DISCUSSION OF PAPERS READ AT THE MAY 1956 MEETING

A REVIEW AND COMPARISON OF WORKMEN'S COMPENSATION EXPERIENCE IN NEW YORK STATE AND WISCONSIN

FRANK HARWAYNE

Volume XLIII, Page 8

DISCUSSION BY W. W. GREENE

This paper demonstrates conclusively that compensation loss cost in Wisconsin per $100 of payroll is substantially lower than the corresponding loss cost in New York in spite of the fact that Wisconsin benefits are over all approximately on a par with New York benefits. It is further demonstrated that this difference in pure benefit cost can be accounted for by lower accident frequency in Wisconsin only to a minor, though by no means neligible, extent.

The fact that differences between states as to actual loss cost are not always or even generally consistent with law differentials based on theoretical valuation of the respective benefit schedules was recognized very early in American compensation rate making, and realization that this situation existed led to the use of what were then termed "reduction factors" or "experience differentials." In deriving these experience differentials the technique initially employed was identical with that used by Mr. Harwayne in his paper.

As far as I know, the present syllabus and recommendations for study do not make any reference to experience differentials. This is doubtless entirely justified by the fact that these devices are no longer used in rate making. However, since Mr. Harwayne has worked out experience differentials between New York and Wisconsin, it would appear to be in order to direct students, or at least those who have a leaning toward the historical perspective, to the part which these gadgets once played in practical rate making.

Undoubtedly Benedict Flynn had in mind the disparity which oft exists between theory and reality when in 1914 he recommended that the first New York compensation rates reflect a differential of 1.90 to be applied to Massachusetts pure premiums, as opposed to a differential of 2.58 which the writer had computed on basis of a strictly desk-chair comparison of benefit schedules, there being of course no actual New York experience at that time. As I recall it, Ben's disagreement with my recommendation was due in large part to his opinion that in the initial stages of the New York Law, employees would not fully avail themselves of its benefits. Evidently subsequent events did not justify his expectation (see Leon Senior's reference

to this matter in his address at the 25th Anniversary Meeting of this Society, P.C.A.S. XXVI pp. 154-155), for, according to Leon the New York 1914-16 experience justified the 2.58 multiplier. Apparently the New York Law was several years old when it was born! In the years which immediately followed it was customary when combining experience from a number of states to employ a factor for the "aging of the act" in conjunction with theoretical law differentials. This procedure constituted recognition, however incomplete, that comparative loss costs cannot be measured solely by a study of benefit provisions.

In the summer of 1918 the writer had occasion to observe a striking example of the difference between theoretic differentials and those based on experience. The theoretic law differential for New Jersey was 98% (Ratio of theoretic New Jersey cost to theoretic cost of the original Massachusetts Act), yet the application of Massachusetts pure premiums to New Jersey payrolls indicated a "direct" experience differential of 64%. The "inverse" calculation indicated 69% (P.C.A.S. VI p. 11).

The writer may have gotten the idea of experience differentials from the late Dr. E. H. Downey. In the Pennsylvania rate revision of 1918 explicit recognition was made of the probable permanency of discrepancies between theoretic differentials and differentials based on experience, for reduction factors based on comparison of actual costs were used to some extent in converting the experience of other states to the Pennsylvania level (P.C.A.S. V p. 243 et seq. "The Revision of Pennsylvania Compensation Insurance Rates, 1918").

In the New Jersey rate revision of 1920 the experience of several states was combined by use of experience differentials. To the best of the writer's recollection these experience differentials were computed by the use of a formula set forth in a paper presented to this Society on November 21, 1919, entitled "Upon Combining Compensation Experience From Several States" (P.C.A.S. VI p. 10 et seq.). This formula, which was referred to by Dr. Downey as "mathematical hocus pocus," provoked some rather spirited discussion, which is to be found in the same volume of the Proceedings beginning with page 310. The use of experience differentials was evidently standard National Council procedure as late as 1926, according to "The 'Permanent' Rate Making Method Adopted by the National Council on Compensation Insurance" (P.C.A.S. XII p. 253). The co-authors, Bill Roeber and the writer, wisely put quotation marks around the word "Permanent!"

Experience differentials or reduction factors (as they were sometimes called) served a useful purpose in the early days of compensation rate making, but I thought these once familiar and friendly gimmicks had passed into the "limbo of forgotten things" until I read Mr. Harwayne's paper!

I hope that these remarks, though not necessarily of any immediate practical value, may prove entertaining to those who are interested in the ancient history of our business.

DISCUSSION OF PAPERS READ AT THE NOVEMBER 1956 MEETING

THE RATE LEVEL ADJUSTMENT FACTOR IN WORKMEN'S COMPENSATION RATEMAKING

MARTIN BONDY

Volume XLIII, Page 106

DISCUSSION BY M. G. McDONALD

The evaluation by Mr. Bondy of the effectiveness of the Rate Level Adjustment Factor in New York has stimulated considerable research on the part of the industry as well as in several State Departments.

The National Council completed an analysis of the policy year experience for eight states which indicated the incurred loss ratio came closer to the permissible in 24 policy years out of 42 (57%). This data includes results as of the first reporting, which were not considered conclusive. However, omitting first reporting indications, the incurred loss ratio came closer to the permissible in 19 cases out of 34 (56%). The adjusted loss ratio, as defined in Mr. Bondy's paper, was closer to the permissible in 43% and 44% of the cases respectively. Tests in Massachusetts indicate similar results.

When the rate level adjustment factor was first introduced in Massachusetts in early 1950, the neutral zone idea was incorporated in the formula, and the use of the neutral zone was disapproved by the Deputy Commissioner who conducted the Hearing. The rate level adjustment factor employing the neutral zone was .987, without it .977.

A test indicates Mr. Bondy's "New" Rate Level Adjustment Formula would have produced slightly better rate levels in Massachusetts than the formula employed. However, in the case where the "New" formula produces a higher rate level than the old, most Departments will be reluctant to approve of the change.

In those states where the present methods produce loss ratios deviating substantially from permissibles, it is suggested that further study be given to other elements in the rate making which have greater effect than the rate level adjustment factor, such as the cost of law amendments as compared with the valuation employed in the revision and the development of losses.

DISCUSSION BY G. B. ELLIOTT

Mr. Bondy's paper discusses one of the elements in the ratemaking procedure for workmen's compensation insurance as used in New

York and most other states. The Rate Level Adjustment Factor is a subject which has received the continuing study of ratemaking bodies and regulatory authorities alike, particularly during the past ten years. Its history and development have been summarized a number of times, most recently in the 34th Annual Report of the National Council on Compensation Insurance (pages 9-13).

Mr. Bondy's thesis is that the Rate Level Adjustment Factor, as used in the New York ratemaking procedure, has tended to distort, rather than to improve, the ratemaking process, and his paper contains several exhibits in support of this point. The experience used in his analysis was that contained in the New York Board's rate filing of July 1, 1956, namely that of the five policy years beginning July 1, 1948. The experience is broken down into ten six-month periods, and an analysis of this experience indicates why the Rate Level Adjustment Factor has had what might be termed an undesirable influence on the results for some of the periods. For the first six periods the unadjusted loss ratios were at or slightly above the permissible, ranging from .563 to .609. There followed a sharp and continuing drop in loss ratios for the four latest periods: .504, .464, .414 and .411. Thus, the Rate Level Adjustment Factors based on the unfavorable experience of the earlier periods (presumably the calendar year experience was also unfavorable, although this experience was not exhibited) produced an increase in premiums for some of the later periods, making the loss ratios lower than they would otherwise have been. There is some question as to whether the effect of the Rate Level Adjustment Factor would have been as noticeable if this sharp change in loss ratios had not been experienced. It would be of interest if Mr. Bondy or some other member of the Society would apply the same method he has used to analyze the effect of the Rate Level Adjustment Factor on the experience in a number of other states.

