An Improved Method for Experience Rating Reinsurance Treaties using Exposure Rating Techniques

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Abstract

This paper deals with two of the most common disadvantages of standard excess of loss experience rating methods: lack of complete individual claim history and significant changes in the underlying book of business due to shifts in limit profile during the experience period. We develop a methodology to estimate an trend factor by layer of loss based on the unlimited trend factor, the severity distribution and the limit profile. This excess trend factor can then be applied to the nominal losses in the layer, overcoming the problem of having incomplete individual claim detail to exposure rate lower more credible layers. The excess trend is split into its frequency and severity components. We also present a methodology to estimate exposure adjustment factors necessaries to bring the experience of excess layer to the projected limit profile distribution. The impact of shifts in limit profile is also analyzed in terms of its frequency and severity components.
Keywords: Treaty Reinsurance, Experience Rating, Exposure Rating, Excess Trend, Exposure Adjustment.

1 INTRODUCTION

Exposure rating and experience rating are the two most prevalent and widely documented approaches to pricing excess of loss reinsurance contracts. Each method has its own strengths and weaknesses in any given situation, and frequently these methods are used in tandem to price a contract, with the final loss estimate usually being a credibility-weighted average of the two methods. Excess of loss exposure rating relies on a current snapshot of the policies subject to the reinsurance contract. This snapshot will often include some measure of the percentage of policies or premium exposing the reinsurance contract (the "limit profile"), an estimate of the gross losses (i.e. before reinsurance) for the policies exposing the reinsurance contract, and a severity distribution used to allocate the portion of the gross losses to the various layers ceding to the reinsurance contract. Excess of loss experience rating relies on the historical losses of the cedant, adjusted for trend to the current claim cost level and adjusted to the current exposure level. Trending of losses is typically accomplished on a per claim basis, with adjustments made for policy deductibles and limits when this information is available. Exposure adjustments are typically accomplished through on-leveling of premium or through the use of historical exposure information (policy counts, revenue, etc.). There are several drawbacks of relying entirely on the experience method. Two of the most common problems found in practice are incomplete data to experience rate lower layers and significant shifts in the mix of business in the underlying portfolio during the experience period. In this paper we present an improved method based on the mathematics of exposure rating that helps us overcome these two common practical problems. The problem of incomplete claim information is dealt with by calculating trend factors by layer of loss based on the selected unlimited trend factor, the severity distribution and the ceding company's limit profile. The methodology to estimate excess trends had been previously introduced in Miccolis [9] and Lange [5]; however those two articles did not take into account the issue of different policy limits, which is the case with excess of loss treaties. Although most relevant reinsurance pricing articles mentioned that the impact of changes in limit profile should be taken into account when experience rating excess of loss treaties very few has been written about how to quantify these changes. The impact of shifts in limit profile by layer of loss is estimated with the method presented in Section 3.3. The paper is outlined as follows: Section 2 presents a brief summary of the mathematics of exposure and experience rating. Section 3.1 outlines the assumptions and notation used throughout the paper. Section 3.2 presents the method developed to estimate the trend factor by layer of loss given the unlimited trend factor. Section 3.3 presents the methodology to estimate the exposure adjustment by layer of loss due to changes in Limits Profile. Detailed worked examples are presented in Sections 3.2.3 and 3.3.2.

2 BACKGROUND

In this section we present a brief overview of the basics principles of first dollar ratemaking as well as excess of loss experience and exposure rating. Sections 2.1 and 2.2 below present the standard methods for loss trending and premium adjustment used in practice and some of the disadvantages of these methods. Section 3 then presents the proposed approach to overcome the drawbacks of standard loss and premium trending methods.

2.1 Loss Trending

Adjusting losses for underlying trends is in essence making adjustments to the loss experience to reflect changes in the expected cost per claim (severity) and the expected frequency between the experience period and the prospective or future period. In the remainder of this paper we assume trend is related to ground-up severity trend only. Adjustments for ground-up frequency trend can be incorporated in a straight-forward fashion, but are not addressed here. In first dollar ratemaking the prospective period usually refers to the period when rates are going to be in effect whereas in reinsurance pricing it refers to the policy or treaty period. In this paper the prospective period is assumed to be the treaty period. McClenahan [8] presents in details the fundamental principles of adjusting losses for severity and frequency trend for ratemaking purposes. In brief the methodology involves estimating from the data (or from other sources) an unlimited trend or a basic limit trend, calculating the time difference between the average loss date of the prospective period and the average loss date of the experience period and then applying the trend factor to the aggregate losses (unlimited or basic limit losses depending on the situation). In his paper McClenahan [8] also discusses the effect of limits on severity trend and makes the following remark:

