Reinsurance Applications for the RMK Framework

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Abstract

Recent work by Ruhm, Mango and Kreps, known as the RMK Framework, has proven to be a great advance in the theory of risk. The RMK Framework is a way of viewing an allocation problem that focuses on the scenarios of greatest concern and the probability that those scenarios take place. This paper avoids the mathematical details of the model, but instead gives three applications for the RMK Framework, using non-technical language to explain the basic concept.

Keywords. Risk Theory, RMK Framework, Reinsurance

1. INTRODUCTION

Over the last few years, a significant advance has taken place in the theory of risk. The idea has centered around papers by Ruhm/Mango [4], Mango [3] and Kreps [2], and so is becoming known as the RMK Framework.[1]

While these papers have given the underlying theory, widespread acceptance is still slow in coming. The purpose of the present paper is to demonstrate the RMK Framework in a couple of familiar reinsurance applications to illustrate its appeal to the more general audience.

The RMK Framework is not a single method, but rather a framework for viewing the risk/reward problem that gives rise to a family of methods which share consistent mathematical properties. While mathematical elegance and flexibility make RMK very appealing to "technical" actuaries, they actually raise suspicion outside actuarial circles – aren't we once again picking the answer we want and then covering our tracks with complicated formulas?

The surprising answer is that RMK is very much in line with the way insurance management already thinks about its business, and it can be presented in a very transparent fashion.

The key idea is that we concentrate on the scenarios in which the company as a whole could lose money, and then ask which business segments contributed to that loss. This idea will be illustrated using three examples:

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[1] The spark of the idea can be traced back even earlier to Halliwell [1], especially "Appendix E – The Allocation Problem", pages 346-348.

1. Allocation of aggregate stop-loss cost to line of business
2. Allocation of profit commission to policy year (the deficit carry forward problem)
3. Allocation of target profit loads by line of business

The reader seeking a more rigorous mathematical treatment of the RMK Framework is advised to read the original papers. Here we are just illustrating the approach, with the hope that seeing its results in practice will be more convincing than mathematical proofs.

2. EXAMPLE #1: ALLOCATION OF AGGREGATE STOP-LOSS COST TO LINE OF BUSINESS

The first problem that we will review deals with how an insurance company should allocate its ceded premium for reinsurance that applies across multiple lines of business.

In this example, you work for a small insurance company that writes three lines of business. You have purchased reinsurance that protects your overall loss ratio. The reinsurer will cover 20% points of loss ratio in excess of a gross 80% loss ratio (that is, the ceding company will be back on the hook for paying losses above a 100% loss ratio). The cost of this cover is 4% of gross premium.

The profile of the business is as follows:

	Subject Premium	ELR	Coef. of Variation (CV)
Line A	1,250	80.0%	.500
Line B	1,875	80.0%	.500
Line C	2,150	69.8%	1.000
All Lines	5,275	75.8%	.438

We make the additional simplifying assumption that losses for the three lines of business come from independent lognormal distributions, though this is not necessary in practice.

How should the 4% reinsurance charge be allocated to line of business? The simplest approach would be to charge each line of business the same 4%. However, the managers for each line immediately begin arguing about why their line should get less than the 4% charge.

The managers for Lines A and B insist that the charge should be proportional to the variance of their loss distributions, leading to something less than 4% for them. The manager for Line C objects, noting that her ELR is well below the 80% attachment point of the reinsurance, and therefore should be charged less than the other lines.

Who is right? We can answer this question by first posing a different question: What would a scenario look like in which the overall 80% attachment point is pierced – which line(s) of business would have caused it?

We can think of several situations in which the reinsurance would be triggered based on the 80% attachment point being pierced. Obviously, any one line could have an extremely bad year, causing the overall loss ratio to be above 80% even if the other two lines of business were better than expected. There could also be various combinations in which two lines of business were a little worse than expected, but still cause the 80% attachment point to be hit.