It is interesting to note that the removal of the effect of the Rate Level Adjustment Factor, while having a noticeable effect on the loss ratios for some of the individual periods, makes only a minor change in the loss ratio for the entire five-year experience period. That is, the loss ratio on a reported basis for the entire period was .520; after application of the loss development factors and removal of the effect of the Rate Level Adjustment Factor, the loss ratio becomes .528—a difference of only .008

Mr. Bondy's comments with respect to the unreliability of calendar year experience are well-taken. This fact has been recognized, to a greater or lesser degree, by the manner in which such experience has been used in determining indicated changes in rate level. Some years ago the effect of calendar year experience was minimized by the establishment of a "neutral zone", while in more recent years such experience has been given a weight equal to the permissible loss ratio. This procedure was further modified last year in National Council states by assigning equal weights to the calendar year and policy year loss ratios, and it is understood that this modified proce-

dure will be used in the July 1, 1957 rate revision in New York. Even this latest change may not be sufficient to give full recognition to the unreliability of calendar year experience, and Mr. Bondy's suggested change in procedure would seem to warrant careful study. However, in order to make such a study, a more complete exposition of Mr. Bondy's proposal would be helpful, at least to this writer.

It is not clear just how the formula outlined in the paper would be used in actual practice. Mr. Bondy sets forth the following conditions:

$$\begin{aligned} \text{Permissible Loss Ratio} &= .565 \\ \text{Maximum Credibility} &= .40 \\ \text{Maximum RLAF} &= 1.10 \\ \text{Minimum RLAF} &= .90 \end{aligned}$$

To determine the loss ratio necessary to produce the maximum Rate Level Adjustment Factor of 1.10, the following formula is used:

$$1.10 = \frac{.40 \text{ Loss Ratio (Max.)}}{.565} + .60$$

$$\text{Loss Ratio (Max.)} = .706$$

The Neutral Zone is then derived in the following manner:

$$\begin{aligned} \text{RLAF} &= \text{Loss Ratio} - \text{PLR} \pm \text{NZ} + 1 \\ 1.10 &= .706 - (.565 \pm \text{NZ}) + 1 \\ \text{NZ} &= \pm .041 = \pm .040 \text{ (rounded)} \end{aligned}$$

An attempt was made to test the formula by assigning a credibility of .20, keeping the three other conditions constant. However, this calculation produced a maximum loss ratio of .848 and a neutral zone of ± .183. It therefore seems obvious that the formula was not intended to be applied in this way, since the stated objective is to increase the credibility as the deviation from the "normal", or permissible, loss ratio increases; whereas in the calculation just mentioned, the assignment of a lower credibility resulted in a higher loss ratio, that is, a greater rather than a lesser deviation from normal.

Another approach was then tried, based on the assumption that as the credibility increases from 0 to .40 the Rate Level Adjustment Factor increases from 1.00 to 1.10. For example, a Rate Level Adjustment Factor of 1.05 would correspond to a credibility of .20. Inserting these values in the above formula produces the same maximum loss ratio of .706, with a neutral zone of ± .091. These results, too, are rather puzzling and it is hoped that Mr. Bondy may be able to shed some light on just how the formula is intended to be applied.

Some comment seems to be appropriate on the neutral zone produced in the example outlined above. If the permissible loss ratio is .565, a neutral zone of ± .04 means that if the calendar year loss ratio lies between .525 and .605, the Rate Level Adjustment Factor would be unity. In an extreme case, therefore, it would be possible for the loss ratio to increase by 8 points in a single year without any

recognition being given to this fact in the ratemaking procedure. Sooner or later the increase would presumably be reflected in the policy year experience, but since one purpose of the Rate Level Adjustment Factor is to recognize trends beyond the policy year experience, it would appear that such a wide neutral zone would tend to defeat this purpose. This criticism could be eliminated, of course, simply by modifying the formula so as to produce a narrower neutral zone within which the Rate Adjustment Factor would be unity.

It is to be hoped that Mr. Bondy and others interested in the workmen's compensation ratemaking procedure will continue to study the problem of how best to use calendar year statistics, and that his proposed procedure will be thoroughly tested in order to determine whether it will result in improved ratemaking methods.

AUTHOR'S REVIEW OF DISCUSSION

MARTIN BONDY

Mr. Elliott's interesting discussion reveals, among other things, that I have not gone into sufficient detail in describing the Neutral Zone formula. I shall attempt to do this in the following paragraphs.

The most important thing to keep in mind is that certain values are to be fixed in advance and do not change from year to year. This is true of any formula we may decide upon. For example, under the existing Rate Level Adjustment Factor formula, we set the condition that the credibility to be allowed is 50%. This is not a value which will change annually. It is based upon certain underwriting considerations and is expected to remain in force until these no longer apply.

Similarly, in the example given in my paper, I have set two conditions. They are:

1. The *maximum* credibility to be allowed is 40%.
2. The *maximum* effect on rate level produced by the Factor is 10 points.

Given these two fixed conditions, the remaining elements of the formula are automatically derived. To determine the loss ratio which will produce the maximum Factor, the formula cited by Mr. Elliott is used:

$$\text{Max. RLAF} = \text{Max. credibility} \times \frac{\text{Loss Ratio underlying max. RLAF}}{\text{Permissible Loss Ratio}} + (1.0 - \text{Max. credibility}) \times \text{Unity}$$

Substituting

$$1.10 = \frac{.40 \times \text{Loss Ratio underlying max. RLAF}}{.565} + .60$$

Solving

$$\text{Loss Ratio underlying max. RLAF} = .706$$

From this, the Neutral Zone is uniquely determined by using the relationship

$$\text{RLAF} = \text{Loss Ratio} - \text{Permissible} \pm \text{NZ} + 1$$
$$1.10 = .706 - (.565 \pm \text{NZ}) + 1$$
$$\text{NZ} = \pm\ 0.41$$

To summarize, the requirements that maximum Rate Level Adjustment Factor shall equal 1.10 and maximum credibility shall be 40% will produce the following formula:

$$\text{RLAF} = \text{Loss Ratio} - .565 \pm .04 + 1.0$$

The credibilities granted under this formula run according to the following table:

Loss Ratio *	*Credibility* (%) **
.565–.605	0
.615	11
.625	19
.635	24
.645	28
.655	31
.665	34
.675	36
.685	38
.695	39
.705	40

* This is a symmetrical table about .565

$$** \text{ Credibility} = \frac{\text{RLAF} - 1}{\frac{\text{Loss Ratio} - 1}{\text{PLR}}}$$

As Mr. Elliott points out, a Neutral Zone of 4 points on each side of unity may be somewhat insensitive. This is one of the underwriting considerations which must be taken into account in setting up a formula of this type. While I feel that a swing of a few points in calendar year experience is not necessarily significant, nevertheless conditions assigned may be too stringent. It should be kept in mind that this was only used as an example. If the maximum credibility were taken at 50%, the resulting formula would have a Neutral Zone of about 1.5 points on each side of unity.

Concerning Mr. Elliott's comment on the first section of the paper, it appears in order to elaborate on certain points which have not been made sufficiently clear. In the very first place, my thesis is that the Rate Level Adjustment Factor represents *no improvement* in the rate structure. As a matter of fact, in New York for the period exhibited, the consequences of using this Factor were inferior rates. I do not believe that the continuance of the Factor would have been justified if there were neither deterioration nor improvement flowing from it. In that case, it would be sort of a neutral Factor. The Factor was introduced not to be neutral but to be of positive assistance in setting the rate level. Moreover, it should be of the greatest utility in times of stress and change. When conditions are on an even keel, there is no urgent need for the introduction of such a Factor.

