REPORT ON COVARIANCE METHOD FOR PROPERTY-CASUALTY RISK-BASED CAPITAL

Actuarial Advisory Committee to the NAIC P/C Risk-Based Capital Working Group

Report on Covariance Method for Property-Casualty Risk-Based Capital

from the

Actuarial Advisory Committee to the NAIC P/C Risk-Based Capital Working Group

February 26, 1993

Introduction

The Actuarial Advisory Committee to the NAIC Property/Casualty Risk-Based Capital Working Group has developed a recommended method for treating *covariance*. Our technique combines the separately-determined RBC amounts for all of the risk elements, assuming that everything bad doesn't occur at once. The proposal is based on data analysis as much as possible and, we believe, sound judgment otherwise. We have included results from our recent extensive analysis of underwriting risk over 1982-91, from testing on individual companies and from comments on earlier proposals.

This report is organized as follows:

Recommendation	A brief description of the proposed covariance formula; subsequent sections describe its rationale.
Conceptual Background	Discusses why a covariance adjustment is needed. The effect of statistical independence, correlation and the role of diversification. The square root rule.
Selecting Independent RBC Categories	Determines which asset, credit and underwriting risk elements are treated as independent, and thus reduce total RBC.
Correlation Between Lines of Business	Develops simplified covariance formula for diversification by line: the concentration adjustment.
Treatment of Affiliates	Shows why affiliate ownership must be treated differently from other equities in covariance formula.
Numerical Example of Covariance Formula	Illustrates the proposed formula with a simple set of numbers for the inputs.
Exhibits	Provides additional detail supporting the analysis.
Appendix	Provides theoretical background for covariance method.

Recommendation

For treatment of covariance between risks in the RBC calculation, we recommend the following formula to combine the RBC for independent risk categories:

$$\textbf{Total Company RBC} = R_1 + \sqrt{[R_2]^2 + [R_3]^2 + \ldots + [R_7]^2}.$$

The variables in the formula are RBC amounts for seven categories:

RBC Amount	*Risk Category (RBC is added for all items in category)*
R_1	Assets: Stock (common and preferred) of U.S. P/C insurance affiliates
R_2	Assets: Equities *excluding* P/C insurance affiliates
R_3	Assets: Fixed income items
R_4	Credit risk
R_5	Loss & LAE reserve and reserve growth risk, *adjusted for concentration*
R_6	Premium risk and premium growth risk, *adjusted for concentration*
R_7	Size risk

The above concentration-adjusted reserve and premium RBC amounts are

Adjusted Reserve RBC = RBC x [0.7 + (0.3 x Reserve Concentration)].

Adjusted Premium RBC = RBC x [0.7 + (0.3 x Premium Concentration)].

The purpose of the concentration adjustment is to allow for the effect of diversification between lines of business. The reserve concentration is the ratio of the reserve for the largest single line to the reserve for all lines. The premium concentration is a parallel calculation. The specific concentration formula is provided in the section of this report that discusses correlation between lines.

The special treatment of property-casualty affiliate RBC (removed from the equities category and denoted by R_1) outside the square root is to avoid applying the covariance adjustment more than once to an insurer. Otherwise RBC can be severely understated. To further address the affiliate covariance problem, we recommend that the insurer have the ***option of consolidating affiliates*** in determining total RBC.

Conceptual Background

In establishing a risk-based capital formula, a sensible, basic approach is to set the capital requirement for each risk element so that the insurer will be reasonably safe from insolvency due to that particular risk element alone. However, the *total RBC* for an insurer should generally be *less than* the simple sum of the RBC amounts for each risk element.

Diversification is responsible for this reduction to total RBC. Most insurers write several lines of business. It is unlikely that all lines will have adverse results at the same time: for example, property catastrophes are independent of liability losses and adverse workers' compensation reserve development does not always correspond to like movement in auto liability reserves. Similarly, many insurers have a broad portfolio of assets including stocks, bonds and real estate. Often the stock and bond markets will move in opposite directions at the same time, offsetting an adverse impact in one area. Thus, an insurer can reduce its chance of insolvency by diversifying its risk across underwriting and asset categories.

For two items, whose future values are uncertain, to have values unrelated to each other is called *statistical independence*. When two risk elements are independent, an adverse movement in one risk item will correspond, with equal likelihood, to either a positive or negative movement in the other. Clearly, when risk elements are independent, there is less total risk than if they are correlated. Statistical independence, which gives rise to the "law of large numbers", is the cornerstone of insurance. The more independent events insured, the more likely that adverse outcomes will be offset by favorable results.

On the other hand, if the risk elements are perfectly correlated, then the total RBC is the sum of the separate RBC amounts : for example, if loss reserves and stocks had 100% correlation, then an adverse development in loss reserves will always be accompanied by an equally adverse result in the stock market. Note that correlation is a measure of covariance, the ability of two variables to move together (i.e., to "co-vary"). Hence the general technique for combining RBC amounts has become known as the "covariance" adjustment.

As indicated in our *Conceptual Framework* document, a practical mathematical technique for recognizing independence of events computes their total RBC as the *square root of* *the sum of the squares of the individual RBC amounts.* We call this the "square root rule." The Life/Health Risk-Based Capital formula, adopted in 1992, also has a square root rule for combining RBC for separate risk elements. The Appendix develops the theory underlying the square root rule and discusses correlation in greater detail.

Perhaps the most important benefit of a risk-based capital program is to motivate insurers to "do the right thing." This proposal encourages *diversification*, both for investment portfolios and underwriting lines of business. We firmly believe that prompting insurers to spread their risk will be a major benefit to policyholders from a properly-designed RBC approach.

