On the Cost of Financing Catastrophe Insurance

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Abstract

After surveying various instruments used to finance catastrophe insurance, this paper demonstrates a method for analyzing the cost of financing catastrophe insurance with the following instruments: (1) insurer capital; (2) reinsurance; and (3) catastrophe options. The procedure first quantifies the cost of financing in terms of the cost of those instruments. The method then permits searching for a mix of instruments that minimizes the cost.

Using a catastrophe model, we create a distribution of simulated losses for each of fifty insurers that report their exposure to ISO. We then create an illustrative catastrophe index based on the combined simulated losses of the fifty insurers. We perform a sample analyses for three insurers.

The analyses show that the best mix of capital, reinsurance, and catastrophe options depends on how well an insurer's losses correlate with the index – that is, on the basis risk. Some insurers can significantly reduce their cost of financing catastrophe insurance by using catastrophe options. To illustrate the effect on premiums of the cost of financing catastrophe insurance, we convert those costs into risk loads.

1. Introduction

Hurricane Andrew caused $15.5 billion of insured property losses in 1992. And it missed Miami, otherwise losses could have been in the $50 billion range. The Northridge Earthquake resulted in $12.5 billion of losses in 1994. And it was only of magnitude 6.7.

In a recent study[1], ISO used the Risk Management Solutions, Inc. (RMS) catastrophe model to simulate possible catastrophic events for the insurers who report their exposure to ISO. The study concluded that losses from a severe hurricane along the east cost could exceed $150 billion. Similarly a severe earthquake in California could generate losses of $50 billion or more.

Losses from such a megacatastrophe could have severe adverse effects on property/casualty insurers and their policyholders. Many insurers could become insolvent or seriously impaired and, therefore, unable to continue insuring the same volume of business. The recognition of this risk has stimulated industry efforts to address the problem of megacatastrophes. Insurance regulators, legislators, government agencies, investment bankers, and others have also contributed to the public policy debate on this critical issue.

Catastrophe Management

A property/casualty insurer can measure the extent of its catastrophe risk by conducting a portfolio analysis to determine the expected distribution of losses from possible events such as hurricanes or earthquakes. This distribution of losses is created by analyzing the company's catastrophe exposure with a computer simulation model, which provides an estimate of losses that would result from a representative set of catastrophic events. Where potential catastrophe losses are too high, the insurer might take steps to reduce its concentration of exposures. Some insurers have given up some business in overly exposed areas to reduce their catastrophe risk to a more manageable level. An insurer could also diversify its catastrophe risk by writing more exposures in areas where it has a lower concentration of exposures or in areas not subject to catastrophes. A concern about that strategy is that the insurer could be taking on a different risk by writing new business in areas where it lacks expertise and an effective distribution network.

Many insurers have opted for loss-reduction measures such as increasing deductible sizes, imposing special wind/earthquake deductibles and offering discounts for loss mitigation activities by policyholders (such as the addition of storm shutters).

Property/casualty insurers have pursued many loss mitigation efforts, such as the ISO Building Code Effectiveness Grading Schedule (BCEGS). The BCEGS program evaluates a community's building code and its enforcement. Insurers can offer discounts for structures built in municipalities with good enforcement of an effective loss mitigating building code.

Financing Catastrophe Risk

Insurers have also been looking at ways of financing their catastrophe risk. One approach is adding capital to the balance sheet. Many insurers have benefited from recent stock market gains as a source of additional capital. Because of their improved capital positions, some insurers have elected to retain more catastrophe risk.

The surge in catastrophes that began in 1989 with Hurricane Hugo, resulted in an increased demand for reinsurance., The rising demand, in turn, produced substantial price increases which led to the formation of new catastrophe reinsurers. That increase in reinsurer capital coupled with improved catastrophe experience has led to more plentiful and less expensive catastrophe coverage.

Traditional reinsurance is not the only approach to financing catastrophes. Those active in capital markets activities, reinsurers, reinsurance intermediaries and property/casualty

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[1]Insurance Services Office, Inc., *Managing Catastrophe Risk*, May 1996.

insurers themselves have come to recognize the possibility of securitizing risk – that is, using other financial instruments to transfer catastrophe risks to the broader capital markets.

All of the instruments for financing catastrophe risk have a cost, but they also have benefits. It takes sophisticated analysis to find an efficient mix of risk financing instruments that provides the greatest benefit for the least cost. Providing an example of such an analysis is the goal of this paper.

This analysis is part of what casualty actuaries call dynamic financial analysis, or DFA. It is similar to other aspects of DFA because it views the various risk financing instruments as assets, with the returns on these assets being positively correlated to insurer losses.

A key factor for delivering an efficient mix of risk financing instruments is the cost of the individual instruments. This cost ultimately becomes part of the price of insurance. This price will be sensitive to the variation in results – many years with small catastrophe losses and occasional years with very large catastrophe losses. Actuaries have traditionally called this part of the price the risk load. We must expand the definition of traditional risk load to include the various instruments available to finance catastrophe insurance.

The intense competitive forces in the marketplace may cause insurers to focus on short-term operating results at the expense of long-term solidity. This amounts to insurers ignoring the possibility of rare catastrophes in their decision making. Insurers may not adequately reflect risk load in pricing, nor make sufficient provision for catastrophe risk financing.

The capital markets can bring an immense amount of financing into the insurance industry, and perhaps significantly lower the cost of financing for the long term. Our challenge is to figure out how to efficiently bring these resources into the insurance industry.