"Where severity trend has been analyzed based upon unlimited loss data or loss data including limits higher than the basic level, the resulting indicated severity trend

must be adjusted before it is applied to basic limit losses. Because such adjustment will require knowledge of the underlying size-of-loss distribution, it is generally preferable to use basic limit data in the severity trend".[1]

However there are situations in practice where one needs to analyze total limit losses when each loss is subject to a different limit of liability (quota share treaty, for example). In this case neither a basic limit trend nor an unlimited trend is appropriate. Hence one needs to estimate a total limits trend which is consistent with the unlimited trend and the basic limit trend. As stated by McClenahan [8] the development of such trend requires knowledge of the size-of-loss distribution. In Section 3.2.1 we present a methodology to estimate trend factors for different layers of losses based on an unlimited trend factor and the severity distribution. When experience rating excess of loss reinsurance one also needs to adjust losses for frequency and severity trends in order to project the expected loss cost in the layer. To do so one requires individual losses with policy limit and deductible details. The trend factor is applied to each ground up loss (gross of the deductible), each trended loss is capped at the policy limit and the deductible is netted out. The resulting loss is then applied to the reinsurance layer. However, underlying policy information is frequently not available for each individual claim and therefore the standard per claim trending methodology can lead to misestimation of the loss cost in the layer. In Section 3.2.1 we develop a methodology to estimate an excess of loss trend based on the limit profile of the ceding company. This methodology helps us to overcome the problem of lack of policy detail by claim.

2.2 Premium Trending

The second fundamental aspect of ratemaking is adjusting historical premium for rate changes as well as other factors that affect the average premium per exposure over

[1] McClenahan [8] p.110.

time. The first step to adjust for changes in average premium is to adjust for rate changes. The methodology for bringing premium to current rate levels is presented in detail in McClenahan [8] and therefore will not be repeated here. If rates can be accurately measured the difference between historical on-level premium would be due to other factors such as growth, changes in rating plans or simply increases in exposure. Jones [3] presents in detail the methodology to adjust historical premium for trends other than rate changes that affect the average premium per exposure. In his paper he identifies four changes that affect premium levels:

1. Past rate changes.

2. Past rating plan changes.

3. The existence of rating plans which change the premium level over time.

4. Past and expected future shifts in the mix of business.

The primary focus of the methodology presented in Jones [3] is first dollar ratemaking and it is based on estimating a trend factor net of rate changes and then applying it in a similar fashion as loss trend is applied to aggregate losses. In this paper we focus our attention on the fourth item identified in Jones [3] : Past and expected future shifts in the mix of business. However we do so from the reinsurer's view point. When insurance companies change their mix of business they can do so in many ways. Two of the most important changes in mix of business affecting reinsurers are: changes in policy limit and deductible distribution and changes in line of business. In a soft market insurance companies tend to offer higher limits of liability to remain competitive and to maintain certain target premium levels. In a hard market capacity is reduced and rates increase, hence insureds tend to buy lower limits because either higher limits are no longer available or they are prohibitively expensive relative to the additional cover they provide. This change in limit distribution has a significant impact on both premium and losses, and the impact is usually magnified in excess layers often ceded to reinsurers. Similarly when companies change their core business from low severity high frequency lines to high severity low frequency lines premium levels tend to change and this is also reflected in the loss experience. This change in line of business also has a significant impact when pricing excess of loss reinsurance. If one does not appropriately adjust the experience to reflect the difference between expected mix of business and the historical mix of business the loss cost in the excess layers can be materially misestimated. In Section 3.3 we present a methodology to adjust excess of loss experience for changes in limit profile based on the mathematics of excess of loss exposure rating. The methodology can be extended to quantify changes in mix of business.

2.3 Excess of Loss Pricing: an Overview

Exposure and experience rating are the two most prevalent and widely documented approaches to pricing excess of loss reinsurance contracts. Each method has its own strengths and weaknesses in any given situation, and frequently these methods are used in tandem to price a contract, with the final loss estimate being a credibility-weighted average of the two methods. In this section we present a brief overview of the two methods and we introduce the notation that will be used in the remainder of the paper.

