The Use of GAMLSS in Assessing the Distribution of Unpaid Claims Reserves

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Abstract

Motivation. Regression modeling through generalized linear models (GLM) has known increasing popularity in last decades after milestone papers published in actuarial literature, representing one of the most used tools to assess the variability of unpaid claims reserve. Generalized additive models for location scale and shape (GAMLSS) represent an extension of classical GLM framework allowing not only the location parameters but also shape and scale parameters of a relevant number of distributions to be modeled as function of dependent variable like accident and development years. The paper applies GAMLSS to triangles coming from NAIC loss triangle databases in order to assess the distribution of unpaid loss reserve in term of best estimate as well as distributional form.
The results of GAMLSS are critically compared with those of classical stochastic reserving approach. All the analyses will be performed using R statistical software.

Keywords. Reserving Methods; Reserve Variability; Generalized Linear Models; GAMLSS; R software; NAIC Schedule P database.

1. INTRODUCTION

Regression modeling through generalized linear models (GLM) has been successfully applied in dynamic financial analysis (DFA) to assess the variability of claims reserves. In particular, over-dispersed Poisson models (ODP) have become popular due to the equality of the best estimate (BE) arising from its application to the ones coming from the classical chain ladder (CL). Distributions other than Poisson have been applied for estimating unpaid claims reserves like gamma and negative binomial.

GLMs can be used to obtain an estimate of the variability of outstanding claims reserves, decomposed into the amount due to inherent process variability (process variance) and the amount due to the estimation error (estimation variance). The latter element can be estimated either analytically or numerically thanks to the bootstrap approach (see England & Verrall, 1999) for bootstrap in claims reserve framework and (England, 2002) for process error evaluation).

GLM assumptions regarding the conditional distribution of the dependent variable are quite restrictive, however, since the variance of the outcome variables (that are the triangle cells) is expressed as a function (i.e., the variance function) of the mean of the outcome variables. A new class of statistical models has been introduced, generalized additive models for location, scale and shape (GAMLSS), with the aim to provide a flexible regression framework. In particular, it allows one to use separate regression equations for all parameters of the assumed conditional distribution of the dependent variable. In addition, it provides tools to assess the reasonableness of the regression forms (by means of the functional relationship assumed and variables included) as well as the shape of the conditional distribution.

Rigby & Stasinopoulos (2005) provide a theoretical introduction to GAMLSS, whilst Rigby & Stasinopoulos (2010) show applications of GAMLSS from a practitioners' point of view. At the time of this paper's drafting, no paper applying GAMLSS in the loss reserving context had been found within actuarial literature, making Schewe (2012) and Clemente and Spedicato (2013) the only approaches available. An early introduction of the idea can be found in Spedicato (2012), whilst Schewe (2012) and Clemente & Spedicato (2013) provide more comprehensive expositions. The first paper uses the GAMLSS approach to estimate claims reserves of numerous lines of business by using paid-to-premium ratios and compares the reserve uncertainty to the CL method. The second paper focuses on estimating claims reserve and quantifying reserve risk variability. On the other side, many works on applying GLM and generalized additive models (GAM) exist (see Renshaw & Verrall, 1998 for a general reference). Actuarial applications of GAMLSS are indeed very scarce: Stasinopoulos (2007) and Klein et al. (2014) applied GAMLSS in a ratemaking context, whilst an application to capital modeling has been shown in Spedicato (2011).

The application of GAMLSS for loss reserving is beneficial for two reasons. The first is that the regression assumptions are more flexible. For instance, making the conditional variance a function of external predictors (like the accident, development or calendar years) allows a more flexible modeling of the conditional distribution of triangle cells' outcomes and, therefore, better assesses the process variance. The second reason is applying GAMLSS provides valid tools to assess the shape of the distribution of losses that can be tested against numerous alternative distributions. Loss reserving with GLMs has given little attention to the shape of the conditional distribution of triangle's cells. In general, it can be said that all reserving models based on GLMs are particular cases of those that can be implemented under a GAMLSS framework.

