

Refresher Course
VALUE OF RISK REDUCTION

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Value of Risk Reduction

When does it make sense for a firm to incur costs to mitigate risk?

The results of Modigliani-Miller (1958) are still frequently referenced today. In broad outline, MM theory indicates that for a firm owned by diversified investors, any risk that can be diversified against broader holdings is irrelevant to the owners – and thus it is not worthwhile for the firm to incur mitigation costs for such risks. However, this result is based on numerous simplifying assumptions, including the assumption that distressed firms have access to unlimited new capital with no extra costs or conditions. Clearly, this is not the case. Froot et al. (1993) is a widely cited reference for the difference in cost of raising new capital vs. retaining earnings, and its conclusion is that firms with ongoing capital needs should protect their earnings stream through risk transfer. Subsequent empirical studies have supported their ideas. Overall, the assumptions behind the MM framework simply do not hold, so this framework is not appropriate for the practice of Enterprise Risk Management. However, due to information lags many insurance CFOs have been brought up within this background, so actuaries need to understand it and be able to discuss it coherently.

An actuarial theory of firm value was developed in the late 1950's by Bruno de Finetti, a one-time compatriot of Modigliani. The capital assumptions used by de Finetti (1957) are almost exactly opposite those of MM: in de Finetti's model, a firm has no means to raise capital except for retained earnings. While in the real world this assumption generally does not apply to healthy firms, it is a fair approximation of the situation for distressed firms. In this theory, firm value is the present value of future dividends to owners. Avoiding insolvency maintains the firm's earnings ability and so increases its value. Paying a cost to mitigate, offset, or transfer diversifiable (sometimes called "specific" or "idiosyncratic") risk in order to avoid distress may therefore make sense. This is very much in the spirit of the actuarial theory of probability of ruin.

This study note starts with the recent financial theory of risk transfer for firms in general in Section 1, and considers issues specific to the insurance industry in Section 2. The actuarial theory is reviewed in Section 3. These sections provide arguments for why risk reduction may be worth the costs, and discuss implications for risk management. Section 4 discusses efforts that have been made to quantify these effects in practice, and Section 5 presents an illustrative example.

1. General motivations for risk reduction

There are numerous reasons why it can be worthwhile for firms to reduce or transfer specific risk, even at a cost. These include but are not limited to

- Costs of financial distress
- Agency issues
- Regulation and taxation
- Relationships with stakeholders

A growing body of empirical research supports the value of risk management in these contexts.

Financial distress can be costly

Contrary to the assumptions of MM, it can be difficult and costly for distressed firms to raise new capital. Issuing high-yield bonds means a substantial recurring expense. New equity shares usually must be issued at a discount – sometimes a large discount – to the current price; this reduces the already-reduced value of existing shares, unwelcome news to shareholders even if they have the first right to purchase the new shares. Even if the shares are not issued at a discount, the effect is dilutive for existing shareholders. Furthermore, assets (such as subsidiaries) offered for sale may attract less favorable prices than before.

Andrade and Kaplan (1998) examine banks from highly leveraged buyouts that ended up distressed. Although in this study the distress was usually a result of the high leverage, the effects observed are more general. Distressed firms experience reduction in earnings, reduction in capital expenditures, forced sale of assets at depressed prices, and delay in restructuring. These in turn mean curtailed growth and lost opportunities.

In a classic paper on risk transfer, Smith and Stultz (1985) identify costs of financial distress that “leak out of” financial markets – they may end up as profits or salaries for someone (law firms, liquidation bureaus, etc.), but out of the realm of publicly traded firms. For example, many of the costs of bankruptcy fall into this category. They also point out that bonds often have covenants that require specific firm action in the case of financial distress, which can restrict the firm’s flexibility in responding to the distress.