2.3.1 Experience Rating

Experience rating relies on the historical losses of the cedant, adjusted to the prospective claim cost and exposure level. Trending of losses is typically accomplished on a per claim basis, with adjustments made for policy deductibles and limits when this information is available. Exposure adjustments are typically accomplished through on-leveling premium or through the use of historical exposure information (policy count, revenue, etc.). Clark [1] presents a detailed overview of the basics of reinsurance pricing. Below are the basic steps and data requirements to experience rate excess of loss treaties.

1. Individual loss information with policy deductible and limit: each ground up loss (gross of deductible) is trended to the future average date of loss, the deductible is netted out and the loss is capped at the policy limit. The resulting loss is the applied to the reinsurance layer.

2. Losses in the layer are then aggregated by year (treaty year or accident year, depending on the basis of the analysis).

3. Aggregate losses in the layer are then developed to ultimate using excess development factors. In this paper we do not discuss excess layers loss development factors. Pinto and Gogol [11] and Siewert [12] discuss methods for estimating loss development factors for excess layers.

4. Historical subject premium (earned or written) is adjusted for rate changes and for exposure changes to the prospective premium level.

5. The loss cost to the layer by year is calculated by dividing the ultimate trended losses in the layer by the corresponding adjusted premium.

6. An average of the loss cost is taken between appropriate years of experience.

7. The reinsurance rate is developed by loading the loss cost for reinsurer's expenses and profit.

There are several disadvantages to relying entirely on the results of experience rating as outlined above. Some of the problems are related to the availability of the data required to perform the experience rating and some problems are related to the methodology.

Disadvantages of experience rating methods

1. Often reinsurers receive individual loss detail for large losses only, i.e. incurred losses that are greater than a certain value below the attachment point. This

generates the problem of incomplete information for lower layers in particular for older years. For example if we use 8% annual trend and we receive losses greater than \$200,000 since 1995, the smallest trended loss in 2005 terms is $\$200,000 * (1.05)^{10} = \$325,779$. Hence, we could not use this data to experience rate layers with attachments lower than \$325,779. In practice this problem is dealt with by selecting a data limit or threshold and layers that attach below that threshold are not experience rated. This methodology has two disadvantages: all data below the threshold are eliminated from the analysis and one cannot perform experience rating of lower layers (which is more credible as there is more data and results are usually more stable than higher layers). In our example, we would be unable to experience rate a \$200k *xs* \$200k layer using standard loss trending due to the incomplete data. We would also be unaware if losses in that \$200k *xs* \$200k layer were deteriorating in recent years. Figure 1 shows the impact of severity trend on the data limit and the amount of data lost due to having incomplete claim history.

2. Another problem encountered in practice is the lack of policy information for each claim. If this information is not available, applying a trend and not capping at policy limit can significantly overstate the expected loss cost. Similarly if deductible information is not available, applying a trend factor to a loss net of the deductible can significantly understate the expected loss cost.

3. If there have been significant changes in the book of business during the experience period such as expanding or contracting limits, then each experience period is on a different mix basis and therefore it is not appropriate to simply average between years. Furthermore, the projected loss cost would not be a true projection of the expected loss cost in the future period. When experience rating excess of loss layers for a book of business that has experienced significant changes

