EASIER ALGORITHMS FOR AGGREGATE EXCESS

Gary Venter

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Probabilities for aggregate claims can be calculated from frequency and severity probabilities using the characteristic function algorithm of Heckman-Meyers [1983], but the formulas are somewhat difficult to follow and program. Two easier algorithms are presented below. These are particularly efficient when there are only a small number of claims expected and when the severity distribution is fairly flat, as in many reinsurance situations.

The first algorithm is the recursion method introduced into actuarial literature in Panjer [1981] and into the PCAS in Venter [1983]. The severity density r(x) is approximated by a discrete function. That is, some interval size h is established, and claims come in lumps of h, 2h, ... At ih this discrete severity function will be denoted by g_i. The claim frequency distribution is assumed to have the following recursive property: there are constants a and b such that $p_{i+1}=[a+\frac{b}{i+1}]p_i$. For the Poisson, a=0 and b=λ. For the negative binomial with mean $\alpha(1-\beta)/\beta$ and variance $\alpha(1-\beta)/\beta^2$, the constants are $a=1-\beta$ and $b=(1-\beta)(\alpha-1)$. The aggregate density f can be calculated recursively:

$$f_0 = p_0, \quad f_k = \sum_{i=1}^{k}(a+b\tfrac{i}{k})g_i f_{k-i}, \quad k>0.$$

To implement this for a general severity, a rule for doing the discrete approximation is needed. A fairly arbitrary rule which appears reasonable is to match interval probabilities and means for each consecutive pair of points $<g_1,g_2>$, $<g_3,g_4>$, etc. Then the cumulative distribution and limited severity will match the original at every second point. For instance, the probability assigned to the points 3h and 4h would be determined by solving the two equations:

$$g_3+g_4=\int_{2.5h}^{4.5h} r(z)dz \qquad\qquad 3hg_3+4hg_4=\int_{2.5h}^{4.5h} zr(z)dz$$

The solution can be expressed with the aid of two auxiliary functions d and m:

$$d_i=\int_0^{ih} r(z)dz \qquad\qquad m_i=\frac{1}{h}\int_0^{ih} zr(z)dz$$

Note that d_i is just the distribution function. It can be verified that:

$$g_3=m_{2.5}-m_{4.5} + 4(d_{4.5}-d_{2.5})$$
$$g_4=m_{4.5}-m_{2.5} - 3(d_{4.5}-d_{2.5})$$

Also, the impact of the extra interval from 0 to h/2 should be incorporated into g_1 and g_2.

As an example, for the Pareto severity in b,c (mean $\frac{b}{c-1}$), the functions are $d_i=1-(1+\frac{ih}{b})^{-c}$ and $m_i=\frac{b}{h(c-1)}[1-(1+\frac{ih}{b})^{-c}(1+\frac{ich}{b})]$. Note that the h can be eliminated by expressing b in units of h. For c=1, $m_i=(b/h)\ln(1+ih/b)-i/(1+ih/b)$.

The second algorithm uses the same type of frequency distribution but assumes a continuous severity distribution. Panjer also derived a continuous recursion:

$$f(x)=p_1r(x)+\int_0^x (a+b\tfrac{y}{x})r(y)f(x-y)dy, \quad x>0$$

This on the surface appears impractical, but it is in fact a Volterra integral equation of the second type, and can be solved numerically. Methods are in Baker [1977]. The following solution is based on the trapezoid rule. Solutions based on Simpson's rule and quadratic quadrature may be found in Ströter [1985], who also incorporates numerical integration of the severity functions.

A third auxiliary severity function is introduced:

$$v_i=\frac{1}{h^2}\int_0^{ih} z^2r(z)dz$$

For instance, for the Pareto, $v_i=\frac{2(b/h)^2}{(c-1)(c-2)}\left[1-(1+\frac{ih}{b})^{-c}(1+\frac{ich}{b}+\frac{c(c-1)i^2(h/b)^2}{2})\right]$, except for $c=1$,

$v_i=2(b/h)^2\left[\frac{i(.5i+b/h)}{(b/h)(i+b/h)}-\ln(1+ih/b)\right]$ and $c=2$, $v_i=2(b/h)^2\left[\ln(1+ih/b)-\frac{i(2i+b/h)}{(i+b/h)^2}\right]$.

The numerical approximation of f will involve first selecting an interval h, and then recursively approximating f at 0, h, 2h, ... First, set $f(0)=p_1r(0)$. This is not $f_0=p_0$, the discrete probability lump at zero, but is rather the approximation of the aggregate density f at just above 0. Then f(kh) is approximated by:

$$f(kh)=\frac{p_1r(kh)+\sum_{i=1}^{k} f((k-i)h)w_{i,k}}{1-w_{0,k}},$$

where the weights $w_{i,k}$ are defined as follows:

Let $s_i=2m_i-m_{i+1}-m_{i-1}+i[d_{i+1}+d_{i-1}-2d_i]+d_{i+1}-d_{i-1}$, and

let $t_i=2v_i-v_{i+1}-v_{i-1}+i[m_{i+1}+m_{i-1}-2m_i]+m_{i+1}-m_{i-1}$.

Then for $0<i<k$, define $w_{i,k}=bt_i/k + as_i$. Let $w_{0,k}=b(m_1-v_1)/k + a(d_1-m_1)$ and let $w_{k,k}=(b/k)[v_k-v_{k-1}-(k-1)(m_k-m_{k-1})]+a[m_k-m_{k-1}-(k-1)(d_k-d_{k-1})]$.

The error in this approximation is proportional to h^2, so it reduces quadratically as h gets smaller. This knowledge of the error structure is an advantage over other approximations. The fact that the s's and t's are not functions of k simplifies the calculation of the weights, so that a spreadsheet calculation is possible. If h is small enough, $w_{0,k}$ will be between 0 and 1, which is necessary to get reasonable results. This sometimes forces a smaller h than would otherwise be needed.

