D&O Reinsurance Pricing - A Financial Market Approach

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Abstract
The large number of high severity D&O losses of the past few years has affected the D&O market place creating a serious capacity crunch. The pricing of this line of business has increased dramatically while restricting coverage. This paper will present an objective methodology based on financial market theory to quantify the risk of writing a large D&O reinsurance portfolio. The authors propose that the analysis of the strong correlation between D&O class action law suits and the financial performance of companies is the most critical element in evaluating a D&O portfolio for reinsurance coverage. In addition, the authors will present mechanisms of risk transfer to capital markets based on this new methodology to obtain additional capacity.

Keywords. Class Action Law Suits, Copula, Correlation, Credit Ratings, Credit Spreads, D&O Pricing, Merton Model, Reinsurance, Securities Litigation, Stock Volatility

1. INTRODUCTION

The goal of this paper is to propose an objective pricing methodology based on financial market theory to quantify the risk of writing a public D&O reinsurance portfolio. This paper is not designed to provide a final solution to the very complex problem of underwriting D&O reinsurance. The authors wish to initiate a paradigm shift in the thought process on how to price and structure D&O reinsurance portfolios. It is our belief that the D&O reinsurance must be thought of more as a financial product rather than as an insurance product. The most critical risk that is managed by a D&O policy is the effect of a company's financial performance on its Directors & Officers as well as its shareholders. Therefore, the risk quantification must bear elements of financial risk analysis. In addition, the authors argue that the financial markets represent a natural capacity provider for this cover as long as the risk is quantified in a manner acceptable to financial markets.

The traditional pricing of D&O, both primary and reinsurance, has been largely unsuccessful and at least partly responsible (along with poor risk selection and generous terms & conditions) for the current crisis in the D&O industry. A timely analysis by the Willis Re Professional Liability group in November 2003 indicates that the cumulative cash flow for the industry since 1994 is $0.5 billion and is projected to be negative $13.9 billion for the decade once all incurred claims are paid. During the years 2000 – 2004, the D&O

industry was involved in seven of the largest securities class action settlements of all time. They are as follows:

Rank	Corporation	Settlement Amount
1.	Cendant Corporation	$3.5 billion
2.	Citi Bank	$2.65 billion
3.	Lucent	$517 million
4.	Bank of America	$490 million
5.	Waste Management	$457 million
6.	Daimler/Chrysler	$300 million *
7.	Oxford Health	$300 million

* *There is an on-going second lawsuit by a large investor who did not join the class action law suit settlement in 2003. In addition, there is a third law suit by foreign investors who were excluded from the initial class action law suit.*

The future of the D&O industry looks risky and uncertain to many industry veterans. John Keogh, CEO of National Fire Union (a member of the AIG Group), who provided a more alarming view of the future liabilities, stated that the 57 largest outstanding cases have $966 billion in claimed damages (Learning from Litigation – Interview; Advisen Ltd. 2004). A simple 5.0% to 10.0% settlement range on claimed damages and 50% insurability on losses would indicate a cost of $24 to $48 billion dollars for the industry.

The plaintiffs' law firms have consistently applied innovative methods both in the discovery process and in the actual litigation of class action law suits. The material increase in the amount of settlements has given leading law firms more resources to conduct necessary research in order to pursue new ways to litigate. The Securities Class Action Services (SCAS), a subsidiary of Institutional Shareholder Services (ISS) published the rankings of top plaintiffs' law firms based on securities class actions settlements occurring in 2003. The settlement amounts for the top 7 law firms are as follows:

Rank	Law Firm	Settlement Amount
1.	Milberg Weiss Bershad Hynes & Lerach	$2.1 billion
2.	Bernstein Litowitz Berger & Grossman	$950 million
3.	Grant & Eisenhofer	$611 million
4.	Goodkind Labaton Rudoff & Sucharow	$551 million
5.	Barrack Rodos & Bacine	$390 million

6.	Entwistle & Cappucci	$311 million
7.	Chitwood & Harley	$303 million

The two major regulatory reforms in recent history have not had much effect on the industry. The long term effect of the Private Securities Litigation Reform Act (PSLRA) has been minimal until now. The involvement of institutional investors as lead plaintiff has markedly increased according to a report by Cornerstone Research published in May 2004. In addition, the settlement amounts have been higher when the lead plaintiff is an institutional investor. A National Economic Research Associates (NERA) trend analysis in 2003 indicates that there is no material change in number of filings since the passage of Sarbanes-Oxley Act (SOX). However, the NERA analysis finds a clear decrease in dismissals of law suits. It is clear that both reinsurers and primary carriers should think outside the box in order to quantify and manage this risk if both groups intend to be profitable in the long run.