The difficulty and cost of raising capital for a distressed firm implies that firms with greater capital needs will gain more value from risk management. Testing this requires a publicly available proxy for risk management activities as well as an indicator of capital need. Although engaging in hedging transactions is only a small part of most companies' risk management activities, it is a publicly available statistic. Firms with greater capital needs are generally those with better growth prospects; thus, higher market-to-book ratio or higher research and development expenditures might indicate elevated capital need. High leverage ratios could also indicate greater capital need and would increase the frictional costs of financial distress. Several studies [Geczy et al. (1997); Nance et al. (1993); He and Ng (1998); Dolde (1995); Samant (1996)] have found relationships between the use of hedging and these predictor variables. Though not every study has found such effects, there is an increasing amount of empirical evidence for the value of risk management as related to external financing costs.

To understand the costs of financial distress, the concept of agency conflicts is useful.

Agency theory explains potential changes in risk behavior

In the terminology of agency theory, an "agent" is an entity to whom you give control of some portion of your funds for your mutual benefit. For instance, the managers of a company are agents of the shareholders, and the shareholders are agents of the bondholders. The key aspect of agency theory is the potential conflict between agent and principal.

Absence or reduction of "skin in the game" tends to lower the agents' risk aversion relative to the principals they represent. Therefore, shareholders would like to establish a management compensation program that aligns their interests with those of management. In practice, this alignment is imperfect. Usually debtholders have less control over what shareholders do with the debt funding, although given the possible need to issue new bonds in the future the firm would be unwise to act against the bondholders' interests. Reducing risk through hedging or other strategies can be a signal to bondholders, potential creditors, and future investors that the firm will not take excessive risks with their money.

When a firm goes into distress, but remains solvent, shareholders and management may become less risk-averse. The loss of funds that caused the distressed state decreases the shareholders' stake, but not that of the debtholders. Such a highly leveraged situation heightens agency conflicts.

Shareholders may prefer high-risk gambles with the joint funds, which they control but of which they own little. They may in fact decline less risky investment possibilities, because these would help debtholders more than shareholders. This conundrum is referred to as “the underinvestment problem.” A related issue is that the firm’s survival prospects may be enhanced by issuing new shares – but the main beneficiaries of that would again be the debtholders, who have little say in the decision.

Taxation and regulation can provide motivations for hedging

Dolde (1995) reports a positive relationship between tax loss carry forwards and the use of risk management instruments. This indicates that taxes provide an incentive for risk management. Furthermore, when the corporate tax rate is higher at higher incomes, maintaining a stable income might lower total taxes paid over time [Smith and Stulz (1985)]. In such cases, the costs of risk offset to accomplish income stability might enhance long-term earnings. However, there is not a great deal of evidence that firms are in the right place on the tax schedule to make this worthwhile in practice.

In regulated industries, the regulator may specifically require a certain level of risk management or impose other burdens on riskier firms. For example, banks and insurance companies are typically required to satisfy certain leverage ratios. Hedging may be necessary to meet these requirements.

Relationships with stakeholders can be damaged by excessive risk

Smith and Stulz (1985) highlight the effect of risk on stakeholders other than shareholders. These include debtholders, customers, employees, and suppliers. All of these prefer predictable outcomes for their relationships with the firm. Carrying higher risk can drive up costs for all of them, endangering relationships that the firm must maintain for continued success.

2. Insurance-specific issues

In addition to the general considerations discussed above, insurers face special issues. These include:

- Agency theory complications: policyholders as debtholders and/or owners
- Special vulnerability to effects of financial distress
- Reinsurance as the dominant form of hedging

For these and other reasons, using risk management to avoid financial distress appears to increase insurers' market value [Staking and Babbel (1995)].

Agency issues and the influence of policyholders

Mayers and Smith (1990) examine risk transfer specifically for insurers. In addition to the general reasons for risk transfer discussed above, they find issues particular to the insurance industry. For one thing, the principal debtholders are in fact the customers (policyholders). Loss reserves and unearned premium reserves are supported by funds held for payments to policyholders. This complicates the agency relationship between shareholders and debtholders, and gives the debtholders more relative power in the relationship. Insurers must maintain lower levels of risk because the relationship with debtholders is ongoing, and customer relationships would be threatened if their risk attitudes were ignored.

For mutual companies, policyholders are owners as well as debtholders. This puts an interesting spin on the agency issues, further reducing tolerance for risk. In addition, a firm with this structure has very limited access to capital markets and so there are fewer options in case of financial distress.