It may interest the reader to know that shortly after the presentation of the paper, the effects of the Factor were tested in some states other than New York. In the 34 cases where a Rate Level Adjustment Factor formula with no Neutral Zone was used, the rate level was better in 17 cases and worse in 17 cases than if no Rate Level Adjustment Factor had been used. Of the 16 cases where the

Factor fell within the Neutral Zone (4 points), the rates were improved in 5 cases and made worse in 11. This would tend to reinforce the belief that a calendar year rate level falling close to the permissible should not be used as a forecasting device. It has been pointed out that the average reported loss ratio for the entire period (presented in the paper) was .520. The average adjusted loss ratio was .528. These are indeed close. However, it is not surprising that any fairly reasonable and unbiased method would produce answers which, over the long pull, hover about the permissible loss ratio. A better test of the efficacy of the procedure would be to compare the average variation about the permissible from year to year.*

*For the years reviewed, the average variation of the actual loss ratios exceeded that of the adjusted figures.

In conclusion, I should like to offer my sincere thanks to Mr. Elliott for pointing out the shortcomings in those areas which required clarification of presentation.

CURRENT RATE MAKING PROCEDURES FOR AUTOMOBILE LIABILITY INSURANCE

PHILIPP K. STERN

VOLUME XLIII, PAGE 112

DISCUSSION BY T. E. MURRIN

A paper on automobile liability insurance ratemaking has been long overdue in appearing in the Proceedings of the Casualty Actuarial Society. Although the need for a paper on this subject has been felt for many years by students particularly, it is welcome as a basic reference for insurance men as well. That the task of writing the paper fell to Mr. Stern is a happy coincidence because of his native ability for clarity of expression and logical discussion. His paper, augmented by many illustrative exhibits, is a valuable contribution to the Society Proceedings.

Mr. Stern's stated purpose of composing a descriptive presentation for the student without any evaluation of ratemaking procedures disarms the reviewer to some extent because controversial matters are thereby avoided. As this paper will be a source of information for students principally, my observations are intended primarily to clarify what Mr. Stern has left unexplained or unsaid rather than criticize what he has said. In reading the paper I tried to keep myself in the place of the student and not read between the lines or recall unsaid things that are familiar to most members of this Society.

As he mentioned in his opening paragraph, Mr. Stern explains many technical terms that appear in the paper, but not always the

first time they occur. In reading the paper I found many terms, which are common to the jargon of our business used without any definition or explanation, such as, transaction reports, summarized reports, statistical program, specified car basis and Fleet Plan.

In discussing ratemaking statistics at the beginning of his paper, Mr. Stern rightly explains the importance of ratemaking statistics, citing applicable language of the rate regulatory statutes regarding statistics and statistical plans. The function of statistics would have been brought into sharper focus I believe, if in his opening statement that the loss portion and the expense portion of the rates are based on experience, Mr. Stern had referred to the provision in the rate regulatory laws providing generally, that in determining rates "Due consideration shall be given to past and prospective loss experience within and outside this state, to catastrophe hazards, if any, to a reasonable margin for underwriting profit and contingencies, to dividends, savings or unabsorbed premium deposits allowed or returned by insurers to their policyholders, members or subscribers, to past and prospective expenses both countrywide and those specially applicable to this state, and to all other relevant factors within and outside this state." Only in the last sentence of the paper did he refer to the basic criteria for rates, namely, that rates shall be adequate, not excessive and not unfairly discriminatory.

My most serious criticism centers on Mr. Stern's presentation of expense provisions in the manual rates and the expected loss ratio. It is unfortunate that Mr. Stern made only a brief reference to this phrase of the ratemaking process as he indicates earlier in his paper. He states " 'the expected loss ratio' . . . represents the portion of the premium dollar available for losses . . . after the requirements for expenses including a stated provision for underwriting profit and contingencies are met." Would that this were literally true! For the benefit of the student Mr. Stern should have accorded fuller discussion to this important element in the manual rates which accounts for a sizeable portion of the premium dollar. In addition to providing for loss payments, the premium dollar also provides for the expenses of selling, underwriting and servicing insurance policies and for taxes. The provision for underwriting profit and contingencies is only theoretical. There will always be losses and expenses but not so with the margin for underwriting profit and contingencies. The nominal margin for underwriting profit and contingencies is solely theoretical because if the losses and expenses combined exceed the premiums, there is no profit and the difference must come out of company surplus.

Mr. Stern correctly points out that expense provisions are determined from countrywide data as reported in the Insurance Expense Exhibit and then unfortunately proceeds to show the New York provisions as being different from the provisions applicable in other states except for the production cost allowance, without making any comment on the differences. In addition, the unallocated loss adjustment item is shown as an expense and expressed as a percentage of

premium in the breakdown of the premium dollar in New York. In the exhibit of "standard" provisions, the unallocated loss adjustment item is expressed in terms of losses and is relegated almost to obscurity in the footnote applicable to the expected loss and loss adjustment ratio. I am afraid that the student will have considerable difficulty in understanding this important element in the ratemaking procedure and be unable to comprehend that the difference between the expected loss ratio in New York and the higher expected ratio applicable to other states, is due almost entirely to the fact that the former ratio excludes, and the latter ratio includes, unallocated loss adjustment expenses. Those familiar with automobile liability ratemaking in New York and other states know that unallocated loss adjustment is handled with losses and expressed as an expense item in terms of premium only in New York and is treated as a function of losses and included with them in ratemaking in all other states. The slight differences in the provisions for administration, inspection, audit and bureau, between New York and other states reflect the unique New York requirement that this provision be reduced slightly to offset the additional dollars that would be collected for these items if the extra assessments for administering the Security Fund and the Safety Responsibility Act were loaded in the rates as a flat percentage. Also it should be pointed out that the basic provision for underwriting profit and contingencies in New York is 3.5% which is less than the standard provision of 5.0% effective in 43 other states and the District of Columbia.

Mr. Stern goes into considerable detail in discussing the current private passenger plan and the preceding plans. In order to understand the change in classification differentials in his example, it was admittedly necessary for Mr. Stern to explain briefly the present plan and the immediate preceding plan. The tremendous amount of additional detail which is not relevant to the topic of the paper will confuse the student, and what is worse, might discourage him from reading the full paper. It would have been far more preferable in my opinion to eliminate the detailed discussion on differences in various classification plans and devote that space to a fuller treatment of the important element of expense provisions.

In his discussion of the statewide rate level Mr. Stern mentions that incurred losses in excess of basic limits are excluded from the experience used in basic manual ratemaking. In his definition of excess losses he also touches the fundamental distinction between basic limits and excess limits losses. Nowhere in his paper however, does he explain clearly that basic limits rates (whether for 5/10 limits or 10/20 limits) are based on the experience in the state for this portion of the coverage on all policies and that the excess limits coverage above basic limits is reviewed separately, at longer intervals than for basic limits coverage, on essentially a countrywide basis in accordance with the applicability of the excess limits tables.

Perhaps it would have been better to add the words "in New York" to the title of the paper and eliminate all reference to what is done

outside of New York because the differences are essentially matters of detail and pointing them up in the paper can confuse rather than clarify the matter in the minds of students. For example, territory relativities, are based on the three latest years in New York and Mr. Stern refers to the use of five years in other states in his discussion of territorial combinations. Furthermore, the section on statewide rate level is unduly complicated by the discussion of earned factors and loss development for the increment of coverage between $5,000/10,000 and $10,000/20,000 in New York. In this connection, Mr. Stern also seems to subordinate the importance of the development of claim costs and claim frequencies to the rate at which exposures and premiums are earned in the development of the earned factor. In addition, the fact that New York State is treated as two states (the three boroughs of New York City and the balance of the state) and the added complication of reflecting the offset for the Preferred Risk Rating Plan in the development of present average rates will add to the bewilderment of the uninitiated.

While Mr. Stern specifically noted many exceptions to the general procedure that he was discussing, for the most part they were so minor that they could have been omitted without hindering the students' understanding of the subject.