As the actuary, we can list out many possible loss scenarios in which the reinsurance is triggered. Further, for each of these scenarios, we can compare each line's actual loss ratio to the 80% attachment point to see how much it contributed to the overall loss. Given a loss distribution for each line of business (and our independence assumption), it is also easy to assign relative probabilities to each of these scenarios. A reasonable allocation scheme will simply be a probability-weighted average of all the scenarios.

This thought process is what we have been calling the "RMK Framework." For ease of illustration, it is best thought of using a simulation model. The steps are as follows:

1. Simulate losses for each line of business.
2. For each line of business, calculate the difference between the actual loss and the 80% attachment point.
3. For all lines combined, calculate the difference between the actual loss and the 80% attachment point. Store this scenario if the answer is positive.
4. Repeat steps 1-3 many times.

5. For each of the scenarios in which overall losses were above 80%, cap the total loss at 20% of the total all-lines premium (this is the reinsurer's limit). Lower the contributions from the individual lines proportionately when the cap applies.[2]

6. Average all of the simulated scenarios.

This procedure is shown on Exhibit 1a. In this example, only twenty scenarios have been generated, though a realistic calculation would require many more simulations.

A great advantage of this method is that we can bring the simulated scenarios back to the line of business managers and defend the allocation by pointing to the scenarios that caused the reinsurance to be triggered.

In fact, we can note several advantages of this way of framing the allocation problem:

- It is easy to explain to the business managers.
- It works directly with a simulation model that may have been created already for other purposes. In fact, if we had created a dependence or correlation structure between the lines of business, the method would still be applied with no changes.
- The answer does not depend on whether two of the lines of business are grouped together or are kept separate.[3]

After discussing Exhibit 1a with company management, a number of refinements or alternatives could be proposed.

One reaction may be that under some scenarios we actually allocate a negative dollar amount to some lines of business. This may in fact be very reasonable, since we are then saying that a "good" line is subsidizing a "bad" line of business; there is no theoretical reason to disallow negatives. However, that may not be acceptable on a practical basis given that it would create potential difficulties in explaining negative ceded premium to external audiences. To illustrate the flexibility in the RMK Framework, we can modify the method so that the charge is allocated in proportion to total loss dollars, eliminating the negative allocations. This is shown on Exhibit 1b.

This flexibility is a strength in viewing RMK as a decision-making framework and not as a

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[2] In each example, the factor that accomplishes this reduction is labeled *L(x)*, in order to be consistent with Kreps' notation for risk measures.

[3] This characteristic is the "additive" in Kreps' "Additive Co-Measures" label.

rigid allocation method.

3. EXAMPLE #2: ALLOCATION OF PROFIT COMMISSION TO POLICY YEAR

For our second example, we assume that you are now a reinsurance actuary pricing an excess-of-loss treaty that includes a profit commission that is calculated on a three-year block. The effective date for the third year is coming up shortly, and you need to know the expected profit commission under the proposed terms. The difficulty is that the first two years are still very immature and, while they appear to be profitable, the results are far from certain. The question is how to estimate the value of this uncertain carry forward of results from prior years.

We are faced with the problem of estimating the overall expected profit commission for the three-year block and then also the allocation problem of assigning the expected commission to the individual policy years.

The profit commission formula is calculated as follows.

$$\text{Profit Commission} = (\text{Reinsurance Premium} - \text{Expense} - \text{Actual Loss}) \cdot \text{Profit\%}$$

where Expense = 20% of Reinsurance Premium
Profit % = 35%

As in the example for the aggregate stop-loss reinsurance program, we begin by simulating a number of loss scenarios. For the profit commission problem, however, we are simulating losses for the same business but for three different policy periods. We could potentially complicate this model by simulating only unpaid losses for the first two years, and also by building in some year-to-year correlation structure. Such complication would not change the way we will be performing the allocation, but it would change the numbers in the scenarios that we examine.

