The Complement of Credibility

by Joseph A. Boor

ABSTRACT

THE COMPLEMENT OF CREDIBILITY

This paper explains the most commonly used complements of credibility and offers a comparison of the effectiveness of the various methods. It includes numerous examples. It covers credibility complements used in excess ratemaking as well as those used in first dollar ratemaking. It also offers six criteria for judging the effectiveness of various credibility complements. One criterion, statistical independence, has not previously been covered in the actuarial literature. This paper should explain all the common credibility complements to the actuarial student.

THE COMPLEMENT OF CREDIBILITY

Many actuarial papers discuss credibility. Actuaries use credibility when data is sparse and lacks statistical reliability. Specifically, actuaries use it when historical losses have a large error around the underlying expected losses (average of the distribution of potential loss costs) the actuary is estimating. In those circumstances, the statistic that receives the remainder of the credibility can be more important than the credibility attached to the data. For example, if the ratemaking statistic varies around the true expected losses with a standard deviation equal to its mean, it will probably receive a very low credibility. So, the vast majority of the rate (in this context, expected loss estimate) will come from whatever statistic receives the complement of credibility. So, it is very important to use an effective statistic for the complement of credibility. This paper will discuss fundamental principles to use in choosing the complement. And, it will discuss several methods actuaries use regularly.

I. Fundamental Principles- What Should The Actuary Consider?

There are four types of issues that any actuary must consider when choosing the complement: practical issues; competitive market issues; regulatory issues; and, statistical issues.

A. Practical Issues

The easiest statistic to use is one that is readily available. For example, the best possible statistic is next year's loss costs. Unfortunately, that statistic is not available (otherwise, companies would not need actuaries). The actuary must choose from the statistics that are available to him. Since some statistics require more complicated programming or expensive processing than others, some statistics are more readily available than others.

Ease of computation is another factor to consider. If a statistic is easy to compute, it is often easier to explain to management and customers. Since few actuaries have unlimited budgets, they usually weigh the time involved in computing a very accurate statistic against the accuracy improvement it generates. Also, when computations are easy to do there is less chance of error.

B. Competitive Market Issues

Rates are rarely made in a vacuum. Generally, whatever rate the actuary produces will be subject to market competition. If the rate is too high, competitors can undercut the rate and still make a profit. That will cost the actuary's employer customers and profit opportunities. If the rate is too low, the employer will lose money. So, in mathematical terms, the rate should be unbiased (neither too high nor too low over a large number of loss cost estimates) and accurate (the rate should have as low an error variance as possible around the future expected losses being estimated). Hence the complement of the credibility should help make the rate as unbiased and accurate as possible.

C. Regulatory Issues

Usually, rates require some level of approval from insurance regulators. The classic rate regulatory law requires that rates be 'not inadequate, not excessive, and not unfairly discriminatory.' The principles of adequacy and non-excessiveness imply that rates should be as unbiased as possible.

Those principles could be stretched to imply that rates should be accurate. The argument goes as follows. Highly inaccurate rates create a much greater risk of insolvency through random inadequacies. The law is concerned with inadequacy because it seeks to prevent insolvencies. So, law suggests rates should be as accurate as possible. For most purposes, actuaries interpret 'unfairly discriminatory' in the ratemaking context as 'unbiased'. Many believe that if a rate truly reflects a class's probable loss experience, it is fair by definition.

The actuary can mitigate regulatory concerns by choosing a complement that has some logical relationship to the loss costs of the class or individual being rated. That means it is easier to explain a high rate for a class or individual in light of the related loss costs.

D. Statistical Issues

Clearly, the actuary must attempt to produce the most accurate rate that is practical. If the complement of the credibility is accurate in its own right and relatively independent of the base statistic (which receives the credibility), the resulting rate will be more accurate. The rationale involves statistical properties of credibility-weighted estimates. As Appendix A shows, if the optimum credibility for two unbiased statistics is used, then the prediction error of the credibility-weighted estimate is

$$\frac{\tau_1^2\tau_2^2(1-\rho^2)}{\tau_1^2+\tau_2^2-2\rho\tau_1\tau_2},$$

where

τ_1^2 is the average squared error (inaccuracy) of the base statistic as a stand-alone predictor of next years' loss costs;
τ_2^2 is the average squared error (inaccuracy) of the complement of the credibility as a stand-alone predictor of next year's mean loss costs;
ρ is the correlation (interdependence) between the first statistic's prediction error (error in predicting next year's mean loss costs) and the second statistic's prediction error.

Reviewing that error expression shows that greater inaccuracy in either the base statistic *or* the complement of credibility will yield greater inaccuracy in the resulting prediction. The expression is symmetric in the two errors. So, the accuracy of the complement of credibility is just as important as the accuracy of the base statistic.

The benefits of independence are more subtle. As it turns out, independence is most important when credibility is most important. That is independence is most important for the intermediate credibilities (Z between 10% and 90%). Following Appendix B, that occurs when the largest standard predicting error ($\sqrt{\text{inaccuracy}} = \tau$) is within two to three times[1] the smaller error. Consider the following graphs of the total prediction error by correlation for τ_2 = one, two, and three times τ_1.

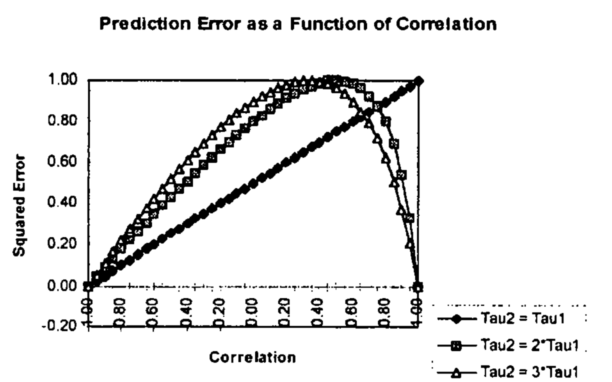

Figure 1

As you can see, the predictions are generally best when there is actually a negative correlation between the two errors (that is, they offset). But, that rarely occurs in practice. Generally, the complement of credibility will have some weak correlation with the base statistic. In that range the prediction error is clearly lowest as the correlation is smaller. Further, the graph beyond the maximum error (correlations near unity) is misleading. Appendix B shows that the downward slope near unity brings negative credibilities. Those negative credibilities are clearly outside the general actuarial philosophy of credibility. So, a complement of credibility is best when it is statistically independent (that is, not related to) the base statistic.