2. HISTORY AND CURRENT STATUS OF D&O INSURANCE AND REINSURANCE

The history of United States D&O insurance dates back to the 1930s when Lloyd's of London was the main, perhaps only provider of the product. In the 1960s several American insurance companies offered D&O insurance. However, for the most part, Lloyd's underwriting guidelines, claims control procedures, and contract wording were used by the entire industry. In the 60s there were two policies for D&O insurance: a policy covering the corporate reimbursement for indemnification to directors and officers (current Side B); and a policy covering the liability of directors and officers that are not reimbursed by the corporation (current Side A). Eventually, the two policies were combined to form the policy we have today with Sides A and B.

D&O insurance had a profitable run in the 1960s and 70s. However, by late 1970s the claim frequency and severity increased dramatically. In addition, rates decreased and additional coverage was offered due to competition from new entrants into the D&O arena. By mid 1980s, the D&O market was in a severe crisis as several primary companies either markedly reduced the limits or entirely eliminated the product line. Meanwhile, many reinsurers either reduced their capacity or completely left the market. The ensuing hard market in the late 1980s produced significant rate increases, coverage reductions and very specific exclusions. The industry became profitable again. Once again new entrants including captive insurers entered the D&O market and brought in much needed capacity to a profitable segment of the insurance industry. Unfortunately by the early 1990s, the market softening started again. The D&O primary and reinsurance rates were reduced and more coverage was offered. The key turning point in the expansion of coverage was the offering of entity coverage (side C) in mid 1990s for the most part without charging any additional premium to cover the additional risk that was being assumed. The aggregate losses began to rise due to increased frequency and severity. The increase in severity was caused mainly by the shareholder claims based on federal securities laws. However, the continued influx of capacity kept the rates low, limits high and coverage terms and conditions generous for an entire decade.

The high profile financial scandals such as Adelphia, Enron, Tyco International and Worldcom in the last few years were a powerful signal to the D&O market that tough market conditions are inevitable. The hard market was evident in the treaty year 2001 reinsurance renewals as reinsurance capacity was not easily available. Today, both primary and reinsurance prices have materially increased, while nearly 50% of the capacity has left the market. It is clear that the pricing compared to the coverage may have reached the hard market of the early 1990s, however, the cash flow of the industry is expected to be on the negative by billions of dollars once current claims are fully paid.

3. CURRENT D&O REINSURANCE PRICING METHODS AND RELATED ISSUES

Traditional actuarial methods provide experience and exposure rating techniques to price excess reinsurance for D&O policies.

<u>Experience Rating</u>

Experience rating compares primary company developed and trended losses to subject premium adjusted to future rates and exposures. The individual losses are trended for inflation and other influences and then distributed by excess layers. The excess loss development factors are applied to layered and summarized losses. Then, the trended and developed losses are divided by adjusted subject premiums. Various averages are computed in order to obtain the final loss cost. None of these steps is trivial especially in the case of

D&O reinsurance where economic and legal environments change materially in a short time period and thus, history is not a reasonable indicator of the future performance of the portfolio. Below is a partial list of disadvantages in using experience rating of this coverage:

- Change in mix of business in the last several years. There is a shift toward higher attachment points and limits, as well as a change in the mix of risks.
- Change in legal environment and claims consciousness. It is difficult to obtain appropriate loss development factors based on historical experience.
- Trend is affected by economic as well as non-economic factors, such as legal environment, and is not readily available.
- The pricing of high excess layers could be subject to the "free cover" problem. If one intends to overcome this problem with curve fitting, tail adequacy is a complex issue that requires special analysis.
- In both improving and deteriorating underwriting environments, the indications based on experience rating show a material lag.

In general, the approach of looking back at the recent history and pricing a volatile and at times catastrophic product line such as D&O is destined for failure. An indication based on historical experience could not project expected loss costs with reasonable accuracy for the reasons outlined above.

Exposure rating

In current D&O exposure rating, industry data is used to obtain severity distributions. The increased limits factors (ILFs) are computed using these distributions. The amount of time required to gather data and develop necessary severity distributions present an inherent lag in the indications developed using traditional exposure rating. For a volatile product line such as D&O, the lag contained in the traditional exposure rating could produce material uncertainties in the indicated premium need. A partial list of disadvantages related to current exposure rating is presented below:

- The fundamental assumption in the exposure rating that the base pricing being adequate, may not be appropriate for D&O at any point in time. The difference in ILFs is applied to the underlying premium to estimate the reinsurance premium amounts. The adequacy of underlying premium has been highly questionable for a long period of time.
- Due to lack of excess loss data, the credibility of industry severity curves is questionable. The current ILFs used by the practitioners may not reflect the string of class action law suits that the industry faced recently.

- Company specifics might be different from industry, requiring the use of detailed classifications such as private vs. public, large vs. small, IT vs. biotech and so on.
- The industrywide data set may not be rich enough to produce ILFs recognizing differences between various sectors.