For each of the simulated scenarios, we calculate a profit or loss for each policy year by comparing the actual loss with the available funding premium (reinsurance premium net of the 20% expense allowance). For scenarios in which the three-year block produces a profit, we multiply each year by the 35% profit-sharing amount. For scenarios in which the three-year block does not produce a profit, we do not include a commission payment.

By taking an average over all of the simulated scenarios, we then have an expected profit commission for the three-year block and also the contribution from each of the three policy periods. Exhibit 2 shows the numbers for a sample of simulated values.

4. EXAMPLE #3: ALLOCATION OF TARGET PROFIT LOADS BY LINE OF BUSINESS

Finally, we turn to the application that was the basis for developing the RMK Framework in its original context: the question of setting profit loads for individual lines of business (or product types).

While it is generally acknowledged that profit loads should be based on the risk inherent in the business written, there has not been much of a consensus on how to define that "risk."

From a stockholders perspective, the risk that matters most is the risk that losses will eat into the capital invested in the company (i.e. that capital will be "consumed"). We will therefore begin with this question – in what scenarios do actual losses exceed the pure premiums actually collected, such that our company loses value?

Following the same example used for the basket aggregate application, we will assume that our company writes three lines of business with the expected losses given in Exhibit 3a. We will also add the information that $2 million of capital is invested in the company.[4]

For each loss scenario generated via simulation, we can readily observe how much capital is taken, and which line(s) of business are most responsible for causing the loss. The capital consumed by each line of business is simply the difference between its actual loss and its expected loss (or pure premium) within a given scenario. In cases where the total loss exceeds the available capital, we simply reduce all lines proportionally. In Exhibit 3a, the factor that accomplishes this reduction is labeled the "Riskiness Leverage Ratio" or $L(x)$, following Kreps' notation.

By averaging together all of the simulated scenarios, we can produce an "expected" amount of capital that is consumed. This could alternatively be described as the stockholder's expected downside result. It is reasonable to allocate our target profit loads proportionally to each line's contribution to this amount.

As stated previously, other risk measures can be used as variations within the RMK

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[4] It may be noted that the amount of capital in the company acts in a manner similar to the limit that the reinsurer provided in the stop-loss example. Once this amount is exhausted, the stockholder is no longer responsible for additional loss payments.

Framework. The stockholders may be interested, for example, in minimizing the variance of the company's results; and setting an overall profit load as a percent of this variance. The allocation scheme then simply changes the Riskiness Leverage Ratio, *L(x)*, to be proportional to the difference between actual and expected results for each scenario. Exhibit 3b shows the results with this change. The resulting allocation is equivalent to setting profit loads in proportion to the covariances of losses by line of business.[5]

5. RESULTS AND DISCUSSION

The RMK Framework is a very clear way of addressing an allocation problem. In addition to its useful mathematical properties, the chief advantage is that it allows decision making to take place with the most significant loss scenarios given the closest consideration.

This paper has deliberately been restricted to simplified examples, but the framework can easily be adapted to larger simulation models and to include risks other than nominal value losses. It should also be clear that the RMK Framework does not itself depend on a particular correlation structure among the variables being simulated; it works with the simulated output regardless of the complexity of the model generating the simulations.

All of the examples in this paper have assumed that a simulation model is used to generate the loss scenarios being reviewed. This also does not need to be the case. The same theory can be applied if a finite number of loss scenarios are selected by the business managers, with subjective weights assigned to each scenario.

6. CONCLUSIONS

The Ruhm/Mango/Kreps (RMK) Framework has been shown to be a very useful way of addressing a variety of insurance allocation problems. This paper has not established any new mathematical theory, but has attempted to show that the RMK Framework is intuitive and transparent for use by actuarial and non-actuarial decision makers.