Many studies support the idea that insurers benefit from managing risk for the sake of policyholders. Empirical evidence indicates that policyholders are not willing to pay as much for insurance from a less stable or weakly-capitalized insurer [Sommer (1996)]. The profit load insureds are willing to pay decreases as the ratio of insurer capital to assets declines, and also decreases as the volatility of that ratio increases. The price discount that insureds demand for accepting a higher expected cost of insurer default is greater than the economic value of the default put value [Phillips, Cummins and Allen (1998)], perhaps because insureds do not diversify this risk. Insurer security affects buying decisions for homeowners' insurance [Grace et al. (2004)]. Furthermore, growth rates are higher for insurers with greater financial strength as measured by rating agencies [Epermanis and Harrington (2006)].

Insurers are particularly vulnerable to financial distress

In general, riskier firms tend to lose market share and shrink in relative market capitalization during periods of downturn in their respective industries [Opler and Titman (1994)]. The insurance industry is not unique in this regard. However, the insurance product is a promise of payment contingent upon certain future events. When an insurer encounters financial distress, its promise of future payment becomes less valuable. Empirical studies and case histories all suggest that a distressed insurer can lose many of its customers. This can create motivation to fight for market share by cutting price; and, as observed above, those customers who remain are likely to expect discounted premium costs, setting into motion a vicious cycle. The costs of financial distress for an insurer therefore include substantial loss of future earnings potential, and its viability may be threatened.

Equity markets tend to react quite adversely to an insurer's financial distress. In many cases, the resulting reduction in market capitalization is a significant multiple of the drop in book value that resulted in the financial distress. This also contributes to the vicious cycle that may imperil the insurer's ability to continue as a going concern.

Reinsurance reporting offers a window into insurers' risk management

Reinsurance – the contractual agreement by which a reinsurer undertakes to offset a specified portion of insurance losses – is the dominant means of hedging insurance risk. Because reinsurance is recorded in financial statements, the insurance industry is more transparent than others in its use of risk transfer.

Mutual insurers make greater use of reinsurance; this is consistent with their ownership by policyholders and their lack of access to capital markets [Mayers and Smith (1990)]. Smaller insurers, and insurers with lower ratings from rating agencies, also buy more reinsurance; this is in accord with the previous discussion of the costs of financial distress and avoiding agency issues with policyholders.

3. The actuarial theory of the value of insurance risk reduction

A direct way to quantify the value of risk transfer is to create a model of firm value that responds to risk issues. De Finetti (1957) formulates the value of a firm as the present value of all future dividends paid to shareholders. He does not allow for any access to capital markets once the firm has been set up, so distress can be very costly and bankruptcy ends the dividend stream. Thus, risk does affect firm value in this model.

Gerber and Shiu (2006) present a well-developed form of de Finetti's model. They use a fairly general severity distribution that could approximate many actual distributions. They focus on the problem of determining a dividend-paying strategy that would optimize firm value for a pre-determined underwriting portfolio. It turns out that the optimal strategy is to pay no dividends if capital is below a certain target level, and pay out any capital beyond that level. The optimization is rather complex, using a dynamic programming approach [Bellman (1954)].

Other authors [Bather (1969); Asmussen et al. (2000)] have extended this model to include the possibility of buying reinsurance. More recently the actuarial and financial paths have come together by bringing in the possibility of refinancing in the capital markets [Peura (2003); Major (2007) also includes the effects of policyholder risk aversion]. The cost of distress financing is an input for such a model. One study on this is Myers and Majluf (1984). Agency dynamics between policyholders and shareholders are another needed input. Panning (2006) argues in general for using financial value as the basis of risk-transfer decisions in insurance, and illustrates with a simple conceptual example.

Insurers and reinsurers have started to use models like this to optimize value by optimizing the level of capital and risk. The next section provides an overview of various methods.