The objective of the paper is twofold: (1) to introduce theoretically GAMLSS as a possible modeling tool for assessing the distribution of loss reserves and (2) to show a practical application on NAIC Schedule P triangles (NAIC DB). The remainder of the paper proceeds as follows: Section 2 discusses the general framework of GAMLSS and the proposed method for claims reserve evaluation, Section 3 describes a practical application on Schedule P databases, Section 4 reports main results and Section 5 drafts conclusions.

2. BACKGROUND AND METHODS

2.1 Introduction to GAMLSS

GLM and GAM proposed to assess loss reserve distribution lead to restrictive modeling for the variance of the response variable since the variance only depends on the mean as expressed within the variance function. Rigby and Stasinopoulos claim that this is true for skewness and kurtosis as well. Thus the authors developed a new model which allows explicit modeling of these moments rather than keeping implicit dependence on the mean. They also relaxed the requirement of a distribution from an exponential family by allowing more general distributions.

GAMLSS is a general class of univariate regression models where the exponential family assumption is relaxed and replaced by a general distribution family. The systematic part of the model allows all the parameters of the conditional distribution of the response variable Y_i ($i = 1,2, \dots, n$) to be modeled as parametric or non-parametric functions of explanatory variables. This means that an actuary can model not only the expected claim payment but also its process variance as a function of accident, development and/or calendar year using a regression expression.

Let $\theta^T = (\theta_1, \theta_2, \dots, \theta_p)$ the p parameters of a probability density function $f_{Y_i}(y_i|\theta_1)$ modeled using an additive model. $\theta_i^T = (\theta_{i,1}, \theta_{i,2}, \dots, \theta_{i,p})$ is a vector of p parameters related to explanatory variables, where the first two parameters $\theta_{i,1}$ and $\theta_{i,2}$ are usually characterized as location μ_i and scale σ_i. The remaining parameters, if any, are characterized as shape parameters. In a reserving context, this framework means that any cell of the triangle can be modeled by any distribution, where the parameters are derived by regression equations of accident and development years. The current R implementation of the software allows distribution up to 4 parameters to be modeled under this framework.

Under this condition, we can derive the following model (when p = 4):

$$\begin{cases} g_1(\mu) = \eta_1 = X_1\beta_1 + \sum_{j=1}^{J_1} Z_{j,1}\,\gamma_{j,1} \\ g_2(\sigma) = \eta_2 = X_2\beta_2 + \sum_{j=1}^{J_2} Z_{j,2}\,\gamma_{j,2} \\ g_3(\nu) = \eta_3 = X_3\beta_3 + \sum_{j=1}^{J_3} Z_{j,3}\,\gamma_{j,3} \\ g_4(\tau) = \eta_4 = X_4\beta_4 + \sum_{j=1}^{J_4} Z_{j,4}\,\gamma_{j,4} \end{cases} \tag{2.1.1}$$

where μ, σ, ν, τ are vectors of length n, X_k are known design matrices, $\beta_k^T = (\beta_{1,k}, \beta_{2,k}, \dots, \beta_{j,k})$ are parameters vector, $Z_{j,k}$ are known design matrices for the random effects and $\gamma_{j,k}$ are random vectors.

In particular, the previous equations imply that the moments of response variable in each cell can be expressed directly as a function of covariates after a convenient parameterization. This since regression equations can be used to model each parameter as a function of covariates and since the moments of any distribution can be expressed as functions of its own parameters. Each linear predictor η_k consists of a parametric component X_k and an additive random component. Instead of random effects, smooth functions may be used as in GAM. Cubic splines, penalized splines, varying coefficients and random effects offer a maximum degree of flexibility since they allow more complex scenarios than GLM (or GAM) to be modeled.

Currently the GAMLSS R package supports more than 60 distributions, non-linear and non-parametric relationships (e.g. cubic splines and non-parametric smoothers) and random effect modeling. See (Rigby & Stasinopoulos, 2010) for more details on the R package. Nevertheless for most real world applications, two-parametric distributions should sufficiently approximate the dependent variable distribution of interest. This means that for the reserving analysis in this paper we will consider only two-parametric distribution families. For example, reserve analysis with up to four parameters can be found. in Schewe (2012).