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[5] This is not the only situation in which RMK is equivalent to a covariance allocation. For example, if the losses are modeled using a multivariate normal distribution, then any choice of risk-measure r(x) will equal the covariance allocation. The full theory on necessary conditions for the two methods to produce equivalent results has not yet been worked out.

Exhibit 1a **Allocation of Excess Loss to LOB for a "Basket Aggregate"**

	Total Loss by Line of Business (simulation results)				Loss Compared to Attachment Point (AP)				20.0%		Allocated Excess Loss			
Scenario	Line A	Line B	Line C	Total	Line A	Line B	Line C	Total	excess of 80.0%*	Capped Portion**	Line A	Line B	Line C	Total
Formula:	X_1	X_2	X_3	X	X_1-AP_1	X_2-AP_2	X_3-AP_3	X-AP	r(x)	L(x)		$[X_i-AP_i]\cdot L(x)$		
1	428	1,926	2,254	4,608	-572	426	534	388	**388**	1.000	-572	426	534	388
2	516	773	1,267	2,556	-484	-727	-453	-1,664	**0**	0.000	0	0	0	0
3	1,277	657	532	2,466	277	-843	-1,188	-1,754	**0**	0.000	0	0	0	0
4	1,182	2,525	822	4,529	182	1,025	-898	309	**309**	1.000	182	1,025	-898	309
5	825	781	1,822	3,428	-175	-719	102	-792	**0**	0.000	0	0	0	0
6	1,564	1,075	307	2,945	564	-425	-1,413	-1,275	**0**	0.000	0	0	0	0
7	1,009	904	685	2,598	9	-596	-1,035	-1,622	**0**	0.000	0	0	0	0
8	1,162	1,449	1,699	4,310	162	-51	-21	90	**90**	1.000	162	-51	-21	90
9	513	2,486	1,278	4,277	-487	986	-442	57	**57**	1.000	-487	986	-442	57
10	742	1,444	6,813	8,998	-258	-56	5,093	4,778	**1,055**	0.221	-57	-12	1,124	1,055
11	992	730	945	2,666	-8	-770	-775	-1,554	**0**	0.000	0	0	0	0
12	1,298	756	2,482	4,535	298	-744	762	315	**315**	1.000	298	-744	762	315
13	796	1,597	488	2,882	-204	97	-1,232	-1,338	**0**	0.000	0	0	0	0
14	566	1,648	930	3,144	-434	148	-790	-1,076	**0**	0.000	0	0	0	0
15	647	1,259	674	2,580	-353	-241	-1,046	-1,640	**0**	0.000	0	0	0	0
16	1,242	2,054	4,197	7,494	242	554	2,477	3,274	**1,055**	0.322	78	179	798	1,055
17	1,214	864	433	2,510	214	-636	-1,287	-1,710	**0**	0.000	0	0	0	0
18	916	836	2,134	3,887	-84	-664	414	-333	**0**	0.000	0	0	0	0
19	1,545	679	1,379	3,602	545	-821	-341	-618	**0**	0.000	0	0	0	0
20	382	2,148	1,826	4,356	-618	648	106	136	**136**	1.000	-618	648	106	136
Average	941	1,329	1,648	3,919	-59	-171	-72	-301	**170**		-51	123	98	**170**

* r(x) = MIN[MAX(X-AP,0),20.0%xPremium]
** L(x) = r(x)/[X-AP]

	Line A	Line B	Line C	Total
Premium	1,250	1,875	2,150	5,275
ELR	80.0%	80.0%	69.8%	75.8%
Mean	1,000	1,500	1,500	4,000
Std Dev	500	750	1,500	
C.V.	0.500	0.500	1.000	
LN Mu	6.7961835	7.20164861	6.9666468	
LN Sigma	0.47238073	0.47238073	0.83255461	

