DISCUSSION BY RICHARD I. FEIN AND JEFFREY T. LANGE

In their paper "Underwriting Individual Drivers: A Sequential Approach," Cozzolino and Freifelder have formulated a model of an aspect of the underwriting process. Like any mathematical model, it is an abstraction of reality and must omit many details of the actual process. This simplification may make some aspects of the model unrealistic; on the other hand, reducing one aspect of the process to the bare essentials does provide insights which would be otherwise obscured. As long as the reader recognizes the limitations inherent in the use of any model, the authors' technique is valuable in assessing risk selection rules and in evaluating merit rating schemes. In order to illustrate the practical value of what may appear to be only a theoretical exercise, we have developed two applications of the authors' model.

Their paper is particularly timely with the changes which are being brought about by No-Fault insurance. First, underwriters must consider changes in their risk selection criteria. Second, some rating techniques, such as the Safe Driver Insurance Plan (SDIP), are being changed or even eliminated by Insurance Departments in No-Fault states. The elimination of such rating tools would imply an adjustment in underwriting standards. If the authors' model can be adapted to describe a company's current underwriting actions (with regard to past driving record), then the parameters of the model may be adjusted to the No-Fault situation (e.g., revised claim frequency) so that alternative underwriting decision rules may be evaluated.

Before accepting the conclusion of a model, the reader must decide whether the underlying assumptions are valid for his situation and whether the mathematics have been correctly carried out. In our opinion, the latter condition is satisfied. The former condition must be examined by each reader before he proceeds to adapt the model to his situation. To aid the reader in gaining an understanding of the model, we have drawn a decision tree depicting the authors' example. (See Figure 1.) The "averaging-out" step, in Raiffa's[1] terminology, is simply the expected value of future returns; the "folding back" is the comparison to zero where the probabilities are from the appropriate negative binomial with the updated parameters.

[1]Raiffa and Schlaifer, *Applied Statistical Decision Theory*, The MIT Press, 1961.

The dotted line in Figure 1 separates the single year decision from the multiperiod adaptive decision, in this case based upon two years. In the former the "insure-don't insure" decision is based solely on the sign of the first year expected return. In this case it was negative and hence the risk is rejected.

In the multiperiod case we encounter a probability step with each branch corresponding to the probability of 0, 1, 2, . . . accidents given the parameters (a, b). At the start of the second year we encounter decision boxes which compare the "do not insure" decision with the one year expected value given the updated parameters based on the experience of the preceding year. In this case the multiperiod decision in the first year is to insure since the expected return in the second year exceeds the expected loss in the first year, thus yielding a positive two year expected value:

$$-1.48 + (.907)(5.52) + (.093)(0) = 3.53 > 0$$

The authors' illustration actually considered the three year adaptive decision although the content of the argument is essentially contained in the two year process.

We may now illustrate how the decision model may be used in a practical application: testing the consistency of underwriting rules with the underlying accident probabilities. We may visualize a kind of underwriting path such that if the tracing of the experience of the risk on the tree leaves the path the resulting decision is not to insure. In other words, the path goes through all of the points of profitable (multiperiod) expectations. This may be easily converted into a set of decision vectors which will contain the admissible experience for continued rating. In the case of the example, there is only one such vector, namely (0, 0), corresponding to no occurrences during each of the first two years. In general, for a horizon m, the set of admissible decision vectors would be of the form

$$\{(K_1, K_2, \ldots, K_{m-1}; K_i \geq 0, i = 1, 2, \ldots m - 1\}$$

where

$$V_{m-i}(a + i, b + k, + \ldots + K_{m-i} > 0$$

The same kinds of decision rules could be obtained using $V_m(a,' b')$, that is, using the m year expected return at every step.

Such a decision rule can be thought of as an "underwriting rule" or risk selection criterion. In the simple example cited above, the resulting criterion was to insure if no accidents occur during the experience period but don't insure if there are accidents. (The example does not postulate the existence of a Safe Driver Insurance Plan, as may be the case in some No-Fault states.) Of course, a different choice of parameters would lead to a different, probably more complex, criterion.

As an illustration of a different parameter, one may observe that the ratio of the premium to the average claim cost has an influence on the decision process. In fact, the smaller the ratio, the more sensitive the profit function is to changes in the parameters (a, b). For example—if the average cost is reduced to $400 (more representative of the physical damage line), it can be shown that it is profitable (from a multiperiod view) to insure the risk even if he has incurred two accidents. Since different parameters lead to significantly different decision (underwriting) rules, one might anticipate that the use of the model would challenge underwriting rules for certain classes.

Since the premium and average claim cost vary by line, the decision process should be evaluated by line and the algebraic sum of the expectations should be the determining criterion when the policy includes several lines. Certainly, a different set of parameter values for each line will be used for the same class of risks, further emphasizing the differences in the underlying processes.