In application of these approximations, two functions are often calculated: the cumulative distribution F and the excess ratio (portion of loss dollars excess of x) R. With μ the aggregate mean, this is given by $R(x)=\frac{1}{\mu}\int_x^\infty (z-x)f(z)dz$, and so $\mu R(x)=\mu-\int_0^x zf(z)dz-x[1-F(x)]$. For the continuous case, care must be taken to include the point mass at zero, $f_0=p_0$, in the integral. Thus, using trapezium, $F(kh)=f_0+.5h(f(0)+f(kh))+h\sum_{i=1}^{k-1}f(ih)$, and $\mu R(kh)=\mu-.5hkhf(kh)-h\sum_{i=1}^{k-1}ihf(ih)-kh(1-F(kh))$. In the discrete approximation formula, $\mu R_k=\mu-\sum_{i=1}^{k}if_i-k(1-F_k)$, where F_k is just $\sum_{i=0}^{k}f_i$.

As an example, a Pareto severity with b=1,000,000 and c=3 was used, which has a mean of 500,000, along with a Poisson frequency. The Pareto was chosen with reinsurance in mind, as excess losses from a Pareto are themselves Pareto distributed. The exhibits compare the discrete and continuous approximations for intervals h of 100,000, $33{,}333\frac{1}{3}$, 10,000, and $3333\frac{1}{3}$ and λ's of .2, 1, and 5. For the continuous approximation, the errors in the distribution function seem generally smaller than those in the excess ratio. In both cases, the numerical integration is contributing to the error, and a better integration method may help. For the smallest h, the distribution function appears to be accurate to five places, and the errors appear to be increasing by a factor of 10 for each larger h, which agrees with the order of h^2 theory. The errors in the excess ratio also appear to increase proportionally to h^2, but they are considerably larger, especially for the higher probabilities, than those for the distribution function.

The discrete approximation was actually better in some ways. Because it is an exact calculation given the discrete severity, the F and R functions are reasonable for h up to 100,000. For even higher h's, the severity approximation can be negative for g_2, however. The main disadvantage of the discrete method is that the discretizing process used seems to make the estimated F_i a closer approximation to the true $F_{i+.5}$ than it is to the true F_i. To illustrate this for $\lambda=5$, an exhibit is included which compares the discrete approximation with h=100,000 at 500,000, 1,000,000, etc. to the continuous approximation with $h=3333\frac{1}{3}$ at 550,000, 1,050,000, etc. The coarser interval with the discrete approximation seems close enough for most purposes, if the half-shift in the interval is acceptable. For some reason, the half shift problem does not seem to arise with the excess ratios.

A significant advantage of the discrete form is that it works easily with limited severity, i.e., a point mass at the top of the severity distribution. Since a continuous severity density is required for positive values for the continuous approximation, it is not clear how it could be adapted for limited severity.

In order to see if an improved numerical integration method would improve the accuracy of the continuous approximation, an adjustment to the trapezoid rule was developed. Rather than approximating the function by line segments, as in the trapezoid, quadratic polynomials were fit through each combination of three consecutive points, and the area under these polynomials used to approximate the integral. The result is just a slight adjustment to the trapezoid rule: in calculating $\int_0^{ih} u(z)dz$ in steps of width h, add $(h/24)[u(2h)-u(0)+u(ih-h)-u(ih+h)]$ to the trapezoid approximation. The continuous approximation exhibits were redone using this quadratic integration rule. The errors at each h level were reduced substantially by this adjustment.

The solution of the integral equation is not necessarily optimal, and the continuous approximation may perform better with other approximations.

References

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Illana Leiser and Jack Barnett provided invaluable assistance in solving the integral equation and programming the exhibits, with the usual rejoinder.

COMPARATIVE APPROXIMATIONS
Distribution Function

	Continuous Severity Model				Discrete Severity Model			
Intrvl→ (000)	100	33 1/3	10	3 1/3	100	33 1/3	10	3 1/3
0	0.81873	0.81873	0.81873	0.81873	0.81873	0.81873	0.81873	0.81873
500	0.94157	0.94036	0.94022	0.94021	0.94585	0.94206	0.94076	0.94039
1000	0.97502	0.97362	0.97346	0.97345	0.97529	0.97407	0.97363	0.97351
1500	0.98774	0.98628	0.98612	0.9861	0.98693	0.98637	0.98618	0.98613
2000	0.99357	0.99208	0.99191	0.9919	0.99229	0.99203	0.99194	0.99191
2500	0.99658	0.99508	0.99491	0.9949	0.99512	0.99497	0.99492	0.9949
3000	0.99828	0.99678	0.99661	0.99659	0.99672	0.99663	0.9966	0.9966
3500	0.9993	0.99781	0.99763	0.99762	0.9977	0.99764	0.99763	0.99762
4000	0.99996	0.99846	0.99829	0.99827	0.99832	0.99829	0.99828	0.99827
4500	1.0004	0.9989	0.99873	0.99871	0.99874	0.99872	0.99871	0.99871
5000	1.0007	0.9992	0.99903	0.99901	0.99903	0.99902	0.99901	0.99901