Aggregate Limit: 20.0%
Attachment Point (AP): 80.0%

Exhibit 1b **Allocation of Excess Loss to LOB for a "Basket Aggregate"**

Removing Potential for Allocation of Negative Values

	Total Loss by Line of Business (simulation results)				Loss Compared to Attachment Point of 0				20.0% excess of	Percent of Total	Allocated Excess Loss			
Scenario	Line A	Line B	Line C	Total	Line A	Line B	Line C	Total	80.0%*	Loss**	Line A	Line B	Line C	Total
Formula:	X_1	X_2	X_3	X	X_1-0	X_2-0	X_3-0	X-0	r(x)	L(x)		$[X_i$-0] · L(x)		
1	428	1,926	2,254	4,608	428	1,926	2,254	4,608	**388**	8.4%	36	162	190	388
2	516	773	1,267	2,556	516	773	1,267	2,556	**0**	0.0%	0	0	0	0
3	1,277	657	532	2,466	1,277	657	532	2,466	**0**	0.0%	0	0	0	0
4	1,182	2,525	822	4,529	1,182	2,525	822	4,529	**309**	6.8%	81	172	56	309
5	825	781	1,822	3,428	825	781	1,822	3,428	**0**	0.0%	0	0	0	0
6	1,564	1,075	307	2,945	1,564	1,075	307	2,945	**0**	0.0%	0	0	0	0
7	1,009	904	685	2,598	1,009	904	685	2,598	**0**	0.0%	0	0	0	0
8	1,162	1,449	1,699	4,310	1,162	1,449	1,699	4,310	**90**	2.1%	24	30	36	90
9	513	2,486	1,278	4,277	513	2,486	1,278	4,277	**57**	1.3%	7	33	17	57
10	742	1,444	6,813	8,998	742	1,444	6,813	8,998	**1,055**	11.7%	87	169	799	1,055
11	992	730	945	2,666	992	730	945	2,666	**0**	0.0%	0	0	0	0
12	1,298	756	2,482	4,535	1,298	756	2,482	4,535	**315**	6.9%	90	52	172	315
13	796	1,597	488	2,882	796	1,597	488	2,882	**0**	0.0%	0	0	0	0
14	566	1,648	930	3,144	566	1,648	930	3,144	**0**	0.0%	0	0	0	0
15	647	1,259	674	2,580	647	1,259	674	2,580	**0**	0.0%	0	0	0	0
16	1,242	2,054	4,197	7,494	1,242	2,054	4,197	7,494	**1,055**	14.1%	175	289	591	1,055
17	1,214	864	433	2,510	1,214	864	433	2,510	**0**	0.0%	0	0	0	0
18	916	836	2,134	3,887	916	836	2,134	3,887	**0**	0.0%	0	0	0	0
19	1,545	679	1,379	3,602	1,545	679	1,379	3,602	**0**	0.0%	0	0	0	0
20	382	2,148	1,826	4,356	382	2,148	1,826	4,356	**136**	3.1%	12	67	57	136
Average	941	1,329	1,648	3,919	941	1,329	1,648	3,919	**170**		26	49	96	**170**

	Line A	Line B	Line C	Total
Premium	1,250	1,875	2,150	5,275
ELR	80.0%	80.0%	69.8%	75.8%
Mean	1,000	1,500	1,500	4,000
Std Dev	500	750	1,500	
C.V.	0.500	0.500	1.000	
LN Mu	6.7961835	7.20164861	6.9666468	
LN Sigma	0.47238073	0.47238073	0.83255461	

Aggregate Limit: 20.0%
Attachment Point (AP): 80.0%

* r(x) = MIN[MAX(X-80.0%xPremium,0),20.0%xPremium]
** L(x) = r(x)/[X-0]