It is as important to recognize the degree of correlation between risk elements as it is to recognize the risk of any individual items. Ignoring the covariance adjustment to RBC could substantially harm a well-diversified insurer.

Selecting Independent RBC Categories

In order to establish a practical application of the square root rule, one must select a limited number of independent risk categories, recognizing that few risk elements are either truly independent of all others or are perfectly correlated with them. In some cases where there was a perceived independence or correlation between risk elements (e.g., reinsurance credit risk and loss reserve risk) we chose to ignore the relationship because the correlation was weak or the items were rather small for a typical insurer, and thus the effect on total RBC was minor.

Exhibit 1 shows that the square root rule tends to overstate the true amount of RBC for independent risk elements. Thus, if risk elements are almost independent (i.e., are weakly correlated), which is likely in practice, then the square root rule will be an even better approximation. See Exhibit 2 (discussed below) for an example of this.

Asset vs. Underwriting Risk

In general, we felt that non-insurance asset risk (including credit risk) was independent of underwriting risk (reserves, premium, size and growth risk). A notable exception is the relationship between bond duration and reserve duration (the interest rate risk); we will provide a separate recommendation on this topic.

Independent Asset Categories

The major asset categories likely to produce enough RBC for a material covariance reduction if independent are stocks, bonds and ceded reinsurance. We believe that ceded reinsurance risk is largely independent of the other assets since we could find no a priori reason why reinsurance defaults should be highly correlated with investment returns (rather, they should be related to adverse underwriting performance).

As indicated in Exhibit 2, based on long-term historical data, the correlation between stock and bond returns is a rather weak 14%. Ignoring the correlation understates combined RBC by a maximum of about 6%. However, the square root rule itself is an approximation that *overstates* RBC, so the errors tend to cancel. Thus, it is reasonable to use the simple square root rule and to assume no correlation between stocks and bonds.

Independent Underwriting Categories

The major underwriting risk categories are loss & LAE reserves, premiums, growth and size (both reserves and premiums).

Based on our extensive study of underwriting risk, we have concluded that reserve and written premium risk are not very well correlated. Here we define risk as the volatility (standard deviation) of the present value of reserve or premium deficiency. The reserve deficit is measured at the end of each year, while the premium shortfall is determined in the *following* year. Note that, at any point in time, the risk in premiums is related to upcoming exposure, since premium adequacy for the evaluation year is already incorporated in the reserve RBC.

Exhibit 3 shows that, from 1982 to 1991, the industry all-lines composite premium and reserve risk elements had only a 26% correlation. In fact, many of the individual lines show a negative association. However, because the historical period includes only one complete underwriting cycle (the next one may behave differently), one must be careful not to attach much credibility to the correlation of any particular line. Thus, we have included a correlation measure that weights each line equally with the all-lines composite. Also, Exhibit 3 shows that the number of years between the worst premium and reserve deficiency varies dramatically by line; with a strong premium/reserve correlation these would all be the same. Since the correlation is weak, and the square root rule overstates RBC, for the sake of simplicity, we have treated these two components as being independent.

Based on our judgment, we have determined that reserve growth risk is highly correlated with reserve risk, and therefore have included it with the reserve RBC category. Similarly, the premium growth risk is put with the premium risk.

Also, we believe that size risk is independent of either reserves or premium, but premium and reserve size risk are highly correlated. Thus, size risk for both should be a single independent RBC category.

Correlation Between Lines of Business

Our underwriting risk analysis has provided a way to measure the covariance between lines of business (an earlier proposal was based on judgment). To simplify the formula while recognizing the relationship between lines within loss reserve and written premium risk categories, we have developed an adjustment that depends on the *concentration* by line of business. It is applied separately to loss & LAE reserves and to written premiums:

Adjusted Reserve or Premium RBC = RBC x [0.7 + 0.3 x Concentration],

where

$$\text{Reserve Concentration} = \frac{\text{Loss\&LAE reserve for largest line (Page 10 /[Col 5 + 6])}}{\text{Total loss \& LAE Reserve}}$$

$$\text{Premium Concentration} = \frac{\text{Net Premium Written for largest line (Page 8 /Col 4)}}{\text{Total Net Premium Written}}.$$

The concentration adjustment reduces the RBC for insurers having a diversified book of business: a monoline insurer would get no reduction to its RBC, but the average insurer (about 30% concentration in both Workers Compensation reserves and PP Auto Liability premium) would get around a 20% reduction (before applying the square-root calculation). The reduction is limited to 30%.

Exhibit 4 derives the concentration adjustment from the average correlation between results for the Schedule P lines of business. We used P/C industry data from 1982-1991 for this analysis. For both reserves and premium, the average correlation between lines is about 40%, a number too low to lump all lines into a single independent category without adjustment, and too high to require independent line categories (to do this would greatly complicate the formula, anyway). Therefore we recommend this intermediate path of using the concentration adjustment.

Treatment of Insurance Affiliates

When the RBC for holding an affiliate is the ownership percentage of the affiliate's RBC, one cannot assume that this asset risk is independent of the other RBC categories. To illustrate, if an insurer creates a subsidiary that is a scaled-down version of the original company, then the results of the sub will be perfectly correlated with that of the parent. Thus, the square root rule should not apply: using it for affiliate RBC applies the covariance reduction *twice* (or more, if there are several layers of ownership), when only once is warranted. Exhibit 5 illustrates this point.

In Case 1, the original insurer (now the consolidated group) carves out a subsidiary one-third the size of the group. The group's RBC is $3,699—which should be identical to the parent's RBC, since the risk of the entire enterprise cannot change by shifting its assets and liabilities back and forth between sub and parent. The sub's RBC is $1,233, which is one-third of the group RBC. This is proper, because the sub is identical to the group, but a third its size.