4. PROPOSED PRICING METHODOLOGY

The true risk that is covered by D&O insurance is financial risk. Today, more than ever, the trigger for this coverage is linked to the financial performance of the entity. Whenever a public company declares bankruptcy, it is a given that there will be at least one law suit against its directors and officers. In addition, a material drop in stock price while the rest of the sector is performing well or re-statement of previously declared income increases the company's probability of being sued, dramatically. There are other reasons such as misstatements on income or growth, regulatory investigations of accounting and other fraud, SEC investigations on improper activities, prior M&A deals and IPO activity that would spur D&O law suits. It is not clear how the passing of Sarbanes-Oxley (SOX) would affect the frequency of law suits. However, the general expectation by the experienced underwriters is that the Sarbanes-Oxley act would cost the industry more money in the long run. Admittedly, directors and officers of a public company could be sued by a competitor, customer or an employee just as well as by shareholders. It is the frequency and the severity of shareholder law suits that are alarming the whole D&O industry. Therefore, the new pricing methodology is based on the premise that the D&O risk must be quantified and managed as a financial risk.

The insurance pricing theory enables actuaries to estimate averages and standard deviations of a portfolio of risks. The governing theory is the law of large numbers. In financial market theory, the main focus is on risk differentiation. The quality of each risk, the expected loss given default, and the dependencies between each element of risk are individually evaluated and quantified. The portfolio analysis in this process is based on the individual evaluations of each risk.

The new methodology uses credit ratings as a base to establish the expected financial performance of a public company. The credit risk represented by the credit ratings is not directly applied in this methodology. Moreover, the expected financial performance of the public entity is further adjusted using credit spreads, volatility of the stock price and credit spread change and other underwriting adjustments.

The following formula presents the basic methodology:

$f(\mathrm{L}) = f(\mathrm{M, F, L, C})$ where

$f(\mathrm{L})$: Distribution of D&O losses

M: Market Cap of the company

F: Frequency of law suits as a function of default rates, credit spreads, volatility of the stock price and/or credit spreads, regulatory investigations, prior M&A or IPO activity, number of shareholders owning 5.0% or more of the outstanding stock

L: Loss as a function of the market cap

C: Correlation within and between sectors

The goal is to apply this formula to a portfolio of risks simultaneously in a simulation environment and model a distribution of D&O losses that is produced by a correlated multi-variate distribution. The authors use Monte-Carlo simulation to produce the necessary loss distribution because a closed form solution to tackle this problem is not yet developed.

<u>Market Capitalization</u>

The exposure base is the most current market capitalization of the company. It can be argued that the limits are a valid exposure base. The authors argue that market capitalization is a reasonable and perhaps a superior selection as the exposure base. The reasons are as follows:

- It is an independent exposure base that is publicly available and easily verifiable
- It is an objective exposure base that is not dependent on the company management (as opposed to ceded limits)
- There is a reasonable and consistent relationship between the market cap and corresponding losses (refer to Appendix A to see the graph based on an internal Willis analysis)

<u>Number of law suits</u>

The base number of law suits is generated using the publicly available credit ratings such as Moody's and S&P. The fundamental assumption is that each default corresponds to a

D&O law suit. It is possible that some defaults may not trigger class action law suits against directors, however, there are other defaults that would trigger both class action law suits and individual investor law suits. In addition, there are other events such as a drop in stock price that would trigger a class action law suit. The authors expect that class action and investor law suits per given portfolio will exceed the number of credit defaults. Thus, the critical idea is to increase the default rates beyond what is represented by credit ratings in order to capture the frequency of law suits that are above and beyond the frequency of defaults.

There are several critical adjustments that are made to the frequency parameter at this juncture:

- The Moody's and S&P credit ratings are adjusted to reflect the credit outlook of each security and the minimum of the adjusted ratings is selected.
- The credit spreads are used to indicate a credit rating for each company based on default rates indicated by the spreads. Each company's credit rating is further down graded if the credit rating indicated by the spreads is lower than the ratings adjusted for the outlook.
- The volatility of the financial performance is measured using two parameters: volatility of the credit spreads and volatility of the stock price. Based on the volatility index a downgrade of the adjusted credit rating may be recommended.
- If the company is under a regulatory investigation the credit rating has to be adjusted downward to reflect the increased frequency of a law suit.
- If there are institutional investors owning more than 5.0% of the outstanding stock, a downward adjustment of the credit rating is recommended.
- If there has been any M&A activity or an Initial Public Offering during the past three years by the company, a downward adjustment of the credit rating is recommended.

The downgraded credit ratings replace the original credit ratings for the companies in the portfolio prior to simulation. A mathematical model based on this approach requires a thorough calibration process to determine the appropriate level of downward adjustments that are necessary to produce an appropriate number of law suits during the simulation process.