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[1] Since Boor[1] shows that credibility is roughly proportional to the relative τ^2's, these examples cover credibilities between 10% and 90%. That range covers mosts instances where credibility matters most.

E. Summary of Desirable Qualities

The previous sections show six desirable qualities for a complement of credibility:

- Accuracy as a predictor of next year's mean loss costs;

- Unbiasedness as a predictor of next year's mean subject expected losses;

- Independence from the base statistic;

- Availability of data;

- Ease of computation; and

- Explainable relationship to the subject loss costs.

II. First Dollar Ratemaking

First dollar (that is, not pricing losses excess above a very high deductible) ratemaking credibility complements are affected by a common characteristic of first dollar ratemaking. First dollar ratemaking generally uses historical loss data for the base statistic. And, in first dollar ratemaking the historical losses are usually roughly the same magnitude as the true expected losses. The regulatory quality of an explainable relationship to the subject loss costs is more important for first dollar than excess ratemaking.

There are a wide variety of techniques actuaries use to develop credibility complements. The following pages discuss some of the major methods in use.

A. Loss Costs of a Larger Group Including the Class -- Classic Bayesian Credibility

The most basic credibility complement comes from the most classic casualty actuarial technique ... Bayesian credibility. In Bayesian credibility actuaries are typically either making rates for a large group of classes or making rates for a number of large insureds that belong to a single class. The classes (or individual insureds) do not contain enough exposure units for their historical loss data to reliably predict next year's mean loss costs. So, actuaries supplement the class's historical loss data by credibility weighting them with the loss costs of the entire group.

In mathematical terms, we use

$$Z(L_c/E_c) + (1-Z)(\sum_i L_i / \sum_i E_i);$$

where

L_c is the historical loss costs for the subject class, c;

E_c is the historical exposure units for class c;

L_i is the historical loss costs for the ith class in the group;

E_i is the historical exposure units for the ith class in the group; and

Z is the credibility.

(For the rest of this paper P_c will denote the historical loss rate for class c (L_c/E_c). P_g will do the same for the group's historical loss cost rate.)

<u>Complement's Qualities</u>

This complement has problems in two areas, accuracy and unbiasedness. The group mean loss costs may be the best available substitute. And they may be unbiased with respect to all the information the actuary has when making the rate (e.g., historical loss data -- the real means remain unknown). But, the actuary should believe that the true expected class losses will take a different value than the group expected losses. So, this method contains an intrinsic bias and inaccuracy that is unknown.

This complement generally has some independence from the base statistic. As long as the base class does not predominate in the whole group, the process errors of all the other classes should be independent from that of the base class. And the error created by using the group mean instead of the class mean is independent of the base class process variance (error). To the extent that the actuary uses the same loss development, trend, and current level factors on the class and group, the error from those factors is interdependent between the class and group loss costs. But, you could view the ratemaking process as first estimating undeveloped, untrended historical expected losses at previous rates; then applying adjustment factors. In the first part of that process, the predicting errors are nearly independent.

This complement performs well on availability and ease of computation. Generally, actuaries compute the group mean and group rate indication as the first stage of the pricing process for the entire line of business.

As long as all the classes in the group have something in common that puts them in the group, that forms the logical connection between the class's loss costs and those of the group. However, that does not totally eliminate controversy from this credibility complement. Customers may often complain that they are treated 'just like everyone else' when their historical losses are below average. Overall, this has an average degree of relationship to the expected subject losses.

Choosing the Larger Group

An actuary should be careful when choosing which larger group to use. For example, given a choice between using same class data from other states (provinces) or other class data from the same state, the actuary should consider: Whether the differences by state in loss levels are more significant than the differences between class costs in the same state? (Usually, class differences are larger); Can the other state's class data be adjusted to reflect the base state loss levels? (reducing bias); Is there a group of classes in the state that the actuary would expect to have about the same loss costs? (small bias.) All those factors merit consideration. The actuary should attempt to find the larger group statistic that has the least expected bias.

Example

Consider the data table below.

Data for Bayesian Credibility Complement

		Last Year's Data			Last Three Year's Data		
Rate Group	Class	Exposures	Losses	Pure Premium	Exposures	Losses	Pure Premium
A	1	100	5000	\$50	250	16000	\$64
	2	300	20000	\$67	850	55000	\$65
	3	400	19000	\$48	1100	55000	\$50
	Subtotal	800	44000	\$55	2200	126000	\$57
B	Subtotal	600	29000	\$48	1700	55000	\$32
C	Subtotal	500	36000	\$72	1400	120000	\$86
D	Subtotal	800	75000	\$94	2300	200000	\$87
Total		2700	184000	\$68	7600	501000	\$66

Table 1

If one is making rates for class 1 in rate group A, one must first consider whether to use the one year or three year historical losses. One must consider that the three year pure premiums will be less affected by process variance (year-to-year fluctuations in experience due to small samples from the distribution of potential claims). On the other hand, sometimes the exposure base is large enough to minimize process variance and societal events are causing pure premiums to change (changes in the potential losses one is sampling from). In that situation the one year pure premiums are preferable.

Suppose one chooses the one year pure premium ($50) for historical data. Then using the three year pure premiums of the class ($64) for the complement would be inappropriate. That is because the three year pure premiums are heavily interdependent with the one year class pure premium. Also, presumably the actuary has already decided that the three year data is biased because of changes in loss cost levels. So, the actuary believes the three year data does not add accuracy to the prediction. For the same reasons, the three year rate group and grand total pure premiums would be inappropriate complements.

The next decision is between the rate group and grand total pure premiums. The choice between these involves a tradeoff between bias reduction and process variance reduction. The rate group data should reflect risks that are more similar to class 1. So, it should have less bias. On the other hand, the grand total data is spread over more risks, so it has less process variance. This example makes the choice difficult. The one year and three year rate group pure premiums are very similar ($55 versus $57). But the other rate groups show more pronounced inconsistencies (i.e. $32 versus $48 for rate group B). The grand total shows it has little process variance. But it appears to contain roughly $15 of bias. The one year rate group A pure premium ($55) is probably the best choice.