4. Quantifying the value of risk transfer for insurers

As we have seen in the prior three sections, there is strong evidence that offsetting risk via hedging or reinsurance can provide value to a firm – and particularly to an insurer. However, quantifying the benefits of risk transfer for insurers is still an emerging discipline. As yet there is no broad consensus and further work in this area is needed. Some possible approaches include:

- Simple multiplier methods

- Efficient frontier comparison
- Cost of allocated risk capital
- Estimates of firm value under different strategies

Simple multiplier methods can provide a rough estimate

In the financial literature, the calculation of the value of risk management generally starts by quantifying historical distress costs for distressed firms. Andrade and Kaplan (1998) estimate these costs as 10% - 23% of pre-distress capital for their over-leveraged banks, and then multiply by the probability of going into distress. Using distress probabilities risk-adjusted for market risk reactions produces a much greater impact – Almeida and Philippon (2008) show that the cost to the bank shareholders after risk adjustment can be more than three times as large as the cost calculated ignoring this. The market value of corporate bonds can be viewed as coming from default probabilities adjusted to reflect market risk attitudes (and liquidity); therefore, an approximation to the necessary probability transformation could be derived from the firm's bond ratings.

For distress of various levels, it would be possible to estimate the loss of future earnings capacity using the historical impact on actual earnings of insurers in financial distress, or using market capitalization as a proxy. For insurers in distress, the market cap reaction is often a multiple of the financial loss, which is similar to what Almeida and Philippon found for banks. The probability of distress could be estimated from internal models and then risk-adjusted. This would enable comparison of the expected cost of distress to the costs of reinsurance or other risk management strategies.

Efficient frontier comparisons are useful, but where along the frontier is best?

Under current market practice, some portions of this approach are typically considered when reviewing potential reinsurance alternatives. The first step is using a simulation model to compute the probability distribution of financial results under each proposed reinsurance program. From the simulated results, estimates of the probability of various levels of distress can be estimated. For instance, “distress” could be defined as failure to achieve estimated earnings, suffering negative earnings, capital falling below twice the regulatory target, or capital falling below the desired rating-agency target. The percentage of capital lost at various probability levels can be tabulated across programs. The cost of each reinsurance alternative can be measured as expected payments to rein-

urers less expected recoveries.

Such analysis allows an “efficient frontier” comparison to be made for each distress threshold. A reinsurance program is inefficient if a less costly program, or linear combination of programs, gives a more favorable result at the selected threshold. Different distress thresholds can have different sets of efficient reinsurance alternatives, and this method does not clarify how to select among efficient alternatives. However, it may be possible to eliminate a number of possibilities via efficiency considerations. At this point management might be able to select its favored reinsurance program from the remaining alternatives using other criteria.

Cost of allocated risk capital offers one metric

One such criterion, which also offers a way to quantify the benefit of risk transfer in a single number, is cost of allocated risk capital; see Exley and Smith (2006) for a comprehensive overview of the theory of capital costs and application to financial firms.

In this approach, an economic capital model is applied to the simulated results net of reinsurance alternatives. The difference in cost of the reinsurance programs can then be compared to the difference in cost of risk capital.

This begs the question of which capital measure is most appropriate. Commonly used capital measures include Value at Risk (VaR) and Tail Value at Risk (TVaR). With these and other measures based on the tail of the probability distribution, there is the additional question of what threshold probability to select.

One approach could be to use the current program as a benchmark, and seek to maintain a comparable ratio of VaR / TVaR levels to capital. This would indicate the company’s required capital under each program. Alternatively, capital metrics that are not tail-based can be used. Naturally, results may vary depending on the capital standard selected.

More robust models of firm value take the next step

Directly addressing the question of value added by any risk management strategy requires a more complete model of firm value. Models such as those described in [Major (2007) and Panning (2006)] reflect risk issues. However, such models can be complex and the inputs themselves must be quantified. This is an emerging approach.

5. Illustrative Example

Let's consider a concrete example to illustrate the concepts discussed in the previous section. In this example, a firm that writes earthquake insurance is considering the benefit of its current reinsurance program as compared to three possible alternatives. In the coming year 201X, the firm expects to write \$100M of premium, with expected losses of \$18.6M and acquisition expenses of \$12.5M. Due to the extreme volatility of its business, the firm holds \$200M of capital.