Including the sub's RBC "inside" the square root (Cov-Adjusted Total 1) gives $2,757 for the Parent's RBC—an amount 25% too low. However, placing the sub's RBC "outside" the square root (Cov-Adjusted Total 2), which assumes that the sub's results depend on the parent's results, yields the correct RBC for the parent.

A third, theoretically correct treatment of affiliate covariance (Cov-Adjusted Total 3) is to consolidate the six independent RBC categories (R_2 through R_7) for parent and affiliates and then apply the square root rule to the six consolidated RBC categories. This gives the RBC of the consolidated insurer—a result that doesn't depend on the ownership structure.

Case 2 shows that only the consolidation method works when the sub is *not a proportionate scaling* of the parent. Here the "inside" method still produces a very low parent RBC, but the "outside" formula gives slightly (by 3%) too much parent RBC. Note that the "inside" formula will *always* give a parent RBC that is too low and the "outside" version will *always* give the correct or higher (although not by much for typical affiliates) parent RBC.

Because the "outside" formula is much easier to use than consolidation in calculating RBC, we recommend it for computing a company's total RBC. However, we also recommend that the insurer have the *option* of consolidating (up to the ownership level) all affiliates in determining total RBC.

Treatment of life insurance subsidiaries is difficult, since there is some correlation with P/C parent results through asset risk. But, we believe that, overall, life affiliates are more independent than dependent, and thus their RBC should be included with equities (R_2) "inside" the square root.

Illustrative Example Using Covariance Formula

Exhibit 6 illustrates the calculation of our recommended RBC covariance method: suppose that a hypothetical insurer owning a subsidiary has the following amounts of RBC by risk category before the covariance calculation:

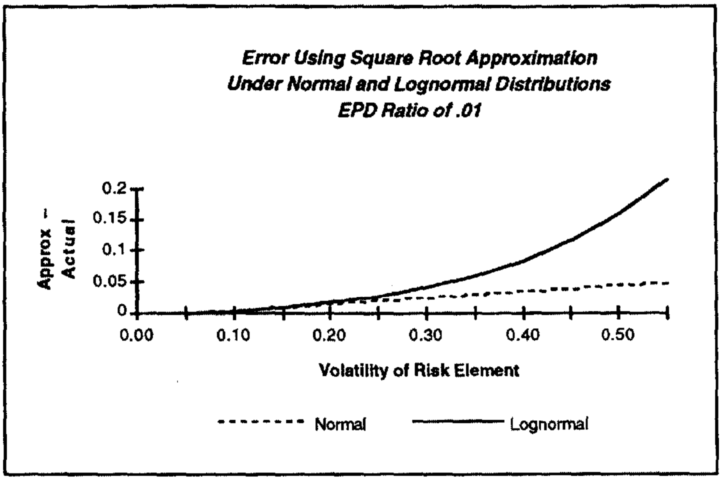
	Parent	*Subsidiary*
Affiliate ownership	$100	$0
Equities	200	60
Fixed income assets	100	0
Credit risk	50	0
Reserve risk	300	90
Premium risk	200	30
Size risk	50	0

The reserve and premium concentrations are 50%, and 40% respectively for both parent and sub. Thus, the sub's RBC is $100 (see Exhibit 6b) and the parent's RBC is $543 (from Exhibit 6a) using the recommended square root rule with the affiliate RBC added after the square root is taken. Applying the consolidation option reduces the insurer's RBC slightly to $542.

Exhibit 1

Error in Using Square Root Approximation

The square root rule approximates the true amount of capital required when two risk elements are independent. The graph below shows the error in this simplification under either the normal or lognormal probability distribution, for two equal-sized independent risk elements having the same standard deviation.



The volatility of the risk element is the ratio of the standard deviation to the mean. The solvency standard chosen for this comparison is an expected policyholder deficit ratio of 1%. The EPD ratio is the average insolvency cost per dollar of obligation to policyholders. This idea is developed in our Conceptual Framework document.

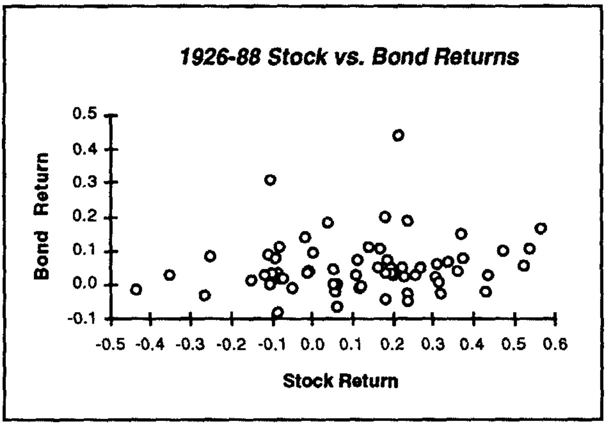
The error is defined as the approximated ratio minus the true ratio of capital to the risk element. Since the error is positive, the square root rule *overstates* the true amount of RBC, assuming that the risk elements have these probability distributions (we believe that these are reasonable choices for most RBC items).

For details on the error calculation and derivation of the square root rule, see the Appendix.

Exhibit 2

Stock vs. Bond Correlation

The scatter diagram below depicts the 1926-1989 stock and bond returns (based on Ibbotson & Associates data). The correlation between them is 14%.