One could also consider using the three year pure premium for historical losses. That does not preclude using the one year rate group data as a complement. Using the one year rate group A pure premium would simply assume that the entire rate group A exposures were sufficient to minimize process variance. So, it may be appropriate to use one year data as a complement to three year data.

B. Loss Costs of a Larger Related Class

Actuaries sometimes use the loss costs of a larger, but related class for the complement of credibility. For example, if a company writes very few picture framing stores but writes a large number of art stores, the actuary may choose to use the art store loss costs for the framing store complement of credibility. He may or may not make some adjustments to the art store loss costs to make them more applicable to framing stores. For example, he may wish to adjust for the minor woodworking exposure. Actuaries pricing General Liability often use this 'base class' (meaning the larger related class in this context) approach.

Complement's Qualities

This approach has qualities similar to the large group complement. It is biased (though the bias and its direction are unknown) and so it is inaccurate. The more the actuary adjusts the related class loss data to match the loss exposure in the subject base class; the more the bias is reduced. The independence may be slightly less if the factor relating the classes generates high losses for the two classes simultaneously. But the actuary must be careful that this seeming independence is not just a simultaneous shift in the expected losses (which is not prediction error, it is an increase in expected losses). It is usually the latter.

This complement does not fare quite as well as the group mean in other categories. Data is not as readily available for this complement as the group mean. But, if the company writes some related class, data should be available and already computed for that class's rates.

The computations involved in adjusting related class data may be more difficult. Any loss cost adjustments will require some extra work. Since there is some relationship between the base class and the related class (they must be related some way by definition), explaining this complement may be easier than explaining the larger group complement.

Example

Consider the case of the framing stores. Suppose the actuary wishes to estimate a fire rate for framing stores and already has a well-established rate for art stores. Perhaps the actuary sees that the only visible difference in exposure is the presence of substantial wood and sawdust. So he might choose to add a judgmental 10% of the excess of the fire rate for lumberyards over the fire rate for art stores.

C. Harwayne's Method

Harwayne's method[3] uses a specific type of data from a related class. Usually it is also a case of using loss costs from the larger group. In Harwayne's method actuaries use countrywide (excepting the base state being reviewed) class data to supplement the loss cost data for each class. But we adjust countrywide data to remove overall loss cost differences between states (or provinces).

The process is as follows. First we determine what the total countrywide average pure premium would be if the countrywide data had the same percentage mixture of classes (class distribution) as the base state. The result reflects the base state class distribution but probably reflects the differences in overall loss costs differences between states.

Next, actuaries use that difference in overall loss costs to adjust the countrywide class data to match the base state overall loss cost levels. We determine the ratio of overall state loss costs to overall (all classes) adjusted countrywide loss costs. Then we multiply that ratio times the countrywide base class loss costs to get the complement of credibility.

That is Harwayne's basic method. In a variant form, actuaries may adjust each state's loss costs individually to the base state level to eliminate biases due to different state distributions between classes (Harwayne used this variant). Then, actuaries compute the average class complement by weighting the individual state's adjusted loss costs. In another variant, actuaries adjust other state's historical loss ratios by class to match the base state's overall loss ratio. In either variant, the basic principles are the same.

<u>Formulas</u>

The simplified formula for Harwayne's method is as follows. Let

$L_{c,s}$ denote the historical losses for class c in the base state s;
$E_{c,s}$ denote the associated exposure units;
$P_{c,s}$ denote the state pure premium for class c
$L_{i,j}$ denote the historical losses for an arbitrary class 'i' in some state j; and
$E_{i,j}$ will denote the associated exposure units; and
$P_{i,j}$ will denote the state j pure premium for class i.

First, actuaries compute the countrywide pure premium adjusted to the state class distribution. The first step is to compute the "state s" average pure premium (rate)

$$P_s = \sum_i L_{i,s} / \sum_i E_{i,s}.$$

The next step is to compute the countrywide rates by class

$$P_i = \sum_{j \neq s} L_{i,j} / \sum_{j \neq s} E_{i,j}.$$

Then, actuaries compute the countrywide rate using the state s distribution of exposures

$$\overline{P} = \sum_i E_{i,s} P_i / \sum_i E_{i,s}.$$

So, the overall pure premium adjustment factor is

$$F = P_s / \overline{P}.$$

And the complement of the credibility for class c is assigned to $F \times P_c$.

Harwayne's more complicated (and more accurate) formula replaces the overall adjustments to countrywide data with separate adjustments for each state. That is, actuaries compute state overall means with the base state ("s") class distribution.

$$\overline{P}_j = \frac{\sum_m E_{m,s} P_{m,j}}{\sum_m E_{m,s}}$$

Then, we compute individual state adjustment factors

$$F_j = P_s / \overline{P}_j .$$

And then we adjust each state's class c historical rates using the F_j's. That is, we compute the adjusted "state j" rates

$$P'_{c,j} = F_j P_{c,j}.$$

and then we weight them with the countrywide distribution between states

$$\text{Complement} = C = \frac{\sum_j E_{c,j} P'_{c,j}}{\sum_j E_{c,j}}.$$

The result is Harwayne's more complicated complement of credibility.

<u>Complement's Qualities</u>

This complement has very high statistical quality. Because Harwayne's method uses data from the same class in other states and attempts to adjust for state-to-state differences, it is very unbiased. It is also reasonably accurate as long as there is sufficient countrywide data to minimize process variance. Since the loss costs are from other states, their prediction errors (remaining bias) should be fairly independent of the base class process error in the base state. One exception might be where there is an across-the-board jump in all class's loss costs in state s that alter the adjustment to the state experience level. But, across-the-board jumps usually flow through into the next year's expected losses, so they are rarely prediction errors.

This complement has a mixed performance on the less mathematical qualities. Data are usually available for this process. But the computations do take time and are complicated. Thankfully, they do bear a much more logical relationship to class loss costs in individual states than unadjusted countrywide statistics. On the other hand, this may be harder to explain because of complexity.

Example

Consider the data below. It is for Harwayne's method on class 1 in state S.