<u>Loss as a function of the Market Cap</u>

The historical relationship between losses from law suits and the market capitalizations prior to the law suits were examined using a database containing about 1200 cases. The results are presented in Appendix A (Figure 1). The loss as a percentage of market capitalization seems to decrease at a decreasing rate as the market capitalization increases. It is beyond the scope of this paper to further analyze the shape of this curve. Given sufficient data, it would be a useful exercise to learn at what threshold (if any), the size of loss as a percentage of market capitalization begins to increase. The volatility around these severity numbers is recognized during the simulation by introducing a random distribution. There are several examples of very large settlements (Cendant above $3 billion and Citi Bank at $2.65 billion) that warrant a material variation around average loss severities by market cap during the simulation process.

Correlation within and between sectors

It is clear from the past experience that the D&O law suits are correlated. For example, the explosion of law suits in the IT industry due to IPO laddering or accounting scandals in many high flying public companies make it evident that the correlation among law suits exists and it is material. It is also possible that large law firms do research on industries and sectors as a whole when long term strategies to bring law suits are designed and planned. In our analysis, we have projected material correlation within industry sectors and a nominal amount of correlation between sectors. It is extremely important to recognize the potential for correlated loss events when generating aggregate D&O losses. As stated at the beginning, our goal is to quantify the risk of writing a portfolio of D&O losses as a reinsurer. It is important to know the average loss so that the basic pricing can be completed. However, it is more important to know the variability around the average loss because reinsurance is bought for the most part to control that variability. If correlation assumptions are not included in the analysis, the tail of the loss distribution would not reflect the true nature of this risk (i.e. the tail would be too weak to predict a reasonable range of future expected losses.) In addition, if capital is allocated as a function of the 99^{th} percentile loss of the loss distribution, then the loss distribution must reflect correlated events to truly reflect the size and the probability of a very large aggregate loss. The technical aspects of the building of a correlation matrix are presented in appendix C where the authors attempt to obtain defaults in a correlated multivariate environment.

5. REINSURANCE AND CAPITAL MARKET PRODUCTS

The output from the modeling described in Section 4 is a distribution of D&O losses that could be used to structure and price the portfolio, to understand the effects of aggregations, and to allocate capital based on parameters determined by the analyst and the client. The authors do not intend to discuss issues relating to the parameter risk involving the development of the loss distribution proposed in this paper, but would like to caution practitioners to be aware of its presence in this type of financial modeling.

<u>Quota Share Reinsurance</u>

It is a fairly simple process to model commissions and other expenses once the distribution of losses is determined to a certain level of credibility. The calculation of loss ratios and combined ratios would follow. The developed loss distribution allows the analyst to estimate not only the mean and standard deviation of the D&O portfolio but also gives an opportunity to estimate higher moments (skewness and kurtosis).

If the capital allocation assumptions are agreed upon (for example, requiring capital to cover the unexpected portion of a 1 in 100 year loss), then the return on allocated/indicated capital calculation is a straight-forward process based on the developed loss distribution.

<u>Excess of Loss Reinsurance</u>

The gross distribution developed from simulation should be layered (per name/per account) according to the limits, aggregate limits, retentions and other conditions of the reinsurance contract to obtain the excess of loss distribution. Then, the pricing of the excess layer and the development of risk/reward measures for the reinsurance transaction becomes a straightforward exercise. In addition, the understanding of the aggregate losses within layers is a valuable insight in both pricing and risk management since aggregate limits and reinstatement premiums can be computed in an efficient manner. Exhibit 1 contains a sample D&O portfolio and exhibit 2 contains the output from modeling exercise.

The calculation of return on indicated capital becomes a routine procedure due to the availability of both gross and net loss distributions. It is an interesting exercise to see the changes in loss costs, indicated capital and return on capital as certain names (accounts) are added and subtracted from the portfolio. The allocation of capital to the portfolio based on marginal cost of capital needed to write the risk is a reasonable and appropriate methodology. An efficient portfolio optimization tool that uses the above developed distribution could add tremendous value to the entire process.

Stop Loss Cover (Structured similar to a CDO)

The reinsurance portfolio of the primary D&O carrier should be constructed by offering pre-determined limits such as $25 million up to $100 million to its clients. By limiting the number of accounts in the portfolio to about 200 names, one can construct a portfolio around $10 billion. Then, the portfolio can be structured in tranches (of say, $200 million) to be sold to reinsurers, hedge funds and other investors. The primary carrier should retain, for instance, the first tranche of $200 million thus, increasing the quality of risk in higher tranches. The cost for higher tranches should decrease materially as reinsurers, hedge funds and investors are further removed from the risk of a loss. The analytical method that is outlined in the paper lends to determining the quality of the risk, the variation around the mean and various percentiles for specified tranches. By developing a distribution of aggregate losses in the way proposed in this paper, primary insurance companies will be able to present an objective and an independent methodology to quantify the risk of writing this type of cover.