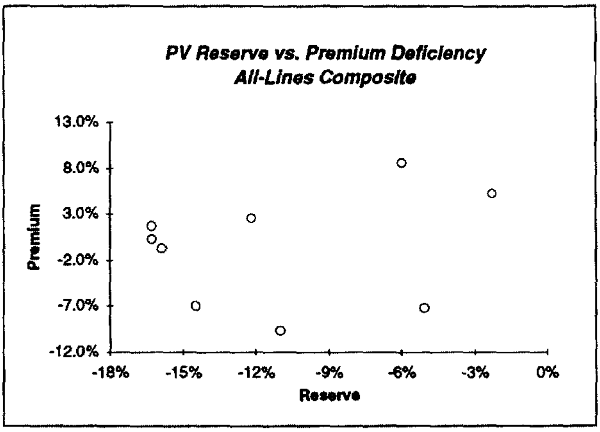
Using the square root rule and incorporating the correlation, the combined RBC for these two risk elements is $C = \sqrt{C_S^2 + C_B^2 + 2(.14)C_S C_B}$,where C_B and C_S are the RBC amounts for stocks and bonds. The maximum error in assuming a zero correlation is a 6.3% understatement of the total RBC, occurring when stock and bond RBC are equal. For an 8-to-1 ratio of stock to bond RBC, the error is only 1.7%.

However, the square root rule itself is an approximation that tends to *overstate* the amount of RBC needed (see Exhibit 1). For example, assume a 1% expected insolvency cost, a normal distribution for asset variability and annual standard deviations of 5% for bond annual returns and 15% for stocks (based on the Ibbotson data). *Including* the above effect of omitting the correlation, the square root rule still overstates the true RBC. The maximum overstatement, occurring with equal amounts of stocks and bonds is 2.3% of the assets. The net overstatement is 3.8% for the lognormal distribution.

Recognizing the above offsetting factors and the importance of simplicity, it is reasonable to use the simple square root rule, assuming no correlation between stocks and bonds.

Exhibit 3

Correlation Between Present Value of Reserve and Premium Deficiency 1982-91 U. S. P/L Industry Results



	Correlation*		Worst-Year
	Raw Value	Weighted	Gap**
Homeowners	-0.14	0.06	7
PP Auto Liab	0.81	0.54	-1
Comm Auto Liab	0.24	0.25	-2
Workers Comp	0.64	0.45	-7
CMP	0.56	0.41	0
Products Liability	0.59	0.42	1
Other Liability ex PL	0.66	0.46	0
Med Mal	0.76	0.51	-2
Special Liability	-0.35	-0.05	-5
Comb 2-Yr Lines	0.38	0.32	-8
International	-0.53	-0.14	4
Property Reins AC	-0.33	-0.04	-5
Casualty Reins B	0.52	0.39	-1
Casualty Reins D	-0.31	-0.02	-2
Reins Intl	-0.07	0.10	-1
All-Lines Composite	**0.26**	**0.26**	**-1**

*Year-end resv deficiency vs. following year prem deficiency. Weighted value uses 50% all-lines average and 50% raw value.

**[Year of worst prem deficiency] - [year of worst resv def] - 1. Perfect correlation would be zero.

Exhibit 4a

Calculating the Concentration Adjustment

For this analysis, we used the data underlying our proposed reserve and premium RBC factors. We have segmented the risk into an industry component, which measures year-to-year variation for all companies, and a company component, which measures variation within a year between all companies. These two risks are assumed to be independent, so their total is computed using a square root rule.

Exhibit 4b summarizes the calculation of the average correlation between lines (ρ in the exhibit) for reserves: it is about 42%. Here we have used the 1985 (a representative year for the period used) reserve volume to weight the line results.

Exhibit 4c performs a parallel calculation for premium, giving an average correlation of about 43%.

We have rounded both of the correlation measures to 40%. Translating the correlations to a concentration adjustment assumes that

(1) the insurer has n lines of business of equal size with concentration
$C = 1/n$ = [volume of the largest line] ÷ [total volume] and

(2) the RBC is the same for each line.

Assumption (1) overstates RBC and (2) understates RBC, so the net effect is nearly exact.

Thus, the concentration adjustment factor is $\sqrt{\rho + (1-\rho) \times C}$,

or approximately $\sqrt{\rho} + (1-\sqrt{\rho}) \times C$.

Using $\rho = 0.40$ for both reserves and premium, we get $\sqrt{\rho} = 0.63$; to compensate for the small correlation between reserves and premiums, we have boosted this to 0.70 in the proposed formula:

Adjusted reserve or premium RBC = [.7 + (.3 x *C*)] x [*unadjusted RBC*].

Exhibit 4b

Correlation Between Lines: 1982-91 U. S. P/L Industry
Loss & LAE Reserves

Line of Business	1985 Volume	Std Dev of Deficiency Company	Industry	Total
	V_i	sc_i	sd_i	st_i
Homeowners	4,999	23.0%	6.2%	23.8%
PP Auto Liab	28,015	15.0%	2.8%	15.3%
Comm Auto Liab	9,216	14.0%	4.4%	14.7%
Workers Comp	31,254	14.0%	6.5%	15.4%
CMP	9,813	18.0%	9.6%	20.4%
Other Liability ex PL	18,263	20.0%	16.3%	25.8%
Products Liability	4,496	29.0%	19.1%	34.7%
Med Malpractice	11,281	26.0%	11.7%	28.5%
Special Liability	1,591	21.0%	3.8%	21.3%
Comb 2-Yr Lines	11,295	28.0%	6.9%	28.8%
International	88	30.0%	6.5%	30.7%
Property Reins AC	1,387	33.0%	16.4%	36.9%
Casualty Reins B	5,394	18.0%	15.5%	23.8%
Casualty Reins D	6,910	24.0%	13.6%	27.6%
Reins International	17	30.0%	12.6%	32.5%
Total	144,019			
Average $\sigma_A = \left(\sum sx_i V_i\right)/\left(\sum V_i\right)$*	σ_A	**18.9%**	**8.7%**	**21.2%**
				$st_i = \sqrt{sc_i^2 + sd_i^2}$