2. A Survey of the Instruments Used in Financing Insurance

Raising Insurer Capital

An insurer always has the option of raising sufficient capital to cover its potential losses, but to raise capital, the insurer must increase its net income to justify this capital. There is also the lost opportunity since the capital committed to an insurer is not available for another venture.

Compared with other industries, property/casualty insurance has not generally achieved high historic returns. Competition from the large number of suppliers has been a major contributing factor. Furthermore, regulation has in some cases also acted to keep insurance rates below actuarially indicated levels.[2]

If an insurer has a heavy concentration of exposures in catastrophe-prone areas, the amount of capital needed can be relatively large compared with the insurer's existing surplus. Furthermore, the additional capital may only be needed occasionally when catastrophe losses are unusually large – perhaps every 100 years. Committing a large amount of additional capital to cover infrequent losses is extremely inefficient and virtually impossible to sustain in a highly competitive marketplace.

Those considerations drive an insurer to seek alternatives to raising capital.

Reinsurance

The capital of US reinsurers was $13.2 billion in 1992. It grew to $26.2 billion by the end of 1997. With the increased demand for reinsurance following the catastrophes in the early 1990s, new offshore reinsurers provided additional capacity. But that capacity is also relatively small compared with the size of potential catastrophe losses.

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[2] Insurance Services Office, *Risk and Returns: Property/Casualty Insurance Compared with Other Industries*. December 1995.

Reinsurers provide modest layers of coverage which are usually sufficient to protect small insurers but not larger insurers.

The availability of reinsurance varies considerably over the life of an insurance cycle. The price may also vary substantially depending on supply and demand as well as recent experience.

Reinsurance pays for the primary insurer's losses that exceed certain amounts, or on a quota share basis. The reinsurance coverage follows the fortunes of the primary insurer. On the other hand, reinsurance can also have high and variable transaction costs for the customized coverage provided.

It is important to remember that a reinsurer may not be able to meet its obligation if a large catastrophe occurs.

One possible solution to the problem of large catastrophes is proposed legislation under which the federal government would provide excess reinsurance. The trade-off for providing this coverage may be increased regulation.

Securitization

The property/casualty insurance industry does not have enough capital to handle a very large catastrophe. By contrast, the broader capital markets have trillions of dollars to invest. The returns on many of these investments are correlated – that is their value is influenced by the same economic conditions. To diversify their portfolios, investors are always looking for investment opportunities not correlated with the economy. Catastrophe risk is independent of the economic conditions that affect other financial instruments.

Many types of financial instruments to transfer catastrophe risk have emerged in recent years. They treat catastrophe risk in various fashions, but all offer the investor a way to profit in exchange for accepting some risk.

Catastrophe bonds have already gained a level of acceptance with several successful deals. A catastrophe, or contingency, bond represents a loan (principal) over a specified term in exchange for fixed interest payments. The occurrence of a qualified catastrophic event during the term of the bond may result in the reduction or elimination of interest payments and for some bonds the loss of some or all of the principal that the investor has loaned to the insurer. If no qualifying catastrophe occurs, the investor receives his principal plus interest. The interest rate usually reflects a premium to reward the additional risk.

Catastrophe bonds generally reflect the catastrophe experience of the insurer selling the bond, although covered losses can be based on an index of industry catastrophe losses. If an industry index is used, then the bond may not mirror the catastrophe experience of the selling insurer.

Securitization of risk has also involved contingent equities. In an agreement developed by Aon Corporation, called a CatEPut℠, an insurer purchases the option of selling a prearranged amount of its stock if a qualifying catastrophe occurs.

This arrangement provides the insurer with immediate access to equity in the event that a loss impairs its surplus. The additional equity increases the likelihood that the insurer will maintain its ratings and will be able to continue its business operations virtually uninterrupted in the wake of such a loss. The seller of the CatEPut℠ has the option to eventually convert the preferred shares to common stock. The insurer can refinance and redeem the shares at any time[3]. Also, there is a provision that the investor does not have to purchase the stock if the catastrophe results in a serious impairment of the insurer, in other words, if the investor's capital infusion would not be sufficient to continue the financial viability of the insurer.

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[3] Reported by William Jewett "Converging Roles Within the Insurance and Finance Marketplace" at the web site: http://www.centrere.com/insights/converge.htm on April 3, 1998.

A third kind of securitization deal involves trading options on a catastrophe index. The index is based on the catastrophe experience of (at least a sample of) the property/casualty industry. An insurer or reinsurer can purchase catastrophe call options that are exercisable if the catastrophe index exceeds a specified strike price. When the index value exceeds the strike price, the contract pays either a specified flat amount, or the amount by which the index exceed the strike price.

These options are traded on an exchange. For example, the Property Claims Service (PCS) index is traded on the Chicago Board of Trade (CBOT). The Guy Carpenter Catastrophe Index (GCCI) is traded on the Bermuda Commodities Exchange (BCE). In addition to public trading, these indices may also be used in private placements. The Risk Management Solutions (RMS) catastrophe index, which is based on the RMS catastrophe model, is used for this specific purpose.

From an individual insurer's perspective, a critical element when considering the use of a catastrophe index is basis risk – that is, how well the index correlates with the insurer's experience. For example, an insurer with exposure concentrated in a small geographic area may suffer high losses if a catastrophe occurs in that area. But that catastrophe may not trigger options based on a national index. An insurer can improve the potential correlation by purchasing options based on smaller geographic areas, such as regions, states or even ZIP-codes, that match the insurer's own portfolio.

