

SUGGESTIONS FOR A STANDARD SYSTEM OF
NOTATION IN CASUALTY ACTUARIAL WORK

BY

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Casualty actuarial science has progressed by now to the point where a standardized system of notation in formulas, applicable as far as possible to all casualty lines of insurance, is feasible and desirable: feasible because the basic terms that actuaries deal with in their work are no longer changing rapidly, desirable because the absence of a standardized notation leads to wasted time and effort on the part of both the author and the reader. In recent years the variety of symbols encountered not only in papers presented to this Society, but more particularly in memoranda submitted to the actuarial committees of the respective casualty ratemaking organizations has tended to become confusing, making it necessary for the reader to familiarize himself with each set of symbols individually. In fact, the final spur to my writing of this paper, which had long been carried in mind as one among several projects to be accomplished in the uncertain future, was a memorandum I received in which the goose-egg familiarly understood to represent zero was used to represent the experience rating off-balance factor, with resulting formulas to make one's head spin.

There have been two attempts, to my knowledge, to standardize our notation. In the early years of this Society a committee was organized to accomplish this work, but the committee was discontinued because it was felt the science was too young at that time to permit the establishment of a notation that could remain reasonably stable. Then Mr. Perkins at the November meeting of the Society in 1920 suggested a standard notation for the workmen's compensation line;¹ but, although certain of the symbols therein suggested are now generally accepted, his system considered as a whole was so complex that it is not commonly used, a fact that was anticipated at that time by the discussions in the *Proceedings*.

Mr. Michelbacher, in his discussion of Mr. Perkins' paper,

¹ A Suggested System of Standard Notation for Actuarial Work in Workmen's Compensation Insurance, by Sanford B. Perkins, Vol. VII, p. 36.

made a plea for a "*simple and universal system of notation*", one applicable to all lines of insurance.

In this paper are presented my suggestions toward the establishment by the Society of such a notation. Three general criteria should be kept in mind in establishing a standard notation: (1) simplicity, (2) universality, that is, applicability to all casualty lines, (3) foundation upon symbols already generally accepted and used.

Actuarial terms in the casualty field may be roughly classified in two groups, basic terms and delimiting terms. The basic terms may stand by themselves, the delimiting terms must qualify some basic term. Thus "premium" and "loss" and "loss ratio" are basic terms, whereas "normal" and "\$50 deductible" are delimiting terms. A simple distinction symbolically between these two groups is to make the delimiting symbols in all cases subscripts to the basic symbols.

BASIC SYMBOLS

Among those symbols universally in use at present, the retention of P for premium should meet with no objection, likewise R for rate, L for actual losses, Z for credibility and K for the constant in the experience-rating credibility formula. The only one of these symbols used currently with possible ambiguity is R , which sometimes means an amount, sometimes a decimal. If we restrict the use of R to an amount, a rate in dollars, cents or mills as the case may be, we shall entirely avoid ambiguity.

There are many different bases of exposure, but if a single symbol be adopted to represent the number of units of coverage exposed for one year the confusion of variations in exposure-base will be avoided. Mr. Perryman in his recent paper on credibility² has used the symbol n with this definition, and it is here proposed to follow his suggestion. In the workmen's compensation line, for example, if n were the number of units of coverage, the premium could be expressed as $n \cdot R$, and the payroll would equal $100n$.

In the same paper Mr. Perryman represented the yearly accident frequency per unit of coverage by q . The number of claims is equal to nq , and we need no other symbol by which to represent the number of claims.

²Some Notes on Credibility, by F. S. Perryman, Vol. XIX, p. 65.

The symbol N has long been used in discussions of workmen's compensation problems to represent the number of employees. This is a term little used now, but in view of the fact that it occurs so frequently in the papers which present the development of the workmen's compensation merit rating procedure, it is desirable to include the symbol N in any notation summary.

In experience-rating discussions occurs the term "adjusted losses", the sum of the actual losses multiplied by the credibility plus the expected losses multiplied by the complement of the credibility. The symbol A is most frequently used for this term. For expected losses the symbol ordinarily used is E . But ambiguity enters here, because E appears on the respective Tables A in the workmen's compensation manual and on the workmen's compensation experience-rating form to represent the expected (or permissible) loss ratio. E should therefore be retained to represent the expected loss ratio, and another symbol adopted for expected losses. Since both initial letters of the words in the term "expected losses" are unavailable, any new letter would not be easily connected with the term. My suggestion is that the composite symbol EP be used for expected losses. This introduces no new symbol, is already used to some extent, and has the further advantage of indicating symbolically the derivation of the expected losses.