All-Lines Composite	σ_C	**14.0%**	**4.5%**	**14.7%**
Independent Std Dev $\sigma_I = \sqrt{\sum [sx_i V_i]^2} / \left(\sum V_i\right)$*	σ_I	**6.2%**	**3.1%**	**6.9%**
Correlation Coefficient $\rho = \left(\sigma_C^2 - \sigma_I^2\right)/\left(\sigma_A^2 - \sigma_I^2\right)$	ρ	**49.1%**	**16.5%**	**42.1%**

*where "x" denotes "*c*", "*d*" or "*t*"

Exhibit 4c

Correlation Between Lines: 1982-91 U. S. P/L Industry
Premium

Line of Business	1985 Volume	Std Dev of Deficiency Company	Industry	Total
	V_i	sc_i	sd_i	st_i
Homeowners	13,843	11.7%	8.8%	14.6%
PP Auto Liab	26,439	10.5%	4.8%	11.5%
Comm Auto Liab	6,485	21.5%	13.2%	25.2%
Workers Comp	15,889	14.6%	7.6%	16.5%
CMP	9,592	19.1%	20.0%	27.7%
Other Liability ex PL	6,927	32.7%	22.5%	39.7%
Products Liability	1,327	43.2%	19.2%	47.3%
Med Malpractice	2,262	23.8%	17.3%	29.4%
Special Liability	1,906	31.6%	10.4%	33.3%
Comb 2-Yr Lines	37,188	22.0%	4.9%	22.5%
International	39	25.0%	18.6%	31.2%
Property Reins AC	1,430	32.0%	23.3%	39.6%
Casualty Reins B	2,791	24.0%	23.5%	33.6%
Casualty Reins D	3,881	30.0%	17.1%	34.5%
Reins International	16	25.0%	22.1%	33.4%
Total	130,015			
Average $\sigma_A = \left(\sum sx_i V_i\right)/\left(\sum V_i\right)$*	σ_A	**18.8%**	**9.5%**	**21.5%**
				$st_i = \sqrt{sc_i^2 + sd_i^2}$

All-Lines Composite	σ_C	**14.0%**	**6.2%**	**15.3%**
Independent Std Dev $\sigma_I = \sqrt{\sum [sx_i V_i]^2} / \left(\sum V_i\right)$*	σ_I	**7.5%**	**3.1%**	**8.1%**
Correlation Coefficient $\rho = \left(\sigma_C^2 - \sigma_I^2\right)/\left(\sigma_A^2 - \sigma_I^2\right)$	ρ	**47.1%**	**35.8%**	**42.6%**

*where "*x*" denotes "*c*", "*d*" or "*t*".

Exhibit 5

Covariance Calculation for Insurance Affiliates
Alternative Versions

Case 1: **Parent and Sub have Proportionate RBC**	*RBC Amount* **Consolidated Group**	**Subsidiary**	**Parent**	**Error**
Affiliate Stock (RS)	0	0	1,233	
Bonds (RB)	1,200	400	800	
Reserves (RR)	3,000	1,000	2,000	
Premium (RP)	1,800	600	1,200	
Total* Before Covariance Reductio	6,000	2,000	5,233	
Cov-Adjusted Total 1 $= \sqrt{RS^2 + RB^2 + RR^2 + RP^2}$	3,699	1,233	2,757	-25%
Cov-Adjusted Total 2 $= RS + \sqrt{RB^2 + RR^2 + RP^2}$	3,699	1,233	3,699	0%
Cov-Adjusted Total 3 *[Parent RBC = Consolidated RBC]*	3,699	1,233	3,699	0%

Case 2: **Parent and Sub have Nonproportionate RBC**	*RBC Amount* **Consolidated Group**	**Subsidiary**	**Parent**	**Error**
Stock (RS)	0	0	2,475	
Bonds (RB)	1,200	360	840	
Reserves (RR)	3,000	2,100	900	
Premium (RP)	1,800	1,260	540	
Total Before Covariance Reduction	6,000	3,720	4,755	
Cov-Adjusted Total 1	3,699	2,475	2,817	-24%
Cov-Adjusted Total 2	3,699	2,475	3,820	3%
Cov-Adjusted Total 3	3,699	2,475	3,699	0%

*For simplicity, this example excludes RBC for equities, credit risk and size.

Exhibit 6a

Example of Covariance Calculation for Risk-Based Capital
Separate Treatment of Affiliate RBC

A: Summary Calculation				Squared
		Base	Adjusted	Adjusted
Risk Element		RBC	RBC	RBC
Equities excl P/C Affiliates	R2	250	250	62,500
Fixed Income	R3	50	50	2,500
Credit	R4	50	50	2,500
Reserves & Resv Growth	R5	400	**340**	115,600
Premium & Prem Growth	R6	140	**115**	13,179
Size (Reserve & Premium)	R7	10	10	100
Subtotal		**900**	815	196,379
Square Root				**443**
P/C Affiliate Stock	R1	**100**		**100**
Total		**1,000**		**543**

B: Adjusted Underwriting RBC Calculation

		Base RBC	Adjusted RBC
(1)	Reserves & Resv Growth	400	**340**
(2)	Premium & Prem Growth	140	**115**
(3)	Reserve Concentration	0.500	
(4)	Reserve Conc Adjustment	**0.850**	
(5)	Premium Concentration	0.400	
(6)	Premium Conc Adjustment	**0.820**	

Notes

(1) Adjusted RBC = Base RBC x (4).
(2) Adjusted RBC = Base RBC x (6).
(3) Ratio of largest line net Loss & LAE reserve to total all lines reserve.
(4) Equals .7 + .3 x (3).
(5) Ratio of largest line net premium earned to total all lines NPE.
(6) Equals .7 + .3 x (5).