In addition to determining underwriting criteria consistent with accident probabilities, the model may be used to check the logical consistency of premium charges resulting from a merit rating scheme. This is naturally dependent upon the appropriate reevaluation scheme employed. A scheme based on some chosen "marginal profit" defined below is possible. For example, using the well worked illustration and the profit function $\pi = P\text{-}Cn$, we may determine P so that V_3 (13.5, 1.37) = O. We call such a P the Marginal Premium *(MP)*. In this case *MP* is $95. Suppose the first year passes, and no accidents occur. We may then determine the appropriate *MP* with respect to V_2(14.5, 1.37), which in this case is $92; if based upon V_3(14.3, 1.37), the three year criterion, the *MP* is $89. While here the difference is slight, it is conceivable that the difference may border on a competitive disadvantage and so the choice of reevaluation could be significant.

In the case that a single accident occurs, recall that using the "insure—don't insure" scheme, we would not insure the risk. We find the *MP* with respect to $V_3(14.5, 2.37)$ (note the updated parameters) to be \$155, an increase of 68%. However, after only one year of claim free experience, the premium drops to \$145, an increase of 53% over the base. A second year of claim free experience indicates a further reduction to a premium of \$137 or 44% above the base. The *MP* does not, in fact, return to the original value of \$95 after three years (compare SDIP) and it takes ten years of accident free experience to return to that level. Under SDIP, the premium remains constant for the three years after the accident, in contrast to the premium variation indicated by the model.

MP has an additional use—given several classifications, and the availability of negative binomial factors, one may determine the necessary differentials among the classes, based upon the ratios of the associated *MP's*.

Models are particularly useful in situations in which actual data are not available as is the present case with the introduction of No-Fault. However, further refinements of the model may be necessary to provide added realism. Even in its present form, we believe it provides insight into the consistency of certain underwriting criteria. In addition, it could be used to construct a theoretical test of surcharges under a merit rating scheme, such as the Safe Driver Insurance Plan.

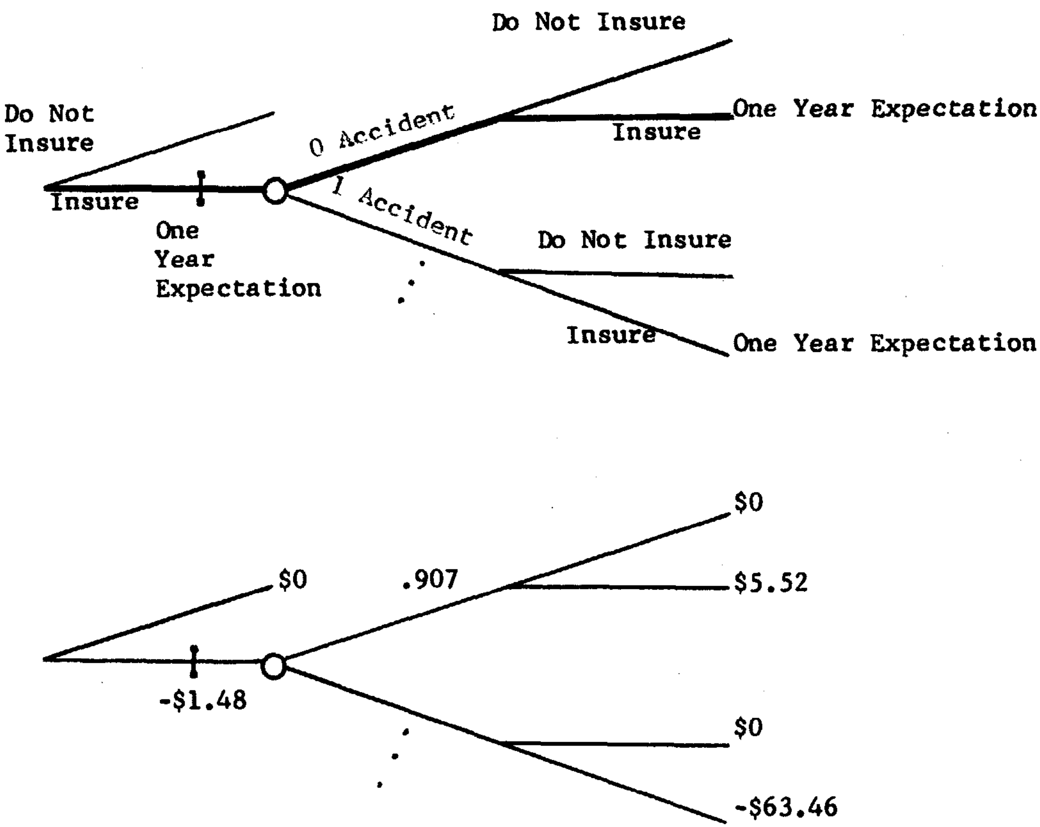
Do Not Insure
Do Not Insure
0 Accident
One Year Expectation
Insure
Insure
1 Accident
One Year Expectation
Do Not Insure
Insure
One Year Expectation
$0
$0
.907
$5.52
-$1.48
$0
-$63.46

FIGURE 1