Exhibit 2 **Allocation of 3-Year-Block Profit Commission to Year**

	Total Loss by Policy Year (simulation results)				Profit or Loss						Allocated Profit Commission			
Scenario	2002	2003	2004	3 Yr Total	2002	2003	2004	3 Yr Total	Profit Comm*	PC %**	2002	2003	2004	3 Yr Total
Formula:	X_1	X_2	X_3	X	P_1-E_1-X_1	P_2-E_2-X_2	P_3-E_3-X_3	P-E-X	r(x)	L(x)		$[P_i$-E_i-$X_i]$ · L(x)		
1	989	845	1,451	3,285	51	299	-192	157	**55**	35.00%	18	105	-67	55
2	1,013	897	1,610	3,519	27	247	-351	-77	**0**	0.00%	0	0	0	0
3	1,987	956	1,641	4,584	-947	188	-382	-1,142	**0**	0.00%	0	0	0	0
4	554	1,096	427	2,076	486	48	832	1,366	**478**	35.00%	170	17	291	478
5	1,719	790	2,150	4,658	-679	354	-891	-1,216	**0**	0.00%	0	0	0	0
6	1,409	524	646	2,579	-369	620	612	864	**302**	35.00%	-129	217	214	302
7	969	881	2,165	4,014	71	263	-906	-572	**0**	0.00%	0	0	0	0
8	611	546	851	2,008	429	598	407	1,434	**502**	35.00%	150	209	143	502
9	660	756	855	2,271	380	388	403	1,171	**410**	35.00%	133	136	141	410
10	1,362	1,109	895	3,366	-322	35	363	76	**27**	35.00%	-113	12	127	27
11	1,552	1,105	1,210	3,866	-512	39	49	-424	**0**	0.00%	0	0	0	0
12	684	1,166	538	2,388	356	-22	721	1,055	**369**	35.00%	124	-8	252	369
13	1,193	1,111	927	3,232	-153	33	331	211	**74**	35.00%	-54	11	116	74
14	1,473	619	899	2,992	-433	525	359	451	**158**	35.00%	-152	184	126	158
15	613	1,081	1,369	3,064	427	63	-111	378	**132**	35.00%	149	22	-39	132
16	625	1,368	1,647	3,640	415	-224	-388	-197	**0**	0.00%	0	0	0	0
17	813	934	648	2,395	227	210	610	1,047	**367**	35.00%	80	73	214	367
18	555	1,326	2,289	4,171	485	-182	-1,031	-728	**0**	0.00%	0	0	0	0
19	882	2,001	561	3,444	158	-857	698	-2	**0**	0.00%	0	0	0	0
20	717	416	650	1,783	323	728	608	1,659	**581**	35.00%	113	255	213	581
Average	1,019	976	1,171	3,167	21	168	87	276	**173**		25	62	87	**173**

* r(x) = MAX(P-E-X,0)x35.0%
** L(x) = r(x)/[P-E-X]

	2002	2003	2004	3 Yr Total
Premium	1,300	1,430	1,573	4,303
ELR	76.9%	74.1%	71.4%	71.2%
Mean Loss	1,000	1,060	1,124	3,062
Std Dev	500	530	562	
C.V.	0.500	0.500	0.500	
LN Mu	6.7961835	6.85445241	6.91272132	
LN Sigma	0.47238073	0.47238073	0.47238073	