Applying a GAMLSS model in a reserving exercise involves both selecting the distribution of the dependent variable (for example weibull or a lognormal) and the functional relationship between the parameters of the dependent variable distribution (say μ and σ, if a two-parametric family has been chosen) and the independent variables (say accident and development years). The R package that implements the GAMLSS models provides various instruments to aid the selection of both the functional form and the distribution assumption. (Rigby & Stasinopoulos, Lancaster Booklet, 2010) paper provides an introduction to GAMLSS regression modeling in which the interested reader can find both theoretical details and applied GAMLSS modeling examples. The main instrument to evaluate the reasonableness of GAMLSS is the analysis of normalized quantile residuals (NQR). Normalized randomized quantile residuals (see Dunn & Smyth, 1996) are used to check the adequacy of a GAMLSS model and, in particular, its distribution component. The residuals are given by $\hat{r}_i = \Phi^{-1}(u_i)$, where Φ^{-1} is the inverse of the cumulative distribution function of a standard normal distribution and $u_i = F(y_i|\hat{\theta}_i)$ is derived by applying the estimated cumulative distribution to y_i. If the model is specified correctly the NQR should follow a Gaussian distribution. Apart from model checking, the normality properties have been used for bootstrapping GAMLSS models as shown in forthcoming section.

2.2 GAMLSS Applications to Loss Reserve Analysis

Focusing now on claim reserving analysis, we consider a generic loss development triangle of dimension (I, J) with rows ($i = 1, \dots, I$) representing the claim accident years (AY) and columns (with $j = 0, \dots, J$) describing the development years (DY) for payments. It needs to be emphasized that the number of columns may differ from the number of rows, for example, because of a tail in the payment development.

Following an approach similar to Renshaw & Verrall (1998), we can now define $P_{i,j}$ as the incremental paid claims and identify the incremental paid claims as response variables of the following structure:

$$\begin{cases} E[P_{i,j}] = g_1^{-1}(\eta_{1,i,j}) \\ \sigma^2[P_{i,j}] = g_2^{-1}(\eta_{2,i,j}) \end{cases}. \tag{2.2.1}$$

If a model for the distribution of incremental paid claims is found on historical data $P_{i,j}$ $(i + j \leq I)$, the model can be applied to predict future payments $P_{i,j}$ $(i + j > I)$. The key advantage of GAMLSS compared to GLMs is that $\sigma^2[P_{i,j}]$ can explicitly be modeled within a statistical framework, instead of relying on the GLM variance function assumption.

The ODP model is one of the most used approaches by actuarial practitioners when performing stochastic reserving under a regression framework. Within this framework it is assumed that each triangle cell $P_{i,j}$ follows a Poisson with parameter $\lambda_{i,j}$.

In addition, it is assumed that:

a. $E[P_{i,j}] = \lambda_{i,j}$ can be modeled using a log-linear regression, for example, as a function of AY and DY dummy indicators: $E[\lambda_{i,j}] = exp(\alpha + \beta_i + \gamma_j)$, where α may be parametrized to a baseline accident/development period level.
b. An over-dispersion parameter ϕ exists such that $var[P_{i,j}] = \phi \cdot E[P_{i,j}] > E[P_{i,j}]$ holds for all i, j.

Taking into account the nature of data, however, other distributions may be more appropriate and provide a better fit to the underlying data than a Poisson.

One of the aims of this paper is the investigation of which is the most appropriate distribution for $P_{i,j}$ in a real-world scenario. Triangles from the NAIC DB will be used as the basis of investigation. A GAMLSS with a two-parametric distribution will be fit to the data and the effect of the covariates on the first parameter μ will be examined.

In a second step, we will verify if the second parameter σ can be held constant or can be expressed as a function of either AY or DY within a regression structure similar to the one for μ.

A third step will be to use the GAMLSS to estimate claim reserves and variability of claim reserves. After a suitable conditional distribution and a regression structure for the location and scale parameters has been chosen, the GAMLSS can be applied to the lower part of the triangle to obtain a best estimate of reserves and variability of the estimates, as further detailed in Schewe (2012) and Clemente & Spedicato (2013).