As I have mentioned earlier in this discussion, my remarks were intended to supplement what Mr. Stern has said and clarify some parts of his paper in the hope that students would benefit. They were not intended to criticize Mr. Stern's careful handling of a technical and complicated subject. The Proceedings of our Society are richer by the addition of his fine paper which will be beneficial to students and others who consult the Proceedings for knowledge.

DISCUSSION BY E. T. BERKELEY

This paper, like Mr. Marshall's recent paper on Workmen's Compensation rate-making, has been written primarily for actuarial students, particularly those who are preparing themselves for Part IV (b) of the Associateship Examinations of the Society, which covers the general principles of rate-making.

Papers of this type are very welcome for they make readily accessible to the students authentic information relating to the fundamental rate-making methods utilized in two of the major classes of business. Mr. Stern deserves a vote of thanks for the time and thought he obviously had to devote to the preparation of this paper and the excellent product he succeeded in turning out.

Mr. Stern develops a logical explanation of the various steps in the rate-making process as respects bodily injury and property damage rates, drawing upon the latest New York rate revision for illustrative exhibits. He explains the source of statistics, defines terminology and then sets forth the basic steps in a rate revision in detail, namely:

A. Determination of state-wide rate level.

B. Development of rate-level change by territory.
C. Calculation of classification rates.

Thus, the conscientious reader should succeed in acquiring a satisfactory knowledge of the method used by the National Bureau of Casualty Underwriters and the Mutual Insurance Rating Bureau for the making of automobile liability rates in a state like New York.

When I first went through this paper I thought it might have been improved by the inclusion of comments on the evolution of the various procedures, the reason for the adoption of particular methods and similar phases of the subject, but a second and more careful reading convinced me that the material presented by Mr. Stern is adequate for the immediate needs of the reader concerned with the general principles of rate-making. Later on, after a few years of general experience in the business and dealing with actuarial problems, the student should have developed a more mature viewpoint, permitting a fuller appreciation of the historical aspects of the rate-making procedure, which should be covered properly and more effectively, I believe, in another paper.

In such a paper, besides some of the matters just mentioned, there could well be included for the benefit of both the casual reader and the student alike a discussion of questions similar to the following, which might have occurred already to the alert and inquisitive mind after reading Mr. Stern's paper:

1. Can the reasons for a developed loss ratio higher or lower than the expected loss ratio be determined and appropriate changes made in the rate-making procedure for future revisions?
2. How reliable are the rates in a state where the member companies of the rate-making organization write only a small portion of the total business?
3. If the provision for underwriting profit and contingencies is required entirely for contingencies, do the companies still make a profit, from interest earnings?

I have been asked questions of this sort numerous times and I feel sure my experience is not unique. Since this indicates a general need for answers other than the discussion of general principles to be found in the Society's references for study in connection with a few such questions, the value of an integrated presentation in a sequel to Mr. Stern's paper becomes apparent.

MONTH OF LOSS DEFICIENCY RESERVES FOR AUTOMOBILE BODILY INJURY LOSSES INCLUDING RESERVES FOR INCURRED BUT NOT REPORTED CLAIMS

DAVID A. TAPLEY

VOLUME XLIII, PAGE 166

DISCUSSION BY N. M. VALERIUS

As a given interval of time moves off into the past, the accidents happening in that time become reported to the insurance company, are estimated as to cost, re-estimated if necessary, and are eventually settled, or closed without payment. In the process, the aggregate incurred loss to the company from those accidents firms up into the ultimate figure. Mr. Tapley's thesis is that this comes about according to a development pattern, primarily dependent on the company's claim practices, that can be studied and relied on for estimating final incurred cost for other later periods of time whose losses have not yet matured.

The paper offers an unorthodox approach to the problem of reserves for incurred but not reported claims. In the first place, it emphasizes that "the interplay of loss transactions" must be recognized, that is, the offset of late reported claims, reopenings and individual claim reserve increases against reserve reductions, settlements, and claims closed without payments. In other words, it attacks all pluses and minuses with one statistical treatment.

The traditional approach has been to have the statistical, actuarial or accounting departments, that are responsible for the annual statement, estimate the incurred but not reported losses and to hold the claim department responsible for adequacy of reserves of reported cases. Did this custom of divided responsibility give basis for the odd statement in the paper concerning two early treatments of the subject, "neither . . . advanced the premise that the incurred but not reported claim reserve together with reserves for reported losses would offset the total liability of the company for losses incurred but not disposed"? In spite of mandatory schedules of recent years that exhibit and emphasize total incurred loss developments, from all sources, in particular Part 5 of Schedule P of the annual statement, the divided approach persists.

In the second place, the method is unorthodox in operating with month of loss where others operate with year of loss. Furthermore, it has the unexpected result that only very recent months require any reserve for unknowns, that is, in the author's company.

The development pattern is found to be such that only the losses of the last three (shifting to four recently) accident months need any deficiency reserve in addition to individual accident cost estimates. For all more mature accident months, the reserves for known

cases are good estimates of the known and the hidden future liabilities.

The method at the time of writing the detailed description was specifically as follows:

Expected Incurred cost of month just ending = case reserves ÷ .500
Expected Incurred cost of month prior = case reserves ÷ .760
Expected Incurred cost of month next prior = case reserves ÷ .840

Expected Incurred cost of all previous accident months — paid losses plus case reserves.

Under this method, it is necessary to maintain month of loss analyses, of course. These provide valuable current run off tests for any company and are more and more feasible to maintain as electronic equipment comes into use.

The development pattern would preferably be applied, as it is derived, as the pattern of the developing known incurred cost, that is, projection factors would be applied to the sum of paid losses and case reserves instead of using alternate factors applied to case reserves alone. But "it is procedurally difficult to obtain fully detailed data appropriate to the current month of loss in the short time available before monthly closing entries must be made." Remember that breakdown by month of loss is required. Only reported reserves are available in time.

Furthermore, one must estimate the paid losses of the three last accident months, as the actual figures are not available soon enough, in order to derive the deficiency reserve from the equation,

Expected incurred cost — case reserves — estimated paid = deficiency reserve.

The expected paid losses are derived from the development pattern as 6%, 8%, and 12% of ultimate losses for the first, second, and third months respectively. We have, as the combined result:

Deficiency reserve of month just ending = 88% of case reserve
Deficiency reserve of month prior = 21.1% of case reserve
Deficiency reserve of month next prior = 4.8% of case reserve

It interests this reviewer to find Mr. Tapley's new method under the necessity to base hidden liability reserve on the case outstanding. Down through the years in our own company, as we have been criticized from time to time for basing our incurred but not reported reserve on the case outstanding, it has been our clincher that these figures come first to hand and other bases would be too late. It is fair to say now, in passing, that the time of arrival is being affected by the new machines and, in our case, there may be consequent changes.

In an addendum within the paper, the author states that recent higher average claim costs have moved the point of stability out beyond the fourth month. It is noted also that there are large fluctuations in the monthly losses and so in the hidden liability reserve re-

quirement. Possible causative factors are discussed. The author hopes longer acquaintance with the monthly analyses will help to explain what happens.

The author feels quite definitely that an improvement in estimating losses has been achieved. Nevertheless his company continues to keep incurred but not reported estimates at hand. I concur in his feeling that the month of loss analysis separates the loss data into convenient packages from which much can be learned as to loss behaviour. We have been running similar analyses over the same period as a part of information to management and for comparison I show here in the same way as the author's January 1954 losses at the bottom of the first page, the reported incurred losses for the January 1954 month of loss, excluding our New York Office and Massachusetts business.