Poisson: Lamda = 0.2
Pareto : b = 1,000,000 & c = 3

COMPARATIVE APPROXIMATIONS
Distribution Function

	Continuous Severity Model				Discrete Severity Model			
Intrvl→ (000)	100	33 1/3	10	3 1/3	100	33 1/3	10	3 1/3
0	0.36788	0.36788	0.36788	0.36788	0.36788	0.36788	0.36788	0.36788
500	0.70744	0.70544	0.70521	0.70519	0.72602	0.71195	0.7072	0.70586
1000	0.84686	0.84411	0.84379	0.84376	0.85315	0.84683	0.84468	0.84407
1500	0.91393	0.91081	0.91045	0.91042	0.91523	0.91199	0.91089	0.91058
2000	0.94914	0.94583	0.94545	0.94542	0.94802	0.94627	0.94567	0.9455
2500	0.96886	0.96545	0.96506	0.96502	0.96653	0.96551	0.96517	0.96507
3000	0.98051	0.97704	0.97664	0.97661	0.97751	0.9769	0.97669	0.97663
3500	0.9877	0.98421	0.9838	0.98377	0.98434	0.98395	0.98382	0.98378
4000	0.99233	0.98881	0.98841	0.98837	0.98874	0.98849	0.9884	0.98838
4500	0.99541	0.99188	0.99147	0.99143	0.99168	0.99151	0.99145	0.99144
5000	0.99752	0.99398	0.99357	0.99354	0.99371	0.99359	0.99355	0.99354
5500	0.99901	0.99546	0.99505	0.99502	0.99514	0.99505	0.99502	0.99502
6000	1.00009	0.99653	0.99612	0.99609	0.99617	0.99611	0.99609	0.99608
6500	1.00088	0.99732	0.99691	0.99687	0.99694	0.99689	0.99688	0.99687
7000	1.00148	0.99791	0.9975	0.99747	0.99752	0.99748	0.99747	0.99747
7500	1.00193	0.99837	0.99796	0.99792	0.99796	0.99793	0.99792	0.99792
8000	1.00229	0.99872	0.99831	0.99827	0.9983	0.99828	0.99827	0.99827
8500	1.00257	0.999	0.99859	0.99855	0.99857	0.99856	0.99855	0.99855
9000	1.00279	0.99922	0.99881	0.99877	0.99879	0.99878	0.99877	0.99877
9500	1.00297	0.9994	0.99899	0.99895	0.99896	0.99895	0.99895	0.99895
10000	1.00312	0.99955	0.99914	0.9991	0.99911	0.9991	0.9991	0.9991

Poisson: Lamda = 1
Pareto : b = 1,000,000 & c = 3

COMPARATIVE APPROXIMATIONS
Distribution Function

Intrvl→ (000)	Continuous Severity Model 100	33 1/3	10	3 1/3	Discrete Severity Model 100	33 1/3	10	3 1/3
0	0.00674	0.00674	0.00674	0.00674	0.00674	0.00674	0.00674	0.00674
500	0.09445	0.09478	0.09481	0.09482	0.10537	0.09869	0.09599	0.09521
1000	0.22777	0.22873	0.22884	0.22885	0.2428	0.2336	0.23028	0.22932
1500	0.37055	0.3722	0.37239	0.3724	0.38654	0.37707	0.37381	0.37287
2000	0.50177	0.50406	0.50432	0.50434	0.51697	0.50844	0.50557	0.50475
2500	0.61275	0.61556	0.61588	0.61591	0.62643	0.61929	0.61692	0.61625
3000	0.70204	0.70528	0.70564	0.70567	0.71407	0.70835	0.70647	0.70594
3500	0.77173	0.77528	0.77568	0.77571	0.78223	0.77778	0.77633	0.77592
4000	0.82507	0.82886	0.82929	0.82933	0.8343	0.8309	0.8298	0.82949
4500	0.86546	0.86943	0.86987	0.86991	0.87368	0.8711	0.87027	0.87004
5000	0.89586	0.89996	0.90042	0.90046	0.90329	0.90135	0.90073	0.90055
5500	0.91869	0.92289	0.92336	0.9234	0.92553	0.92407	0.9236	0.92347
6000	0.93586	0.94012	0.9406	0.94065	0.94225	0.94116	0.9408	0.9407
6500	0.9488	0.95312	0.95361	0.95365	0.95487	0.95404	0.95377	0.95369
7000	0.9586	0.96297	0.96346	0.9635	0.96442	0.9638	0.96359	0.96354
7500	0.96607	0.97047	0.97096	0.97101	0.97171	0.97123	0.97108	0.97103
8000	0.9718	0.97622	0.97672	0.97676	0.9773	0.97694	0.97682	0.97678
8500	0.97623	0.98066	0.98116	0.9812	0.98163	0.98134	0.98125	0.98122
9000	0.97967	0.98412	0.98462	0.98466	0.98499	0.98477	0.9847	0.98468
9500	0.98237	0.98683	0.98733	0.98738	0.98764	0.98746	0.98741	0.98739
10000	0.98451	0.98898	0.98948	0.98952	0.98973	0.98959	0.98955	0.98953
10500	0.98621	0.99069	0.99119	0.99123	0.9914	0.99129	0.99125	0.99124
11000	0.98758	0.99206	0.99256	0.99261	0.99274	0.99265	0.99262	0.99262
11500	0.98868	0.99317	0.99367	0.99372	0.99383	0.99376	0.99373	0.99373
12000	0.98959	0.99408	0.99458	0.99463	0.99472	0.99466	0.99464	0.99463
12500	0.99033	0.99482	0.99533	0.99537	0.99545	0.9954	0.99538	0.99538
13000	0.99094	0.99544	0.99594	0.99599	0.99605	0.99601	0.996	0.996
13500	0.99145	0.99595	0.99646	0.9965	0.99656	0.99653	0.99651	0.99651
14000	0.99188	0.99638	0.99689	0.99694	0.99698	0.99695	0.99694	0.99694
14500	0.99225	0.99675	0.99726	0.9973	0.99734	0.99732	0.99731	0.99731
15000	0.99255	0.99706	0.99756	0.99761	0.99764	0.99762	0.99762	0.99762
15500	0.99282	0.99732	0.99783	0.99787	0.9979	0.99789	0.99788	0.99788
16000	0.99304	0.99755	0.99806	0.9981	0.99813	0.99811	0.99811	0.99811
16500	0.99324	0.99774	0.99825	0.9983	0.99832	0.99831	0.9983	0.9983
17000	0.99341	0.99791	0.99842	0.99847	0.99849	0.99848	0.99847	0.99847
17500	0.99355	0.99806	0.99857	0.99861	0.99863	0.99862	0.99862	0.99862
18000	0.99368	0.99819	0.9987	0.99874	0.99876	0.99875	0.99875	0.99875
18500	0.9938	0.99831	0.99881	0.99886	0.99887	0.99887	0.99886	0.99886
19000	0.99389	0.99841	0.99891	0.99896	0.99897	0.99897	0.99896	0.99896
19500	0.99398	0.99849	0.999	0.99905	0.99906	0.99905	0.99905	0.99905
20000	0.99406	0.99857	0.99908	0.99912	0.99914	0.99913	0.99913	0.99913