Exhibit 6b

Example of Covariance Calculation for Risk-Based Capital Consolidation Method

A: Calculation For Subsidiary

Risk Element		Base RBC	Adjusted RBC	Squared Adjusted RBC
Equities excl P/C Affiliates	R2	60	60	3,600
Fixed Income	R3	0	0	0
Credit	R4	0	0	0
Reserves & Resv Growth	R5	90	77	5,852
Premium & Prem Growth	R6	30	25	605
Size (Reserve & Premium)	R7	0	0	0
Total		**180**	161	10,057
Square Root				**100**
Reserve Conc Adjustment		**0.850**		
Premium Conc Adjustment		**0.820**		

B: Consolidated Calculation

Risk Element		Base RBC*	Adjusted RBC	Squared Adjusted RBC
Equities excl P/C Affiliates	R2	310	310	96,100
Fixed Income	R3	50	50	2,500
Credit	R4	50	50	2,500
Reserves & Resv Growth	R5	490	417	173,472
Premium & Prem Growth	R6	170	139	19,432
Size (Reserve & Premium)	R7	10	10	100
Total		**1,080**	976	294,105
Square Root				**542**
Reserve Conc Adjustment		**0.850**		
Premium Conc Adjustment		**0.820**		

*Sum of Subsidiary RBC (above) and Parent RBC (Exhibit 6a)

Appendix

CORRELATION AND INDEPENDENCE OF RISK ELEMENTS

Excerpted from

"Risk Measurement for Property-Liability Risk-Based Capital Applications"

by Robert P. Butsic

1992 Casualty Actuarial Society Discussion Paper Program.

Having demonstrated how risk-based capital for each risk element can be calculated separately by treating each element as a mini-insurer, we now need a way to *combine* the risk capital for the separate elements. As shown next, we cannot simply add their required capital amounts together unless the risk elements are highly correlated with the proper sign.

A Numerical Illustration

For example, suppose that we have a line of business with riskless assets and risky losses, which can have only two possible realizable values. The values and their probabilities are given below. The desired EPD (expected policyholder deficit) ratio is 1%. The risk-based capital needed for this degree of protection is easily calculated at $2,900:

Single Line	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
	6,900	2,000	.6	2,000	0
	6,900	7,000	.4	6,900	100
Expected Value	6,900	4,000		3,960	**40**
Capital:		2,900			
Capital / Loss:		.725			
EPD Ratio:		**.01**			

Now suppose that we have another line of business with an identical loss distribution, but directly correlated with the first: if a $2,000 loss amount occurs for the first line, the same amount occurs for the second line; similarly, a $7,000 amount will occur concurrently for both lines. The effect of combining the two lines is the same as if we now had a single line twice as large as the original single line:

Two Correlated Lines	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
	13,800	4,000	.6	4,000	0
	13,800	14,000	.4	13,800	200
Expected Value	13,800	8,000		7,920	**80**
Capital:		5,800			
Capital / Loss:		.725			
EPD Ratio:		**.01**			

Now suppose that the two lines are statistically *independent*: the value of the loss for one line does not depend on the value for the other. Then we have the following possible total losses with their associated probabilities:

Amount		Probability	
4,000	= 2,000 + 2,000	**.36**	= (.6)(.6)
9,000	= 2,000 + 7,000 or 7,000 + 2,000	**.48**	= (.6)(.4) + (.4)(.6)
14,000	= 7,000 + 7,000	**.16**	= (.4)(.4)

Adding the two $2,900 risk-based capital amounts and using the above combined losses and probabilities, we can determine the EPD for the total of the two lines:

Two Independent Lines	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
	13,800	4,000	0.36	4,000	0
	13,800	9,000	0.48	9,000	0
	13,800	14,000	0.16	13,800	200
Expected Value	13,800	8,000		7,968	**32**
Capital:		5,800			
Capital/Loss:		.725			
EPD Ratio:		**.004**			

Notice that the $32 expected deficit for the combined lines is less than the sum of the individual expected deficits ($80). This produces a 0.4% protection level, compared to the 1% value for the separate pieces. To reach the same 1% level as before, we need *less* capital than obtained by adding the separate amounts of risk-based capital:

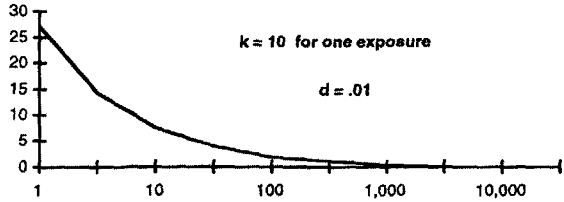
Two Independent Lines	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
	13,500	4,000	0.36	4,000	0
	13,500	9,000	0.48	9,000	0
	13,500	14,000	0.16	13,500	500
Expected Value	13,500	8,000		7,920	**80**
Capital		5,500			
Capital/Loss		**.687**			
EPD/Loss		**.01**			

As shown here, we only need $5,500 in capital, which is $480 less than the $5,980 needed when the losses are correlated. The capital ratio to loss drops from .725 to .687.