Methods used to derive the insurance loss model are outside the scope of this discussion, as are the mechanics of simulating and applying specific reinsurance structures to the modeled losses. Instead, we will focus on the output produced by the simulation model: a distribution of financial outcomes on a gross basis as well as net after each of the reinsurance alternatives. The expected values are as follows:

Expected Underwriting Profit & Loss Summary for Projected Year 201X (\$M)

	<u>Gross</u>	<u>Current</u>	<u>Option 1</u>	<u>Option 2</u>	<u>Option 3</u>
(1) Gross Premium	100.0	100.0	100.0	100.0	100.0
(2) Reinsurance Premium	-	60.8	55.1	49.5	40.2
(3) = (1) – (2) Retained Premium	100.0	39.2	44.9	50.5	59.8
(4) Gross Losses	18.6	18.6	18.6	18.6	18.6
(5) Reinsurance Recoveries	-	11.0	10.0	9.0	7.6
(6) = (4) – (5) Retained Losses	18.6	7.7	8.6	9.6	11.0
(7) Gross Expenses	12.5	12.5	12.5	12.5	12.5
(8) Reinsurance Commission Received	-	-	-	-	-
(9) = (7) – (8) Retained Expenses	12.5	12.5	12.5	12.5	12.5
(10) = (3) – (5) – (9) Retained Underwriting Result	68.9	19.0	23.8	28.4	36.3
(11) = (2) – (5) – (8) Net Cost of Reinsurance	0.0	49.9	45.1	40.5	32.6

Because insurance losses are driven by infrequent earthquakes, in most years the insurer suffers no losses – but when losses occur they can be very severe. The probability distribution of underwriting results is shown below, along with selected statistics from the simulation that will be used in later calculations.

Distribution of Modeled 201X Underwriting Profit

<u>Percentile</u>	<u>Return Period</u>	<u>Gross</u>	<u>Current</u>	<u>Option 1</u>	<u>Option 2</u>	<u>Option 3</u>
50.00%	2	87.5	26.7	32.4	38.0	47.3
75.00%	4	87.5	26.7	32.4	38.0	47.3
80.00%	5	83.9	23.1	28.8	34.4	43.7
90.00%	10	57.9	(2.9)	2.9	8.5	17.7
95.00%	20	(15.3)	(18.8)	(24.1)	(32.9)	(45.4)
98.00%	50	(161.1)	(37.4)	(42.7)	(51.5)	(68.2)
99.00%	100	(300.3)	(56.0)	(63.0)	(71.7)	(85.9)
99.50%	200	(468.6)	(82.2)	(94.8)	(112.8)	(110.8)
99.60%	250	(532.4)	(101.2)	(113.7)	(131.8)	(132.0)
99.75%	400	(669.8)	(211.3)	(217.7)	(228.9)	(241.6)
99.80%	500	(729.7)	(274.6)	(281.2)	(291.9)	(302.9)
99.90%	1,000	(951.2)	(483.1)	(488.6)	(500.3)	(518.2)
	Mean	68.9	19.0	23.8	28.4	36.3
	Probability of Negative UW Profit	5.6%	10.5%	9.6%	8.9%	7.9%
	Probability of UW Profit \leq -100M	3.26%	0.41%	0.47%	0.66%	0.72%
	E[Profit Profit \leq -100M]	(332.2)	(206.8)	(215.8)	(189.5)	(193.8)
	1-in-100 VaR	300.3	56.0	63.0	71.7	85.9
	1-in-250 VaR	532.4	101.2	113.7	131.8	132.0
	1-in-100 TVaR	566.3	187.4	196.7	209.3	217.9

If the company buys no reinsurance at all, the probability of fully exhausting surplus in the coming year is between 1% and 2%. A one-year ruin probability in excess of 1% is likely to find disfavor with regulators, so it is likely that the company needs to engage in risk transfer for regulatory reasons – as well as for the reasons discussed above. Customers might expect the likelihood of an earthquake loss to be in the 1% to 2% range, and if the insurer is bankrupt with the same probability, they might not want to buy insurance. The table above indicates that the current structure reduces the one-year ruin probability to approximately 0.25%, i.e. complete exhaustion of surplus is roughly a 1-in-400 year event.