In order to assess the variability of the claims reserves, the following bootstrap-like approach can be used:

1. Fit a GAMLSS model M on an incremental paid claims triangle using a suitable distribution function and development year and accident year as covariates. The functional relationship between the location and scale parameters and their predictors could be modeled using dummy variables or more sophisticated functional relationships such as polynomials or splines. This approach would be similar to a classical ODP modeling approach for development triangles, but here not only a regression for the expected value of the cell but also for its variability would be done. The estimated parameters will be used to derive BE reserves and to model the process variance. Note that the application is not bound to incremental paid claims triangle but incremental incurred claims triangle could be used as well.

2. In order to allow for prediction error, it is proposed to adapt the bootstrap algorithm proposed in Renshaw & Verrall (1998) to GAMLSS model:

    a. Compute the normalized quantile residuals, $\hat{r}_{i,j} = \Phi^{-1}\left(F\left(P_{i,j}|\hat{\theta}_{i,j}\right)\right)$.

    b. Generate N upper triangles of residuals $\hat{r}_{i,j}^{k}$, with $k = 1, \dots, N$ by replacement.

    c. Derive N upper triangles of pseudo incremental payments from the GAMLSS model by the inverse relation: $\hat{P}_{i,j}^{k} = F^{-1}\left(\Phi\left(\hat{r}_{i,j}^{k}|\hat{\theta}_{i,j}\right)\right)$

d. Refit the GAMLSS model M on N triangles in order to assess model variance

e. For each cell of the lower part of each triangle, simulate the outcome $P_{i,j}$ from the process distribution with mean and variance depending by the fitted GAMLSS

3. The sum of lower triangle part cells values as predicted by the GAMLSS model corresponds to the reserve.
4. The N values derived at step 3 represent the simulated distribution of claims reserve.
5. The main moments, that is, the best estimate and a measure of loss variability, can be estimated by such distribution.

Clemente & Spedicato (2013) applied this approach to the classical Taylor-Ashe triangle (Taylor & Ashe, 1983) finding a Gamma distribution with development year as covariate to best fit the payment pattern within a reasonble set of choices. This paper will apply the outlined approach on generic NAIC loss triangles, using various distributions and shows how to derive with the BE reserve and its variability.

3. RESULTS

3.1 Process variance analysis

The first part of the analysis investigates whether, when performing loss reserving under a regression modeling framework, a statistical distribution may be deemed the most appropriate using a statistical goodness-of-fit criterion. For this purpose, a generalized Akaike information criterion (GAIC) will be used to compare GAMLSS. It is obtained by adding to the fitted global deviance a fixed penalty for each degree of freedom in the model.

Furthermore, to address the question stated above, various GAMLSS models have been fit on NAIC Schedule P loss triangles following the approach outlined as follows:

a) The incremental claim payments are expected to vary both by accident and development year. A second and third structure has been defined, allowing the scale parameter to vary by either accident or development year.

b) Accident and development years enter the GAMLSS regression as dummy variables in all our analyses.

c) The following distributions were tested: Poisson (POI), negative binomial (NBI), gamma (GA), Weibull (WEI), lognormal (LNORM) and inverse Gaussian (IG). Whilst the GAMLSS R package can handle more than 60 different distributions, the relatively limited choice is driven by the authors' aims to introduce the approach and to restrict the analysis to the most used distributions within current actuarial practice. For each distribution, the three regression structures mentioned above were implemented. Note that the two discrete distributions, Poisson and negative binomial, are being used for a continuous random variable for the same reasons outlined in England & Verrall (1999). Recall that it is shown that a GLM reserve estimate under an ODP framework is equal to a chain-ladder reserve estimate.

d) Each combination of regression structure and distribution has been fit to each triangle of the NAIC. For each triangle, we selected the model with the lowest value of GAIC criterion among those for which the GAMLSS algorithm was able to estimate parameters.

e) The conditional distributions and parameter assumptions of the reference models have been tabulated for all the NAIC DB lines of business (product liability, other liability, medical malpractice, workers compensation, commercial auto and private passengers auto).