Potential Future Development: Option Pricing based on the Wang Transform

The "Wang Transform" introduced by Shaun Wang (2002) can be applied here using the probabilities derived from the aggregate distribution and estimating the "Market Price of Risk" (a.k.a. Sharpe Ratio (λ)) based on the underlying market data of the companies in the portfolio. One clear difference in this methodology compared to Shawn Wang's methodology for insurance risks is that one can compute the Market Price of Risk based on the underlying data by following the approach proposed in this paper. This is a material advantage of treating D&O as a financial product as opposed to an insurance product.

6. CONCLUSION

We have presented an objective and independent methodology to quantify the risk of writing a large public D&O reinsurance portfolio. This is a starting point rather than a final solution to a very complex problem. It is our sincere hope that with our work, we have started a paradigm shift in the thought process on how to assess risk vs. reward in writing D&O reinsurance. Please note that any model, however sophisticated, will not replace good old fashioned underwriting required prior to and during the risk selection process. In the final analysis, this methodology and future reinsurance pricing models based on the methodology must be viewed as tools designed to enhance the total underwriting process.

Appendix A: Model and Related Adjustments

Our D&O model consists of 2 main parts.

1. The adjustments to the initial credit ratings (presented in Section 4)
2. The simulation engine containing final ratings, severity curves and the correlation matrix

Comments on adjustments to credit ratings:

- The adjustment based on the credit spreads is predicated on the formula derived from the "Reduced Form Approach" by Lubochinsky (2002). The formula is as follows:

  Spread (S) = $d*(1 – R)*(1+r)/[1- d*(1 – R)]$, where d: indicated default rate; R: recovery rate; r: risk free rate

The value for d represents the new adjusted credit rating for the security.

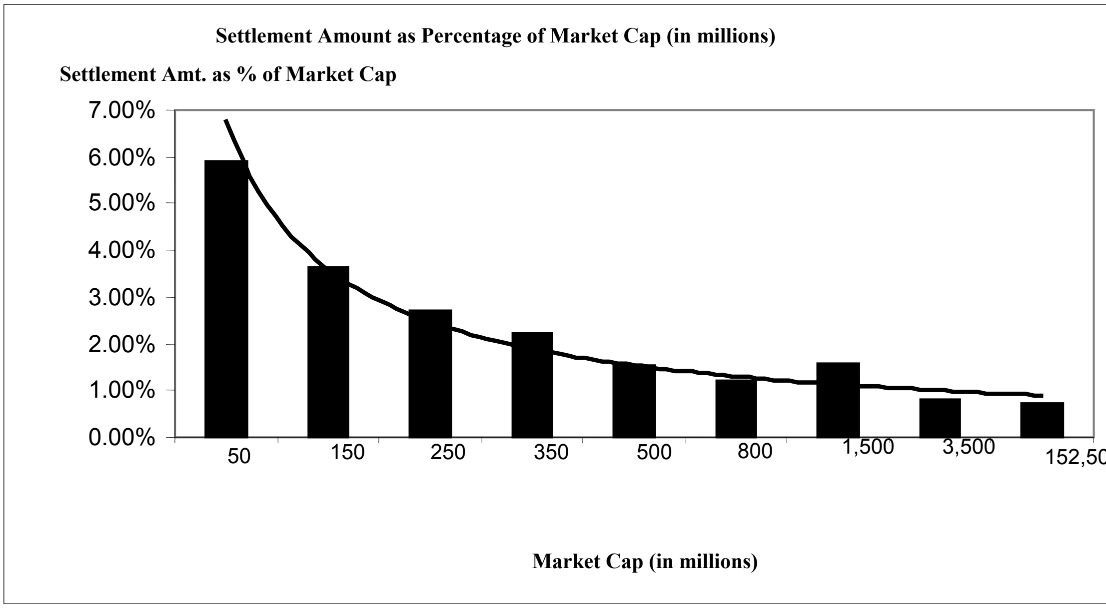
- The most difficult adjustment is to determine the necessary down grades based on stock and credit spread volatility. There are many securities litigation suits that are based on sudden drops in stock price, income and growth not matching the stated stable numbers predicted by the management and re-statement of income due to poor financial performance as well as outright fraud. Therefore, it is extremely important to capture the volatility of the stock price as a predictor of future law suits.

    1. Compare the β (volatility of the stock) to the β for the industry sector. For example, the volatility of a technology company stock should be measured against the rest of the technology sector not against the general market. The model contains confidence interval that set the downgrades to one, two or three notches.
    2. The volatility of the credit spreads is compared to the average movement of the spreads for the industry. The proprietary confidence intervals determine the extent of the downgrades to the previously adjusted credit ratings.