Poisson: Lamda = 5
Pareto : b = 1,000,000 & c = 3

COMPARATIVE APPROXIMATIONS
Excess Ratio

	Continuous Severity Model				Discrete Severity Model			
Intrvl→ (000)	100	33 1/3	10	3 1/3	100	33 1/3	10	3 1/3
0	1	1	1	1	1	1	1	1
500	0.47807	0.47169	0.47112	0.47108	0.46927	0.4709	0.47106	0.47107
1000	0.27994	0.27061	0.26987	0.26982	0.26924	0.26975	0.26981	0.26981
1500	0.18639	0.1741	0.17317	0.17311	0.17284	0.17308	0.1731	0.17311
2000	0.13637	0.12099	0.11988	0.11981	0.11968	0.11979	0.1198	0.1198
2500	0.10747	0.08894	0.08763	0.08755	0.08747	0.08753	0.08754	0.08754
3000	0.08996	0.06825	0.06674	0.06664	0.06659	0.06663	0.06663	0.06663
3500	0.07911	0.0542	0.05247	0.05237	0.05233	0.05235	0.05236	0.05236
4000	0.07239	0.04425	0.04232	0.04221	0.04218	0.04219	0.0422	0.0422
4500	0.06835	0.03699	0.03486	0.03473	0.0347	0.03471	0.03472	0.03472
5000	0.06614	0.03155	0.02921	0.02907	0.02905	0.02905	0.02905	0.02905

Poisson: Lamda = 0.2
Pareto : b = 1,000,000 & c = 3

COMPARATIVE APPROXIMATIONS
Excess Ratio

Intrvl-> (000)	Continuous Severity Model 100	33 1/3	10	3 1/3	Discrete Severity Model 100	33 1/3	10	3 1/3
0	1	1	1	1	1	1	1	1
500	0.56701	0.56467	0.56444	0.56442	0.56232	0.56425	0.5644	0.56442
1000	0.35211	0.34824	0.34788	0.34785	0.34686	0.34776	0.34784	0.34784
1500	0.23478	0.22924	0.22873	0.22868	0.22817	0.22864	0.22868	0.22868
2000	0.16647	0.1591	0.15842	0.15836	0.15807	0.15833	0.15835	0.15836
2500	0.12471	0.11541	0.11454	0.11448	0.1143	0.11445	0.11447	0.11447
3000	0.09821	0.08691	0.08586	0.08577	0.08566	0.08575	0.08576	0.08576
3500	0.08093	0.06758	0.06633	0.06623	0.06615	0.06621	0.06622	0.06622
4000	0.06945	0.05403	0.05257	0.05246	0.0524	0.05244	0.05244	0.05244
4500	0.06176	0.04424	0.04259	0.04246	0.04241	0.04244	0.04244	0.04244
5000	0.05663	0.03701	0.03515	0.035	0.03496	0.03498	0.03498	0.03498
5500	0.05328	0.03154	0.02948	0.02932	0.02928	0.0293	0.0293	0.0293
6000	0.0512	0.02734	0.02507	0.02489	0.02486	0.02487	0.02487	0.02487
6500	0.05005	0.02406	0.02159	0.02139	0.02136	0.02137	0.02137	0.02137
7000	0.04958	0.02146	0.01879	0.01857	0.01854	0.01855	0.01855	0.01855
7500	0.04964	0.01938	0.0165	0.01628	0.01624	0.01625	0.01625	0.01625
8000	0.0501	0.01771	0.01462	0.01438	0.01435	0.01435	0.01435	0.01435
8500	0.05087	0.01635	0.01306	0.01279	0.01276	0.01276	0.01276	0.01276
9000	0.0519	0.01523	0.01174	0.01146	0.01142	0.01143	0.01143	0.01143
9500	0.05312	0.01432	0.01062	0.01032	0.01029	0.01029	0.01029	0.01029
10000	0.05451	0.01357	0.00966	0.00935	0.00931	0.00931	0.00931	0.00931