Date of Evaluation	*Reported Loss Reserves*	*Cumulative Paid Losses*	*Reported Losses*	*Ratios*
1-31-54	$ 822,996	$ 18,375	$ 841,371	.555
2-28-54	1,140,857	88,122	1,228,979	.810
3-31-54	1,228,486	203,943	1,432,429	.944
4-30-54	1,179,911	337,372	1,517,283	1.000
5-31-54	1,148,967	414,795	1,563,762	1.031
6-30-54	1,104,188	493,144	1,597,332	1.053
9-30-54	945,993	699,203	1,645,196	1.084
12-31-54	725,093	924,237	1,649,330	1.087
3-31-55	559,652	1,055,277	1,614,929	1.064
6-30-55	431,865	1,159,005	1,590,870	1.048
9-30-55	380,732	1,207,913	1,588,645	1.047

This paper is stimulating and informative. It does seem to the reviewer, however, that the writer has been too optimistic in his appraisal, being based on so short an acquaintance with the method and its results. He should certainly continue to produce the usual incurred but not reported reserve alongside the subject method, as he is doing.

A theoretical appraisal may be stated as follows, setting down first a restatement of the thesis:

1) The case basis incurred value (paid losses plus estimated unpaid losses) for claims reported up to the point of stability (the end of the 4th or 5th month starting with the month of the accidents) follows a fairly definite pattern, this pattern being characterized by a maximum at the point of stability with some tailing off thereafter.

2) The incurred losses which have not emerged before the point of stability but come to light later are matched by the redundancy in the reserves of known cases being currently closed.

It therefore remains only to determine during the first 3 (or 4)

months a "deficiency reserve," intended to bridge the gap between the case basis value of known claims and the case basis value of known claims at the end of the 4th (or 5th) month. The basis for determining such "deficiency reserves" is to apply a factor to the incurred value of known claims at the end of the first, second, third, etc. months, this factor being determined from a study of past relationships between the values of then known claims at the ends of these early months and the value at the end of the 4th (or 5th) month. Actually the factor is applied to outstanding value of known claims, not the incurred value, because of procedural difficulties.

Mr. Tapley suggests that the pattern of the total incurred value of known claims for this company may not be valid for other companies and that the data from which he derives his "deficiency reserves" during the first 3 or 4 months may also not be valid for other companies. He does not, however, mention what appears to be a basic theoretical flaw in his method, namely, that there is no logical relationship between the patterns of development of the incurred value of reported losses and of the value of incurred but unreported losses. It is implicit in his theory that the incurred value of reported losses be overstated at the end of the 4th or 5th month by the then value of incurred but unreported claims.

Is there a logical reason why this relationship should exist? It is obvious that the basis for this supposed relationship would be destroyed if the claims adjusters who establish the value of known claims refine their estimating processes to a point where the value at the end of the 4th or 5th month is a true ultimate value. In other words, the whole fabric would be destroyed by a change in the estimation practices followed by the adjusters.

Similarly we may criticize the projection of the "deficiency reserve" during the first months from the incurred value of known claims. There is no necessary logical relationship. In fact, any tendency to delay unduly the reporting of claims would tend to decrease reported claims and thereby decrease "deficiency reserve" just at a time when the "deficiency reserve" ought to increase.

While this method of developing "deficiency reserves" and of assuming that incurred but not reported reserves are taken care of by over-estimate of reported reserves after the 4th or 5th month may be valid while conditions continue to follow the pattern they have displayed recently, in general, the deduction of laws from observed phenomena is only valid when it is reasonable to assume that logical relationships exist among the observed phenomena.

DISCUSSION BY L. J. SIMON

Mr. Tapley's paper is very interesting and clearly presented. A second reading is strongly recommended because it will reveal a number of subtle points that may have been slighted in the first reading. The philosophy of the deficiency reserve approach is quite stimulating to the imagination and should provoke some interesting dis-

cussions among actuaries as well as within companies. People concerned with claims procedures and those concerned with developing figures for the financial statements of the company are often not actuaries, and this approach will take a great deal of salesmanship on our part.

My remarks will be chiefly directed to the area of statistical experimentation and testing of hypothesis based on the data presented in Mr. Tapley's paper. To restate the author briefly, the month of loss deficiency reserve is the amount of reserve needed to complement case reserves and paid losses in order to give a complete evaluation of incurred losses for an accident month. The deficiency is due to the company not having complete information on losses which have occurred either because the loss has not been investigated thoroughly enough to permit an accurate case basis estimate to be made, or because the loss has not been reported as yet. To establish this month of loss deficiency reserve one must somehow estimate or predict the total loss for the given accident month, called "base" loss (also referred to by some as "ultimate" loss.) Then by deducting payments to date and case basis reserves outstanding from the predicted "base" loss one arrives at the deficiency reserve for the month in question.

The first phase of this discussion will be to develop regression equations which will be usable in predicting the deficiency reserve for a given month at each stage of its development. The second phase will be an analysis of variance to test the homogeneity of the years and the months. Let me hasten to add immediately that these techniques are by no means suggested as a mathematical substitute for the oft discussed "judgment". Rather they are designed to give us more facts and clearer guides to the correct answer. With more complete actuarial analyses of the facts, we have a fuller knowledge and hence can make better judgments.

To illustrate the use of regression equations in predicting the unknown values, let's define

Y = the "base" loss

P_i = the amount paid to date on a month, i is the age or stage of development and equals 1, 2, . . .

R_i = the case reserves as of the end of the i^{th} month of development where i equals 1, 2, . . .

D_i = the deficiency reserve as of the end of the i^{th} month of development where i equals 1, 2, or 3.

These symbols will carry superscripts of one prime to indicate the predicted value as of the end of month 1, two primes for the month 2 prediction, and three primes for month 3. For example, Y'' is the predicted "base" loss for a certain accident month where the prediction is made just after the close of month 2. As another example, P_3 is the amount paid through the close of month 3 on a certain accident month. In this notation the author uses the following formulae in making his deficiency reserve predictions:

$$Y' = \frac{R_1}{.500} \text{ and } P_1' = .06Y' \qquad \text{so } D_1' = Y' - P_1' - R_1 = .8800R_1$$

$$Y'' = \frac{R_2}{.760} \text{ and } P_2'' = .08Y'' \qquad \text{so } D_2'' = Y'' - P_2'' - R_2 = .2105R_2$$

$$Y''' = \frac{R_3}{.840} \text{ and } P_3''' = .12Y''' \quad \text{so } D_3''' = Y''' - P_3''' - R_3 = .0476R_3$$

To establish equations similar to these using least squares principles, the monthly data for 1954 from Tapley's Exhibits II, V, and VI was recorded in Table I rounded off to thousands. Table II contains the summary statistics, and the only symbol not defined there is N, the number of months in the sample.

We are setting out with an objective of getting the best predictions of Y, P_1, P_2, and P_3 which we recognize will be made by those variables which most closely correlate with them. These correlation coefficients were calculated from the general formula:

$$r = \frac{\Sigma xz}{\sqrt{\Sigma x^2 \cdot \Sigma z^2}}$$

As a test of significance (see reference 1, page 193) :

$$t = \frac{r(N-2)}{\sqrt{1-r^2}} \qquad \text{with } (N-2) \text{ degrees of freedom.}$$

The results are as follows:

To be used for month:	*Variables*	*r*	*t*	*Probability that r is zero is less than:*
1	R_1 & Y	.855	5.21	.001
1	R_1 & P_1	.751	3.59	.01
2	R_2 & Y	.969	12.42	.001
2	(P_1+R_2) & Y	.702	3.12	.02
2	P_1 & Y	.974	13.53	.001
2	P_1 & P_2	.478	1.72	.2
2	R_2 & P_2	.333	1.12	.3
2	(P_1+R_1)&P_2	.462	1.65	.2
2	(P_1+R_2)&P_2	.344	1.16	.3
3	R_3 & Y	.964	11.48	.001
3	P_2 & Y	.470	1.68	.2
3	(P_2+R_2)& Y	.980	15.56	.001
3	(P_2+R_3)& Y	.990	22.00	.001
3	P_2 & P_3	.946	9.18	.001
3	R_3 & P_3	.310	1.03	.4
3	(P_2+R_2)&P_3	.676	2.90	.02

At month 1 we have no choice but to predict Y and P_1 by using R_1. Fortunately the correlations are fairly good and are statistically significant. At month 2, however, we have R_1, P_1, and R_2 available so predictions can be made using these singly or in combination. To predict Y, (P_1+R_2) is quite good, but R_2 will be used by itself because it is easier to use and has nearly the same correlation with Y. There is no really good way to predict P_2 so one could just use the mean of the observed values. As a matter of personal judgment, it is felt preferable to use P_1.