The maximum of the two sets of downgrades is selected as the adjustment for this step.

The chart below (courtesy of Thomson Financial) presents a comparison of the movement of IBM's stock price against industry indices.

Willis Analytics Figure 1



Appendix B: Credit Spreads

The credit ratings published by the ratings agencies provide a measure of financial health of a public company. However as demonstrated in cases such as Enron, there is a certain time interval between change of the company's financial situation and change of its credit rating. Credit spreads (the difference between yields on risk free bonds and corporate bonds) are available instantaneously and reflect bad news well ahead of credit ratings. According to Schonbucher (2003), credit spreads contain the market's opinion on the default risk of the obligor. They provide an objective, market based early warning instrument for changes in the default risk of the obligors. Thus, credit spreads, though volatile, provide a more timely measure of a company's debt paying ability, hence, financial health. There is an intense discussion in current literature regarding the kind of information that is embedded in spreads. Customarily, value of a spread is expressed as a sum of default spread and residual spread, (Lubochinsky (2002)). The first component, default spread, is a direct measure of risk of default. According to the "reduced form" approach (Lubochinsky (2002)), the default spread is proportional to the risk of default without recovery. However, default spread may not always be the main component of the spread. For example, it is shown that for AAA entities only 5% of credit spreads is attributable to risk of default (see Delianedes, Geske (2001)). The residual spread is influenced by taxes, jumps, liquidity, and market risk factors.

Appendix C: Correlation

A simple definition of linear correlation:

Correlation is the degree to which two or more quantities are linearly associated. In a two-dimensional plot, the degree of correlation between the values on the two axes is quantified by the so-called correlation coefficient.

According to Li (1999): the linear correlation of default for two securities *i* and *j*, ρ_{ij} satisfies the following equation

$$\rho_{ij} = \frac{Cov(i,j)}{\sqrt{Var(i)\cdot Var(j)}} = \frac{Cov(i,j)}{\sqrt{u_i(1-u_i)u_j(1-u_j)}} \qquad (1),$$

where u_i, u_j are corresponding default probabilities. The approach the authors used to incorporate correlation into the simulation engine is described below. Any attempt to simulate credit events without giving appropriate regard to the effects of correlation would severely underestimate the tail of the distribution.

How to build a correlation matrix for simulation

It is necessary to compute $Cov(i,j)$ using the within and between correlations assumptions determined at the outset of the analysis. The authors use Merton's approach to calculate $Cov(i,j)$. The Merton approach to the firm's value suggests that a default occurs when the value of assets is below certain threshold (Merton (1974)). In other words, default takes place when a random variable representing firm's assets X_i (with CDF $P(X_i)$) is below a certain level. Two companies are in default if $X_i < P^{-1}(u_i),\ X_j < P^{-1}(u_j)$. Then the covariance equals to

$$P(P^{-1}(u_i), P^{-1}(u_j)) - u_i u_j$$

In order to generate credit events (defaults), Monte Carlo simulation is applied. A set of independent normal random variables are transformed to a correlated standard normal random variables by introducing a correlation matrix to the process. The correlated standard normal random variables are compared to the thresholds based on default rates. The correlation matrix needs to be decomposed based on the Cholesky decomposition prior to creating a matrix of correlated standard random variables. There are two key adjustments that are necessary to obtain a reasonable set of outcomes. They are outlined below.

<u>The Merton Adjustment</u>

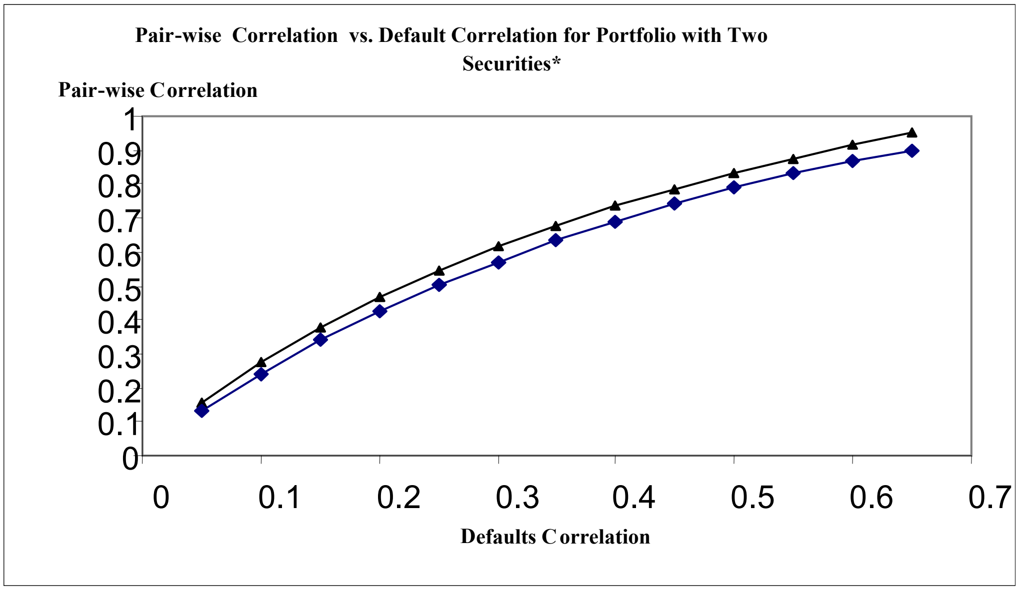
The most straightforward approach in the calculation of a copula in the above equation is to assume that X_i, X_j are normally distributed. Then, one obtains for coefficient of correlation (Pugachevsky (2002))