For the analysis of reserve variability, a triangle was picked at random (group 226, private passengers auto) and the upper and lower parts of the incremental paid loss triangle were arranged. Traditional reserving models were estimated with the aid of Gessman, Zhang, & Murphy (2013) R package as well as various GAMLSS reserving models. The underlying best estimates have been compared with the subsequent payments shown in the lower triangle and released within the NAIC DB package. The models' reserve standard errors have been compared as well.

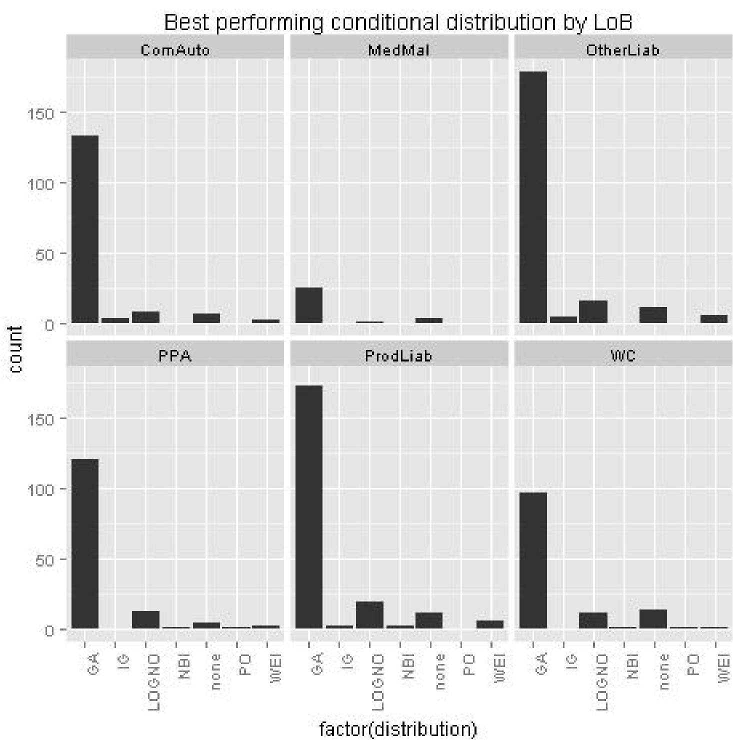

Figure 1: Best performing conditional distribution by line of business (LoB)