There is little reason for an additional symbol to represent the expense loading. None is in common use, and since the expense loading is the complement of the expected loss ratio, it may be adequately represented by $1.0 - E$.

For a merit-rating decimal modification of a rate, that is, the ratio of the modified rate to the base rate, the usual symbol is M . Various symbols have been used to represent the discount, or the complement of M . For simplicity, since discount is so infrequently encountered, it may be expressed in terms of M , as $1.0 - M$.

No common symbol is now in use for representing a change in rate level. This term is most frequently encountered in connection with the adjustment of premiums in those lines in which rate revisions are based upon loss ratios rather than upon pure premiums, notably the burglary and plate glass lines. But it also occurs in discussions of workmen's compensation rate-level formulas such as have recently been occupying the attention of

actuaries. The symbol, a , provides a convenient representation of the change in rate level. If a symbol is needed to represent a rate level in terms of some base level, $1 + a'$ may be used. Thus, if we start with a base level of 1.000, and increase rates first by 10%, then by 20%, in two successive years, we have the following relationships, using subscripts $1, 2$ for the two revisions.

$$\begin{aligned} 1.10 &= 1 + a_1, & 1.20 &= 1 + a_2 \\ 1.10 &= 1 + a'_1, & 1.32 &= (1 + a'_1)(1 + a_2) = 1 + a'_2 \end{aligned}$$

I have thus far purposely avoided mention of two of the most important terms, pure premium and loss ratio, because at present neither is commonly represented either by a single letter or by a composite symbol.

The usual abbreviation for pure premium is *p.p.*, for loss ratio, *L.R.* or *l.r.* These abbreviations have commonly been carried over into formula notation, but properly should be retained as abbreviations only. If represented by a composite symbol, either

term would introduce a fraction, $\frac{L}{n}$ in the one case, $\frac{L}{P}$ in the

other. Occasionally (borrowing possibly from the life insurance field) Greek letters have been used, π for pure premium, ρ for loss ratio. My own feeling is that the basic symbols, at least, should all be such that the ordinary office typewriter can handle them, and that we should therefore keep Greek letters out of our notation to facilitate the preparation and duplication of memoranda. The letters p and r , for pure premium and loss ratio respectively, have been used at times. Because of the almost universal use of *p.p.* and *L.R.* it would appear not feasible to adopt exclusively the symbols p and r , but I suggest the recognition of both the symbols and the abbreviations for formula notation, with an attempt on the part of Society members to make the use of p and r customary.

The off-balance factor, in connection with experience rating, has been too much booted about by the symbol-makers. The most widely used notation apparently is *O.B.* or its variant form *O-B.* One solution would be to recognize *O.B.* and *B* as variant symbols, with the encouragement of the simpler symbol *B* on the part of Society members. *B* formerly represented a similar factor appearing on the 1923 experience rating forms for workmen's

compensation but this older factor is now discontinued in every state, so that no confusion should result.

Various types of correction factors arise in actuarial discussions. The symbols C and F appear to be the common representations of them, but I suggest the retention of F as a general symbol, adaptable to any particular situation, because the symbol C is best used for another purpose. In workmen's compensation insurance there is need for a symbol to represent loss constants and expense constants, and other constants appear in other lines. For loss constant, LC has been much used, in fact so much used one is hesitant to suggest a change. But for clarity, two-letter basic symbols should be avoided entirely unless they imply an actual multiplication of the quantities represented by the two individual letters. The adoption of C for constant, with qualifying subscripts where they are needed, will eliminate the difficulty.

No detailed consideration is given here to symbols for those terms peculiar to the accident and health business, nor for annuity calculations in the workmen's compensation line. It is my understanding that the former have become fairly well standardized through the agency of the Actuarial Society of America. And I am not well qualified to determine the latter, which have been covered in detail by Mr. Perkins in his paper cited above. Both additional groups of symbols should be included in any standard system established by this Society.

For discussion of deductible or excess coverages, three other basic symbols must be introduced. The decimal portion of the losses which is retained by the assured under such coverage is usually denoted by k . Expenses are divided into two portions; expense which remains a fixed amount per unit of exposure, and expense which remains a constant percentage of the rate, thus varying in the amount per unit of exposure as the rate varies. That decimal portion of the premium at manual rates which represents the fixed expenses is denoted by f . That decimal portion of the premium represented by the variable expenses, constituting a fixed percentage of the premium, whatever premium may be developed, is denoted by v .