$$\rho_{ij} = \frac{N^{(2)}(N^{-1}(u_i), N^{-1}(u_j), \rho_{ij}^M) - u_i u_j}{\sqrt{u_i(1-u_i)u_j(1-u_j)}} \qquad (2).$$

In equation (2), $N^{(2)}$ is the cumulative bivariate normal distribution function with pair-wise correlation coefficient ρ_{ij}^M and $N^{(-1)}$ is the inverse of standard normal distribution. The matrix ρ_{ij}^M is determined numerically from Eq.(2) and is used in the loss simulation. Figure 2 contains graphs of ρ_{ij}^M for two arbitrarily selected probabilities of default as a function of events correlation ρ_{ij}. After pair-wise correlation coefficients are computed, the simulation engine can produce a correlated multi-variate distribution. According to Pugachevsky (2002), the main advantage of this method is that it is easy to define correlations between random variables in a simulation environment. The fact that this method does not project the time of default is a weakness in general, however, it is not a major issue for the D&O reinsurance pricing methodology that the authors propose. Schonbucher (2003) presents a model using a generalization of the Archimedean Copula as opposed to the Normal copula that is applied in this model to capture the timing of default.

It is evident from Fig. 2 that the resulting correlation ρ_{ij}^M is materially higher than the discrete events correlation ρ_{ij}. It should be noted that the use of events correlation ρ_{ij} in simulation would lead to substantial underestimation of the correlation effect.

Willis Analytics Figure 2



*Default probabilities are (0.1, 0.05) and (0.05, 0.05) for upper and lower lines, respectively.

How to make the correlation matrix positive-definite

The resulting correlation matrix obtained from Eq. (2) is not necessarily positive-definite. The positive-definiteness is a requirement that guarantees the ability to decompose the correlation matrix after the application of Merton adjustment. There are several known techniques that would help transform the correlation matrix into a positive definite matrix. The authors chose the approach suggested by Rebonato and Jackel (1999) to revise the matrix ρ_{ij}^{M}. The adjustment procedure involves three steps. First, eigenvalues and eigenvectors of the matrix of pair-wise correlations Σ^2 are defined,

$$\Sigma^2 S = \Lambda S ,$$

where Λ, S are matrices of eigenvalues and eigenvectors, respectively. Second, zero or negative eigenvalues are replaced by very small positive numbers. Third step involves the production of the correlation matrix using modified eigenvalues $\lambda^{'}$ and eigenvectors of initial correlation matrix. Taking into account that diagonal elements of the correlation matrix have to be equal to one, the resulting modified matrix equals

$$\sqrt{T} S \Lambda^{'} S^{T} \sqrt{T'}$$

where the matrix T is required for the normalization.

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Glossary

Call Option An option contract giving the owner the right (but not the obligation) to buy a specified amount of an underlying security at a specified price within a specified time.

Collateralized Debt Obligation (CDO) is an investment-grade security backed by a pool of bonds, loans, and other assets. Investors bear the credit risk of the collateral. Multiple tranches of securities are issued by the CDO, offering investors various maturity and credit risk characteristics

Copula A function that joins univariate distribution functions to form multivariate distribution functions.

Credit Risk is the risk due to uncertainty in a counterparty's (also called an obligor or credit's) ability to meet it's obligations. Because there are many types of counterparties, from individuals to sovereign governments and many different types of obligations, from auto loans to derivatives transactions, credit risk takes many forms

Credit Spread for a bond equals to difference between yield on a risky bond and yield on a default-free government bond with a similar maturity

Market Capitalization (Market Cap) is the total dollar value of all outstanding shares

Private Securities Litigation Reform Act (PSLRA) was enacted by the Congress in 1995 to discourage "meritless" securities class action litigation. The Act introduced a "Hightened Pleading Standard" requiring plaintiff to "state with particularity the facts giving rise to a strong inference that the defendant acted with the required state of mind". Automatic Stay of Discovery provides that discovery is stayed when a defendant files a dispositive motion to dismiss in a securities fraud claim. That allows defendant not to produce any documents that trial lawyers have demanded while the court decides the motion to dismiss