Many investors favor the use of an industry index because the losses are not a function of an individual insurer's underwriting and claim settlement practices. Furthermore, the provisions of an option contract are standardized. This increases liquidity, as standardized contracts are easier to trade than customized contracts. Because of standardization, options can have smaller transaction costs than reinsurance or catastrophe bonds which require individual analysis and negotiation.

Catastrophe options provide certain challenges that insurers must recognize. As noted earlier, basis risk provides a measure of how well catastrophe options will meet an insurer's need to hedge risk. An insurer may collect substantial funds on catastrophe options when its actual catastrophe losses are small. More importantly, an insurer may collect little or no funds on catastrophe options but still suffer a substantial catastrophe loss. An insurer must carefully analyze basis risk before deciding if catastrophe options are a good way of hedging catastrophe risk.

Another critical element in the success of securitization is the regulatory acceptance of catastrophe options and other securitization instruments as reinsurance – an offset to an insurer's direct losses. Some insurers have established offshore companies to reinsure their catastrophe risk. The insurers then sell catastrophe bonds or use other financial instruments to finance the offshore reinsurers.

Rating agencies' evaluation of an insurer's financial strength is a critical element in attracting and retaining business. If rating agencies do not view an insurer's securitization measures as financially sound, the insurer may receive a poor rating – and therefore suffer a loss of business. Consequently, rating agencies' acceptance of a catastrophe securitization approach may be important to its success.

3. The Cost of the Instruments Used in Financing Insurance

So far, this paper has surveyed the various instruments available to finance catastrophe risk. The remainder of the paper will focus on one promising form of securitization – options on a catastrophe index – and see how insurers can combine them with capital and reinsurance to finance catastrophe risk.

We classify the various instruments for financing catastrophe insurance into the following elements:

1. Insurer Capital – This is money put up by investors in the insurance company. The company can use its capital to pay losses if current income is insufficient.
2. Reinsurance – This is money provided by outside entities that agree to pay losses in accordance with a predetermined function of the insurer's loss. Some securitization deals fall into this category.
3. Catastrophe Options – This is money provided by outside entities that agree to pay money contingent on the occurrence of a catastrophic event recorded on an index. That payment may or may not correspond with the insurer's loss. That is, catastrophe options do present basis risk.

Each instrument has a cost and a benefit. The insurer's problem is to find the combination of instruments that provides adequate financing for the least cost.

We define:
The cost of financing insurance =

the expected loss (net of reinsurance recoveries and recoveries from catastrophe options)
+ the cost of capital
+ the cost of reinsurance
+ the cost of catastrophe options

Our purpose in using reinsurance and catastrophe options is to reduce the expected loss and the cost of capital – and ultimately the cost of financing insurance.

Although this definition covers the insurer's entire operation, we will focus on catastrophes. Thus, our discussion of the cost of financing insurance will reflect *only the catastrophe losses*, with one exception – the cost of capital. The insurer's other assets and liabilities affect that cost. This discussion will ignore the remaining elements of the insurer's operation.

Quantifying the Cost of Financing Insurance

To perform this analysis, we will need to quantify the cost of financing insurance in terms of the probability of a catastrophic loss. We give some sample costing formulas below. The formulas have the advantage of being simple, but they are by no means unique or necessary to the examples given below.

For any random variable, Z, we define:

μ_Z = the expected value of Z

σ_Z = the standard deviation of Z.

See the appendix for the formulas for the various means and standard deviations used below.

Quantifying the Cost of Capital

We employ a probabilistic capital requirements formula as the starting point for this methodology. In the United States, insurers are not subject to an official probabilistic capital requirements formula. However, most actuaries believe that capital requirements should have probabilistic input. Actuaries generally accept the idea of a formula, but any particular formula will spark a debate. While we use one such formula here, an insurer can use another formula that suits the needs and perceptions of its management.

Let X be a random variable representing the insurer's total loss, net of recoveries from reinsurance and catastrophe options. Our formula for the cost of capital is:

$$\text{Cost of Capital} = K \times T \times \sigma_X$$

where:

T is a factor reflecting the insurer's risk aversion; and

K is the required return needed to attract sufficient capital.

We can link T to the insurer's probability of insolvency. For example, if we assume the insurer's losses follow a normal distribution, a choice of T = 2.32 corresponds to a one-in-one-hundred chance of insolvency. If the insurer is more risk averse, or if it feels that the distribution of insurer results is unusually skewed, the insurer can select a higher value of T.

The insurer will select K so that its rate of return is close to that obtained by other investments with similar risk. K will vary with market conditions.

In the examples below, we will let

$$X = X_O + X_C$$

where:

X_C = All catastrophe losses net of recoveries from reinsurance and index contracts; and

X_O = All other net losses.

When we partition X is this manner, the formula for the cost of capital becomes

$$\text{Cost of Capital} = K \times T \times \sqrt{\sigma^2_{X_O} + \sigma^2_{X_C}}$$

under the assumption that X_O and X_C are independent.

Quantifying the Cost of Reinsurance

The cost of catastrophe reinsurance depends upon market conditions. After a large catastrophe, the demand for reinsurance usually rises and reinsurer capital falls. Therefore, catastrophe insurance is in short supply and the reinsurance available fetches a high price. High prices attract new capital to reinsurers, and prices generally fall until the next catastrophe occurs.

The benefit of the reinsurance treaty is to reduce the insurer's cost of capital by reducing its expected loss, μ_{X_C}, and its standard deviation of loss, σ_{X_C}.