Here is a point where actuaries will disagree among themselves because there is no clear evidence of what choice should be made.

At the close of month 3 we know R_1, R_2, R_3, P_1, and P_2 and wish to predict Y and P_3. For P_3, there is no question that P_2 is the best predictor. To predict Y, the strongest correlations are with R_3, R_2, (P_2+R_2) and (P_2+R_3) in that order. Because (P_2+R_2) is available from last month's punched card runs and correlates very highly with Y, it will be selected. Notice how judgment plays a part in this process, but how clearly the actuarial tools guide its use. The general form of a regression equation is:

$$Z' = \overline{Z} + \frac{\sum xz}{\sum x^2} (X - \overline{X})$$

This form of regression will be called Method A and produces a probable error of

$$.700\sqrt{\frac{\Sigma z^2-\frac{(\Sigma xz)^2}{\Sigma x^2}}{N-2}}$$

where .700 is used instead of the customary .674 because we only have 10 degrees of freedom.

Another regression equation may be established which is similar to the type used by the author in that no constant is involved and the line thus passes thru the origin. This will be called Method B and the general form of the regression equation is:

$$Z' = \left(\frac{\Sigma XZ}{\Sigma X^2}\right) X$$

and the probable error of prediction is:

$$.697\sqrt{\frac{\Sigma Z^2-\frac{(\Sigma XZ)^2}{\Sigma X^2}}{N-1}}$$

where .697 is used due to having 11 degrees of freedom.

The results of the two methods are:

	The Best Least Squares Equation		*Probable Error of Prediction*			
			In Units		As a Percent of Mean Prediction	
To Predict	Method A	Method B	Method A	Method B	Method A	Method B
Y'	$1.5895R_1+727.96$	$2.0770R_1$	166.50	186.09	5.5%	6.1%
P_1'	$0.0374R_1-9.49$	$0.0310R_1$	5.68	5.76	12.6	12.8
Y''	$1.4146R_2-248.53$	$1,3095R_2$	79.31	82.79	2.6	2.7
P_2''	$3.0011P_1+92.78$	$4.9340P_1$	47.45	50.25	20.8	22.1
Y'''	$1.2935(P_2+R_2)-261.35$	$1.1928(P_2+R_2)$	63.88	68.38	2.1	2.2
P_3'''	$1.0939P_2+130.96$	$1.6145P_2$	20.25	35.65	5.3	9.4

To conclude the discussion of single variable regression equations, it appears that method B produces very nearly as good results as method A and has the advantage of simplicity and logical clearness. It would, undoubtedly, be the method to use in the practical situation.

Having progressed this far one cannot help but wonder what would occur if every possible shred of loss and reserve evidence were used at each stage to make the best possible linear multiple regression prediction of the value D itself. Rather than going through all the calculations necessary to get the needed sums of squares and cross products, the formulae for D make it just a matter of algebraic manipulation to get the values.

By definition,

$$D_1 = Y - P_1 - R_1$$
$$D_2 = Y - P_2 - R_2$$
$$D_3 = Y - P_3 - R_3$$

At the end of month 1 only R_1 is available so we proceed as before to produce $D_1' = .5521R_1 + 737.45$ with a probable error of 164.65 which is 10.7% of the mean prediction. To protect against the undesirable effects of being under-reserved it might be advisable to cover one probable error by applying a 10% additional "safety factor" to D_1'.

At the end of month 2 we have R_1, P_1, and R_2 available for predicting D_2''. The most complex linear combination envisioned is:

$$D_2'' = aR_1 + bP_1 + cR_2 + d(R_1+P_1) + e(R_2+P_1) + f(R_2-R_1) + g$$

There is considerable overlap here, but it is planned to eliminate all variables that do not contribute significantly to the regression. A multiple regression equation such as this can be solved by a number of methods. Personal preference led to the use of Doolittle's method (see reference 2, page 327) because it provides a systematic way to test the statistical significance of the regression coefficients and eliminate those that are not significant. The solution is rather tedious and will not be presented here, but the resultant equation is:

$$D_2'' = .0057R_1 + 5.6855R_2 + .0570R_2 + .0184(R_1+P_1) + .0628(R_2+P_1) + .1785(R_2-R_1) + 240.11$$

However, most of these coefficients have no statistical significance whatsoever. Eliminating non-significant variables one by one resulted in $D_2'' = 4.9312P_1 + .2023R_2 - 204.08$ with a probable error of 59.66 or 12.2% of the mean prediction. The multiple correlation coefficient is .793 and the test of significance on the regression coefficients results in probabilities of less than .10 that the coefficients equal zero. This is not very encouraging, but they are the best available. Here again it is suggested that a safety factor be employed of, say, 12%.

At the end of month 3 we know P_1, P_2, R_1, R_2, and R_3. The following equation was tried, $D_3''' = hP_2 + iR_3 + j(P_2+R_2) + k(P_2+R_3) + m(R_2+P_2-R_1-P_1) + n$.

Proceding as before, the equation was solved, but the results showed no significant regression coefficients. The most promise was held by P_2 which has a probability of .15. The equation is: $D_3'' = .4293P_2 + 46.88$ with a probable error of 41.22 or 28.5% of the mean prediction. Here again a loading of about 30% would be applied to the prediction as a "safety factor".

It is interesting to note how the size of the probable error decreases as more information becomes available, but the size of the prediction decreases even faster so that our percentage error becomes quite large.

In closing the phase on least squares regression equations as a means of predicting, it would be well to pinpoint the two primary advantages over the judgment method (where one looks at a series of factors and selects one that looks reasonable) or the simple arithmetic method (adding up a series of factors and dividing by N).

The first advantage is that it provides a statistical method of selecting among the various criteria available for predicting, thus allowing the actuary to reject those which are of no significance and permitting him to select the best among the remaining indicators. Secondly, the range of error in the prediction may be specified using this method and the actuary has a clear concept of the likely fluctuation in his prediction.

The second phase of the analysis of Mr. Tapley's data dwelt on testing the two hypotheses (1) there is no difference in loss amounts between the years 1954 and 1955 and (2) there is no difference among the various means of the months January through July. These hypotheses may both be tested by an Analysis of Variance and for this purpose the "base" losses from 1954 and 1955 for the months January through July were arrayed as shown in Table III.

There are several excellent sources for information on Analysis of Variance along with working models (see reference 3, page 24) and this is one of the simpler types. Summarizing the results from Table III in a convenient table:

ANALYSIS OF VARIANCE TABLE

Variance	*Degrees of Freedom*	*Sum of Squares*	*Mean Square*	*F*
Between years	1	922,631	922,631	31.40
Between months	6	659,570	109,928	3.74
Error	6	176,304	29,384	
TOTAL	13	1,758,505		

The hypotheses were tested in the following manner:

(1) Hypothesis: There is no significant difference between years. $F = \frac{922,631}{29,384} = 31.40$. Enter F table with $n_1 = 1$ and $n_2 = 6$. The .01 value of F is 13.74 and therefore we reject the hypothesis.

(2) Hypothesis: There is no significant difference between the months. $F = \frac{109,928}{29,384} = 3.74$. Enter F table with $n_1 = 6$ and $n_2 = 6$. The .05 value of F is 4.28 and therefore we accept the hypothesis.