Data for Harwayne's Method

State "s"	Class "c"	Exposure "E"	Losses "L"	Pure Premium "P"
S	1	100	200	2.00
	2	180	600	3.33
	Subtotal	280	800	2.86
T	.1	150	550	3.67
	2	300	1200	4.00
	Subtotal	450	1750	3.89
U	1	90	200	2.22
	2	220	900	4.09
	Subtotal	310	1100	3.55
All	1	340	950	2.79
	2	700	2700	3.86
	Total	1040	3650	3.51

Table 2

For Harwayne's full method, one first computes

$$\overline{P}_T = \frac{100 \times 3.67 + 180 \times 4.00}{100 + 180} = 3.88\text{, And}$$

$$\overline{P}_U = \frac{100 \times 2.22 + 180 \times 4.09}{100 + 180} = 3.42\,.$$

Then, one computes the state adjustment factors $F_T = 2.86/3.88 = .737$ and $F_U = 2.86/3.42 = .836$. The next step is to compute the other state's adjusted class 1 rates $P'_{1,T} = .737 \times 3.67 = 2.70$ and $P'_{1,U} = .836 \times 2.22 = 1.86$. The last step is to weight the two state's adjusted rates with their class 1 exposures to produce

$$C = \frac{2.70 \times 150 + 1.86 \times 90}{150 + 90} = 2.39.$$

That is Harwayne's complement of the credibility.

D. Trended Present Rates

In some cases, most notably countrywide rate indications, there is no larger group to use for the complement. So, actuaries use present rates adjusted for inflation (trend) since the last rate change. If there was a difference between the last actuarial indication and the charged rate, we build that in too. Essentially, this test allows some credibility procedure to dampen swings in the historical loss data yet still forces the manual rates to keep up with inflation.

Formula for the Complement

The formula for this complement of credibility is

$T^t \times R_L \times P_L \div P_C$, where

T is the annual trend factor, expressed as one plus the rate of inflation (this will usually be the same as the trend factor in the base indication);

t is the number of years between the original target effective date of the current rates (not necessarily the date they actually went into effect) and the target effective date of the new rates (This will often be different than the number of years in the base class trend. It is also usually different than the number of years between the experience period and the effective date of the new rates);

R_L represents the loss costs presently in the rate manual;

P_L represents the last indicated pure premium (loss costs); and

P_C represents the pure premiums actually being charged in the current manual. This may differ from R_L because P_L and P_C may be taken over a broader group.

Complement's Qualities

This complement is not as desirable as the previous complements but sometimes it may be the only alternative. It is less accurate for loss costs with high process variance. That is because that process variance is presumably reflected in last year's rate. That is why it is primarily used for countrywide indications or state indications with voluminous data. It is unbiased in the sense that pure trended loss costs (i.e., with no updating for more current loss costs) are unbiased. Since it includes no process variance, it is fairly independent from the base statistic.

On the less mathematical side, this statistic performs fairly well. Everything an actuary needs to compute it is already in the base rate filing. So, it is available and easy. There is one exception to this. Should you wish to analyze the effects of rate changes the company did not achieve at the level of individual classes, this may require more data than companies typically maintain. This statistic is also very logically related to the loss costs being analyzed. After all, the present rates are based on this complement.

Example

Consider the following data for 1996 policy rates:

Present pure premium rate -- $120;

Annual inflation (trend) -- 10%;

Amount requested in last rate change -- +20%;

Effective date requested for last rate change -- 1/1/94;

Amount approved by state regulators -- +15%;

Effective date actually implemented -- 3/1/94.

The complement of the credibility would be

$$C = \$120 \times 1.1^2 \times \frac{1.20}{1.15} = \$152.$$

E. Rate Change from the Larger Group Applied to Present Rates

This complement is very similar to the Bayesian complement. But it does not have the substantial (though unknown) bias of the Bayesian complement. That is because the true class expected losses may be very different from the large group expected losses. This larger group test uses the large group rate change applied to present rates instead of the large group historical loss data (Bayesian complement). Presumably, present rates are an unbiased predictor of the prior (i.e., before changes reflected in current ratemaking data) loss costs. And, as long as both rates need reasonably small changes, any bias in the overall larger group rate change as a predictor of the class rate change should be small. Also, using large group rate changes instead of straight trend allows the rate to mirror broad changes in loss cost levels that may not be reflected in trend.

Example

An example may help to illustrate how eliminating bias improves rate accuracy over time. In the graph below the group experience was simulated by successively applying N(10%,0.25%) (normal distribution with a mean of 10% and a standard deviation of $\sqrt{0.25\%} = 5\%$) trends to a value starting at one. The true class expected losses were set at exactly half the group expected losses each and every year (a slightly unrealistic assumption). The historical class losses have a standard deviation of one-third the true expected losses for the class. A detailed chart of the values actually simulated is in Appendix C.

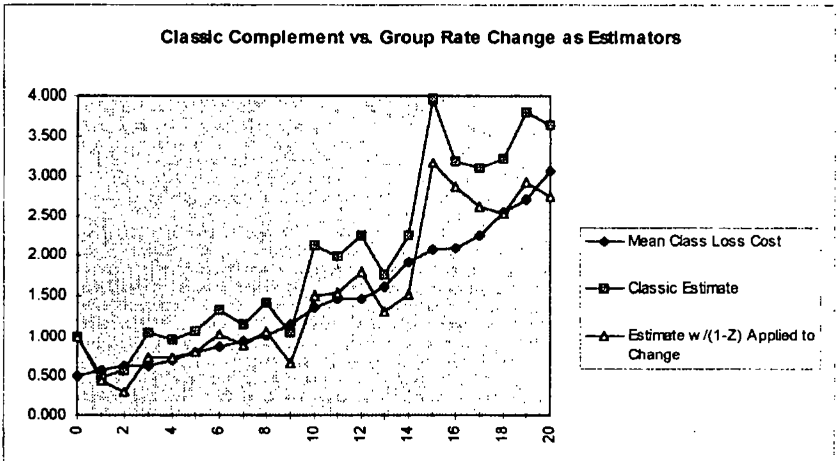

Figure 2

As the graph shows, the classic complement results in rates with consistent bias above the true expected losses. The complement based on applying group changes to present rates starts too high but very quickly becomes unbiased. It is almost always a better estimate.