To develop a strategy for using reinsurance, an insurer needs to know its reinsurance costs. Those costs depend upon the retention and the limit of the reinsurance treaty, and each reinsurer has its own prices.

Let X_R be a random variable representing the reinsurance recovery. We will use the following formula for the cost of reinsurance in the examples below:

$$\text{Reinsurance Cost} = (\mu_{X_R} + \lambda \cdot \sigma_{X_R}^2) \times (1 + e)$$

where λ is a risk load multiplier, and e is an acquisition expense factor.

Quantifying the Cost of Catastrophe Options

In this paper, we will work with binary options on a catastrophe index. The holders of those options exercise them for a fixed amount, such as $1,000, when the index exceeds a predetermined strike price. Otherwise the options expire worthless.

To the seller of such options, the expected return should be competitive with other available investments of comparable risk. One way of gauging comparable risk is the analysis of bond defaults. For example, Moody's Investors Service has a web site that publishes bond default rates and interest rate spreads. In browsing Moody's web pages one finds the following statements about default rates:

- "Moody's trailing 12-month default rate for speculative-grade issuers ended 1997 at 1.82% -- up from last year's 1.64%, but well below its average since 1970 of 3.38%."
- "Moody's expects its speculative-grade 12-month default rate to rise toward the 2.5% level in 1998."[4]

With respect to interest rate spreads, Moody's states the following:

- "The spread of the median yield-to-maturity of intermediate-term speculative-grade bonds over seven-year US Treasuries climbed just 3 basis points to 267 basis points -- 92 basis points below its January 1993 to January 1997 average of 359 basis points."[5]

When comparing speculative-grade bonds to catastrophe options, the investor might consider the following:

- The projected 12-month default rate of speculative-grade bonds is 2.5%.
- We can estimate the probability of exercising the catastrophe options (as we will show below). We can compare that probability with estimated default rates for bonds.
- Catastrophe options can require posting a 100% margin at the time of sale. The money in the margin account earns a risk-free rate of return. Thus, the price of the option should be comparable to the interest rate spread for a bond of comparable risk over risk-free investments.
- The average spread of speculative-grade bonds over intermediate-term risk-free investments is about 3.5%. The spread could be lower over a 12-month term, but it should not be lower than the projected default rate.

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[4] The web site URL is http://www.moodys.com/defaultstudy/index.html. We obtained this quote on April 3, 1998.

[5] The web site URL is http://www.moodys.com/economic/1QDFLT97.htm. We obtained this quote on April 3, 1998.

- The exercise of a catastrophe option is not correlated with the other economic risks. That fact makes the catastrophe options more attractive to investors and should lower their price.

With all this information, one can compare the posted price of catastrophe options with bonds of equivalent risk. Investors will have varying interpretations of the information, but our point is that information relevant to the pricing of catastrophe options is publicly available.

4. An Illustrative Example

As an illustration of the kind of analysis investors and insurers can do, we used a catastrophe model to quantify the cost of financing insurance in terms of the costs of attracting capital, buying reinsurance, and buying catastrophe options. We compared the insurer's losses – generated by the catastrophe model – to the benefits provided by the various instruments.

To do the analysis, we took a sample of fifty insurers that report their personal lines exposure to ISO. We then analyzed the personal lines exposure for each of the fifty insurers using a hurricane model provided by Risk Management Solutions, Inc.[6] The analysis provided loss estimates and annual rates of occurrence for about 9,000 events for the insurers in the sample. We created "index" events by summing the losses for each event over all the insurers. We then multiplied the loss for each event by a factor that set the largest event equal to 100.

We then produced Table 4.1 below. The table contains the illustrative index values and the model-generated losses for one of the fifty insurers from the sample. We produced a similar exhibit for each of the fifty insurers.

[6] All hurricane loss estimates incorporated in this paper were developed by ISO's use of Risk Management Solutions' (RMS) proprietary IRAS hurricane technology. However, development of the individual company exposure data and the analyses were performed by ISO. Therefore the loss projections and conclusions presented in this paper are the responsibility of ISO.

With information like that provided in the exhibit, we can adjust insurer losses for any recoveries from a reinsurance contract or from catastrophe options. Since the model gives us the probability[7] of any loss and/or recovery, we can calculate any summary statistics needed to determine the cost and benefits of the various instruments used in financing insurance.

Table 4.1
Illustrative Index and Insurer Information

Event	Event Probability	Illustrative Index Value	Direct Insurer Loss
1	0.000001210	100.000	1,212,550,269
2	0.000001210	89.041	1,509,161,589
3	0.000001810	87.558	1,303,694,653
4	0.000007020	83.480	761,956,629
5	0.000007020	83.197	734,137,782
6	0.000004660	82.153	735,660,852
7	0.000007910	80.948	1,004,861,128
8	0.000050600	80.548	1,071,076,934
9	0.000007020	79.187	688,269,904
10	0.000001810	77.481	1,652,933,116
11	0.000002590	76.217	741,327,246
12	0.000005760	75.547	654,930,780
13	0.000009060	75.175	1,450,085,508
14	0.000022900	75.108	1,148,344,417
15	0.000001210	75.046	1,003,713,967
16	0.000007020	74.142	718,320,849
17	0.000000460	73.670	612,322,934
18	0.000002590	72.964	607,625,092
19	0.000000767	72.303	1,035,338,915
20	0.000000460	72.180	564,886,456
21	0.000001810	72.050	1,269,991,504
22	0.000021000	71.547	921,203,300
23	0.000000738	71.478	582,199,078
24	0.000018700	71.246	757,962,586
25	0.000000202	70.661	1,078,827,927
26	0.000001210	70.567	1,017,469,903
27	0.000001210	70.289	1,162,380,661
28	0.000001810	68.992	1,273,618,722
29	0.000007250	68.731	966,395,280
30	0.000007020	68.640	598,955,192
⇓	⇓	⇓	⇓

[7] Event probabilities can be calculated from the RMS model output. The RMS model provides annual rates of occurrence for individual events.