The conclusion reached then is that the seven months are homogeneous but there is a significant difference between years. Our own knowledge and experience in the field tells us that this difference between years might be attributed to an increased volume of business or due to an increase in loss costs on the line of insurance. The author was kind enough to furnish me with the fact that between the two years the average increase in earned exposure was 12.1%. The effect of this increase was eliminated from the data by dividing each X_{2t} by 1.121 and again running the analysis of variance. The results are:

ANALYSIS OF VARIANCE TABLE

Variance	*Degrees of Freedom*	*Sum of Squares*	*Mean Square*	*F*
Between Years	1	88,166	88,166	3.38
Between Months	6	588,229	98,038	3.76
Error	6	156,602	26,100	
TOTAL	13	832,997		

The hypotheses were tested in the following manner:

(1) Hypothesis: There is no significant difference between years. $F = \frac{88,166}{26,100} = 3.38$. Enter F table with $n_1 = 1$ and $n_2 = 6$.

The .05 value of F is 5.99 and therefore we accept the hypothesis.

(2) Hypothesis: There is no significant difference between the months. $F = \frac{98,038}{26,100} = 3.76$. Enter F table with $n_1 = 6$ and $n_2 = 6$. The .05 value of F is 4.28 and therefore we accept the hypothesis.

This analysis shows that when we take account of the increase in exposure, the entire group of data may be considered homogeneous both as to month and as to year.

A myriad of other statistical questions arise as a result of this paper. Can a smaller company utilize these techniques? If we had accurate earned exposure on a monthly basis and could calculate accurate pure premiums, would we still find the data to be homogeneous? Could a method such as this be used in lieu of establishing case reserves, especially in lines with a smaller variance than bodily injury? What could be done if a company were not so fortunate as to hit a stability point at month 4, but instead had quite variable results over a long period? In smaller companies, would earned exposure and earlier information on paid losses be available soon enough and wouldn't it improve the predictions considerably? Could claim count and reserve count be introduced to additionally sharpen the prediction? Many of the answers are self-evident, but may serve to stimulate other approaches and variations in technique. Three questions of a more imponderable nature are (1) Will actuaries be sufficiently persuasive within their own companies to establish this method as an increase in accuracy and an expense saver if it is used in lieu of punched cards; (2) Will rate makers be able to establish it as an integral part of rate making methods and thus relieve company tabulating departments of bothersome detail and simultaneously increase accuracy; (3) Will supervisory officials accept such a formula approach especially if it is used in lieu of case basis reserves? Let's

work for the best and retain the highest possible standards in this field of endeavor.

TABLE I RAW DATA

		Y	P_1	P_2	P_3	R_1	R_2	R_3
1954	January	2645	44	172	315	1220	2096	2232
	February	2601	28	130	264	1076	2045	2219
	March	2592	51	248	389	1282	1950	2078
	April	2529	40	147	290	1454	2051	2160
	May	3188	31	195	338	1453	2592	2777
	June	2808	35	364	507	1430	2116	2207
	July	3052	38	195	379	1454	2349	2485
	August	2948	48	198	336	1505	2183	2421
	September	3205	49	328	445	1332	2336	2472
	October	3523	55	207	385	1582	2598	3001
	November	3313	46	204	341	1592	2578	2860
	December	4081	74	343	570	2078	3006	3279
TOTAL		36485	539	2731	4559	17458	27900	30191

TABLE II SUMMARY STATISTICS

MEANS

(General Notation: $\overline{X}$)

Y	P_1	P_2	P_3	R_1	R_2	R_3
3040.4167	44.9167	227.5833	379.9167	1454.8333	2325.0000	2515.9167

SUMS OF SQUARES AND CROSS PRODUCTS OF VALUES

(General Notation: ΣXZ)

	Y	P_1	P_2	P_3	R_1	R_2	R_3
Y	113,243,231						
P_1	1,682,350	25,873					
P_2	8,486,401	127,658	687,045				
P_3	14,148,456	212,290	1,109,222	1,819,603			
R_1	54,144,698	809,217	4,068,661	6,790,734	26,068,562		
R_2	86,362,034	1,277,818	6,438,376	10,758,395	41,304,743	65,952,172	
R_3	93,647,797	1,387,077	6,954,194	11,636,199	44,770,274	71,489,635	77,556,739

SUMS OF SQUARES AND CROSS PRODUCTS REDUCED TO DEVIATIONS ABOUT THE MEANS

(General Notation: $\Sigma xz = \Sigma XZ - \overline{N}.\overline{X}.\overline{Z}$)

	Y	P_1	P_2	P_3	R_1	R_2	R_3
Y	2,313,628.92						
P_1	43,565.42	1,662.92					
P_2	183,023.08	4,990.58	65,514.92				
P_3	287,196.42	7,514.92	71,669.58	87,562.92			
R_1	1,065,103.83	25,061.83	95,511.17	158,148.88	670,081.67		
R_2	1,534,409.00	24,643.00	88,801.00	158,720.00	714,893.00	1,084,672.00	
R_3	1,854,577.42	30,997.92	83,225.58	116,139.92	847,400.83	1,295,560.00	1,598,698.92

TABLE III RAW DATA AND SOME CALCULATIONS

Month t	*Years*			
	X_{1t}	X_{2t}	$_s\Sigma X_{st}$	$\overline{X}_{.t}$
X_{s1}	2645	3023	5668	2834.0
X_{s2}	2601	2834	5435	2717.5
X_{s3}	2592	3343	5935	2967.5
X_{s4}	2529	3453	5982	2991.0
X_{s5}	3188	3540	6728	3364.0
X_{s6}	2808	3254	6062	3031.0
X_{s7}	3052	3562	6614	3307.0
$\Sigma_t X_{st}$	19415	23009	42424	
$\overline{X}_{s.}$	2773.57	3287.00		3030.29

The notation employed designates the value in the s^{th} year and the t^{th} month as X_{st} where $s = 1, 2$ and $t = 1, 2 \ldots, 7$. Means are denoted as $\overline{X}_{s.}$ to indicate that it is the mean of all t values for a particular s and by $\overline{X}_{.t}$ to denote that it is the mean of all s values for a particular t. The grand mean of the entire table will be denoted $\overline{X}..$ Finally, N_s is the number of cases of s (2 in our example), $N_t = 7$ and $N_{st} = 14$. Let's call the sum of squares between years χ_1^2, between months χ_2^2, and the residual or error χ_3^2. Then:

$\chi_1^2 = \sum_s \sum_t (\overline{X}_{s.} - \overline{X}..)^2 = (2773.57 - 3030.29)^2 + (3287.00 - 3030.29)^2 = 922{,}631$ with degrees of freedom $= N_s - 1 = 1$

$\chi_2^2 = \sum_s \sum_t (\overline{X}_{.t} - \overline{X}..)^2 = (2834.00 - 3030.29)^2 + \ldots\ldots + (3307.00 - 3030.29)^2 = 659{,}570$ with degrees of freedom $= N_t - 1 = 6$

$\chi_a^2 = \sum_s \sum_t (X_{st} - \overline{X}_{s.} - \overline{X}_{.t} + \overline{X}_{..})^2 = (2645 - 2773.57 - 2834.00 + 3030.29)^2 + \ldots\ldots + (3562 - 3287.00 - 3307.00 + 3030.29)^2 = 176{,}304$ with degrees of freedom $= (N_s - 1)\ (N_t - 1) = 6$

Finally, as a check on the calculations

$\chi_1^2 + \chi_2^2 + \chi_a^2 = \sum_s \sum_t (X_{st} - \overline{X}_{..})^2 = (2645 - 3030.29)^2 + \ldots\ldots + (3562 - 3030.29)^2 = 1{,}758{,}505$ with degrees of freedom $= N_{st} - 1 = 13$

Other more convenient computational formulae can be developed and actually were employed by the writer.