Formula

This complement has a fairly straightforward formula. It is

$$R_c \times \left\{ 1 + \frac{(P_g - R_g)}{R_g} \right\}, \text{ where}$$

R_c is the present manual loss cost rate for class c;

P_g is the present indicated loss cost rate for the entire group of classes; and

R_g is the present average loss cost rate for the entire group.

Complement's Qualities

This is a significant improvement over the Bayesian complement. It is largely unbiased. If the year-to-year changes are fairly small, it is very accurate over the long term (though often not as accurate as Harwayne's complement in practice). And since the complement is based on group variance, it is fairly independent. Since this requires a group rate change that must be calculated anyway, it is both available and easy to compute. Since it includes the present rate, it has a logical relationship to the class loss costs.

Numerical Example

Consider the following data.

Data for Applying Group Rate Change to Class Data

Class	Exposure	Losses	Indicated Pure Premium	Present Pure Premium	Underlying Losses
1	100	$70,000	$700	$750	$75,000
2	200	$180,000	$900	$920	$184,000
3	300	$200,000	$667	$700	$210,000
Total	600	$450,000	$750	$782	$469,000

Notes : -Both indicated and present pure premiums are at current cost levels.
- Underlying losses are extension of exposures by present premiums
-Total present premium is ratio of total underlying to total exposures.

Table 3

Using this data, the complement for class 1 would be

$$\$750 \times \left(1 + (\$750 - \$782) / \$782\right) = \$719 .$$

F. Competitor's Rates

New companies and companies with small volumes of data often find their own data too unreliable for ratemaking. So their actuaries use competitor's rates for the complement of credibility. They rationalize that if the competitor has a much larger number of exposures, the competitor's statistics have less process error. An actuary in this situation must consider that manual rates reflect marketing considerations, judgment, and the effects of the regulatory process as well as loss cost statistics. So competitor's rates have significant inaccuracies. They are also affected by differences in underwriting and claim practices between the subject company and its competitors. So, competitor's rates probably have systematic bias as well. The actuary will often attempt to correct for those differences by using judgment. But those corrections and their size and direction may generate controversy. However, using competitor's rates may be the best viable alternative in some situations.

Complement's Qualities

Competitor's rates generally have prediction errors that are independent of the subject class loss costs. That is because their errors stem more from inter-company differences that are unrelated to subject company loss cost errors. They are often available from regulators, although the process may take some work. They are harder to use since they usually must be posted manually.

Regulators may complain that competitor's rates are unrelated to the subject company's own loss costs. But, if the company's own data is too unreliable, competitor's rates may be the only alternative.

Example

Consider a competitor's rate of $100. Suppose a Schedule P analysis suggests the competitor will run a 75% loss ratio. Further, suppose one's own company has less underwriting expertise. So, one's company expects 10% more losses per exposure than the competitor. The complement would be $\$100 \times .75 \times 1.1 = \$83$.

G. Loss Ratio Methods

This paper discussed all the previous complements in terms of pure premium ratemaking. But all the methods except the loss costs from a larger related class and competitor's rates also work with loss ratio methods. All the actuary needs to do is consider earned premium to be the exposure base. Replacing the exposure units with earned premium yields usable formulas.

III. Specific Excess Ratemaking

Complements for excess ratemaking are structured around the special problems of excess ratemaking. Since specific excess policies only cover losses that exceed a very high per claim deductible (attachment point), there usually are very few actual claims in the historical loss data. So, actuaries will try to predict the volume of excess loss costs using the loss costs below the attachment point. For liability coverages, the loss development of excess claims may be very slow. That accentuates the sparsity of ratemaking data. Also, the inflation inherent in excess layers is different (usually higher) than that of total limits losses (see [2]). Since the 'burning cost' (historical loss data) is an unreliable predictor, the statistic that receives the complement of credibility is especially important.

A. Increased Limits Factor

When loss costs for the first dollar coverage up to the insurer's limit of liability are available, actuaries may use an increased limits factor approach. Actuaries multiply the 'capped' loss costs by the increased limits factor for the attachment point plus the limit of liability. Then, we divide the result by the increased limits factor for the attachment point. That produces an estimate of loss costs from the first dollar up to the limit of liability. Then we subtract the loss costs below the original attachment point. The remainder estimates the expected losses in the specific excess layer.

Actuaries use a variety of sources for increased limits factors. The Insurance Services Office publishes tables of estimated increased limits factors for products, completed operations, premises and operations liability, and manufacturers and contractors liability. The National Council on Compensation Insurance publishes excess loss pure premium factors that allow actuaries to compute increased limits factors for workers compensation. The Proceedings of the Casualty Actuarial Society may contain tables of property losses by ratio to probable maximum loss. Those can be converted to increased limits factors by using the factors for the ratio of the attachment point to the probable maximum loss (and the ratio of the attachment point plus the limit of liability to the probable maximum loss). Actuaries may compute increased limits factor tables using a company's own data (if the company sells enough specific excess). Actuaries may modify industry tables to reflect their company's loss cost history. Competitor prices may allow actuaries to estimate increased limits factors for obscure coverages. We would consider the ratios between competitor prices for various limits of liability.

Formula

The formula is as follows:

$$\left(P_A \times ILF_{A+L} \div ILF_A\right) - P_A \quad \text{or} \quad P_A \times \left(\frac{ILF_{A+L}}{ILF_A} - 1\right).$$

And in this case

P_A is the loss costs capped at the attachment point (A) (by convention, it usually premium capped at the attachment point multiplied by the loss ratio the actuary projects.);

ILF_{A+L} is the increased limits factor for the sum of the attachment point and the limit of liability(L); and

ILF_A is the increased limits factor for the attachment point.

Complement's Qualities

As long as the insured being rated has a different loss severity distribution than the norm, this complement contains bias. In that fairly likely event, it is also inaccurate. But, actuaries must weigh those facts against the greater inaccuracy of burning cost statistics. When pricing specific excess insurance, actuaries must usually settle for less accurate and potentially biased estimators. That is because there are few highly accurate estimators available.