Illustrative Catastrophe Options

Using the illustrative catastrophe index, we set up illustrative catastrophe options that pay \$1,000 if the largest single event loss in the year exceeds a specified strike price. If no single event exceeds the strike price, the option is not exercised and the buyer receives \$0. In the examples that follow, we consider trades on options with strike prices of 5, 10, 15, . . . , 95, 100. The following table gives the probabilities that each option will be exercised. See the appendix for the formula for calculating those probabilities.

Table 4.2

Strike Price	**Exercise Probability**
0	1.00000000
5	0.16313724
10	0.07855957
15	0.04006306
20	0.02321354
25	0.01387626
30	0.00816229
35	0.00440132
40	0.00296168
45	0.00187601
50	0.00100615
55	0.00070126
60	0.00040197
65	0.00028771
70	0.00018975
75	0.00013880
80	0.00008846
85	0.00001125
90	0.00000121
95	0.00000121
100	0.00000121

The catastrophe options used in this example have a structure similar to those traded on the Guy Carpenter Catastrophe Index (GCCI),[8] with four important differences:

1. The scale of the indices is different. The illustrative index has 100 as its highest value whereas the GCCI has 700 as its highest value.
2. The sets of insurers that make up the indices are different.
3. The illustrative index simply sums the losses for each insurer, whereas the GCCI uses a complex set of rules designed to keep a single insurer from having too much influence at the ZIP-code level.
4. The illustrative index is an annual index, whereas the GCCI is semiannual and overlaps with the normal hurricane season in either one or five months.

The following table gives the costs used in the examples below. To calculate the price of the option, we added 0.035% of the variance of the contract payoff to the expected payoff. We arrived at the 0.035% figure by comparing the exercise probability of an option with a strike price of 20, against the price of a speculative-grade bond, as discussed above.

[8] For information about the options traded on the Guy Carpenter Catastrophe Index, visit the Bermuda Commodities Exchange web site at http://www.bcoe.bm

Table 4.3

Strike Price	Expected Payout	Contract Price
0	1000.000	1000.000
5	163.137	210.920
10	78.560	103.895
15	40.063	53.523
20	23.214	31.150
25	13.876	18.666
30	8.162	10.996
35	4.401	5.935
40	2.962	3.995
45	1.876	2.531
50	1.006	1.358
55	0.701	0.947
60	0.402	0.543
65	0.288	0.388
70	0.190	0.256
75	0.139	0.187
80	0.088	0.119
85	0.011	0.015
90	0.001	0.002
95	0.001	0.002
100	0.001	0.002

Insurer Examples

The following analysis of three insurers shows how those insurers can reduce the cost of financing insurance through the proper use of reinsurance and catastrophe options. The insurers are three members of the sample of fifty insurers that we selected above. We randomly adjusted the losses of each insurer to protect their anonymity.

- Insurer #1 is a medium sized national insurer with exposure that tracks relatively well with the exposure underlying the illustrative index.
- Insurer #2 is a large national insurer with exposure that tracks less well with the exposure underlying the index than Insurer #1.
- Insurer #3 is a regional insurer with exposure that does not track well with that of the index.

We provide summary statistics for the insurers' catastrophe losses.

Table 4.4

	Insurer #1	**Insurer #2**	**Insurer #3**
Expected Catastrophe Loss	34,839,348	95,417,229	2,385,629
Std. Dev. Of Catastrophe Loss	81,044,318	196,767,192	18,098,024
Coef. of Correlation with Index	0.93	0.75	0.35

We now provide the economic assumptions underlying our estimate of the cost of financing insurance. The assumptions made here are not specific to the particular insurer, but we could modify the assumptions and/or make them specific after a discussion with an insurer's management.

The Cost of Financing Insurance

As discussed above, we use the following formula for the cost of insurer capital:

$$\text{Cost of Capital} = K \times T \times \sqrt{\sigma_{X_0}^2 + \sigma_{X_C}^2}$$

with K = 20%; T = 3.00 and σ_{X_0} = the insurer's initial σ_{X_C}. In a real case, we would estimate σ_{X_0} by analyzing the insurer's other assets and liabilities.

In the examples that follow, we use the following formula for the cost of reinsurance:

$$\text{Reinsurance Cost } = (\mu_{X_R} + \lambda \cdot \sigma_{X_R}^2) \times (1 + e)$$

with $\lambda = 1.5 \times 10^{-7}$ and e = 10%. The selected value of λ is close to what ISO uses in its risk load formula for increased limits ratemaking.

If the insurer buys N_S contracts for strike price S at cost C_S, the total cost of the index contracts is:

$$\sum_S N_S \cdot C_S$$

Table 4.3 gives the values of C_S for each strike price, S.

The insurer's management has to make three key decisions to minimize the cost of financing insurance:

1. How much capital should the insurer retain?
2. What layer of reinsurance does the insurer buy?
3. How many index contracts, N_S, does the insurer buy at a given strike price, S?