DELIMITING SYMBOLS

We come to the delimitation of these basic symbols, a process which is at present accomplished by means of subscripts. The commonest of these delimitations in practice occur in the discussions of experience-rating. The use of subscripts n and e to represent the normal and excess portion for experience-rating purposes is universal. Thus we should have K_n and K_e , Z_n and Z_e , L_n and L_e , P_n and P_e , EP_n and EP_e , A_n and A_e . Where printed, these subscripts are best in smaller type than is used in the text, but for typewritten memoranda the ordinary small type can be used with little possibility of confusion. The only duplication involved is in the use of n for units of coverage, but if the exposure symbol should ever occur in the same formula with the subscript n , care may be taken to use operational symbols where the exposure symbol is joined to another symbol as in $p \cdot n$ for pure premium times exposure. If it is necessary to distinguish actual unmodified from actual modified losses, subscripts a and m may be used. If it is further necessary to differentiate between the normal and excess portions of unmodified and modified losses, a very infrequent occurrence, we can borrow from the notation of life contingencies and separate the two subscripts by a colon. Thus $L_{n:m}$ would represent the normal portion of actual modified losses. I believe this is preferable to the use of superscripts or prescripts.

In discussing the public liability experience rating plan, which develops a separate modification for excess limits, it is necessary to distinguish between the excess standard portion and the excess limits portion. Since the notation s/l for standard limits is commonly used, we may extend this to include e/l for excess limits, and e/s for excess standard. Thus $P_{s/l}$ would denote standard limits premium, $M_{e/l}$ the excess limits modification, and $L_{e/s}$ the excess standard losses.

One further differentiation in connection with merit-rating is that between an experience-rate modification and a schedule-rate modification. Now that schedule-rating is virtually making its final stand, the necessity for this differentiation is less pressing, but it still appears feasible. The subscripts E and S may be used. Of course, the small capital cannot be typed, but they will be very infrequently needed. The subscript S can also be used to

denote the merit-rate for garages, as distinguished from the manual-rate and experience-rate.

Whenever premiums and losses are involved in a formula, there is seldom any question as to whether the premiums are earned or written, and the losses paid, outstanding or incurred. The easiest way to treat these variations is to indicate clearly in the text of the discussion what delimitations are involved, or in the very rare instance where subscripts appear to be needed, for the author to set down a system for use in that particular paper or memorandum.

The use of the basic symbol C for constant has been suggested. Where it becomes necessary to delimit this symbol, subscripts E for expense and L for loss may be used, thus C_E for expense constant and C_L for loss constant.

In workmen's compensation insurance, differentiation is needed at times between various partial pure premiums, partial losses and the like. Mr. Perkins suggested that all these delimiting symbols consist of two letters, and added a C (for cases) where only a single letter was indicated. This procedure, to my knowledge, has never been followed but I believe it to be feasible. A two letter subscript, not separated by colon or bar, will then always denote some subdivision of the experience by type of loss. I have added symbols to distinguish between serious and non-serious. In print these subscripts would appear as small capitals.

Death.....	DC
Permanent total disability.....	PT
Death and permanent total disability.....	DP
Major permanent partial disability.....	MP
Minor permanent partial disability.....	PM
Temporary total disability.....	TT
Temporary partial disability.....	TP
Serious.....	SC
Non-serious.....	NC
Medical.....	MC

In treating deductible or excess coverages, it is necessary to differentiate between the various amounts of assured's retention. This can be easily accomplished by means of subscripts $_{50a}$ for \$50 deductible, $_{250e}$ for \$250 excess, and the like. Thus R_{50a}

would denote the rate for \$50 deductible coverage. k is the same for both deductible and excess coverage, so that k_{100} denotes the decimal portion of the losses amounting to no more than \$100 per claim. Moreover f is constant for any line of insurance for all deductible amounts, so that f_d and f_e serve to differentiate the two values of f for any one line; similarly for v .

There are many other terms, such as law-amendment factor and inspection pure premium, which are only occasionally encountered in formulas. When such terms occur, the author can establish his own notation: their infrequency does not justify their inclusion in a standard system.

The perfectly general rule may be adopted that, when in the interest of clarity it becomes necessary to use two subscripts, these should be separated by a colon. Such occurrences should always be very rare. The simpler the notation is kept the better, both for the author and for the reader. Ordinarily, the text in which a formula is set has already established the exact meaning of the symbols used so that no delimitation of their application by subscript appendages is needed.