But what *value*, if any, does risk transfer offer to the purchaser in this case? In order to apply the methods discussed in Section 4, we must have a criterion for “distress.” As revealed in Sections 1 and 2, financial distress can set in well before capital is fully exhausted. In this example we assume that a net underwriting loss of \$100M or more (i.e. a loss of 50% of capital) puts the company in distress, but any situation less severe than this does not create distress. (In practice, this would be quite a high pain threshold: an insurance company is likely to suffer at least some effects of distress if 10% – 20% of surplus is depleted.)

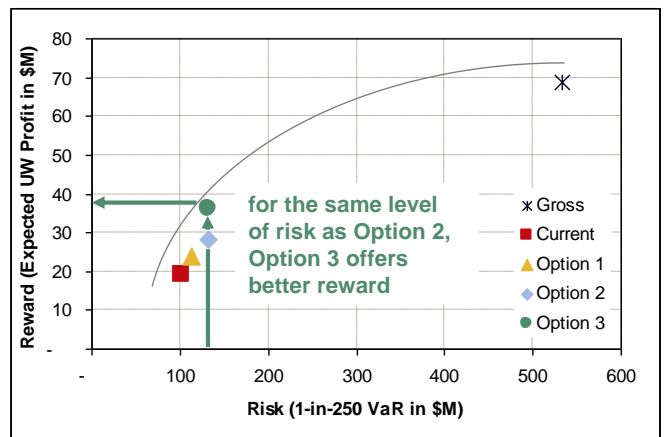
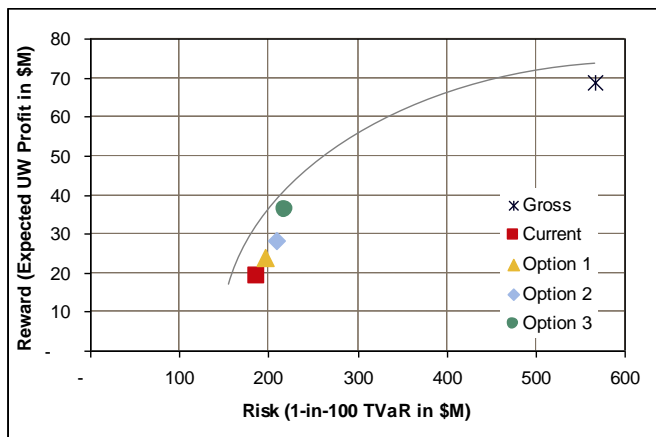
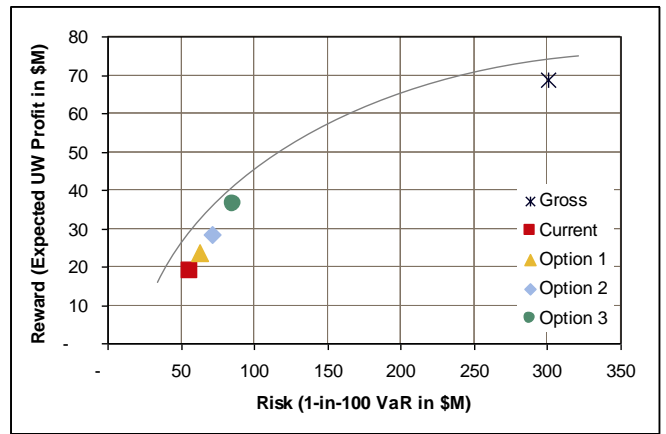
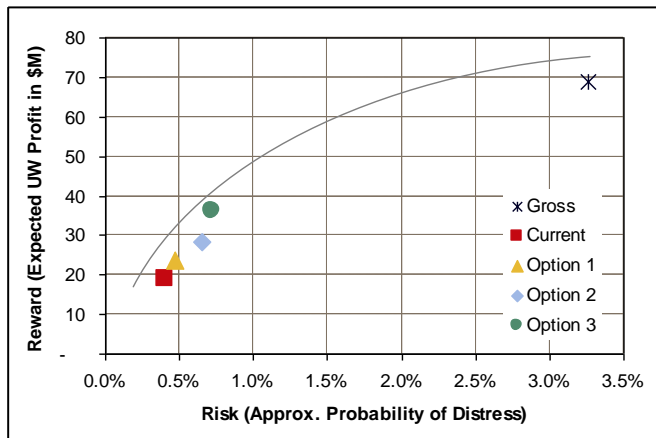
Review of efficient frontier comparisons