Now, for a given reinsurance layer and a given set of index contracts, we can calculate the quantities μ_{X_R}, $\sigma^2_{X_R}$, μ_{X_C}, and $\sigma^2_{X_C}$ using formulas given in the appendix.

Thus our expression for the cost of financing insurance becomes

$$\mu_{X_C} + K \times T \times \sqrt{\sigma^2_{X_O} + \sigma^2_{X_C}} + (\mu_{X_R} + \lambda \cdot \sigma^2_{X_R}) \times (1+e) + \sum_S N_S \cdot C_S$$

We seek to minimize this expression by choosing the right layer of reinsurance and the right numbers, N_S, of catastrophe options.

We do not now have an analytic solution to this minimizing problem. That is because of the effort involved in deriving one and because we do not feel that the assumptions we made in calculating the cost of financing insurance are final.[9] Instead, we used a numerical search algorithm, Excel Solver™. As it is difficult to ascertain that the numerical search solution is indeed the optimum, we should characterize the results as "the best solution we could find."

In order to reduce the computing time, we restricted the reinsurance retention and limit to multiples of $1,000,000 and the number of catastrophe options to multiples of 100. In addition we forced the number of catastrophe options to be the same for each of the

[9] For an analytic solution to a simpler problem, see "A Buyer's Guide to Options on a Catastrophe Index" by Glenn Meyers. The paper has been accepted for publication in the *Proceedings of the Casualty Actuarial Society*.

following groups of strike prices: 5, 10, 15, and 20; 25, 30, 35 and 40; 45,50, and 55; 60, 65, and 70; 75, 80, and 85; and 90, 95, and 100.

The search for the minimum cost of financing insurance produced the following results:

Table 4.5

Contract	Number of Index Contracts		
Range	Insurer #1	Insurer #2	Insurer #3
5-20	47,400	93,100	0
25-40	74,400	118,100	6,300
45-55	59,500	67,900	0
60-70	47,600	28,600	0
75-85	81,400	545,100	0
90-100	37,200	634,800	0
		Reinsurance	
Retention	73,000,000	457,000,000	54,000,000
Limit	13,000,000	36,000,000	105,000,000

The elements of the cost of financing insurance are as follows:

Table 4.6

Best Solution Obtained for the Cost of Financing Insurance

	Insurer #1	Insurer #2	Insurer #3
Expected Net Loss	16,315,629	62,086,995	1,464,410
Cost of Capital	47,905,407	143,662,761	12,914,922
Cost of Reinsurance	2,132,070	1,848,530	1,726,342
Cost of Index Contracts	22,252,015	42,409,101	249,427
Cost of Financing Insurance	88,605,121	250,007,387	16,355,100

We compared the "best solution" with two alternative solutions:

Table 4.7

Cost of Financing Insurance without Reinsurance or Index Contracts

	Insurer #1	Insurer #2	Insurer #3
Expected Net Loss	34,839,348	95,417,229	2,385,629
Cost of Capital	62,095,747	166,962,499	15,356,683
Cost of Reinsurance	0	0	0
Cost of Index Contracts	0	0	0
Cost of Financing Insurance	96,935,095	262,379,728	17,742,312

Table 4.8

Cost of Financing Insurance after Dropping the Smallest Element from the Best Solution

	Insurer #1	**Insurer #2**	**Insurer #3**
Expected Net Loss	17,945,994	63,198,145	1,648,555
Cost of Capital	48,508,962	145,045,517	13,023,441
Cost of Reinsurance	0	0	1,726,342
Cost of Index Contracts	22,252,015	42,409,101	0
Cost of Financing Insurance	88,706,971	250,652,763	16,398,337

We can make two observations:

- The introduction of catastrophe options and reinsurance can significantly reduce the cost of financing insurance. In the examples the cost was reduced by 8.6 % for Insurer #1, 4.7% for Insurer #2, and 7.8% for Insurer #3.
- The role of catastrophe options was more significant for the insurers whose catastrophe losses were better correlated with the index. Conversely the role of reinsurance was more significant for the insurer whose catastrophe losses were poorly correlated with the index.

The Marginal Cost of Financing Catastrophe Insurance

The examples illustrate that reinsurance and catastrophe options can significantly reduce the cost of financing insurance. However the analysis does not address the question of how much the insurer needs to build the cost of financing into its premiums. Actuaries usually refer to that cost as the risk load.[10]

To answer the question, we calculate the cost of financing insurance, with and without the catastrophe lines. We call the difference between those costs the marginal cost of

[10] See "The Competitive Market Equilibrium Risk Load Formula for Catastrophe Ratemaking" by Glenn Meyers, *Proceedings of the Casualty Actuarial Society LXXXIII*, 1997, for background on risk loads for catastrophe ratemaking. That paper goes beyond the current paper by allocating the risk load to individual insureds. However it accounts only for the cost of capital, and does not account for reinsurance and catastrophe options.

financing catastrophe insurance. If the insurer can recover that cost in the premiums it charges, it should write the insurance.