The reason for the reduced capital requirement through independence of risk elements is the *law of large numbers*. The spread of realizable values (relative to their mean) is reduced when independent elements are combined. The following graph depicts the diminishing capital needed to provide a 1% protection level for losses arising from independent normal exposures (having a standard deviation to mean ratio (k) of 10 for a single exposure):

Figure 1

Capital / Loss For Independent Normal Exposures



This illustrates that if losses are truly independent of each other, a small line of business will need a relatively large amount of capital, while a larger one requires much less capital. In reality, however, there is a limit to the risk reduction allowed by the law of large numbers. The mean or other parameters of the loss distribution are rarely known with certainty, introducing *systematic*, or parameter risk affecting all exposures. Thus, an insurer with a very large homogeneous book of business will still be subject to considerable uncertainty, and consequent capital needs.

Correlation Under the Normal Distribution

Although the preceding numerical example illustrates the capital reduction due to independence of risk elements, one must be careful not to generalize regarding the degree of reduction.[1] More robust conclusions can be reached by analyzing a continuous probability model, such as the normal distribution.

The normal distribution has the important property that sums of normal random variables are themselves normal random variables with additive means and easily-computed variances. For two assets (A_1 and A_2), two liabilities (L_1 and L_2), or an asset and a liability (A and L), we have

[1]For example, using a 10% EPD Ratio, the capital requirement drops to $2,000 for the single line of business. The *combined* capital need drops to $1,000 for the two independent lines—less capital than for a single line. This effect is due to using a discrete probability distribution with a limited range of outcomes.

	Mean	Variance
Two Assets	$A = A_1 + A_2$	$\sigma^2 = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2$
Two Liabilities	$L = L_1 + L_2$	$\sigma^2 = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2$
Asset and Liability	$C = A - L$	$\sigma^2 = \sigma_A^2 + \sigma_L^2 - 2\rho\sigma_A\sigma_L$

Here σ_1 and σ_2 denote the standard deviations of risk elements 1 and 2 (either assets or liabilities) and σ the total SD of combined risk elements (for assets minus liabilities, the SD of the capital). For the asset and liability combination, σ_A is the total asset SD and σ_L the total liability SD. The correlation coefficient between risk elements is ρ.

With perfect positive correlation ($\rho = 1$), we have $\sigma = \sigma_1 + \sigma_2$ for risk elements on the same side of the balance sheet or $\sigma = \sigma_A - \sigma_L$ for assets and liabilities. With perfect negative correlation ($\rho = -1$), $\sigma = \sigma_1 - \sigma_2$ and $\sigma = \sigma_A + \sigma_L$. When the elements are independent, $\rho = 0$, and thus $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$ and $\sigma = \sqrt{\sigma_A^2 + \sigma_L^2}$ for the two cases.

The formula for the EPD ratio with normally distributed combined risk elements is identical to that for individual elements as presented earlier:

$$d = \frac{D}{L} = k\,\varphi\!\left(\frac{-c}{k}\right) - c\,\Phi\!\left(\frac{-c}{k}\right).$$

Here c is the capital to loss, k is the total standard deviation divided by the total expected loss L, D is the total expected policyholder deficit, $\varphi(\bullet)$ and $\Phi(\bullet)$ are the respective standard normal density and cumulative distribution functions. The lognormal EPD ratio for combined risk elements is identical to the earlier formula for the separate risk elements.

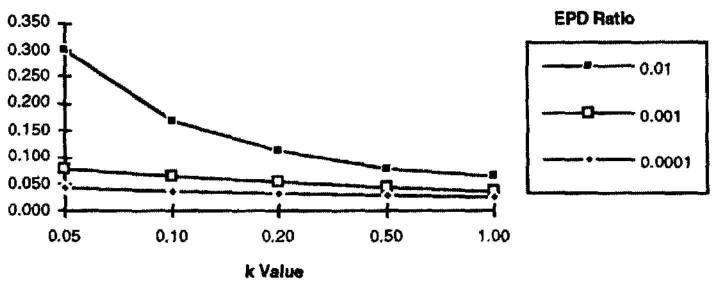
As indicated earlier, for the normal and lognormal distributions the relationship between c and k is approximately linear for a fixed EPD ratio d. Since $c = -d$ when $k = 0$ (no risk), we have $c \cong ak - d$ for some constant a. Under the assumption that we desire a high level of protection (d less than 1% or so), we can further simplify the relationship to $c \cong ak$.

Since the total capital C equals cL and the total SD σ equals kL, it follows that if $c = ak$, then $C = akL = a\sigma$. Therefore, the risk-based capital for the total of separate risk elements is proportional to their combined standard deviation. Risk capital for perfectly correlated items can be added (or subtracted, depending on whether the correlation is positive or negative or whether the items are on the same side of the balance sheet). Risk capital for independent (and partially correlated items) can be combined according to the square root of the sum of the squares of their standard deviations, plus twice the product of their SD's and the correlation coefficient.We will refer to this as the *square root* rule.

The graph below shows the relative error in using the square root rule, for two independent risk elements of the same size and standard deviation:

Figure 2

Relative Error Using Square Root Rule for Equal Independent Normal Risk Elements



This graph shows that the error decreases as the EPD ratio decreases and as the risk increases. For a reasonable (i.e., .001) protection level, the error is less than 10%. To illustrate, suppose that we have two independent lines of business each with a \$1,000 expected loss and \$200 SD. For a .001 EPD ratio, each requires \$438 of capital in isolation. When the lines are combined, Equation (6) produces a capital ratio of .292, or \$584 in capital when applied to the \$2,000 expected total losses. The square root rule produces $\$619 = 438\sqrt{2}$, which is about 6% more than the exact calculation[2] yields.

