DISCUSSION BY DONALD A. JONES

Two events that occurred in November 1969 stick in my mind. The University of Michigan football team upset the Ohio State team to become Big Ten champions and the Casualty Actuarial Society and the Society of Actuaries co-sponsored a research seminar based on the decision analysis work of Howard Raiffa and Robert Schlaifer. These concurrent events share the same cell in my memory because my attendance at the latter prevented my witnessing the former!

The Cozzolino and Freifelder approach to underwriting individual drivers is an excellent example of the Raiffa and Schlaifer decision analysis and would have been a highlight of the 1969 research seminar. Some of the seminar presentations were repeated at a 1970 spring meeting of the Society of Actuaries and hence are part of Volume XXII of the *Transactions of the Society.*

When a theoretical model is suggested for an application, the robustness of the model is an important question; i.e., how sensitive are the results to the assumed distribution and its parameter values? To explore this question, I used the authors' gamma-Poisson model with seven pairs of values for the gamma distribution parameters (a, b) chosen so that the mean b/a would equal the authors' 0.10148. These values were $(13.5 \times 10^j, 1.37 \times 10^j)$ for $j = 0$ (the authors' values), ± 1, ± 2, and ± 3. The results for these different parameter values are summarized in Table 1.

Since the standard deviation of the gamma distribution is $\sqrt{b}/a$, the homogeneity of the driver population increases with j. This may be observed in column (4) of Table 1, which shows the standard deviations ranging from 2.742 for $j = -3$ down to 0.00274 for $j = 3$.

As the homogeneity of the driver population increases, we should anticipate the expected value of sample information, *EVSI*, (Column (8), Table 1) and the expected value of sequential decisions over the three year period, $V_3(a, b)$, (Column (11), Table 1) to decrease. Such is the case with these expected values, even being zero for $j = 1$, 2, and 3.

Overall the values in Table 1 show a consistent and monotone pattern that indicates that the gamma-Poisson model reacts well to parameter changes. Perhaps more exploration between $j = 0$ and $j = 1$ would be illuminating.

Let us turn to exploring robustness with respect to the assumed distribution. It would be nice if we could "linearize the problem" to arrive at an analysis that would be distribution-free in the sense of depending on only the first two moments of the underlying distribution. Such is a common approach to credibility theory. Since expected values of truncated random variables, which are the objective of this analysis, depend on more characteristics of the assumed distribution than just the first two moments, linearization is not feasible.

Thus, for a brief exploration of the robustness of the authors' model with respect to the distribution assumption, I calculated the corresponding values under the following assumptions:

(1) The Poisson distribution for the number of accidents was replaced by a Bernoulli distribution:

$$P(n|p, t) = \binom{t}{n} p^n (1-p)^{t-n}, n = 0, 1, \cdots, t$$
$$E(n|p, t) = pt$$

(2) The gamma distribution for λ which described the population heterogeneity was replaced by a beta distribution for p:

$$g(p|c, d) = \frac{\Gamma(c+d)}{\Gamma(c)\Gamma(d)} p^{c-1}(1-p)^{d-1} \qquad 0 < p < 1$$

$$E(p|c, d) = \frac{c}{c+d}$$

For these distributions, the number of accidents in "time" t (the parameter in the above binomial distribution), unconditional on p, for one driver is

$$P(n|c,d, t) = \binom{t}{n} \frac{\Gamma(c+d)}{\Gamma(c)\,\Gamma(d)} \frac{\Gamma(n+c)\,\Gamma(t+d-n)}{\Gamma(t+c+d)} \qquad n = 0, 1, \cdots, t$$

$$E(n|t) = tc/(c+d)$$

$$\text{Variance } (n|t) = tcd(t+c+d)/[(c+d+1)(c+d)^2]$$

For the numerical example I set $t = 1$ (the maximum number of accidents per year), $c = 1.37$, and $c + d = 13.5$, which gave the authors' expected value for n. The corresponding calculations for this model are shown in the "Beta-Bernoulli" line of Table 1. You can see that these values are very close to those given by the authors' distribution, which is probably no surprise since both models give nearly the same marginal distribution for n.

I will close with a couple of less quantitative observations. First, the justification for $\pi_1 = P - cn_1$ might be put in familiar analytical form for casualty actuaries. The underwriting profit from one individual for one year is the random variable

$$\pi_1 = P - (X_1 + X_2 + \cdots + X_{n_1}),$$

where the X_i's are the claim amounts. Under the authors' condition (b) that decisions will be based on expected values, we have $E(\pi_1) = P - E(X_1 + X_2 + \cdots + X_{n_1})$. This last term has been evaluated as $E(X_i)E(n)$ under the authors' condition (a) elsewhere in *PCAS* LV, page 179.

As a resident of Michigan, which has the country's newest No-Fault law, I would ask this gathering of actuaries if the authors' fine paper will be the swan song of individual merit rating theory in the *Proceedings?*

TABLE 1

j	a (1)	b (2)	$E[\lambda]$ (3)	$\sigma[\lambda]$ (4)	$R(a, b)$ (5)	$P(0\|a, b, 1)$ (6)	$EVPI$ (7)	$EVSI$ (8)
−3	.0135	.00137	.10148	2.742	−1.48	.99410	99.41	98.07
−2	.135	.0137	.10148	.867	−1.48	.97125	97.12	85.40
−1	1.35	.137	.10148	.274	−1.48	.92687	92.69	38.65
0	13.5	1.37	.10148	.0867	−1.48	.90674	90.67	5.00
1	135.	13.7	.10148	.0274	−1.48	.90384	90.38	0
2	1350.	137.	.10148	.00867	−1.48	.90353	90.35	0
3	13,500.	1370.	.10148	.00274	−1.48	.90350	90.35	0
Beta-Bernouilli			.10148	.0793	−1.48	.89852	89.85	4.96

j	$V_1(a+2, b)$ (9)	$V_2(a+1, b)$ (10)	$V_3(a, b)$ (11)	$V_2(a, b)$ (12)	$EVSI_3$ (13)	$R_3^*(a, b)$ (14)	$V_3^L(a, b)$ (15)
−3	99.32	197.88	195.23	96.58	100.12	98.64	72.21
−2	93.58	180.71	174.03	83.92	91.59	90.11	63.75
−1	59.10	98.00	89.36	37.17	53.67	52.19	30.89
0	11.61	16.12	13.13	3.52	11.09	9.61	3.42
1	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
Beta-Bernoulli	11.61	16.03	12.92	3.48	10.93	9.45	3.36