Expense %	20.00%
% Returned	35.00%

Exhibit 3a **Capital Consumption**

	Total Loss by Line of Business (simulation results)				Loss Compared to Expected Loss					Riskiness	Capital Consumed			
Scenario	Line A	Line B	Line C	Total	Line A	Line B	Line C	Total	**Capital Consm'd***	Leverage Ratio**	Line A	Line B	Line C	Total
Formula:	X_1	X_2	X_3	X	X_1-μ_1	X_2-μ_2	X_3-μ_3	X-μ	r(x)	L(x)		[X_i-μ_i] · L(x)		
1	682	1,246	1,013	2,941	-318	-254	-487	-1,059	**0**	0.000	0	0	0	0
2	435	1,555	1,641	3,631	-565	55	141	-369	**0**	0.000	0	0	0	0
3	415	2,840	643	3,898	-585	1,340	-857	-102	**0**	0.000	0	0	0	0
4	1,624	813	1,237	3,674	624	-687	-263	-326	**0**	0.000	0	0	0	0
5	443	569	1,125	2,137	-557	-931	-375	-1,863	**0**	0.000	0	0	0	0
6	2,174	1,033	1,760	4,967	1,174	-467	260	967	**967**	1.000	1,174	-467	260	967
7	841	953	1,169	2,963	-159	-547	-331	-1,037	**0**	0.000	0	0	0	0
8	994	1,924	1,191	4,109	-6	424	-309	109	**109**	1.000	-6	424	-309	109
9	705	1,006	2,003	3,714	-295	-494	503	-286	**0**	0.000	0	0	0	0
10	564	1,194	1,374	3,132	-436	-306	-126	-868	**0**	0.000	0	0	0	0
11	524	857	2,007	3,389	-476	-643	507	-611	**0**	0.000	0	0	0	0
12	1,087	1,101	1,952	4,140	87	-399	452	140	**140**	1.000	87	-399	452	140
13	1,310	2,462	3,231	7,002	310	962	1,731	3,002	**2,000**	0.666	206	641	1,153	2,000
14	384	958	854	2,196	-616	-542	-646	-1,804	**0**	0.000	0	0	0	0
15	750	3,653	1,086	5,489	-250	2,153	-414	1,489	**1,489**	1.000	-250	2,153	-414	1,489
16	442	1,883	1,060	3,386	-558	383	-440	-614	**0**	0.000	0	0	0	0
17	1,619	908	754	3,281	619	-592	-746	-719	**0**	0.000	0	0	0	0
18	582	1,982	1,854	4,418	-418	482	354	418	**418**	1.000	-418	482	354	418
19	1,091	1,069	3,851	6,011	91	-431	2,351	2,011	**2,000**	0.995	90	-428	2,338	2,000
20	1,808	2,564	501	4,873	808	1,064	-999	873	**873**	1.000	808	1,064	-999	873
Average	924	1,528	1,515	3,968	-76	28	15	-32	**400**		85	173	142	**400**

* r(x) = MIN[MAX(X-μ,0), 2,000]
** L(x) = r(x)/[X-μ]

	Line A	Line B	Line C	Total
Premium	1,250	1,875	2,150	5,275
ELR	80.0%	80.0%	69.8%	75.8%
Mean (μ_i)	1,000	1,500	1,500	4,000
Std Dev	500	750	1,500	
C.V.	0.500	0.500	1.000	
LN Mu	6.7961835	7.20164861	6.9666468	
LN Sigma	0.47238073	0.47238073	0.83255461	

