to construct a consistent set of Tables. Such a set of Table L's would provide insurance charges that fully reflect the effect of premium size, entry ratio and per accident limit.

Acknowledgements

The adjustments to current level, the construction of the California Table M and the idea of producing a Table L are due to Lester B. Dropkin. Mr. Dropkin also provided advice and criticism during the construction of Table L and the writing of this article. Miles R. Drobisch and Robert E. Meyer made important actuarial contributions throughout the entire project.

APPENDIX

Table of Symbols

A = The actual unlimited losses

A^* = the actual limited losses

$E\{\cdot\}$ is the expectation operator

E = $E\{A\}$

F = the cumulative distribution function of A/E

F^* = the cumulative distribution function of A^*/E

k = the loss elimination ratio

$\phi^*(r)$ = the Table L charge

$\psi^*(r)$ = the Table L savings

R = the retrospective premium

B = the basic premium ratio

P = the standard premium (before expense gradation)

C = the loss conversion factor

G = the maximum premium

H = the minimum premium

L_G = the actual limited losses that will produce the maximum premium

r_G = L_G/expected loss

L_H = the actual limited losses that will produce the minimum premium

r_H = L_H/expected loss

L = the losses which will produce the retrospective premium

I* = the net Table L insurance charge

D = the expense gradation, expressed as a ratio to P

$\phi(r)$ = the Table M charge

$\psi(r)$ = the Table M savings

$\Delta\phi(r)$ = the increment on the Table M charge due to superimposing a per accident limit

A superscript ^{M} and subscript $_i$ refer to the ith row of the Table M tabulation, for a particular size group.

P_i^M = the standard premium

A_i^M = the actual unlimited losses

E_i^M = the estimated unlimited losses

E_i^M = the expected unlimited losses

N_i^M = the number of risks

$S_{1,i}^M$ = Sum 1

$S^M_{2,i}$ = Sum 2

r^M_i = the adjusted ratio

ϕ^M_i = the Table M charge

A superscript ^{L} and subscript $_i$ refer to the ith row of the Table L tabulation, for a particular size group.

P^L_i = the standard premium

A^L_i = the actual limited losses

E^L_i = the estimated unlimited loss

E^L_i = the expected unlimited loss

N^L_i = the number of risks

$S^L_{1,i}$ = Sum 1

$S^L_{2,i}$ = Sum 2

ϕ^L_i = the Table L charge

Exhibit I

FACTORS USED TO DEVELOP POLICY YEAR 1969 PREMIUM AND LOSSES TO 4/1/74 RATE AND BENEFIT LEVEL

A. Policies effective 1/1/69-9/30/69

Premium:	1.312*		
Losses:	Indemnity		
		Death	2.086
		Perm. Total	2.016
		Major	1.411
		Minor	1.169
		Temporary	1.434
	Medical		1.063

B. Policies effective 10/1/69-12/31/69

Premium:	1.341*		
Losses:	Indemnity		
		Death	2.086
		Perm. Total	2.016
		Major	1.411
		Minor	1.169
		Temporary	1.434
	Medical		1.043

C. The factor to be applied to the adjusted premium to derive the adjusted expected losses is .635.

*These factors reflect only the changes due to experience and benefit levels.