This complement's error is fairly independent of the burning cost error. This complement tends to contain a systematic (parameter-type) error rather than the process error inherent in burning cost. It is dependent on burning cost only to the extent that both are highly related to the losses below the attachment point.

Very few specific excess statistics are readily available or easy to compute. Considering the alternatives, the availability of industry increased limits tables (in the United States) makes this the easiest specific excess complement to compute. Also, the data for this test is available as long as premiums or loss costs capped at the attachment point are available.

The excess loss cost estimates this complement produces are more logically related to the losses below the attachment point than those above. That can be controversial with customers. But that is a common problem with excess insurance pricing. However, burning cost is unreliable in isolation. And that problem is common to all excess complements.

Example

Consider the following table of increased limits factors.

Increased Limits Factors

Limit of Liability	Increased Limits Factor
$50,000	1.00
$100,000	1.50
$250,000	1.90
$500,000	2.50
$1,000,000	3.50

Table 4

Suppose one wishes to estimate the layer between $500,000 and $1,000,000 given losses capped at $500,000 of $2,000,000. The complement using increased limits would be

$$C = \$2{,}000{,}000 \times \left(\frac{3.5}{2.5} - 1\right) = \$800{,}000.$$

B. Lower Limits Analysis

Sometimes the historical losses near the attachment point may be too sparse to be reliable. So an actuary may wish to base his complement on basic limits losses, where the basic limit is some fairly low loss cap. In this case the formula is almost exactly the same as that of the previous analysis. The actuary simply multiplies the historical basic limits losses by a difference of increased limits factors. Specifically, he multiplies basic limits losses by the difference between the increased limits factor for the attachment point plus the limit of liability and the increased limits factor for the attachment point. The result is the complement of credibility.

Formula

The formula is

$P_b \times \left(ILF_{A+L} - ILF_A\right)$; where

P_b represents the historical loss data with each loss capped at the basic limit (b); and

ILF_{A+L} and ILF_A are as before.

Alternately, the actuary might choose to use a low capping limit (d) that is different from the basic limit underlying the increased limits table. Then, the formula would be

$$P_d \times \left(\frac{ILF_{A+L}}{ILF_d} - \frac{ILF_A}{ILF_d}\right).$$

Complement's Qualities

Actuaries must usually use judgment to decide whether loss costs capped at the attachment point or some lower limit are more accurate and unbiased predictors of the excess loss. Estimates made using the lower cap are more prone to bias. That is because using losses far below the attachment point accentuates the impact of variations in loss severity distributions. But, when there are few losses near the attachment point, historical losses limited to the attachment point may be unreliable and inaccurate predictors of future losses. So, using higher loss caps may produce even more inaccurate predictors of excess losses.

By an argument similar to that of the previous test, this complement's errors are fairly independent of those of burning cost.

Generally, this complement features more available statistics and a slightly greater complexity. Basic limits losses may need to be coded for statistical reporting. So, they may be readily available for this complement. On the other hand, since insureds and reinsureds may place a higher priority on accounting for the total losses they retain, they are not as available as losses limited to the attachment point. The calculations are no more complicated for basic limits analysis than retained limits (attachment point) analysis. The only exception would where actuaries must manually compute the loss costs between basic limits and the attachment point from a claims list.

As with the straight increased limits factor approach, this complement may generate controversy with customers because it is not based on actual burning cost.

Example

Suppose an actuary is estimating the losses between \$500,000 and \$1,000,000 and the actuary feels he can only rely on historical losses limited to \$100,000. The estimated historical losses limited to \$100,000 are \$1,000,000. Then, using the increased limits factors from Table 4, he would calculate the complement at

$$C = \$1,000,000 \times \left(\frac{3.5}{1.5} - \frac{2.5}{1.5}\right) = \$666,667.$$

C. Limits Analysis

The previous approaches work well when losses limited to a single capping point are available, but sometimes they are not. Reinsurance customers generally sell policies with a wide variety of policy limits. Some of the policy limits will fall below (not expose) the attachment point. Some limits may extend beyond the sum of the attachment point and the reinsurer's limit of liability. In any event, each subject (first dollar) policy limit will require its own increased limits factor.

So, actuaries analyze each limit of coverage separately. Generally, we assume that all the limits will experience the same loss ratio. So, we multiply the all limits combined (total limits) first dollar loss ratio times the premium in each first dollar limit to estimate the loss costs for that limit. Then, we perform an increased limits factor analysis on each first dollar limit's loss costs separately. The formula is as follows:

Formula

$$LR_T \times \sum_{d \geq A} W_d \frac{\left(ILF_{\min(d,A+L)} - ILF_A\right)}{ILF_d}\text{; where}$$

LR_T is the estimated total limits loss ratio;

The 'd' are all the policy limits the customer sells that exceed the attachment point ($\geq$ A); and

each W_d is the premium volume the customer sells with policy limits of 'd'.

The ILF's have the same meaning as previously.

Complement's Qualities

Actuaries use this approach because it may be all that is available. Reinsureds may be unable to split their historical losses any more finely than losses that would have pierced the cover in the past versus all other losses. Since the total limits loss costs (which are almost always available, at least as an estimate) may include claims beyond the layer, it may be impossible to calculate losses limited to the attachment point. In any event, if some of the reinsured's policy limits are below the attachment point, they do not expose the layer and should be excluded from an increased limits factor calculation. So, this may the only available complement with low bias.

It is biased and inaccurate to the same extent that the previous increased limits factor-based complements were biased or inaccurate. It is more time-consuming to compute (unless the alternative is computing limited claims from claims lists). And it generates the same controversy as the other methods since it is not the same as the actual burning cost.

Example

Suppose an actuary is estimating the losses in a layer between \$250,000 and \$500,000. Breakdowns of losses by size are unavailable. But, the actuary believes the loss ratio of the customer's entire business to be 70%. He does have a breakdown of premiums by limit of liability. Using that breakdown and the increased limits factors from Table 4, he computes the losses in the layer below.

Limits Analysis for Layer Between $250,000 and $500,000

Limit of Liability	Premium	Times 70% Loss Ratio	Increased Limits Factor	% in Layer	Loss in Layer
$250,000	$ 600,000	$ 420,000	1.9	0.00%	$ -
$500,000	$ 300,000	$ 210,000	2.5	24.00%	$ 50,400
$1,000,000	$ 300,000	$ 210,000	3.5	17.14%	$ 36,000
Total	$ 1,200,000	$ 840,000			$ 86,400

Table 5

So, he estimates the losses in the layer at $86,400.