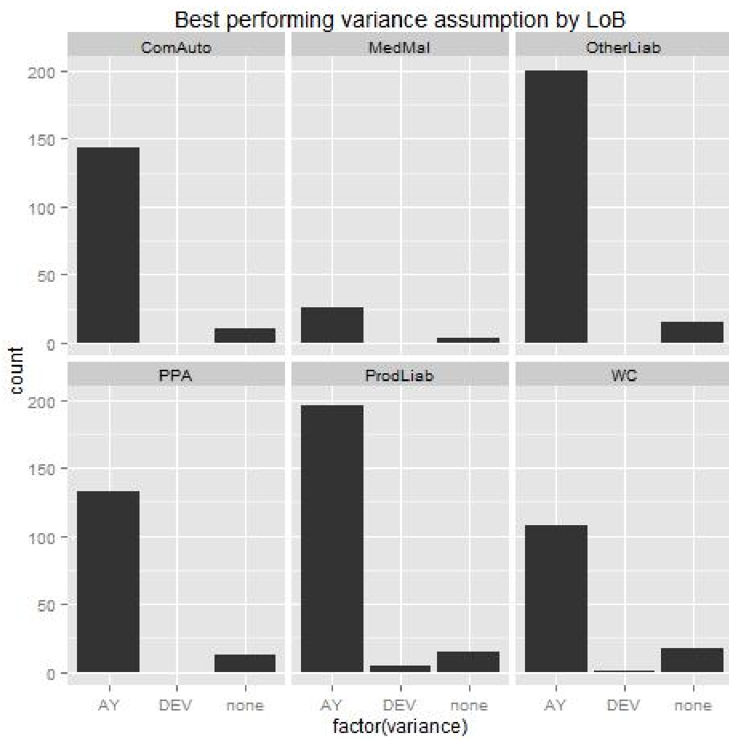

Figure 2: Best performing shape parameter assumption by LoB

Various GAMLSS have been fit on NAIC DB varying the conditional distribution assumption as well as the variance dependency by either accident or development years or neither of them. The best performing model as measured by the goodness of fit has been selected. In other words, for each group's triangle, within each line of business (LoB), a "best" model has been selected. It has been defined by a conditional distribution assumption (tabulated on Figure 1) and a shape parameter assumption (tabulated on Figure 2). The gamma distribution supersedes by far the other distributions as the most appropriate distribution by AY. The lognormal and Weibull distributions are a distant second and third, respectively. Assuming the claim payment follows a discrete distribution (Poisson or negative binomial), as was done in earlier GLM reserving approaches, appears to be not supported by empirical data. Similarly, assuming the scale parameter to vary by accident year appears to improve the model fit in terms of GAIC.

The R programming code that replicates the analysis of this section are the first three files listed in the appendix.

3.2 Full distribution of Unpaid Claim Reserves

The approach outlined in the methodology section has been applied in order to estimate the reserve BE and its variability. The exercise has been carried on a group of 266 triangles for the private passenger LoB (henceforth called example triangle or ET). As pertaining to the NAIC Schedule P triangles set, it shows 10 years of development for AYs 1988–1987. The obtained figures have been compared with the actual incremental payments during calendar year 1998–2006 and with the BE and standard deviation implied with other reserving algorithms applied on the same triangle (Mack formula, bootstrap chain ladder with a gamma process distribution, GLM ODP).

Initially various GAMLSS models have been fit on the ET in order to find an appropriate stochastic model for the claim triangle. The selected model assumes a GAMMA conditional distribution, modeling the expected value to depend on both the accident and development years whilst the variance to vary by development year only. Then the unpaid claim distribution (see Figure 3) has been obtained by estimating both process and parameter uncertainty as described in Section 2.2. The green and red lines in Figure 3 represent observed payments in the lower part of the triangle and GAMLSS BE, respectively.

model	Best Estimate	Standard Deviation
Mack	30.065	2.517
BootstrapCL	32.635	141.905
ODP	30.065	6.695
GAMLSS BASE	31.821	14.354

Table1: BE and standard deviations of various loss development models on private passenger, Group 226

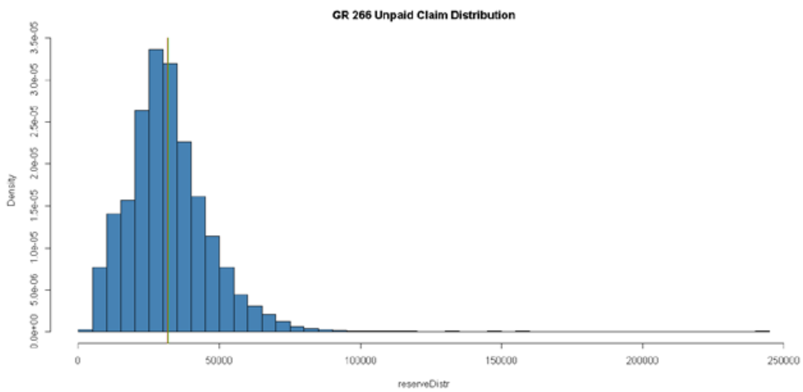

Figure 3: GR 266 Reserve Distribution derived by the GAMLSS model.

The BE and standard deviation of unpaid claims are reported in Table 1. When compared with actual calendar year 1998–2006 payments totaling 31,713, the GAMLSS model BE appears to be the closest to the actual results. However, the standard deviation of the GAMLSS model appears to be quite large (slightly more than double than that given by the ODP and almost seven times than the variability given by the Mack formula, even though far lower than the bootstrapped chain ladder with gamma process variance). It is difficult to explain why the difference in variability is so great when compared with standard models. One possible reason is that when a model that predicts not only the central tendency but also the variability is bootstrapped, the resulting variance is exacerbated.