Exhibit 2

TABLE M TABULATION

Per Accident Limit None
Premium Group $50,000-$74,999

P^M Standard Premium	A^M Unlimited Losses	R^M Ratio $A^M/.635\,P^M$	N^M No. of Risks	S_1^M Sum 1	S_2^M Sum 2	r^M Adjusted Ratio	ϕ^M Table M Charge
52560	613844	18.39	1	1	.00	19.69	.0000
67149	359698	8.44	1	2	9.95	9.04	.0122
55952	252361	7.10	1	3	12.63	7.60	.0155
66066	284590	6.78	1	4	13.59	7.26	.0167
54224	233166	6.77	1	5	13.63	7.25	.0168
62008	257002	6.53	1	6	14.83	6.99	.0182
52908	212986	6.34	1	7	15.97	6.79	.0196
64705	218974	5.33	1	8	23.04	5.71	.0283
60916	199527	5.16	1	9	24.40	5.52	.0300
54882	169679	4.87	1	10	27.01	5.21	.0332
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
129538	83761	1.02	2	297	248.26	1.09	.3052
337975	216228	1.01	6	303	251.23	1.08	.3088
61387	39021	1.00	1	304	254.26	1.07	.3126
257966	161917	.99	4	308	257.30	1.06	.3163
60512	37487	.98	1	309	260.38	1.05	.3201
474696	292505	.97	8	317	263.47	1.04	.3239
125436	76567	.96	2	319	266.64	1.03	.3278
269485	162348	.95	4	323	269.83	1.02	.3317
182042	108794	.94	3	326	273.06	1.01	.3357
115205	67993	.93	2	328	276.32	1.00	.3397
170616	99546	.92	3	331	279.60	.99	.3437
170812	98793	.91	3	334	282.91	.97	.3478
169417	97123	.90	3	337	286.25	.96	.3519
242806	137449	.89	4	341	289.62	.95	.3560
170335	95167	.88	3	344	293.03	.94	.3602
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
427583	24645	.09	7	834	736.71	.10	.9056
241566	12210	.08	4	838	745.05	.09	.9159
118387	5524	.07	2	840	753.43	.07	.9262
354826	13465	.06	6	846	761.83	.06	.9365
517000	16238	.05	9	855	770.29	.05	.9469
251374	6484	.04	4	859	778.84	.04	.9574
421285	8971	.03	7	866	787.43	.03	.9680
186661	2124	.02	3	869	796.09	.02	.9786
69061	253	.01	1	870	804.78	.01	.9893
73929		.00	1	871	813.48	.00	1.0000
53246049	31504086		871				

Exhibit 3

TABLE L TABULATION

Per Accident Limit $25,000
Premium Group $50,000-$74,999

P^L Standard Premium	A^L Limited Losses	R^L Ratio $A^L/.635\,P^L$	N^L No. of Risks	S_1^L Sum 1	S_2^L Sum 2	r^L Adjusted Ratio	ϕ^L Table L Charge
64705	155790	3.79	1	1	.00	4.13	.1244
72430	154658	3.36	1	2	.43	3.66	.1249
119890	233142	3.06	2	4	1.03	3.33	.1257
54882	103120	2.96	1	5	1.43	3.22	.1262
124015	214953	2.73	2	7	2.58	2.97	.1276
52908	90696	2.70	1	8	2.79	2.94	.1279
53381	90782	2.68	1	9	2.95	2.92	.1281
71059	119030	2.64	1	10	3.31	2.88	.1285
54071	90284	2.63	1	11	3.41	2.87	.1287
62370	102733	2.59	1	12	3.85	2.82	.1292
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
135956	88016	1.02	2	268	137.21	1.11	.2960
538802	345005	1.01	9	277	139.89	1.10	.2994
132415	84256	1.00	2	279	142.66	1.09	.3028
500604	314540	.99	8	287	145.45	1.08	.3063
60512	37487	.98	1	288	148.32	1.07	.3099
412211	254086	.97	7	295	151.20	1.06	.3135
249776	152440	.96	4	299	154.15	1.05	.3172
269485	162348	.95	4	303	157.14	1.03	.3209
294239	175722	.94	5	308	160.17	1.02	.3247
246740	145880	.93	4	312	163.25	1.01	.3286
359509	210121	.92	6	318	166.37	1.00	.3325
472991	273365	.91	8	326	169.55	.99	.3365
273313	156216	.90	5	331	172.81	.98	.3405
242806	137449	.89	4	335	176.12	.97	.3447
224497	125549	.88	4	339	179.47	.96	.3489
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
427583	24645	.09	7	834	623.27	.10	.9040
241566	12210	.08	4	838	631.61	.09	.9144
118387	5524	.07	2	840	639.99	.08	.9249
354826	13465	.06	6	846	648.39	.07	.9354
517000	16238	.05	9	855	656.85	.05	.9460
251374	6484	.04	4	859	665.40	.04	.9567
421285	8971	.03	7	866	673.99	.03	.9674
186661	2124	.02	3	869	682.65	.02	.9782
69061	253	.01	1	870	691.34	.01	.9891
73929		.00	1	871	700.04	.00	1.0000
53246049	27161165		871				