D. Fitted Curves

The problem with most of the previous complements is that they do not give special attention to the claims above or near the attachment point. So, they miss differences in loss severity distributions between insureds. But of course that must be counterbalanced against the fact that individual insured's large claims histories usually lack credibility.

By fitting a family of loss severity curves to the distribution, actuaries make the most of the large claim data that is available. If the loss history shows no claims beyond the attachment point but many claims that are very near to the attachment point, a fitted curve will usually reflect that and project high loss costs in the subject layer. On the other hand, if there are few large claims close to the attachment point, the fitted curve will project low loss costs for the layer.

The details of how to fit curves are beyond the scope of this paper(see [4]), but it will provide an outline of how to use fitted curves in practice. After fitting and trending the curve, an actuary will use the curve to estimate what percentage of the curve's total loss costs lie in the subject layer. He may do this by evaluating the difference between the limited mean function $\int_{-\infty}^{L} xf(x)dx + (1 - F(L))L$ at the attachment point and the attachment point plus the limit of liability. He would then divide the result by the total mean (or the mean when claims are capped at the typical policy limit) to get the percentage of the total loss costs that lie in the layer. Multiplying that percentage by the total claims cost yields the estimate of claim costs in the layer (for details, see [4]).

Of course, excess values from curve fits need extensive loss development just like burning costs. Actuaries may use excess loss development factors such as those published by the Reinsurance Association of America, or they may triangulate the fitted loss costs.

Complement's Qualities

This method is generally unbiased (except for concerns that the general shape of a family of curves may predispose the results for the family to estimated costs in particular layers that are either too high or too low.) When there are few large claims, it is more accurate than burning cost. It is often more accurate than increased limits factors simply because it does a better job reflecting any general tendency towards large or small claims. On the hand, fitting curves forces data into a mold that may not fit the data. The actual loss severity distribution will almost certainly look very different from all the members of the family of curves. This 'super-parameter' risk introduces error of its own. The 'super-parameter' risk is totally distinct from process risk, and that makes the complement fairly independent. On the other hand, the presence or absence of burning cost claims in the layer can influence the curve fit heavily. So, this complement has somewhat more dependent (relative to burning cost) errors than the increased limits approaches.

Data availability and computational complexity are problems here. To fit a loss severity curve an actuary must either use a detailed breakdown of all the claims into size ranges or use a listing of every single claim. Usually, that data is not readily available. Further, the processing required to fit curves requires fairly complex mathematical calculations. Besides the fact that complex calculations require special personnel, the complexity makes the results difficult to explain to lay people.

On one hand, this complement uses more of the insured's own data in and near the layer than any other excess complement. On the other hand, its complexity may make that fact difficult to communicate.

IV. Summary

The complement of the credibility deserves at least as much actuarial attention as the base statistic (historical loss data). Actuaries owe special attention to its unbiasedness and accuracy. In some cases, interdependence must be avoided. And any actuarial method must be implemented using reasonable labor on available statistics. Meeting those qualities may require statistics that make less explainable sense to lay people, but explainability must be considered, too.

This paper has detailed several statistics that are commonly used for the complement of credibility. Their use improves many actuarial projections considerably.

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THE ERROR IN CREDIBILITY ESTIMATES

This appendix will show that the error in an optimum credibility weighted estimate is

$$\Phi(x_1, x_2) = \frac{\tau_1^2 \tau_2^2 (1-\rho^2)}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2}.$$

The proof involves three equations from Boor[1]:

(1) $\Phi(x_1, x_2) = Z\tau_1^2 + (1-Z)\tau_2^2 + (Z^2 - Z)\delta_{1.2}^2$ (p.182, the simplified error of the credibility-weighted estimate);

(2) $Z = \dfrac{\tau_2^2 - \tau_1^2 + \delta_{1.2}^2}{2\delta_{1.2}^2}$ (p.183, the formula for the optimum credibility); and

(3) $\delta_{1.2}^2 = \tau_1^2 + \tau_2^2 - 2\mathrm{Cov}(x_1, x_2)$ (p.179, the formula relating $\delta_{1.2}^2$ to the correlation).

In this case τ_1, τ_2, and ρ are the same as they were in the body of the paper (the prediction errors of burning cost and the credibility complement and their correlation); $\Phi(x_1, x_2)$ is the minimum possible average squared prediction error from credibility weighting burning cost (x_1) and the credibility complement (x_2); and $\delta_{1.2}^2$ is the average squared difference between burning cost and the credibility complement.

Simple algebra on (1) allows one to pull out several terms that will create the numerator of (2).

$$\Phi(x_1, x_2) = -Z(\tau_2^2 - \tau_1^2 + \delta_{1.2}^2) + \tau_2^2 + Z^2\delta_{1.2}^2;$$

$$= -Z^2 2\delta_{1,2}^2 + \tau_2^2 + Z^2\delta_{1.2}^2 = \tau_2^2 - Z^2\delta_{1.2}^2.$$

Using the definition of Z (equation (2)) once again with some algebra gives

$$= \tau_2^2 - \frac{(\tau_2^2 - \tau_1^2 + \delta_{1,2}^2)^2}{4\delta_{1,2}^2}.$$

Using (3) and the relationship between the covariance and correlation gives

$$= \tau_2^2 - \frac{(\tau_2^2 - \tau_1^2 + \tau_1^2 + \tau_2^2 - 2\mathrm{Cov}(x_1, x_2))^2}{4\delta_{1,2}^2};$$

$$= \tau_2^2 - \frac{(\tau_2^2 - \tau_1^2 + \tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2)^2}{4\delta_{1,2}^2};$$

$$= \tau_2^2 - \frac{(2\tau_2^2 - 2\rho\tau_1\tau_2)^2}{4\delta_{1,2}^2};$$

$$= \tau_2^2 - \frac{(\tau_2^2 - \rho\tau_1\tau_2)^2}{\delta_{1,2}^2}.$$

Then, more algebra gives

$$= \tau_2^2\left(1 - \frac{(\tau_2 - \rho\tau_1)^2}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2}\right);$$

$$= \frac{\tau_2^2}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2} \times (\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2 - \tau_2^2 + 2\rho\tau_1\tau_2 - \rho^2\tau_1^2);$$

$$= \frac{\tau_2^2}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2} \times (\tau_1^2 - \rho^2\tau_1^2);$$

$$= \frac{\tau_2^2\tau_1^2(1 - \rho^2)}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2} \quad ;$$

and that is the error formula we sought to prove.