Reference 1: Fisher, R. A., Statistical Methods for Research Workers, 10th ed., Edinburgh: Oliver and Boyd Ltd., 1948

Reference 2: Johnson, P. O., Statistical Methods in Research, New York: Prentice-Hall, Inc., 1949

Reference 3: Jackson, R. W. B., Application of the Analysis of Variance and Covariance Method to Educational Problems. Department of Educational Research, University of Toronto, Bulletin No. 11, 1940

AUTHOR'S REVIEW OF DISCUSSION

DAVID A. TAPLEY

Mr. Simon's discussion of regression equations and of the need for continuously testing the homogeneous character of the data we are tabulating is more than just pertinent. It brings into effective focus an understanding of the substantial amount of detailed and continuous testing that is being carried forward in connection with these data. The purpose of these tests is twofold. Initially each monthly tabulation must be reviewed to determine known procedural change effects. Secondly, and only after such validation, it must be combined with prior data and current values for the various factors used procedurally must be re-computed and tested.

It may be observed that these continuing processes of analyses are far too cumbersome and expensive to maintain for the sole purpose of producing deficiency reserves. The additional companywide products of our program include the testing of current loss levels, the testing of total reserve levels, the early determination of loss and procedural trends and other items we need not mention here. In all these connections, the early statistical reflections of change, the knowledge that something is different to a measured degree is often of great assistance.

While actuaries and statisticians may have individual preferences as to the detailed form of these testing processes, Mr. Simon has given a clear indication of their general characteristics. In addition

we generally concur with his stated views on the use of regression equations. At the time the paper was written, we were dissatisfied with the sheer lack of samples available for such purposes. However, the simple factors we employed to obtain "guide" projections have worked out better than we had any statistical reason to expect.

The discussion by Mr. Valerius is quite broad and raises certain fundamental questions. Without indulging in repetitious quotation, several comments are made with the intent of showing that the month of loss reserve concept is particularly susceptible to specific dangers which are inherent in almost any reserve process.

Within any given body of collected loss experience exist the unchangeable loss components underlying every analytic method whether it be based on calendar, policy or accident period. We can vary our methods but we cannot alter the data once it is established in the record, either in total or with respect to any individual component thereof. The several components of incurred losses appear, as Mr. Valerius suggests, quite erratic and independent; that is, there does not seem to be any logical and necessary relationships among them except that they are all components of total incurred losses. Furthermore their separate values are constantly interchanging under development. Unreported losses become reported losses. Reported losses become variable reserves, or payments, or reserves closed without payment. After some extended period they all convert to a single total of claims paid under both the policy and accident period forms of analysis.

The lack of logical relationship among the several components of total incurred losses means that we are utterly dependent upon sound statistical concepts in the evaluation of all loss data, and this particularly is true in dealing with that portion of our total losses which are said to be incurred but not reported. It also means that no single component can be demonstrated as a logical function of, or necessarily to change consistently in relation to, any other component. Thus any formula or procedure which evaluates one component on the basis of data for a second component is not based upon a logical relationship. Instead it is based upon temporary consistencies in observed data that are subject to change.

The lack of logical relationships among the several components of total incurred losses obviously creates particular requirements that must be met by any method designed to evaluate losses and loss components. Let us assume momentarily that:

(1) $X_1, X_2, X_3, \ldots X_n$ represent total incurred losses on either a policy or an accident period basis at succeeding dates of evaluation for a given period of loss, and

(2) $A_1, A_2, A_3, \ldots A_n$ and $B_1, B_2, B_3, \ldots B_n$ represent a division of components of total incurred losses as above,

for every given date of evaluation, $A_i + B_i = X_i$, where there is no

demonstrable relationship between the A and B components and where the matured or X_n value of total incurred losses can only be proven by development.

We have no choice but to assume the stated $A_i + B_i = X_i$ relationships exist. They are inherent in every evaluation of total incurred losses whether for ratemaking or for annual statements.

Similarly we are forced to assume that successive values for X must be maintained as accurately and also as consistently as possible. Any assumption to the contrary makes it difficult to support ratemaking techniques which exclude retrospective adjustment factors.

Now if we attempt to determine successive values for A_i, (incurred but not reported losses) separately from B_i, (known losses), we have only one possible way in which to test the accuracy of the total values so obtained. We must examine the resulting values of X_i, for stability. Under the month of loss concept this testing process is carried forward continuously.

The lack of logical relationships among the several components of loss is the cause of what has been termed an implicit theoretical flaw in our method. Unquestionably, known losses are understated in early periods of development because of the then value of incurred but not reported claims. By the same standard they are understated by the then value of claims that will reopen and by the additional reserves needed on claims reported but not investigated. They are also overstated by the then value of reserves that will eventually close without payment. This type of "flaw" is implicit in the data, not in the method of analysis. Customary methods make little effort to define and evaluate such "flaws". One major objective of the deficiency reserve plan is to prevent such unavoidable "overlappings" from being reflected as large fluctuations in the developing value of total incurred losses and, in the event such variations do occur, to expose them in fullest detail to the eyes of management.

Procedural distortions, such as result from a change in the estimation practices of adjusters, are reflected in the components of loss making up any body of recorded experience. We cannot eliminate them as potential sources of error from the basic data. We can average them over an annual period but this does not prevent their accumulation in one direction in times of trend or change. By adopting less than annual periods of evaluation we, potentially, if not actually, increase the ranges of probable variation due to reduced reliability of the smaller segments of data. In contrast, however, we also benefit from a greater definition of components and more flexibility of method. This facilitates the early discovery of all variations, helps to measure them effectively and assists in indicating any necessary adjustments needed in our procedures. There is unquestionably some minimum size of exposures, losses and reserve need that will not satisfy the test of statistical significance. But this criterion is fundamental to all reserves regardless of the method of analysis. There are obviously many territories, both rating and statistical, that develop less than minimum exposures required to produce acceptable

reliability. Special techniques and procedures have historically been adopted to insure a meaningful and dependable interpretation of the loss experience that is recorded in such areas. Such special techniques and procedures are not unavailable to the interpretation of month of loss analyses.

One final point of clarification is pertinent to these discussions. Under the processes employed for developing policy year losses, it is customary to project first reportings to acceptable maturity. The projection factors so used are primarily designed to accomplish an adequate estimation of total losses incurred even though only about one half of the total exposures have then expired. The basic concepts which govern this phase of the policy year loss development processes are the same concepts which underlie the deficiency reserve program with but a single exception; namely, the deficiency reserve program has no application to losses that will be incurred in the future. Also, in a broad sense, the methods of the policy year development process have been employed under the deficiency reserve program, and these methods have been altered only as required to employ accident periods instead of policy period, and monthly instead of annual analyses. Finally, the determination of static values for all unknown losses at given dates of evaluation are obtained as the sum of such losses appropriate to all immature months of loss. Such total evaluations of unknown losses are subsequently tested by development, and may be readily reconciled to accident year rating data. They make it possible to obtain an accurate check upon adequacy of the sum of the incurred but not reported loss reserves and the known losses as reported in the Annual Statement. Here the deficiency reserve plan provides a test of the adequacy of the estimated total incurred losses determined by the sum of the incurred but not reported reserve and the reported incurred losses. Furthermore, the deficiency reserves so used may be completely reconciled to accident year statistics employed for rate-making.

It may now be seen that the deficiency reserve concept is neither in conflict, nor necessarily in competition, with customary practices. It is equally subject to the vagaries of chance variations in underlying loss components and equally responsive to sound statistical principles. Its usefulness depends entirely upon the extent to which it is adaptable to management needs and the observance of sound practice in using the data so provided. I fully share the concern of Mr. Simon and Mr. Valerius that we safeguard the soundness and high standards of our actuarial processes. I also believe that the accident period form of analysis offers considerable promise in supplementing our existing kinds of experience. It is certainly deserving of fair appraisal and adequate testing. In the light of recent industrywide developments such treatment now appears assured.