Continuing our example, the cost of financing insurance without catastrophe insurance[11] is: $K \times T \times \sigma_{X_o}$. Thus the marginal cost of financing catastrophe insurance becomes

$$\mu_{X_C} + K \times T \times \left(\sqrt{\sigma_{X_O}^2 + \sigma_{X_C}^2} - \sigma_{X_O}\right) + (\mu_{X_R} + \lambda \cdot \sigma_{X_R}^2) \times (1+e) + \sum_S N_S \cdot C_S$$

We summarize the results for the three insurers in our illustrative example:

Table 4.9

The Marginal Cost of Financing Catastrophe Insurance Using the Best Solution

	Insurer #1	Insurer #2	Insurer #3
Cost of Financing without Cats	43,908,324	103,258,865	10,764,807
Cost of Financing with Cats	88,605,121	250,007,387	16,355,100
Marginal Cost of Cats	44,696,797	146,748,522	5,590,293
Marginal Cost/Expected Loss	1.283	1.538	2.343

We do a similar calculation without considering reinsurance or contracts on a catastrophe index.

Table 4.10

The Marginal Cost of Financing Catastrophe Insurance Without Reinsurance or Index Contracts

	Insurer #1	Insurer #2	Insurer #3
Cost of Financing without Cats	43,908,324	103,258,865	10,764,807
Cost of Financing with Cats	96,935,095	262,379,728	17,742,312
Marginal Cost of Cats	53,026,771	159,120,863	6,977,505
Marginal Cost/Expected Loss	1.522	1.668	2.925

Here we see that the proper use of reinsurance and catastrophe options can have a significant effect on premiums, as the marginal cost of financing catastrophe insurance is substantially lower for each insurer using a mix of reinsurance and catastrophe options.

[11] Technically, we should include the expected value of the losses without the catastrophe insurance. But the focus of this paper is on catastrophes, and the expected loss for the noncatastrophe exposure will cancel out when we compute the marginal cost of financing catastrophe insurance.

5. The Next Steps

This paper has taken a first step beyond the insurer capital and reinsurance paradigm, by showing how to incorporate instruments with basis risk to reduce the cost of financing catastrophe insurance. Having taken this first step, there are a number of directions that can be taken. We list a few.

- The insurer could consider buying catastrophe options on a regional or state index, as well as a national index. The additional flexibility could decrease the cost of providing insurance for some insurers – such as Insurer #3 above.
- Returns from catastrophe options could be imbedded within the reinsurance. That is, the reinsurance would cover the difference between the insurer's actual loss and the index recovery.
- We could create a customized index to form the basis of settlement between the insurer and a reinsurer. Such an index would be based on the industry data, but with a customized set of ZIP-codes. With such an arrangement, adverse selection by the primary insurer would no longer be an issue.
- A reinsurer could use the catastrophe options as a hedge for its combined exposure. To do this, the reinsurer would have to combine the exposure of all its treaties and do an analysis similar to that done above. The options could give the reinsurer increased capacity to write more catastrophe coverage.

Appendix

The Calculation of the Statistics for a Maximum Event Index Contract

This appendix gives the formulas for the statistics used in calculating the cost of financing insurance. The calculations are complicated by the fact that the catastrophe index recovery for an event depends upon whether or not the event was the largest event. We solve this by calculating conditional statistics based on the event being the largest – and then calculate global statistics by summing over the conditional probabilities.

We are given n (about 9000) events from the catastrophe model and the index values associated with each event. We assume that the events are independent and that they can only happen once in a year[12]. The events are sorted in decreasing order of the index value. Table A.3 gives the first 30 rows of the of the calculation. The following table gives the formulas used in this exhibit.

Table A.1
Formulas for Table A.3

ith Row of Column	Description and Formula
Event	The ith event specified by the catastrophe model
Index Value	The value of the index if the ith event is the largest
Event Probability, p_i	The probability of the ith event as specified by the catastrophe model
Max Event Probability, $_M p_i$	The probability that the ith event happens and all larger events do not happen $_M p_i = p_i \cdot \prod_{j=1}^{i-1} (1 - p_j)$
Contract Value, v_i	The amount paid by the insurer's portfolio of catastrophe options *given that the ith event is the maximum event*
Direct Insurer Loss, x_i	The loss generated by catastrophe model for the ith event on the insurer's exposure
Reinsurance Recovery, r_i	The amount recovered from the reinsurance contract for the ith event
Event Loss Given Max, e_i	$e_i = x_i - v_i - r_i$

[12] The RMS model provides annual rates of occurrence for events. Because rates are so small, making the assumption that events can only happen once per year is not unreasonable.

Table A.1 – Continued

ith Row of Column	Description and Formula
E[Loss | Event is the Max], E_i	$E_i = e_i + \sum_{j=i+1}^{n} E[(x_j - r_j)]$ $= e_i + \sum_{j=i+1}^{n} (x_j - r_j) \cdot p_j$ $= e_i + E_{i+1} - e_{i+1} + (x_{i+1} - r_{i+1}) \cdot p_{i+1}$
E[Loss^2 | Event is the Max], ${}_2E_i$	${}_2E_i = E_i^2 + \sum_{j=i+1}^{n} \mathrm{Var}[(x_j - r_j)]$ $= E_i^2 + \sum_{j=i+1}^{n} (x_j - r_j)^2 \cdot p_j \cdot (1 - p_j)$ $= E_i^2 + {}_2E_{i+1} - E_{i+1}^2 + (x_{i+1} - r_{i+1})^2 \cdot p_{i+1} \cdot (1 - p_{i+1})$