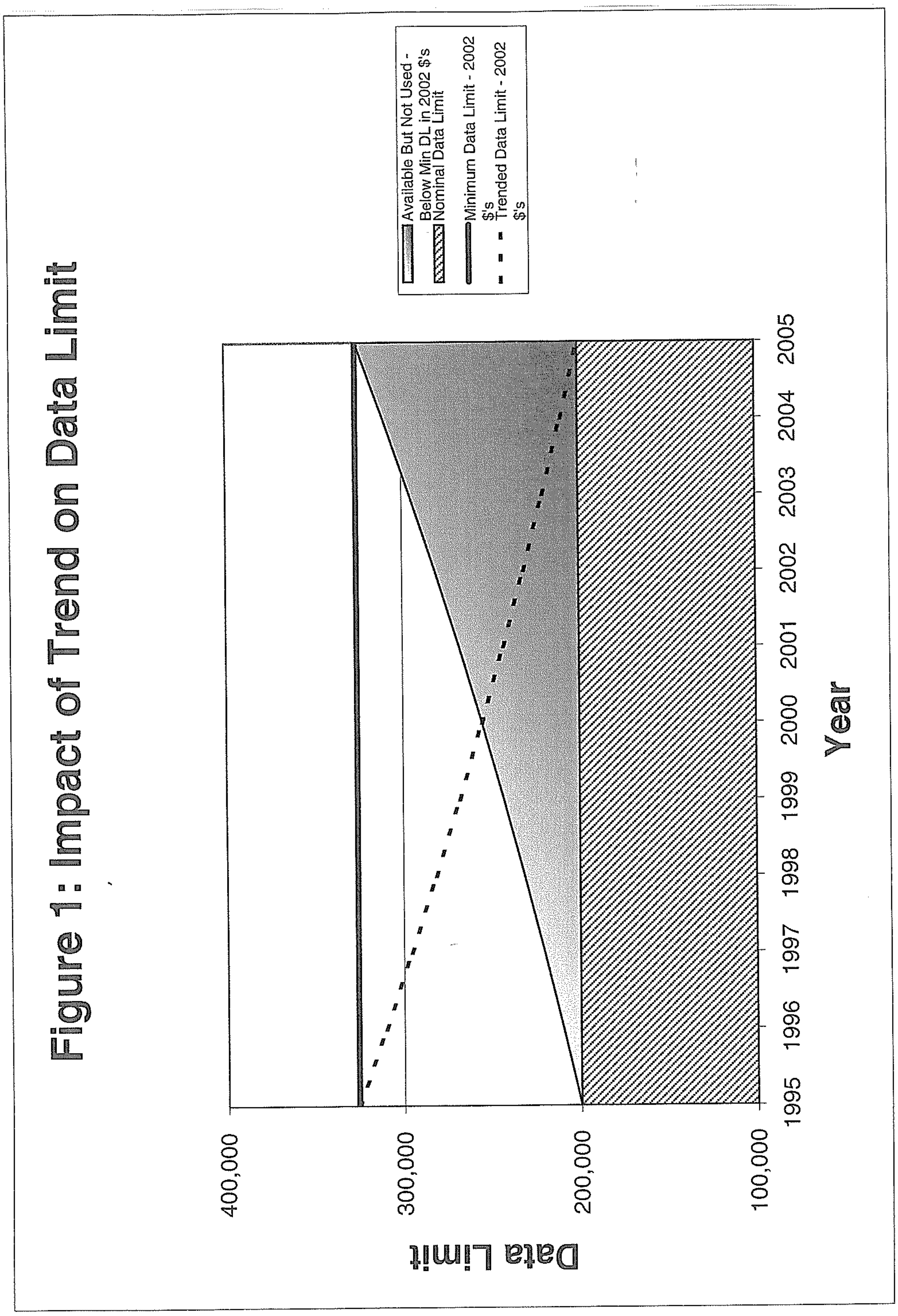

Figure 1 : Impact of Trend on Data Limit
Available But Not Used - Below Min DL in 2002 $'s
Nominal Data Limit
Minimum Data Limit - 2002 $'s
Trended Data Limit - 2002 $'s
Data Limit
400,000
300,000
200,000
100,000
Year
1995
1996
1997
1998
1999
2000
2001
2002
2003
2004
2005

in the mix of business actuaries often load or give credit for these changes in exposure in their final experience rating results. However very little has been written in the literature about quantifying these changes in a more systematic and rigorous way.

In Section 3.2 we present a method that will help us overcome the practical problems in items 1 and 2 above and in Section 3.3 we present a method to help us quantify the impact of changes in limits therefore overcome the problem in item 3 above.

2.3.2 Exposure Rating

The exposure rating method relies on a current snapshot of the policies subject to the reinsurance contract. This snapshot will include some measure of the percentage of policies or premium exposing the reinsurance contract, usually called the "limit profile", and an estimate of the gross losses (i.e. before reinsurance) for such policies. To perform an exposure rating one requires a size of loss distribution (severity distribution), an Increased Limit Factor table or an exposure curve. In the US, many actuaries rely on ISO severity or exposure curves as benchmarks for those lines of business ISO covers. For non-ISO lines of business reinsurers often use ceding company data to fit severity distributions. Outside the US, the use of exposure rating is more cumbersome given the lack of industry benchmark severity distributions by line of business. The objective of the exposure rating method is to estimate the proportion of the loss for the underlying policy that is expected in the excess layer.

Assumptions and Notation:

1. Let X be the random variable representing the ground up cost per claim.

2. Let $f_X(x)$ and $F_X(x)$ denote the probability density function and cumulative density function of X.

3. Assume an underlying policy with limit PL and attachment or deductible D.[2]

[2] In practice deductibles and attachments are not the same and there are many variations on

This is the policy written by the ceding company and we assume the ceding company takes 100% of this policy. In other words there is no coinsurance either with the insured or with other insurers.