Poisson: Lamda = 1
Pareto : b = 1,000,000 & c = 3

COMPARATIVE APPROXIMATIONS
Excess Ratio

Intrvl-(000)	Continuous Severity Model 100	33 1/3	10	3 1/3	Discrete Severity Model 100	33 1/3	10	3 1/3
0	1	1	1	1	1	1	1	1
500	0.80893	0.80901	0.80902	0.80902	0.80845	0.80899	0.80902	0.80902
1000	0.64078	0.64095	0.64097	0.64097	0.64019	0.64092	0.64097	0.64097
1500	0.50077	0.50112	0.50116	0.50116	0.50034	0.50111	0.50116	0.50116
2000	0.3884	0.38905	0.38913	0.38913	0.38839	0.38908	0.38913	0.38913
2500	0.30032	0.30139	0.30151	0.30152	0.3009	0.30148	0.30152	0.30152
3000	0.23223	0.23384	0.23402	0.23403	0.23353	0.234	0.23403	0.23404
3500	0.17999	0.18221	0.18245	0.18248	0.18208	0.18245	0.18248	0.18248
4000	0.13998	0.14287	0.1432	0.14322	0.14292	0.14321	0.14323	0.14323
4500	0.10928	0.1129	0.1133	0.11334	0.11311	0.11333	0.11334	0.11334
5000	0.08561	0.08999	0.09048	0.09052	0.09035	0.09051	0.09053	0.09053
5500	0.06723	0.07239	0.07297	0.07302	0.07289	0.07301	0.07302	0.07302
6000	0.05282	0.05878	0.05944	0.0595	0.05941	0.0595	0.05951	0.05951
6500	0.04139	0.04817	0.04893	0.04899	0.04892	0.049	0.049	0.049
7000	0.03223	0.03983	0.04068	0.04075	0.0407	0.04076	0.04076	0.04076
7500	0.02478	0.03322	0.03416	0.03424	0.0342	0.03424	0.03425	0.03425
8000	0.01865	0.02792	0.02895	0.02904	0.02902	0.02905	0.02905	0.02905
8500	0.01352	0.02363	0.02476	0.02486	0.02484	0.02487	0.02487	0.02487
9000	0.00918	0.02013	0.02135	0.02146	0.02145	0.02147	0.02147	0.02147
9500	0.00545	0.01724	0.01856	0.01867	0.01867	0.01868	0.01869	0.01869
10000	0.0022	0.01483	0.01625	0.01637	0.01637	0.01638	0.01638	0.01639
10500	¯0.00067	0.01281	0.01432	0.01445	0.01446	0.01447	0.01447	0.01447
11000	¯0.00324	0.0111	0.0127	0.01284	0.01285	0.01286	0.01286	0.01286
11500	¯0.00555	0.00963	0.01133	0.01148	0.01149	0.0115	0.0115	0.0115
12000	¯0.00767	0.00837	0.01016	0.01031	0.01033	0.01033	0.01033	0.01033
12500	¯0.00962	0.00727	0.00915	0.00932	0.00933	0.00934	0.00934	0.00934
13000	¯0.01144	0.0063	0.00828	0.00846	0.00847	0.00848	0.00848	0.00848
13500	¯0.01315	0.00545	0.00752	0.00771	0.00773	0.00773	0.00773	0.00773
14000	¯0.01476	0.00469	0.00686	0.00705	0.00707	0.00707	0.00707	0.00707
14500	¯0.01629	0.00401	0.00628	0.00648	0.0065	0.0065	0.0065	0.0065
15000	¯0.01776	0.0034	0.00576	0.00597	0.00599	0.00599	0.00599	0.00599
15500	¯0.01917	0.00284	0.0053	0.00552	0.00554	0.00554	0.00554	0.00554
16000	¯0.02053	0.00234	0.00489	0.00511	0.00514	0.00514	0.00514	0.00514
16500	¯0.02184	0.00187	0.00452	0.00475	0.00478	0.00478	0.00478	0.00478
17000	¯0.02313	0.00144	0.00419	0.00443	0.00446	0.00446	0.00446	0.00446
17500	¯0.02438	0.00105	0.00389	0.00414	0.00417	0.00417	0.00417	0.00417
18000	¯0.0256	0.00068	0.00362	0.00388	0.00391	0.00391	0.00391	0.00391
18500	¯0.0268	0.00034	0.00337	0.00364	0.00367	0.00367	0.00367	0.00367
19000	¯0.02797	0.00001	0.00314	0.00342	0.00345	0.00345	0.00345	0.00345
19500	¯0.02913	¯0.00029	0.00294	0.00322	0.00325	0.00325	0.00325	0.00325
20000	¯0.03027	¯0.00058	0.00274	0.00304	0.00307	0.00307	0.00307	0.00307

Poisson: Lamda = 5
Pareto : b = 1,000,000 & c = 3

COMPARATIVE APPROXIMATIONS

Distribution Function

(000)	Continuous Model h=3,333 1/3		Discrete Model h=100,000		Continuous Model h=3,333 1/3
0	0.00674	0	0.00674	0	0.00674
550	0.10686	500	0.10537	500	0.09482
1,050	0.24324	1,000	0.24280	1,000	0.22885
1,550	0.38634	1,500	0.38654	1,500	0.37240
2,050	0.51648	2,000	0.51697	2,000	0.50434
2,550	0.62585	2,500	0.62643	2,500	0.61591
3,050	0.71352	3,000	0.71407	3,000	0.70567
3,550	0.78176	3,500	0.78223	3,500	0.77571
4,050	0.83392	4,000	0.83430	4,000	0.82933
4,550	0.87337	4,500	0.87368	4,500	0.86991
5,050	0.90306	5,000	0.90329	5,000	0.90046
5,550	0.92535	5,500	0.92553	5,500	0.92340
6,050	0.94212	6,000	0.94225	6,000	0.94065
6,550	0.95476	6,500	0.95487	6,500	0.95365
7,050	0.96435	7,000	0.96442	7,000	0.96350
7,550	0.97165	7,500	0.97171	7,500	0.97101
8,050	0.97726	8,000	0.97730	8,000	0.97676
8,550	0.98159	8,500	0.98163	8,500	0.98120
9,050	0.98497	9,000	0.98499	9,000	0.98466
9,550	0.98762	9,500	0.98764	9,500	0.98738
10,050	0.98971	10,000	0.98973	10,000	0.98952
10,550	0.99138	10,500	0.99140	10,500	0.99123
11,050	0.99273	11,000	0.99274	11,000	0.99261
11,550	0.99382	11,500	0.99383	11,500	0.99372
12,050	0.99471	12,000	0.99472	12,000	0.99463
12,550	0.99544	12,500	0.99545	12,500	0.99537
13,050	0.99604	13,000	0.99605	13,000	0.99599
13,550	0.99655	13,500	0.99656	13,500	0.99650
14,050	0.99697	14,000	0.99698	14,000	0.99694
14,550	0.99733	14,500	0.99734	14,500	0.99730
15,050	0.99764	15,000	0.99764	15,000	0.99761
15,550	0.99790	15,500	0.99790	15,500	0.99787
16,050	0.99812	16,000	0.99813	16,000	0.99810
16,550	0.99831	16,500	0.99832	16,500	0.99830
17,050	0.99848	17,000	0.99849	17,000	0.99847
17,550	0.99863	17,500	0.99863	17,500	0.99861
18,050	0.99876	18,000	0.99876	18,000	0.99874
18,550	0.99887	18,500	0.99887	18,500	0.99886
19,050	0.99897	19,000	0.99897	19,000	0.99896
19,550	0.99905	19,500	0.99906	19,500	0.99905
20,050		20,000	0.99914	20,000	0.99912