FOR CORRELATIONS NEAR UNITY, CREDIBILITY IS NEGATIVE

This appendix will show that whenever the correlation exceeds the point of maximum error, the credibility of one statistic is negative. To explain this principle, reviewing the graph of error by correlation will help.

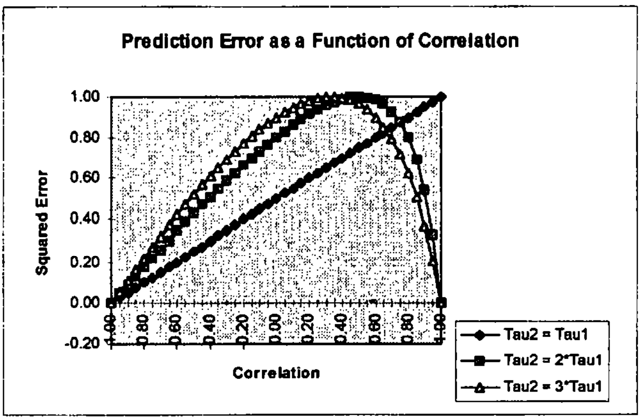

Fig. 1 (reprinted)

As one can see, the prediction error is initially minimized when the correlation is negative. Then it increases until the error is maximized. Then the error decreases again beyond that maximum point. This section will show that the one credibility is actually negative beyond that maximum point.

As it happens, when $\tau_2 \geq \tau_1$, that maximum point is where $\rho = \tau_1/\tau_2$. And all correlations beyond that yield negative credibility for the complement. Alternately, when $\tau_1 \geq \tau_2$, $\rho = \tau_2/\tau_1 \leq 1$ is the point of maximum prediction error. Beyond that, the burning cost's credibility will be negative. But, this appendix must prove that.

It is easy to show that Φ has a maximum where $\rho = \tau_1/\tau_2$. One need only note that the function $\Phi(\rho)$ has a maximum where

$$0 = \frac{\partial\Phi}{\partial\rho} = \frac{2\rho(\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2) - 2\tau_1\tau_2(1-\rho^2)}{(\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2)^2}$$

(using the definition of $\Phi(\rho)$ from appendix I). Using some algebra, that is equivalent to

$0 = 2\rho\tau_1^2 + 2\rho\tau_2^2 - 4\rho^2\tau_1\tau_2 - 2\tau_1\tau_2 + 2\rho^2\tau_1\tau_2$; or

$0 = (\tau_1 - \rho\tau_2)(\tau_2 - \rho\tau_1)$.

So, the maximum is at τ_1/τ_2 or τ_2/τ_1, whichever is less than one.

To show that correlations beyond that maximum point result in negative credibilities, it suffices to show that they fulfill Boor's condition for negative credibility ([1], p.183)

$\tau_2^2 \geq \tau_1^2 + \delta_{1,2}^2$.

But that follows directly from Boor's equation relating the credibility and covariance ([1], p. 179). That is, since

$$\delta_{1,2}^2 = \tau_1^2 + \tau_2^2 - 2\text{Cov}(x_1,x_2) = \tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2,$$

Boor's condition is equivalent to

$\tau_2^2 \geq \tau_1^2 + \tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2$.

Or,

$$\rho \geq \frac{\tau_1}{\tau_2};$$

that is, Boor's condition for negative credibility is fulfilled and fulfilled only for ρ beyond the point of maximum error. So, the correlations near unity yield negative credibilities.

DATA FOR EXAMPLE APPLYING COMPLEMENT TO GROUP RATE CHANGE

(a) Year	(b) Group N(.1,.0025) Trend	(c) Group Loss Cost	(d) Mean Class Loss Cost	(e) Class with Process Variance N(0,((d)/3)^2)	(f) Classic Z	(g) Classic Estimate	(h) Estimate w/(1-Z) Applied to Change
0	0.115	1.000	0.500	0.188	0.692	1.000	1.000
1	0.101	1.115	0.558	0.256	0.692	0.481	0.438
2	0.021	1.228	0.614	0.825	0.692	0.572	0.306
3	0.107	1.254	0.627	0.695	0.692	1.044	0.724
4	0.137	1.389	0.694	0.782	0.692	0.954	0.731
5	0.091	1.579	0.790	1.037	0.692	1.065	0.792
6	0.082	1.723	0.862	0.747	0.692	1.324	1.025
7	0.082	1.865	0.932	1.034	0.692	1.153	0.885
8	0.143	2.017	1.009	0.468	0.692	1.418	1.056
9	0.188	2.305	1.153	1.759	0.692	1.039	0.659
10	0.075	2.739	1.369	1.393	0.692	2.119	1.498
11	0.000	2.945	1.472	1.653	0.692	1.988	1.545
12	0.093	2.946	1.473	0.992	0.692	2.256	1.782
13	0.192	3.220	1.610	1.516	0.692	1.753	1.315
14	0.075	3.839	1.919	3.501	0.692	2.244	1.527
15	0.009	4.128	2.064	2.358	0.692	3.966	3.162
16	0.077	4.167	2.083	2.213	0.692	3.193	2.862
17	0.136	4.487	2.244	2.225	0.692	3.096	2.616
18	0.062	5.096	2.548	2.733	0.692	3.214	2.525
19	0.133	5.411	2.705	2.394	0.692	3.806	2.917
20	0.093	6.128	3.064	2.819	0.692	3.654	2.752

Notes
- Column (g) is [(f)*(previous column (e) + (1-(f))(previous column (c)] * (1+10% trend)
- Column (h) is {(f)*[previous column (e) - previous column (h)] + (1-(f))*[previous column (b) *previous column (c) -1.1*previous column (c)] + previous column (h)}*(1 +10% trend)