In addition, the estimated BE appears to be very sensitive to changing conditional distribution assumptions. Varying either the conditional distribution assumption or the variance form assumption can imply large swings in terms of the BE (see Table 2) as well as in the inherent variability.

model	Best Estimate	Standard Deviation
GA, dev	31.821	14.354
GA, ay	26.614	645
GA, none	34.527	18.671
WEI, ay	26.106	521

Table 2: Loss reserve BE and sigma by changing GAMLSS conditional distribution and variance modeling assumptions.

3.3 Comparison of GAMLSS Reserves Estimate with BLUE Chain Ladder

A final exercise was done comparing the accuracy of reserve estimates using mechanic chain ladder and GAMLSS approach with respect to actual lower triangle part payments (since calendar year 1999). The accuracy was measured by means of the root mean-squared error (RMSE); that is, the square root of the average squared difference between an actual outcome (the lower triangle cells' actual payments) and its estimate (the best estimate). The analysis was performed on the full NAIC DB private passenger auto triangle set, excluding those triangles on which a standard GAMLSS model did not converge. In addition, chain ladder link ratios were estimated using the regression through the origin formula, which has been shown to be a best linear unbiased estimator (BLUE) of the development factors (Murphy, 1994).

Model	RMSE
BLUE Chain Ladder	97.577
GAMLSS	16.276

Table 3: RMSE comparisons between BLUE Chain Ladder and GAMLSS approach

The analysis shows that the correlation between the GAMLSS and chain ladder estimates is very high. In addition, even if neither GAMLSS nor chain ladder systematically outperforms the other, the GAMLSS RMSE is significantly lower than chain ladder value; thus, suggesting GAMLSS could provide sensible reserves estimates.

The R programming code that replicates this analysis are the last two files listed in the appendix.

As a general remark, whilst GAMLSS models allow for a great degree of flexibility, they have not yet been studied extensively. In particular, the actual R implementation is not optimized by means of incorporating C code in the computationally most critical part of the estimation process. In addition, model estimation convergence problems may arise, especially when complex regression structures are used or non-standard conditional distributions are chosen. The following measures were taken in order to overcome such drawbacks:

1. The R code was highly parallelized to take advantage of multicore processors when performing the analysis on the whole NAIC database. The aim is running much larger chunks of computations in parallel.

2. Exception was explicitly handled within the R programming code to avoid analysis interruptions.

In addition, the following adjustments to the data were performed in the data preparation part:

1. Incremental paid data were modeled.
2. Negative increments were zeroed.

DISCUSSION

This paper has investigated the use of GAMLSS regression models for P&C loss reserving. On an overall basis, a mixed result can be drawn. For various reasons, these models, appear to be potentially relevant for loss reserving. By allowing an explicit modeling of the variability by either or both accident and development years, GAMLSS overcome the limitations of standard GLMs. Therefore, they would better model the process variance of loss reserves. On the other hand, the additional parameter that is needed to be modeled reduces the degrees of freedom available. This can be a strong limitation of the model when using triangles of similar size to those provided in the NAIC DB , but in case larger triangles (as quarterly based triangles or 15x15 yearly based triangles requested in Solvency II technical reports) are used, this limitation can be overcome.

The preliminary analyses carried out in the paper have shown that actual triangles of the NAIC DB present a source of variability that departs from the variance function assumption on which standard GLMs are based. In addition, a gamma conditional distribution outperforms, in terms of goodness of fit, other distribution like negative binomial and Poisson that are commonly assumed in standard GLM reserving models.

When a GAMLSS approach has been used to assess P&C loss reserves, BE, and variability on an actual triangle, results have been comparable with those of other reserving methods . In addition, the GAMLSS approach systematically applied on the whole NAIC DB private passenger auto triangle set has shown an RMSE lower than the BLUE chain ladder, when predicted payments (i.e., reserves) have been compared with actual payments. On the other hand, unpaid claim distributions arising from bootstrapping GAMLSS models have been shown to be extremely sensitive with respect to changes in marginal distribution assumptions and cell variance. A final limitation, that needs to be stressed, is that convergence problems arise much more frequently than with standard GLMS. This requires a greater effort in data checking and model selection.