Available Surplus: 2,000

Exhibit 3b **Capital Consumption (Risk Measure = Variance)**

	Total Loss by Line of Business (simulation results)				Loss Compared to Expected Loss						Allocation of Risk Measure			
Scenario	Line A	Line B	Line C	Total	Line A	Line B	Line C	Total	Risk Measure* (variance)	Riskiness Leverage Ratio**	Line A	Line B	Line C	Total
Formula:	X_1	X_2	X_3	X	$X_1-\mu_1$	$X_2-\mu_2$	$X_3-\mu_3$	$X-\mu$	$r(x)$	$L(x)$		$[X_i-\mu_i]\cdot L(x)$		
1	682	1,246	1,013	2,941	-318	-254	-487	-1,059	**112,097**	-105.9	33,626	26,931	51,540	112,097
2	435	1,555	1,641	3,631	-565	55	141	-369	**13,606**	-36.9	20,825	-2,015	-5,204	13,606
3	415	2,840	643	3,898	-585	1,340	-857	-102	**1,036**	-10.2	5,949	-13,637	8,725	1,036
4	1,624	813	1,237	3,674	624	-687	-263	-326	**10,626**	-32.6	-20,345	22,395	8,576	10,626
5	443	569	1,125	2,137	-557	-931	-375	-1,863	**346,977**	-186.3	103,691	173,470	69,816	346,977
6	2,174	1,033	1,760	4,967	1,174	-467	260	967	**93,599**	96.7	113,617	-45,174	25,156	93,599
7	841	953	1,169	2,963	-159	-547	-331	-1,037	**107,592**	-103.7	16,468	56,768	34,356	107,592
8	994	1,924	1,191	4,109	-6	424	-309	109	**1,187**	10.9	-65	4,616	-3,365	1,187
9	705	1,006	2,003	3,714	-295	-494	503	-286	**8,174**	-28.6	8,431	14,131	-14,388	8,174
10	564	1,194	1,374	3,132	-436	-306	-126	-868	**75,398**	-86.8	37,855	26,605	10,938	75,398
11	524	857	2,007	3,389	-476	-643	507	-611	**37,321**	-61.1	29,056	39,254	-30,989	37,321
12	1,087	1,101	1,952	4,140	87	-399	452	140	**1,970**	14.0	1,227	-5,600	6,343	1,970
13	1,310	2,462	3,231	7,002	310	962	1,731	3,002	**901,251**	300.2	92,958	288,719	519,574	901,251
14	384	958	854	2,196	-616	-542	-646	-1,804	**325,314**	-180.4	111,016	97,826	116,471	325,314
15	750	3,653	1,086	5,489	-250	2,153	-414	1,489	**221,579**	148.9	-37,259	320,471	-61,633	221,579
16	442	1,883	1,060	3,386	-558	383	-440	-614	**37,725**	-61.4	34,250	-23,528	27,004	37,725
17	1,619	908	754	3,281	619	-592	-746	-719	**51,683**	-71.9	-44,503	42,525	53,661	51,683
18	582	1,982	1,854	4,418	-418	482	354	418	**17,452**	41.8	-17,468	20,137	14,782	17,452
19	1,091	1,069	3,851	6,011	91	-431	2,351	2,011	**404,318**	201.1	18,225	-86,618	472,710	404,318
20	1,808	2,564	501	4,873	808	1,064	-999	873	**76,262**	87.3	70,579	92,920	-87,237	76,262
Average	924	1,528	1,515	3,968	-76	28	15	-32	**142,258**		28,907	52,510	60,842	**142,258**
											20.32%	36.91%	42.77%	
Premium	1,250	1,875	2,150	5,275										
ELR	80.0%	80.0%	69.8%	75.8%										
Mean (μ_i)	1,000	1,500	1,500	4,000		Risk as % of Variance:		10.00%						
Std Dev	500	750	1,500											
C.V.	0.500	0.500	1.000											
LN Mu	6.7961835	7.20164861	6.9666468											
LN Sigma	0.47238073	0.47238073	0.83255461											

* r(x) = 10.00%x(X-μ)^2
** L(x) = r(x)/[X-μ]

Note: This is now equivalent to Covariance.

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Supplementary Material

An Excel spreadsheet containing each of the exhibits in this paper is available upon request from the author.

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Abbreviations and notations

RMK, for Ruhm/Mango/Kreps

X_i	random variable for losses in Line or Period *i*
$\mu_i = E[X_i]$	the expected value of X_i
$X = \sum X_i$	random variable for the sum of all loss categories in the portfolio
$r(x)$	a function of the total loss in the portfolio, representing the quantity to be allocated
$L(x)$	"Leverage" ratio: a multiplier ensuring the allocation balances to the correct overall amount

Biography of Author

Dave Clark is Vice President & Actuary with American Re-Insurance. His past papers include "LDF Curve-Fitting and Stochastic Reserving: A Maximum Likelihood Approach", which won the 2003 prize for the Reserves Call Paper; and "Insurance Applications of Bivariate Distributions", co-authored with David Homer, which won the 2004 Dorweiler Prize.