An efficient frontier chart compares risk and reward. Various risk measures are available from the table of simulation results: we might consider the probability of distress as well as the VaR and TVaR at different return thresholds. For “reward” we will use net retained underwriting profit.

Comparison of Risk and Reward Measures

	<u>Gross</u>	<u>Current</u>	<u>Option 1</u>	<u>Option 2</u>	<u>Option 3</u>
Interp. Probability of Distress	3.26%	0.41%	0.47%	0.66%	0.72%
1-in-100 VaR	300.3	56.0	63.0	71.7	85.9
1-in-250 VaR	532.4	101.2	113.7	131.8	132.0
1-in-100 TVaR	566.3	187.4	196.7	209.3	217.9
Expected Net UW Profit (\$M)	68.9	19.0	23.8	28.4	36.3

It may be useful to examine the efficient frontiers graphically. In each case, we show risk increasing along the horizontal axis and reward increasing along the vertical axis, so in each case the northwest corner of the graph is the most desirable region (highest reward, lowest risk).



The 1-in-250 VaR risk metric indicates that Option 2 is inefficient compared to Option 3, but this is less clear using the other metrics. And none of these comparisons enables us to choose between Current or Option 1 (less risk, less reward) vs. Option 3 (more risk, more reward).

Cost of allocated risk capital

An allocated risk capital methodology can assist in this choice. We observe that the held capital of \$200M is approximately twice the 1-in-250 VaR and roughly equal to the 1-in-100 TVaR under the current reinsurance structure. We can use these as benchmarks for required risk capital, and calculate the cost of risk capital at (say) 10%.

Comparison of Allocated Risk Capital Costs to Net Cost of Reinsurance

	<u>Gross</u>	<u>Current</u>	<u>Option 1</u>	<u>Option 2</u>	<u>Option 3</u>
Net cost of Reinsurance	-	49.9	45.1	40.5	32.6
Risk Capital Estimate A: 2 x (1-in-250 VaR)	1,064.9	202.3	227.4	263.6	263.9
Capital Cost at 10%	106.5	20.2	22.7	26.4	26.4
Savings in Capital Cost	-	86.3	83.7	80.1	80.1
(Savings in Cap. Cost) – (Net Cost of RI)	-	36.4	38.7	39.7	47.5
Risk Capital Estimate B: 1-in-100 TVaR	566.3	187.4	196.7	209.3	217.9
Capital Cost at 10%	56.6	18.7	19.7	20.9	21.8
Savings in Capital Cost	-	37.9	37.0	35.7	34.8
(Savings in Cap. Cost) – (Net Cost of RI)	-	(12.0)	(8.1)	(4.8)	2.2

Using the first estimate of risk capital, twice the 1-in-250 VaR, any of the reinsurance options offers a savings in capital costs that exceeds the net cost of reinsurance: in other words, using this capital measure any of the purchases is a good buy. The current structure offers somewhat less benefit than Option 1 or Option 2, but Option 3 is superior to all.

Using the second estimate of risk capital, the 1-in-100 TVaR, only Option 3 offers a positive benefit. This is reassuring in that Option 3 is preferred using both methods, but unsettling in that the TVaR capital metric views the other three possibilities as destructive to value while the VaR metric shows them as additive to value.

This leaves us in doubt about the “right” way to allocate capital. Twice the 1-in-250 VaR is in line with some regulatory and rating agency targets. However, this metric disregards the extreme tail of the distribution as well as events with return periods more remote than 1-in-250. The TVaR

method, which at least reflects the shape of the tail of the probability distribution, still requires selection of an arbitrary return threshold. There are many more possibilities for capital allocation in the ERM literature, but these go beyond the scope of this note.