4. Assume the expected gross loss ratio for the underlying book is ELR (before reinsurance).

5. The ceding company's projected subject premium for the prospective period is SP. The subject premium is the premium written or earned to which the reinsurance rate applies, e.g. premium net of facultative insurance or net of commissions. It is usually defined in the treaty wording.

6. We denote by α the proportion of the total subject premium that corresponds to policies with limits PL and deductible D. This is often presented in the limit profile.

7. The reinsurance layer is $L\ xs\ A$, i.e. losses for the underlying policy in excess of A subject to a limit L.

The following definition and notation will be widely used in the remainder of the paper.

Definition 2.1 *Let X be a random variable with probability density function $f_X(x)$ and cumulative distribution function $F_X(x)$. The Limited Expected Value of X up to a limit a, i.e.* $\min(X, a)$, *is given by*

$$E[X \wedge a] = \int_0^a x f_X(x)dx + a(1 - F_X(a)) = \int_0^a (1 - F_X(x))dx. \qquad (2.1)$$

See Klugman, Panjer and Willmot [4]. Note that the notation $X \wedge a$ stands for $\min(X, a)$. The exposure rating method can be expressed in terms of Limited Expected Values, Increased Limit Factors or Exposure Curves, see for example Clark [1]

how deductibles and attachments apply. The notation deductible refers to primary business whereas attachment refers to excess business. For simplicity and consistency we will assume in this paper that a policy with limit PL and deductible D covers losses excess of D up to a limit PL.

and Ludwig [7]. The expected loss cost in the layer for a policy with limit PL and deductible D can be expressed as follows:

$$\text{Loss Cost} = (SP)(ELR)\alpha \frac{E_X[X \wedge T] - E_X[X \wedge B]}{E_X[X \wedge PL + D] - E_X[X \wedge D]}, \tag{2.2}$$

where $T = \min(PL + D, L + A + D)$ (i.e. top of the layer, allowing for the fact that some policies may only partially expose the layer) and $B = \min(PL + D, A + D)$ (i.e. the bottom of the layer, allowing for the fact that some policies may not expose the layer in which case $T = B$ and the Loss Cost is 0). The ratio of expected severity in the layer to expected severity for the underlying policy in equation (2.2) is usually referred to as "the % of loss in the layer". To estimate the total loss cost in the layer for the underlying book of business one adds across all combinations of policy limits and deductibles presented in the limit profile of the ceding company.

3 A METHOD FOR IMPROVING EXPERIENCE RATING TECHNIQUES

3.1 Assumptions and Notation

1. Let X denote the ground up cost per claim for the experience period.

2. Let Y denote the ground up cost per claim for the projected or future period. Hence, $Y = rX$, where r is the trend factor necessary to bring experience losses to future claim cost level.

3. We assume that $F_Y(y)$ the distribution function of Y is given. $E_Y[.]$ will denote the expectation calculated using the distribution function of Y.

4. PL and D are the policy limit and deductible for policies written by the ceding company. We assume there are $p = 1, \ldots, P$ combinations of policy limits and deductibles.

5. Let α_p denote the proportion of premium expected to be written for the pth combination of limit and deductible in the experience period. Let β_p be the proportion of premium written for the pth combination of policy and deductible in the prospective period.

6. Let OLP denote the on-level subject premium for the experience period and SP the projected subject premium for the prospective period.

7. Let ELR denote the expected gross loss ratio (i.e. before reinsurance) for the prospective period.

8. Let N_p and M_p be the number of claims for the ceding company for the pth combination of policy limit and deductible with projected subject premium and on-level subject premium respectively.

9. Let $S_{X,p}$ and $S_{Y,p}$ denote the cost per claim for the pth policy type for the experience and prospective period respectively.

10. The reinsurance layer is $L\ xs\ A$.

11. Let $S_{X,p}(A,L)$ and $S_{Y,p}(A,L)$ denote the loss cost per claim in the layer for experience and future period respectively.

12. Let LC denote the projected expected gross loss cost for the ceding company before reinsurance in the future period, and $LC(A,L)$ the projected expected loss cost in the reinsurance layer.

Using the notation and assumptions above it can be seen that

$$\begin{aligned} LC = &\ (SP)(ELR) = \textstyle\sum_{p=1}^{P} E[N_p]E[S_{Y,p}] \\ LC(A,L) = &\ \textstyle\sum_{p=1}^{P} E[N_p]E[S_{Y,p}(A,L)] \end{aligned} \tag{3.1}$$

where the last equality is the standard result of the Collective Risk Model. It then follows that:

(a) The expected cost per claim for the pth policy type is given by

$$E_Y[S_{Y,p}] = E_Y[Y \wedge PL + D] - E_Y[Y \wedge D]. \tag{3.2}$$

(b) The expected number of claims for the pth policy type is given by

$$E[N_p] = \frac{(SP)(ELR)\beta_p}{E_Y[S_{Y,p}]}, \tag{3.3}$$

i.e. the total loss cost for the pth policy type divided by the expected cost per claim.