What I should like to see as a result of this paper is the development of a standard set of symbols for the casualty actuarial science, possibly through the establishment of a committee of the Society to accomplish this. Mr. Michelbacher, in his discussion of the paper by Mr. Perkins (Vol. VII, p. 405) mentions the committee previously established, as follows:

“Early in the history of our Society some thought was given to this problem, and, in fact, at one time our Council established a special committee on ‘Terms, Definitions and Symbols’ for the express purpose of creating a satisfactory system of notation for casualty insurance. It was found, however, that scientific work in this field had not progressed far enough and had not become sufficiently standardized and it was decided, therefore, to drop the subject and to abolish the committee. Under the circumstances this action was undoubtedly justified, but there is every indication that the intention was not to drop the subject permanently, but rather to await a more appropriate time for the development of a uniform system of notation.”

Surely the time is now ripe for such a development. The life insurance actuarial bodies periodically publish in the front of their proceedings or transactions a revised list of symbols, keep-

ing that list up to date as their science develops. Our Society might well do the same, in an effort to eliminate the confusion caused by the varying notations now in use.

LIST OF BASIC SYMBOLS

Premium.....	<i>P</i>
Actual losses.....	<i>L</i>
Adjusted losses (experience rating).....	<i>A</i>
Expected (or permissible) loss ratio.....	<i>E</i>
Expected losses.....	<i>EP</i>
Rate (manual unless otherwise specified).....	<i>R</i>
Credibility.....	<i>Z</i>
Constant in experience-rating credibility formula.....	<i>K</i>
Units of coverage per year.....	<i>n</i>
Claim frequency (per unit of coverage).....	<i>q</i>
Number of claims.....	<i>nq</i>
Number of employees (workmen's compensation).....	<i>N</i>
Actual loss ratio.....	<i>r</i>
(<i>L.R.</i> and <i>l.r.</i> recognized)	
Pure premium.....	<i>p</i>
(<i>p.p.</i> recognized)	
Off-balance factor (in connection with experience-rating) (<i>O.B.</i> recognized)	<i>B</i>
Constant (per risk).....	<i>C</i>
Correction factor (for general purposes).....	<i>F</i>
Deductible and excess coverages:	
Decimal portion of losses retained by assured.....	<i>k</i>
Decimal portion of manual rate for fixed expenses.....	<i>f</i>
Decimal portion of rate for variable expenses.....	<i>v</i>
Rate level:	
Ratio of rate level to next preceding rate level (Algebraic sum of unity plus decimal change in rate level)	$1 + a$
Ratio of rate level to base rate level.....	$1 + a'$

Deductible or excess coverages:

Deductible.....	d
Excess.....	e
Assured's retention.....	(actual amount)
For example, rate for \$100 deductible.....	R_{100d}
losses, \$50 excess coverage	L_{50e}
decimal portion of losses elimi- nated under either \$500 deduc- tible or \$500 excess coverage....	k_{500}
Decimal portion of excess rate for variable expenses.....	v_e

Note: If two delimiting symbols are needed to qualify a single basic symbol, separate the delimiting symbols by a colon.

For example, normal portion of modified losses..... $L_{N:M}$

INCURRED BUT NOT REPORTED CLAIM RESERVES

BY

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The subject of reserves for incurred but not reported claims has received very scant consideration in our *Proceedings*, nor is there available to the writer's knowledge any written material of consequence on either the theoretical or practical aspects of the subject.

A Committee of the Association of Casualty and Surety Accountants and Statisticians studied the subject in 1927 as respects the fidelity and surety lines and submitted a report recommending that such reserves be determined as a function of premiums in force. Specifically, the Committee recommended the following minimum percentages of in-force premiums: fidelity—10%; surety—3.5%.

The writer, in conjunction with his office associates, has given considerable study to this subject during recent years and has maintained numerous records designed to aid in the calculation of this particular reserve liability. While no claim is made that an entirely complete solution has been reached, the results of our methods have been so generally satisfactory as to encourage a discussion of the subject before this Society.

For the purposes of this paper an incurred but not reported claim is defined as a claim arising out of an event or accident which occurred on, or prior to, a certain date, but notice of which was not received by the home office of the company until after such date. The date we usually associate with this definition is December 31, since this date is of particular significance from the annual statement viewpoint. Unless otherwise stated, the subject will be considered from the standpoint of this date.

The definition submitted is inclusive and specific and covers all situations and practices, in that the governing condition is the fact of notice of the claim being received or not received on or before the particular date. It is assumed that all notices received as of the particular date will be recorded as of such date, although the actual physical recording may take place at a subsequent date—i.e., that notices received up to and including December 31 will