[2]Because the error in using the square root for the normal and lognormal distributions overstates the combined amount of capital needed, a closer fit could be had by using a root higher than two. For instance, in the normal example given, using a 2.4th root (.42 power) gives an exact result.

A parallel calculation using the lognormal distribution shows a 15% error: the true required capital is \$694, compared to \$800 indicated by the square root rule.[3]

The square root rule can be extended to incorporate more than two risk elements. The total capital C is a function of the individual element risk capital amounts C_i and the separate correlation coefficients between each pair of n risk elements (note that the sign of the correlation coefficient depends on which side of the balance sheet the two items reside):

$$C = \left[\sum_{i=1}^{n} C_i^2 + \sum_{i \neq j}^{n} \rho_{ij} C_i C_j \right]^{\frac{1}{2}}.$$

Practical Application of Correlated and Independent Risk Elements

The preceding analysis has shown the effect of correlation between risk elements. Some examples of balance sheet items having varying degrees of correlation are presented in the table below:

Correlation	**Asset/ Asset**	**Liability/ Liability**	**Asset/ Liability**
Positive	Common Stock/ Preferred Stock Common Stock/ Bonds	Loss Reserve/ LAE Reserve	Bonds/ Loss Reserve
Zero	Cash/ Real Estate	Liability Loss Reserve/ Property Unearned Premium Reserve	Common Stock/ Unearned Premium Reserve
Negative	Common Stock/ Put Options	Loss Reserve/ Income Tax Liability Loss Reserve/ Dividend Reserve	Property-Liability Stock/ Loss Reserve Reinsurance Recoverable/ Loss Reserve

[3]The higher capital amounts are a consequence of thicker tail of this distribution, compared to the normal distribution. For the lognormal model, the error increases with increasing risk (k).

In general, reinsurance transactions create a high degree of correlation between ceding and assuming parties. Ownership of insurance subsidiaries (affiliates) or stock also produces highly correlated values. Where it is difficult to determine the numerical correlation between items, a practical approach would be to judgmentally peg the correlation at zero, 1 or −1, whichever is closest to the perceived value.

We can demonstrate the effect of independent and correlated risk elements by constructing a numerical example. The table below shows risk elements from a hypothetical insurer's balance sheet at market values. The capital ratios assume a .001 EPD ratio and are based roughly on empirical data.

	Amount	Capital Ratio	RBC
Stock	200	0.30	60
Bonds	1000	0.05	50
Affiliates	100	0.30	30
Loss Reserve	800	0.40	320
Property UPR	100	0.10	10
Total			**470**

The 30% stock capital factor arises from using the 16.6% standard deviation of 1946 to 1989 annual returns from Ibbotson and Associates (1990). Based on the same source, we have used a 6% annual SD for bonds (the corporate bond SD is 9.8% for a 20-year maturity; adjusting for a more typical property-liability insurer's duration gives a lower value), producing an approximate 5% capital ratio. The loss reserve capital ratio is based on a study of loss ratio variation by Derrig[4] (1986). We have assumed that the affiliate stock risk is the same as for general non-insurance stock, that all the risk elements are lognormally distributed and that the EPD's are discounted at an 8% riskless interest rate. In the loss reserve (equal to the present value of the expected payments), we have also included the loss expenses and the liability portion of losses arising from the unearned premiums.

[4]Derrig used a sample of Workers' Compensation and Private Passenger Auto loss ratios from 51 insurers over the period 1976-1985 (since calendar-year losses were used, the variance should be similar to that for loss reserves). The combined annual variance was .059, which we have judgmentally reduced to .045 reflecting a greater variance in the unpaid loss tail; the variance is lowered when the loss is brought to present value. This produces a capital ratio (to the discounted loss) of about 0.40. Notice that a further adjustment would be needed to convert the capital factor for application to an *undiscounted* loss reserve: using an 18% reserve discount, the required *statutory* surplus is (1 + .40)(1 - .18) - 1 = .15 times the undiscounted reserve.

The sum of the separate risk-based capital amounts is $470. This value assumes that *all* items are fully correlated, ignoring any independence or partial covariance between the items. Now assume that only the following pairs of elements are correlated:

		Correlation Coefficient
Stock	Bonds	0.2
Stock	Affiliates	1.0
Bonds	Affiliates	0.2
Bonds	Loss Reserve	0.4
Affiliates	Loss Reserve	-1.0

The property UPR is independent of all other items. Notice that the bonds/reserve correlation coefficient is positive due the parallel change in value from interest rate movements; since these two items are on opposite sides of the balance sheet, this means that their joint movement will *reduce* total risk.[5] Similarly, the negative sign of the affiliates/reserve correlation coefficient indicates that these opposing items will *increase* total risk when combined.

Applying Equation (7), we have the sum of the squares of the separate risk capital amounts equal to 109,500. The sum of the cross products (each of the above pairs appears twice) of the capital amounts times their correlation coefficients equals 11,800. Thus the approximate total risk capital is $\$348 = \sqrt{121{,}300}$. If all the risk elements were independent, the total required capital would be only $\$331 = \sqrt{109{,}500}$.

The impact of the bond/reserves covariance can be found by setting the correlation coefficient to zero: here the total risk capital increases to $366. Thus, the effect of their correlation is to reduce required capital by $18. Similarly, if the affiliate and reserves values were independent, the required capital would drop by $28 to $320.

A more sophisticated RBC calculation would divide the risk elements into additional categories and might include a provision for the value of future business.

[5]The correlation methodology provides a means of allowing for matching of asset and liability durations. If the durations of fixed maturity assets and loss payments were equal, and the movements in value were due solely to interest rate fluctuations, then a (negative) 100% correlation coefficient would be appropriate.