Table A.2

Cost of Financing Insurance Statistics

Overall Statistic	Formula
E[Reinsurance Recovery], μ_{X_R}	$\mu_{X_R} = \sum_{i=1}^{n} p_i \cdot r_i$
Var[Reinsurance Recovery], $\sigma_{X_R}^2$	$\sigma_{X_R}^2 = \sum_{i=1}^{n} r_i^2 \cdot p_i \cdot (1 - p_i)$
E[Net Catastrophe Loss], μ_{X_C}	$\mu_{X_C} = \sum_{i=1}^{n} {}_M p_i \cdot E_i$
Var[Net Catastrophe Loss], $\sigma_{X_C}^2$	$\sigma_{X_C}^2 = \sum_{i=1}^{n} {}_M p_i \cdot {}_2E_i - \mu_{X_C}^2$

Exercise Probabilities

Let PE_i denote the probability that maximum event catastrophe option at the level of event i will be exercised. The option will be exercised if either the ith or a lower numbered (higher loss) event happens. That is:

$$PE_1 = p_1, \quad PE_i = p_i + PE_{i-1} \cdot (1 - p_i)$$

Table A.3 Preliminary Calculations for the Cost of Financing Insurance Statistics

Event	Index Value	Event Probability	Max Event Probability	Contract Value	Direct Insurer Loss	Reinsurance Recovery	Event Loss Given Max	E[Loss|Max]	E[Loss^2|Max]
1	100.00	0.000001210	0.000001210	1,125,200,000	1,212,550,269	16,000,000	71,350,269	105,039,888	1.06712E+16
2	89.04	0.000001210	0.000001210	1,021,700,000	1,509,161,589	16,000,000	471,461,589	505,149,400	2.33194E+17
3	87.56	0.000001810	0.000001810	1,021,700,000	1,303,694,653	16,000,000	265,994,653	299,680,134	7.95274E+16
4	83.48	0.000007020	0.000007020	939,300,000	761,956,629	16,000,000	(193,343,371)	(159,663,127)	4.17510E+16
5	83.20	0.000007020	0.000007020	939,300,000	734,137,782	16,000,000	(221,162,218)	(187,487,015)	5.30470E+16
6	82.15	0.000004660	0.000004660	939,300,000	735,660,852	16,000,000	(219,639,148)	(185,967,298)	5.23874E+16
7	80.95	0.000007910	0.000007910	939,300,000	1,004,861,128	16,000,000	49,561,128	83,225,155	8.84949E+15
8	80.55	0.000050600	0.000050598	939,300,000	1,071,076,934	16,000,000	115,776,934	149,387,575	2.02818E+16
9	79.19	0.000007020	0.000007019	856,900,000	688,269,904	16,000,000	(184,630,096)	(151,024,174)	3.88460E+16
10	77.48	0.000001810	0.000001810	856,900,000	1,652,933,116	16,000,000	780,033,116	813,636,074	6.19226E+17
11	76.22	0.000002590	0.000002590	856,900,000	741,327,246	16,000,000	(131,572,754)	(97,971,674)	2.23955E+16
12	75.55	0.000005760	0.000005759	856,900,000	654,930,780	16,000,000	(217,969,220)	(184,371,820)	5.20551E+16
13	75.18	0.000009060	0.000009059	856,900,000	1,450,085,508	16,000,000	577,185,508	610,769,915	3.42608E+17
14	75.11	0.000022900	0.000022898	856,900,000	1,148,344,417	16,000,000	275,444,417	309,002,893	8.34181E+16
15	75.05	0.000001210	0.000001210	856,900,000	1,003,713,967	16,000,000	130,813,967	164,371,248	2.37695E+16
16	74.14	0.000007020	0.000007019	774,500,000	718,320,849	16,000,000	(72,179,151)	(38,626,801)	1.07551E+16
17	73.67	0.000000460	0.000000460	774,500,000	612,322,934	16,000,000	(178,177,066)	(144,624,990)	3.68535E+16
18	72.96	0.000002590	0.000002590	774,500,000	607,625,092	16,000,000	(182,874,908)	(149,324,364)	3.85299E+16
19	72.30	0.000000767	0.000000767	774,500,000	1,035,338,915	16,000,000	244,838,915	278,388,677	6.68006E+16
20	72.18	0.000000460	0.000000460	774,500,000	564,886,456	16,000,000	(225,613,544)	(192,064,034)	5.58109E+16
21	72.05	0.000001810	0.000001810	774,500,000	1,269,991,504	16,000,000	479,491,504	513,038,744	2.37731E+17
22	71.55	0.000021000	0.000020997	774,500,000	921,203,300	16,000,000	130,703,300	164,231,531	2.34399E+16
23	71.48	0.000000738	0.000000738	774,500,000	582,199,078	16,000,000	(208,300,922)	(174,773,109)	4.83588E+16
24	71.25	0.000018700	0.000018697	774,500,000	757,962,586	16,000,000	(32,537,414)	976,524	6.73762E+15
25	70.66	0.000000202	0.000000202	774,500,000	1,078,827,927	16,000,000	288,327,927	321,841,651	9.01151E+16
26	70.57	0.000001210	0.000001210	774,500,000	1,017,469,903	16,000,000	226,969,903	260,482,415	5.82464E+16
27	70.29	0.000001210	0.000001210	774,500,000	1,162,380,661	16,000,000	371,880,661	405,391,786	1.45612E+17
28	68.99	0.000001810	0.000001810	726,900,000	1,273,618,722	16,000,000	530,718,722	564,227,570	2.89618E+17
29	68.73	0.000007250	0.000007249	726,900,000	966,395,280	16,000,000	223,495,280	256,997,239	5.66513E+16
30	68.64	0.000007020	0.000007019	726,900,000	598,955,192	16,000,000	(143,944,808)	(110,446,942)	2.59361E+16