Exhibit 4

PERCENTAGE OF LOSSES ELIMINATED BY PER ACCIDENT LIMITATION

Premium Group	No. of Risks	Standard Premium	Unlimited Losses	Percentage of Losses Eliminated by Per Accident Limitation					
				$25,000	$30,000	$35,000	$50,000	$75,000	$100,000
1,312- 1,499	6,662	9,358,863	4,952,782	14.72%	11.28%	8.65%	5.16%	2.77%	1.13%
1,500- 2,499	21,605	41,765,968	21,601,634	11.62	8.95	6.98	4.80	3.73	3.05
2,500- 4,999	20,081	70,432,208	36,033,170	11.07	8.49	6.58	4.49	3.28	2.55
5,000- 7,499	7,919	48,346,275	26,378,640	11.33	8.21	5.93	3.31	1.94	1.23
7,500- 9,999	4,301	37,181,359	20,712,830	10.61	7.57	5.36	2.99	1.78	.93
10,000-14,999	4,412	53,755,635	27,899,522	10.00	7.22	5.21	3.11	2.24	1.77
15,000-24,999	3,704	70,901,480	40,345,585	13.79	11.15	9.21	6.87	5.36	4.33
25,000-49,999	2,627	91,073,043	54,578,452	14.00	11.27	9.30	6.69	4.86	3.68
50,000-74,999	871	53,246,049	31,504,086	13.79	10.87	8.77	6.07	4.65	3.80
75,000-99,999	421	36,319,106	20,427,558	11.31	8.50	6.49	3.94	2.78	2.26
100,000 & over	969	247,044,882	157,077,074	12.65	9.75	7.69	4.94	3.23	2.38
Total	73,572	759,424,868	441,511,333	12.44%	9.61%	7.56%	5.02%	3.52%	2.67%

Note: Limitation of losses includes use of average death indemnity value of $37,400.

Exhibit 5

COMPARATIVE INSURANCE CHARGES FOR SELECTED PREMIUM SIZES

Standard Premium	Entry Ratio	1965 Countrywide Table M	1972 Countrywide Table M	1974 California Table M	1974 California Table L Per Accident Limit of:					
					$25,000	$30,000	$35,000	$50,000	$75,000	$100,000
2,500	1.85	.519		.669	.672	.671	.671	.670	.670	.669
5,000	1.70	.393	.420	.578	.581	.581	.580	.580	.580	.580
7,000	1.64	.351	.376	.525	.528	.529	.528	.529	.528	.528
10,000	1.57	.302	.338	.468	.473	.474	.474	.474	.471	.471
15,000	1.50	.249	.281	.420	.421	.421	.420	.420	.419	.418
25,000	1.43	.212	.221	.355	.347	.347	.346	.346	.348	.347
35,000	1.37	.194	.204	.297	.292	.290	.289	.290	.290	.293
50,000	1.32	.174	.195	.268	.268	.263	.261	.258	.260	.260
67,500	1.27	.164	.186	.234	.243	.236	.231	.230	.229	.229
80,000	1.25	.157	.180	.211	.233	.224	.217	.213	.213	.213
254,948	1.08	.126	.136	.187	.219	.207	.199	.191	.190	.188

Note: The tabulated California Table M and Table L charges were applied at the average risk premium size in the premium group; charges for other premium sizes were derived from these values by linear interpolation.

Exhibit 6

CALIFORNIA TABLE L CHARGE FOR VARIOUS PREMIUM GROUPS
LOSSES LIMITED TO $25,000 PER ACCIDENT