(c) The expected severity in the layer for the pth policy type is given by

$$E_Y[S_{Y,p}(A,L)] = E_Y[Y \wedge T] - E_Y[Y \wedge B]. \tag{3.4}$$

where T and B are as defined in equation (2.2). Note that this is not the conditional expected severity in the layer (i.e. the severity in the layer given that the loss has exceeded the layer attachment). To calculate the conditional expectation, this quantity needs to be divided by $(1 - F_Y(D + A))$.

Thus we can re-write the exposure rating equation (2.2) as follows:

$$\text{Loss Cost} = E[N_p]E_Y[S_{Y,p}(A,L)], \tag{3.5}$$

and therefore

$$LC(A,L) = \sum_{p=1}^{P} E[N_p]E_Y[S_{Y,p}(A,L)]. \tag{3.6}$$

Expressing the exposure rating method in terms of expected frequency and severity will help us present all the methods below in terms of frequency and severity.

3.2 Aggregate Trend

Miccolis [9] and Lange [5] present detailed discussion on the mathematics of Increased Limits Factors and Excess of Loss pricing. Miccolis [9] presents the mathematical foundations of pricing increased limits based on the basic limit loss cost and then ties the concept of increased limits factors to pricing excess of loss coverage. Lange [5] presents the methodology to estimate increased limits factors given loss experience by policy limit. Although in both articles Miccolis and Lange discuss the effect of trend and inflation in excess layers and how the excess trend can be calculated from the unlimited trend factor and the severity distribution, no numerical examples are presented in their discussions and they do not discuss how the calculation of excess trends can help improve excess of loss experience methods. In this paper we present the methodology for calculating excess trends by layer of loss, both frequency and severity excess trends, and how the resulting excess trend could be used for experience rating excess layers. The following results are stated in Miccolis [9] and will be used in the worked examples below.

Result 1 *Let X be a random variable with cumulative distribution function $F_X(x)$. Define $Y = aX$, where $a \geq 0$. Then the following results hold:*

$$F_Y(y) = F_X(y/a) \qquad \textit{and} \tag{3.7}$$

$$E_Y[Y \wedge y] = aE_X[X \wedge y/a] \tag{3.8}$$

In other words given the distribution of X one can easily deduce the distribution of Y by either re-scaling the distribution of X dividing by a or by trending the parameters of the distribution of X. Most of the commonly used loss distribution functions can be expressed in terms of trended parameters. For example if X follows a lognormal distribution with parameters μ and σ then aX also follows a lognormal distribution with parameters $\mu' = \mu + \ln(a)$ and $\sigma' = \sigma$.

3.2.1 Layer Excess Trend

Actuaries are already familiar with the fact that although a ground up or unlimited trend factor may apply to all sizes of loss the leverage effect of inflation varies greatly by layer of loss and is highly dependent on the attachment or retention. In this section we address the problem of consistency between unlimited trend, basic limits trend and excess trend. The method presented below is not new to the CAS, Miccolis [9] describes the leverage effect of inflation for excess of loss layers, whereas Siewert [12] and Pinto and Gogol [11] have used similar techniques to estimate excess of loss development factors based on ground up loss development factors and a severity distribution. The fundamental idea of adjusting losses for trends is to reflect the change of the average loss cost between the experience period and the future period. Therefore, if one can calculate the expected loss cost for the future period and the average loss cost for the experience period, the loss trend will be given by the ratio of these two figures. The methodology presented below is based on this principle. In the methodology presented below we assume that limit profile and mix of business have remained constant throughout the experience period and will remain constant during the prospective period. Thus we are only adjusting for changes in average loss cost.

Steps to calculate excess trend for the layer $L\ xs\ A$ Using the notation outlined in Section 3.1 the following are the steps to estimate the trend factor applicable to the excess layer.

1. Assume the following items for the prospective period are given or can be estimated: ELR, Limits Profile with $p = 1, \ldots, P$ limit and deductible combinations, expected subject premium SP, the severity distribution for the claim cost Y is given by $F_Y(y)$ and the unlimited trend factor between the experience period and the prospective period is r.