Lamda=5

COMPARATIVE APPROXIMATIONS
Distribution Function

Intrvl- (000)	Continuous Severity Model 100	33 1/3	10	3 1/3	Discrete Severity Model 100	33 1/3	10	3 1/3
0	0.81873	0.81873	0.81873	0.81873	0.81873	0.81873	0.81873	0.81873
500	0.9408	0.94024	0.94021	0.94021	0.94585	0.94206	0.94076	0.94039
1000	0.97408	0.97348	0.97345	0.97344	0.97529	0.97407	0.97363	0.97351
1500	0.98677	0.98614	0.9861	0.9861	0.98693	0.98637	0.98618	0.98613
2000	0.99257	0.99194	0.9919	0.9919	0.99229	0.99203	0.99194	0.99191
2500	0.99558	0.99494	0.9949	0.9949	0.99512	0.99497	0.99492	0.9949
3000	0.99728	0.99664	0.99659	0.99659	0.99672	0.99663	0.9966	0.9966
3500	0.99831	0.99766	0.99762	0.99762	0.9977	0.99764	0.99763	0.99762
4000	0.99896	0.99832	0.99827	0.99827	0.99832	0.99829	0.99828	0.99827
4500	0.9994	0.99875	0.99871	0.99871	0.99874	0.99872	0.99871	0.99871
5000	0.9997	0.99905	0.99901	0.99901	0.99903	0.99902	0.99901	0.99901

Poisson: Lamda = 0.2
Pareto : b = 1,000,000 & c = 3

Quadratic Integration

COMPARATIVE APPROXIMATIONS
Distribution Function

	Continuous Severity Model				Discrete Severity Model			
Intrvl→ (000)	100	33 1/3	10	3 1/3	100	33 1/3	10	3 1/3
0	0.36788	0.36788	0.36788	0.36788	0.36788	0.36788	0.36788	0.36788
500	0.70636	0.70527	0.70519	0.70518	0.72602	0.71195	0.7072	0.70586
1000	0.84543	0.84391	0.84377	0.84376	0.85315	0.84683	0.84468	0.84407
1500	0.91237	0.9106	0.91043	0.91042	0.91523	0.91199	0.91089	0.91058
2000	0.94753	0.94561	0.94543	0.94542	0.94802	0.94627	0.94567	0.9455
2500	0.96722	0.96523	0.96504	0.96502	0.96653	0.96551	0.96517	0.96507
3000	0.97886	0.97682	0.97662	0.9766	0.97751	0.9769	0.97669	0.97663
3500	0.98606	0.98398	0.98378	0.98376	0.98434	0.98395	0.98382	0.98378
4000	0.99068	0.98858	0.98838	0.98837	0.98874	0.98849	0.9884	0.98838
4500	0.99376	0.99165	0.99145	0.99143	0.99168	0.99151	0.99145	0.99144
5000	0.99587	0.99375	0.99355	0.99353	0.99371	0.99359	0.99355	0.99354
5500	0.99736	0.99523	0.99503	0.99501	0.99514	0.99505	0.99502	0.99502
6000	0.99843	0.9963	0.9961	0.99608	0.99617	0.99611	0.99609	0.99608
6500	0.99922	0.99709	0.99689	0.99687	0.99694	0.99689	0.99688	0.99687
7000	0.99982	0.99769	0.99748	0.99747	0.99752	0.99748	0.99747	0.99747
7500	1.00028	0.99814	0.99794	0.99792	0.99796	0.99793	0.99792	0.99792
8000	1.00063	0.99849	0.99829	0.99827	0.9983	0.99828	0.99827	0.99827
8500	1.00091	0.99877	0.99857	0.99855	0.99857	0.99856	0.99855	0.99855
9000	1.00113	0.99899	0.99879	0.99877	0.99879	0.99878	0.99877	0.99877
9500	1.00131	0.99917	0.99897	0.99895	0.99896	0.99895	0.99895	0.99895
10000	1.00146	0.99932	0.99911	0.9991	0.99911	0.9991	0.9991	0.9991