Recovery Rate In the event of a default, the recovery rate is the fraction of the exposure that may be recovered through bankruptcy proceedings or some other form of settlement

Sarbanes-Oxley (SOX) Act was passed by Congress in 2002 to "protect investors by improving the accuracy and reliability of corporate disclosures made pursuant to the securities laws, and for other purposes"

Sharpe Ratio (Market Price of Risk) is the difference between the return on a security and the return on a benchmark portfolio divided by the standard deviation of the return on the security; differential return per unit of risk

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Willis Analytics **Exhibit 1**
D&O Reinsurance Pricing

Test Portfolio

Index	Account Name	Market Cap	Sector	Original Rating	Adjusted Rating	Loss as % of Mkt Cap	Std. Dev.
1	Company 1	5,615,101,390	6	A2	Baa1	0.73%	5.00%
2	Company 2	1,247,762,880	3	Baa2	Baa3	1.59%	10.00%
3	Company 3	221,642,688	4	B1	B3	2.73%	10.00%
4	Company 4	210,080,000	1	Ba3	B1	2.73%	10.00%
5	Company 5	196,820,000	7	A3	Baa1	3.64%	10.00%
6	Company 6	166,790,000	4	Ba2	B2	3.64%	10.00%
7	Company 7	162,630,000	8	Aaa	Aa1	3.64%	7.00%
8	Company 8	161,460,000	9	Baa1	Baa2	3.64%	9.00%
9	Company 9	156,520,000	10	A3	Baa1	3.64%	10.00%
10	Company 10	149,890,000	11	A3	Baa2	3.64%	15.00%
11	Company 11	148,200,000	2	B1	B2	3.64%	15.00%
12	Company 12	144,560,000	5	B1	B2	3.64%	15.00%
13	Company 13	136,890,000	1	Caa1	Caa3	3.64%	15.00%
14	Company 14	126,620,000	5	Baa3	Ba1	3.64%	15.00%
15	Company 15	112,710,000	12	Baa1	Baa2	3.64%	15.00%
16	Company 16	108,550,000	13	Aaa	Aa1	3.64%	15.00%
17	Company 17	104,910,000	3	Ba1	B3	3.64%	15.00%
18	Company 18	98,930,000	1	Ba2	Ba3	5.91%	15.00%
19	Company 19	95,680,000	4	Ba3	B1	5.91%	15.00%
20	Company 20	93,340,000	3	A1	A3	5.91%	15.00%

Willis Analytics
D&O Reinsurance Pricing

Exhibit 2

Test Portfolio

Reinsurance Terms	LAYER 1	LAYER 2	LAYER 3	LAYER 4
Per Risk Limit	999,999,999,999	2,000,000	3,000,000	15,000,000
Per Risk Attachment	-	-	2,000,000	5,000,000
Aggregate Limit	999,999,999,999	999,999,999,999	9,000,000	30,000,000
Aggregate Deductible	-	-	-	-

Percentiles	LAYER 1 Losses	LAYER 2 Losses	LAYER 3 Losses	LAYER 4 Losses	LAYER 1 Counts	LAYER 2 Counts	LAYER 3 Counts	LAYER 4 Counts
Mean	4,351,446	1,197,609	1,149,676	1,616,123	0.73	0.73	0.49	0.29
Std Dev	11,230,421	2,291,762	2,447,945	4,665,260	1.35	1.35	1.00	0.69
C.V.	258%	191%	213%	289%	186%	186%	203%	238%
Median	0	0	0	0	0	0	0	0
Min	0	0	0	0	0	0	0	0
Max	221,944,867	27,346,732	32,941,040	59,328,614	16	16	12	10
65.0%	974,793	974,793	-	-	3	3	0	0
70.0%	2,578,211	2,000,000	353,564	-	3	3	1	0
75.0%	6,376,881	2,000,000	3,000,000	712,352	4	4	1	1
80.0%	13,006,533	2,000,410	3,000,000	6,921,037	4	4	1	1
85.0%	23,952,844	2,079,845	3,000,000	15,000,000	5	5	1	1
90.0%	45,372,773	2,960,912	3,000,000	15,000,000	6	6	1	1
95.0%	82,476,651	4,007,874	6,000,000	15,000,000	7	7	2	2
96.0%	94,504,038	4,104,277	6,000,000	16,705,200	8	8	2	2
97.0%	110,950,420	4,532,038	6,000,000	21,781,801	8	8	2	2
98.0%	129,181,731	5,729,388	6,000,000	30,000,000	9	9	2	2
99.0%	157,752,527	6,062,463	8,232,189	30,000,000	10	10	3	2