Supplementary Material

The full R code to replicate numerical results of the paper is available. In particular:

1. 0-loadNaicTriangles.R loads NAIC CSV files.
2. 1-prepare data set 4 modeling.R performs additional preprocessing.
3. 2-regression models on Naic Triangles.R performs additional analyses.
4. 3-GAMLSS reserve variability.R performs analysis of reserve variability on a real triangle
5. 4-Compare ChainLadder and GAMLSS.R applies ChainLadder and GAMLSS on PAP triangles and compares RMSE.

5. REFERENCES

Casualty Actuarial Society. (s.d.). "Loss Reserving Data Pulled from NAIC Schedule P." Downloaded Dec. 31, 2013. Available at http://www.casact.org/research/index.cfm?fa=loss_reserves_data.

Clemente, G.P., G.A. Spedicato (2013). "Claim Reserving Using GAMLSS," *Proceedings of XXXVII Meeting of the Italian Association for Mathematics Applied to Economic and Social Sciences.* Slides available at Slideshare, http://www.slideshare.net/gspedicato/draft-amases.

Dunn, P., G. Smith (1996). "Randomized Quantile Residuals," *Journal of Computational and Graphical Statistics* 5, 236-244.

England, P.D. (2002). Addendum to "Analytic and Bootstrap Estimates of Prediction Errors in Claims Reserving," *Insurance: Mathematics and Economics* 31, 461-466.

England, P.D., R.J. Verrall (1999). "Analytic and Bootstrap Estimates of Prediction Errors in Claims Reserving." *Insurance: Mathematics and Economics* 25, 281-293.

Gessman, M., W. Zhang, and D. Murphy (2013). "ChainLadder R Package." *Statistical methods for the calculation of outstanding claims.*

Heller, G. Z., D.M. Stasinopoulos, R.A. Rigby, P. De Jong (2007). "Mean and Dispersion Modelling for Policy Claims Costs." *Scandinavian Actuarial Journal* 2007(4), 281-292.

Klein, N., M. Denuit, S. Lang, T. Kneib (2014). "Nonlife Ratemaking and Risk Management with Bayesian Generalized Additive Models for Location, Scale and Shape," *Insurance: Mathematics and Economics* 55, 225-259.

Murphy, D.M. (1994). Unbiased Loss Development Factors. *Proceedings of the Casualty Actuarial Society* LXXXI, 154-222.

Renshaw, A., R.J. Verrall (1998). "A Stochastic Model Underlying the Chain-Ladder Technique," *British Actuarial Journal* 4, 903-923.

Rigby, R.A., D.M. Stasinopoulos (2005). "Generalized Additive Models for Location, Scale and Shape" (with discussion). *Applied Statistics* 54(3), 507-554.

Rigby, R.A., D.M Stasinopoulos (2010, May 27). "A Flexible Regression Approach Using GAMLSS in R." Available at http://www.gamlss.org/wp-content/uploads/2013/01/book-2010-Athens1.pdf.

Rigby, R.A., D.M Stasinopoulos (2010, 12 27). "Lancaster Booklet." Available at http://www.gamlss.org/wp-content/uploads/2013/01/book-2010-Athens1.pdf.

Schewe, F. (2012, 11 28). "Reserve Estimation and Analysis with Generalized Additive Models for Location, Scale and Shape." Master thesis.

Spedicato, G.A. (2011, May 8). "Solvency II Premium Risk Modeling under the Direct Compensation CARD Scheme." Doctoral thesis. University La Sapienza, Roma, Italy.

Spedicato, G.A. (2012, April). "P&C Reserving using GAMLSS Models." (p. 94-108).Paper presented at the 2012 Mathematical and Statistical Methods for Actuarial Science and Finance (MAF) Conference in Venice, Italy.

Abbreviations and notations
AY, accident year
BE, Best Estimate
BE, best estimate
BLUE, best linear unbiased estimator
CAS, Casualty Actuarial Society
CL, chain ladder
DY, development years
GAIC, generalized Akaike information criterion
GAMLSS, generalized additive models for mean location and shape
GLM, generalized linear models
NAIC DB, NAIC Schedule P triangles data base
NQR, normalized quantile residuals
ODP, over-dispersed Poisson models
OLS, ordinary least squares

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