Poisson: Lamda = 1
Pareto : b = 1,000,000 & c = 3

Quadratic Integration

COMPARATIVE APPROXIMATIONS
Distribution Function

Intrvl-(000)	Continuous Severity Model 100	33 1/3	10	3 1/3	Discrete Severity Model 100	33 1/3	10	3 1/3
0	0.00674	0.00674	0.00674	0.00674	0.00674	0.00674	0.00674	0.00674
500	0.09457	0.09479	0.09481	0.09482	0.10537	0.09869	0.09599	0.09521
1000	0.22802	0.22876	0.22884	0.22885	0.2428	0.2336	0.23028	0.22932
1500	0.37086	0.37224	0.37239	0.3724	0.38654	0.37707	0.37381	0.37287
2000	0.50211	0.5041	0.50432	0.50434	0.51697	0.50844	0.50557	0.50475
2500	0.61309	0.6156	0.61588	0.61591	0.62643	0.61929	0.61692	0.61625
3000	0.70238	0.70532	0.70565	0.70568	0.71407	0.70835	0.70647	0.70594
3500	0.77205	0.77532	0.77568	0.77571	0.78223	0.77778	0.77633	0.77592
4000	0.82539	0.8289	0.82929	0.82933	0.8343	0.8309	0.8298	0.82949
4500	0.86576	0.86946	0.86988	0.86991	0.87368	0.8711	0.87027	0.87004
5000	0.89615	0.89999	0.90042	0.90046	0.90329	0.90135	0.90073	0.90055
5500	0.91898	0.92292	0.92336	0.9234	0.92553	0.92407	0.9236	0.92347
6000	0.93614	0.94016	0.94061	0.94065	0.94225	0.94116	0.9408	0.9407
6500	0.94908	0.95316	0.95361	0.95365	0.95487	0.95404	0.95377	0.95369
7000	0.95888	0.963	0.96346	0.9635	0.96442	0.9638	0.96359	0.96354
7500	0.96635	0.9705	0.97097	0.97101	0.97171	0.97123	0.97108	0.97103
8000	0.97208	0.97625	0.97672	0.97676	0.9773	0.97694	0.97682	0.97678
8500	0.9765	0.98069	0.98116	0.98121	0.98163	0.98134	0.98125	0.98122
9000	0.97994	0.98415	0.98462	0.98466	0.98499	0.98477	0.9847	0.98468
9500	0.98264	0.98686	0.98734	0.98738	0.98764	0.98746	0.98741	0.98739
10000	0.98478	0.98901	0.98948	0.98952	0.98973	0.98959	0.98955	0.98953
10500	0.98648	0.99072	0.99119	0.99123	0.9914	0.99129	0.99125	0.99124
11000	0.98784	0.99209	0.99257	0.99261	0.99274	0.99265	0.99262	0.99262
11500	0.98895	0.9932	0.99368	0.99372	0.99383	0.99376	0.99373	0.99373
12000	0.98985	0.99411	0.99458	0.99463	0.99472	0.99466	0.99464	0.99463
12500	0.9906	0.99485	0.99533	0.99537	0.99545	0.9954	0.99538	0.99538
13000	0.99121	0.99547	0.99595	0.99599	0.99605	0.99601	0.996	0.996
13500	0.99172	0.99599	0.99646	0.9965	0.99656	0.99653	0.99651	0.99651
14000	0.99215	0.99642	0.99689	0.99694	0.99698	0.99695	0.99694	0.99694
14500	0.99251	0.99678	0.99726	0.9973	0.99734	0.99732	0.99731	0.99731
15000	0.99282	0.99709	0.99757	0.99761	0.99764	0.99762	0.99762	0.99762
15500	0.99308	0.99735	0.99783	0.99787	0.9979	0.99789	0.99788	0.99788
16000	0.99331	0.99758	0.99806	0.9981	0.99813	0.99811	0.99811	0.99811
16500	0.99351	0.99778	0.99825	0.9983	0.99832	0.99831	0.9983	0.9983
17000	0.99367	0.99795	0.99842	0.99847	0.99849	0.99848	0.99847	0.99847
17500	0.99382	0.99809	0.99857	0.99861	0.99863	0.99862	0.99862	0.99862
18000	0.99395	0.99822	0.9987	0.99874	0.99876	0.99875	0.99875	0.99875
18500	0.99406	0.99834	0.99882	0.99886	0.99887	0.99887	0.99886	0.99886
19000	0.99416	0.99844	0.99892	0.99896	0.99897	0.99897	0.99896	0.99896
19500	0.99425	0.99853	0.999	0.99905	0.99906	0.99905	0.99905	0.99905
20000	0.99433	0.99861	0.99908	0.99913	0.99914	0.99913	0.99913	0.99913

Poisson: Lamda = 5
Pareto : b = 1,000,000 & c = 3

Quadratic Integration

COMPARATIVE APPROXIMATIONS
Excess Ratio

	Continuous Severity Model				Discrete Severity Model			
	100	33 1/3	10	3 1/3	100	33 1/3	10	3 1/3
0	1	1	1	1	1	1	1	1
500	0.48036	0.47207	0.47116	0.47108	0.46927	0.4709	0.47106	0.47107
1000	0.28231	0.271	0.26991	0.26982	0.26924	0.26975	0.26981	0.26981
1500	0.18866	0.17448	0.17321	0.17312	0.17284	0.17308	0.1731	0.17311
2000	0.13857	0.12136	0.11992	0.11981	0.11968	0.11979	0.1198	0.1198
2500	0.10963	0.08931	0.08767	0.08755	0.08747	0.08753	0.08754	0.08754
3000	0.0921	0.06862	0.06677	0.06665	0.06659	0.06663	0.06663	0.06663
3500	0.08123	0.05456	0.05251	0.05237	0.05233	0.05235	0.05236	0.05236
4000	0.0745	0.04461	0.04236	0.04221	0.04218	0.04219	0.0422	0.0422
4500	0.07046	0.03735	0.03489	0.03473	0.0347	0.03471	0.03472	0.03472
5000	0.06825	0.03191	0.02925	0.02907	0.02905	0.02905	0.02905	0.02905

Poisson: Lamda = 0.2
Pareto : b = 1,000,000 & c = 3

Quadratic Integration

COMPARATIVE APPROXIMATIONS
Excess Ratio

	Continuous Severity Model				Discrete Severity Model			
	100	33 1/3	10	3 1/3	100	33 1/3	10	3 1/3
0	1	1	1	1	1	1	1	1
500	0.56809	0.56483	0.56445	0.56442	0.56232	0.56425	0.5644	0.56442
1000	0.35343	0.34843	0.3479	0.34785	0.34686	0.34776	0.34784	0.34784
1500	0.2361	0.22943	0.22874	0.22869	0.22817	0.22864	0.22868	0.22868
2000	0.16775	0.15929	0.15844	0.15837	0.15807	0.15833	0.15835	0.15836
2500	0.12596	0.1156	0.11456	0.11448	0.1143	0.11445	0.11447	0.11447
3000	0.09944	0.08709	0.08587	0.08577	0.08566	0.08575	0.08576	0.08576
3500	0.08214	0.06776	0.06635	0.06623	0.06615	0.06621	0.06622	0.06622
4000	0.07065	0.0542	0.05259	0.05246	0.0524	0.05244	0.05244	0.05244
4500	0.06296	0.04442	0.04261	0.04246	0.04241	0.04244	0.04244	0.04244
5000	0.05782	0.03718	0.03517	0.035	0.03496	0.03498	0.03498	0.03498
5500	0.05447	0.03171	0.0295	0.02932	0.02928	0.0293	0.0293	0.0293
6000	0.05239	0.02751	0.02509	0.0249	0.02486	0.02487	0.02487	0.02487
6500	0.05123	0.02423	0.0216	0.02139	0.02136	0.02137	0.02137	0.02137
7000	0.05076	0.02163	0.0188	0.01858	0.01854	0.01855	0.01855	0.01855
7500	0.05082	0.01956	0.01652	0.01628	0.01624	0.01625	0.01625	0.01625
8000	0.05128	0.01788	0.01464	0.01438	0.01435	0.01435	0.01435	0.01435
8500	0.05205	0.01652	0.01307	0.0128	0.01276	0.01276	0.01276	0.01276
9000	0.05308	0.01541	0.01176	0.01146	0.01142	0.01143	0.01143	0.01143
9500	0.0543	0.01449	0.01064	0.01033	0.01029	0.01029	0.01029	0.01029
10000	0.05569	0.01374	0.00968	0.00935	0.00931	0.00931	0.00931	0.00931