2. Using the standard exposure rating method as described in Section 2.3.2 calculate the expected loss cost in the layer for each policy type. We denote this expected loss cost by LC_p, for $p = 1, \ldots, P$.

3. For each policy type calculate the expected cost per claim in the layer at the

future claim cost level and at the experience period claim cost level. Therefore the change in average cost in the layer per policy type is given by:

$$\text{(Excess Trend)}_p = \frac{E_Y[Y \wedge T] - E_Y[Y \wedge B]}{E_X[X \wedge T] - E_X[X \wedge B]} = \frac{E[S_{Y,p}(A,L)]}{E[S_{X,p}(A,L)]}, \tag{3.9}$$

where T and B are the top and bottom of the layer as defined above. Note that $X = Y/r$ and since the distribution of Y is given the expected values on X can be calculated using the distribution of Y re-scaled as stated in Result 1.

4. Then the total layer trend can be calculated as the weighted average of the excess trend per policy type, where the weights are given by the contribution to the total loss cost of each policy type. In other words the total layer trend is calculated as:

$$\text{Layer Trend} = \frac{\sum_{p=1}^{P}(LC_p)\text{(Excess Trend)}_p}{\sum_{p=1}^{P}(LC_p)} \tag{3.10}$$

As discussed in Miccolis [9] , the leverage effect of inflation in the excess layer is controlled by the attachment. The reason for this is that losses in excess of the attachment will increase for two reasons: smaller losses that on an incurred basis did not reach the attachment can potentially exceed the attachment once the trend factor is applied and losses that exceeded the attachment have most of the inflation effect within the layer. Hence, more losses will trend into the layer and losses in the layer become larger. When pricing excess of loss reinsurance one is usually interested not only in estimating the total loss cost but also in estimating how much of the loss cost is due to frequency and how much is severity. Is frequency in the layer increasing or are claims in the layer getting larger? What is the impact of trend in excess frequency and severity? In the following sections we estimate how much of the excess trend calculated in (3.10) is due to frequency and how much is due to severity.

3.2.2 Excess Frequency and Severity Trend

In order to calculate the excess frequency trend in the layer we need to estimate the change in expected claim count excess of the attachment between the prospective period and the experience period. Note that when we discuss excess frequency trend in this context, we are referring to the increased frequency in an excess layer resulting from ground-up severity trend. Ground-up frequency trends are not specifically addressed in this paper, although they can be incorporated into this framework. The following result will be used to determine the excess frequency trend in the layer.

Result 2 *Let N denote the number of claims in a given portfolio of policies and let $F_Y(y)$ denote the claim size distribution. The expected number of losses that exceed a certain threshold A, $N(A)$, is given by:*

$$E[N(A)] = E[N](1 - F_Y(A)). \tag{3.11}$$

Using this result it can be easily seen that the frequency trend is given by the following equation:

$$\begin{aligned} \text{Excess Frequency Trend} &= \frac{\sum_{p=1}^{P} \frac{E[LC_p]}{E[S_{Y,p}]} \Pr(S_{Y,p} > A)}{\sum_{p=1}^{P} \frac{E[LC_p]}{E[S_{Y,p}]} \Pr(S_{X,p} > A)} \\ &= \frac{\sum_{p=1}^{P} E[N_p](1 - F_Y(A+D)) \mathbf{1}_{\{PL>A\}}}{\sum_{p=1}^{P} E[N_p](1 - F_X(A+D)) \mathbf{1}_{\{PL>A\}}}, \end{aligned} \tag{3.12}$$

where $\mathbf{1}_{\{PL>A\}}$ is the indicator function that takes the value of 1 if a policy exposes the layer. The formula in (3.12) can be interpreted as follows: the expected number of claims in the layer given the loss distribution for the prospective period relative to the expected number of claims in the layer given the loss distribution of the experience period. Note that in equation (3.12) we are assuming the same number of claims for the underlying policy, but more claims will penetrate the layer as severity increases. Note that in the formulas above we have assumed constant ground up frequency trend. However, if ground up frequency is also increasing the assumed frequency trend would naturally flow through the calculations in a multiplicative fashion. Equations (3.10) and (3.12) give us the total excess trend factor and the excess frequency trend factor. To estimate the severity trend factor in the layer we simply divide the total layer trend factor by the excess frequency trend factor. Once we have a trend factor by layer we can apply this factor to the **nominal** losses in the layer. In other words, we take the actual incurred losses in the layer, develop them to ultimate with appropriate development factors and then apply the layer trend factor as developed in this section. There are various advantages of this method:

1. When large losses do not include policy and deductible information one does not need to make assumptions about the underlying policy. The limits profile provides the assumptions about the distribution of underlying policies and applies an average trend for the layer in light of this. One simply takes the nominal aggregate losses in the layer and then applies the layer trend factor.