Applying a simple model of firm value

The best way to test the value added by any of the reinsurance strategies is to apply a model of firm value that reflects risk effects. Here we will use a highly simplified version of the method outlined in Panning (2006), and define the firm's value as the risk-adjusted present value of all future earnings. The risk adjustment is implemented by assuming that going into distress (sustaining a net underwriting loss of \$100M or more) is fatal to the firm and eliminates all earnings from that point forward. For simplicity, we assume no growth or change in the portfolio; we further assume that at year's end, any profits are released as a dividend to shareholders and any depletion of capital is replenished at no cost by the shareholders. This means that if the firm does not become distressed, the financial results for each subsequent year are identically distributed. Denote

E = expected annual earnings

d = probability of distress

r = risk-free interest rate

$D = (1 - d) / (1 + r)$ = one-year discount factor

The discount factor D reflects both time value of money and probability of distress. Now the value of the firm can be expressed as

$$\begin{aligned} V &= ED + ED^2 + ED^3 + \dots \\ &= ED (1 + D + D^2 + \dots) \\ &= E \times [D / (1 - D)] \end{aligned}$$

In this framework we call $M = D/(1 - D)$ the perpetuity value multiplier, so $V = E \times M$.

We are now in a position to calculate the value of the firm under each of the different reinsurance strategies. The figures below assume a risk-free interest rate of 1.5% and a 3% investment rate of return. It is assumed that reinsurance premium and expenses are paid at the beginning of the year and losses at the end of the year, so that assets available for investment at the beginning of the

year are the surplus of \$200M, plus retained premium, minus expenses.

Simplified Firm Value Calculation

	<u>Gross</u>	<u>Current</u>	<u>Option 1</u>	<u>Option 2</u>	<u>Option 3</u>
(1) Expected UW Profit	68.9	19.0	23.8	28.4	36.3
(2) Invested Assets = Surplus + Retained Prem – Expenses	287.5	226.7	232.4	238.0	247.3
(3) = (2) × 3.0%	8.6	6.8	7.0	7.1	7.4
Expected Investment Income					
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E = (1) + (3) = Total Earnings	77.5	25.8	30.8	35.5	43.7
d = Probability of Distress	3.26%	0.41%	0.47%	0.66%	0.72%
D = (1 – d) / (1+r)	0.953	0.981	0.981	0.979	0.978
M = D/(1 – D)	20.335	52.250	50.546	46.091	44.772
<hr/>					
V = E × M	1575.9	1348.8	1552.1	1638.1	1956.1
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Benefit to Value	-	(227.1)	(23.8)	62.2	380.2

This model indicates that Option 3 enhances the total value of the firm significantly more than Option 2, while the current structure and Option 1 reduce the value of the firm by transferring too much risk. In other words, the model confirms that Option 3 is more efficient than Option 2 – and also reveals that the firm should position itself further towards the high risk / high reward portion of the efficient frontier.

In this example, the order of the reinsurance alternatives from most to least beneficial is the same whether done using the simplified firm value model, or the 1-in-250 VaR or 1-in-100 TVaR calculations with 10% cost of capital. However, this need not be the case. Replacing the 10% cost of capital with an extremely high value, for example, can render the less risky strategies relatively more attractive under the VaR / TVaR criteria. On the other hand, one might question whether the relatively low risk free rate and the rate of investment return used in the firm value model could be consistent with a much higher cost of capital – although the unusual market conditions prevailing after the 2008 financial crisis suggest this is at least possible. Clearly, the input values deserve careful consideration.

6. Final remarks

Understanding of the value of risk reduction, including risk transfer, has advanced considerably

in recent years. For firms in general and insurers in particular, there are clear reasons why risk reduction can add value. Key issues are avoiding financial distress and the need to re-capitalize. Quantification of the value of risk transfer in particular situations can be done in various ad-hoc ways, but the science is still under development and industry practice varies widely.

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