Poisson: Lamda = 1
Pareto : b = 1,000,000 & c = 3

Quadratic Integration

COMPARATIVE APPROXIMATIONS
Excess Ratio

	Continuous Severity Model				Discrete Severity Model			
	100	33 1/3	10	3 1/3	100	33 1/3	10	3 1/3
0	1	1	1	1	1	1	1	1
500	0.80888	0.80901	0.80902	0.80902	0.80845	0.80899	0.80902	0.80902
1000	0.64072	0.64094	0.64097	0.64097	0.64019	0.64092	0.64097	0.64097
1500	0.50075	0.50112	0.50116	0.50116	0.50034	0.50111	0.50116	0.50116
2000	0.38843	0.38906	0.38913	0.38913	0.38839	0.38908	0.38913	0.38913
2500	0.30038	0.3014	0.30151	0.30152	0.3009	0.30148	0.30152	0.30152
3000	0.23231	0.23385	0.23402	0.23403	0.23353	0.234	0.23403	0.23404
3500	0.18008	0.18222	0.18245	0.18248	0.18208	0.18245	0.18248	0.18248
4000	0.14007	0.14288	0.1432	0.14322	0.14292	0.14321	0.14323	0.14323
4500	0.10937	0.11291	0.1133	0.11334	0.11311	0.11333	0.11334	0.11334
5000	0.0857	0.09	0.09048	0.09052	0.09035	0.09051	0.09053	0.09053
5500	0.06731	0.0724	0.07297	0.07302	0.07289	0.07301	0.07302	0.07302
6000	0.05289	0.05879	0.05945	0.0595	0.05941	0.0595	0.05951	0.05951
6500	0.04147	0.04818	0.04893	0.04899	0.04892	0.049	0.049	0.049
7000	0.0323	0.03984	0.04068	0.04075	0.0407	0.04076	0.04076	0.04076
7500	0.02485	0.03322	0.03416	0.03424	0.0342	0.03424	0.03425	0.03425
8000	0.01872	0.02792	0.02895	0.02904	0.02902	0.02905	0.02905	0.02905
8500	0.01359	0.02363	0.02476	0.02486	0.02484	0.02487	0.02487	0.02487
9000	0.00924	0.02013	0.02135	0.02146	0.02145	0.02147	0.02147	0.02147
9500	0.00551	0.01724	0.01856	0.01867	0.01867	0.01868	0.01869	0.01869
10000	0.00226	0.01484	0.01625	0.01637	0.01637	0.01638	0.01638	0.01639
10500	¯0.00061	0.01282	0.01432	0.01445	0.01446	0.01447	0.01447	0.01447
11000	¯0.00318	0.0111	0.0127	0.01284	0.01285	0.01286	0.01286	0.01286
11500	¯0.0055	0.00964	0.01133	0.01148	0.01149	0.0115	0.0115	0.0115
12000	¯0.00761	0.00837	0.01016	0.01031	0.01033	0.01033	0.01033	0.01033
12500	¯0.00957	0.00727	0.00915	0.00932	0.00933	0.00934	0.00934	0.00934
13000	¯0.01138	0.0063	0.00828	0.00846	0.00847	0.00848	0.00848	0.00848
13500	¯0.01309	0.00545	0.00752	0.00771	0.00773	0.00773	0.00773	0.00773
14000	¯0.0147	0.00469	0.00686	0.00705	0.00707	0.00707	0.00707	0.00707
14500	¯0.01624	0.00401	0.00628	0.00648	0.0065	0.0065	0.0065	0.0065
15000	¯0.0177	0.0034	0.00576	0.00597	0.00599	0.00599	0.00599	0.00599
15500	¯0.01911	0.00285	0.0053	0.00552	0.00554	0.00554	0.00554	0.00554
16000	¯0.02047	0.00234	0.00489	0.00511	0.00514	0.00514	0.00514	0.00514
16500	¯0.02179	0.00188	0.00452	0.00475	0.00478	0.00478	0.00478	0.00478
17000	¯0.02307	0.00145	0.00419	0.00443	0.00446	0.00446	0.00446	0.00446
17500	¯0.02432	0.00105	0.00389	0.00414	0.00417	0.00417	0.00417	0.00417
18000	¯0.02554	0.00069	0.00362	0.00388	0.00391	0.00391	0.00391	0.00391
18500	¯0.02674	0.00034	0.00337	0.00364	0.00367	0.00367	0.00367	0.00367
19000	¯0.02792	0.00002	0.00314	0.00342	0.00345	0.00345	0.00345	0.00345
19500	¯0.02908	¯0.00028	0.00294	0.00322	0.00325	0.00325	0.00325	0.00325
20000	¯0.03022	¯0.00057	0.00274	0.00304	0.00307	0.00307	0.00307	0.00307

Poisson: Lamda = 5
Pareto : b = 1,000,000 & c = 3

Quadratic Integration