2. The approach provides consistency between unlimited trend and trend at various layers of loss. This is not only useful for experience rating excess layers, but it also helps quantify the differences in ground-up trend for two books of comparable business with differing limits.

3. Since we are not trending individual losses it is no longer necessary to select a data limit or threshold, hence all data available can be used in the analysis. Since no data are eliminated we could perform an experience rating for lower layers and then use the severity distribution to extrapolate losses to higher layers. This method has the advantage that experience is usually more stable in lower layers than higher layers.

The principal disadvantage of the method is that it is more difficult to explain to a non-technical audience. The typical underwriter may not understand why a loss that has settled for policy limits continues to receive a trend adjustment, which is one of the results of using this approach. This is necessary under this approach, as it counteracts the under-trending of a loss that has barely penetrated the layer attachment.

The following section provides a worked example to illustrate step by step the trending methodology presented above.

3.2.3 Worked Example

All Tables related to this example are presented in the appendix. Assume that a ceding company writes limits between $250,000 and $5,000,000 with projected written premium distribution as in Table 1. We assume in this example that all policies are primary and there are no deductibles. We assume that a reinsurer is interested in pricing an excess of loss treaty for this ceding company for underwriting year 2005. The loss distribution for this line of business for underwriting year 2005 is assumed to follow a lognormal distribution with parameters μ=9.31 and σ=2.29. The annual unlimited trend factor for this line of business is assumed to be 8%. Therefore, the loss distribution for losses in 2000 values is also a lognormal distribution with parameters $\mu' = \mu - \ln(1.08^5) = 8.93$ and $\sigma' = \sigma = 2.29$. In this example we work out the trend factor for various layers to be applied to experience losses for underwriting year 2000. The unlimited trend factor between 2000 and 2005 is given by $1.08^5 = 1.47$. Table 1 presents the limited expected value or expected severity for each policy in 2005 and 2000 loss cost values. As per our notation in Section 3.2.1 we have $E[S_{Y,p}] = E_Y[Y \wedge PL]$ represents the expected severity in 2005 values and $E[S_{X,p}] = E_X[X \wedge PL]$ the severity in 2000 values, since there are no deductibles.

Table 2 shows the results of applying the standard exposure rating method using the limit profile as in Table 1 and the distribution with 2005 parameters and an expected gross loss ratio of 60%. (We have assumed a loss ratio for completeness, though it can be seen that the loss ratio cancels in all the equations in Section 3.2.1 and 3.2.2). The results shown in Table 3.2 were calculated using equation (2.2) and adding across policy limits.

Table 3 shows the excess trend per policy across layers using equation (3.9). For example, the trend factor for the layer \$250,000 xs \$250,000 is zero for policies with limit of \$250,000 as they do not expose the layer, and the trend factor is the same for all other policies as the remaining policies fully exposed the layer. The trend factor in this case is calculated as follows:

$$\frac{E_{2005}[Y \wedge 500,000] - E_{2005}[Y \wedge 250,000]}{E_{2000}[X \wedge 500,000] - E_{2000}[X \wedge 250,000]} = \frac{64,416 - 48,539}{50,191 - 38,900} = 1.248.$$

We also note from Table 3 the trend factor is different for policies that partially expose the layer. We note this in the case of the \$750,000 policy limit in the layer \$500,000 xs \$500,000 and all policies in the \$5MM xs \$0.

The total trend in Table 3 is calculated as the weighted average of the trend factor per policy with the corresponding exposure rating loss cost as given in Table 2. From Table 3 we observe the effect of capping at policy limits. For example, if we had not taken into account the effect of policy limits the trend factor in the first \$5MM would have been 1.389 instead of 1.328. We also observe the leverage effect of loss trend in the various layers, for example the annual trend factor for the layer \$4MM xs \$1MM is 9.59% compared to an unlimited ground up trend factor of 8%. If we were analyzing aggregate gross losses for the ceding company using a trend factor of 8% would be inadequate since we would not be taking into account the fact that the majority of the business is written at lower limits. In this case it would be more adequate to use a trend factor of 5.83% which takes into account the limit profile of the ceding company's portfolio. Table 4 shows the frequency trend in the various layers. The second column of Table 4 shows the expected number of claims by policy limit. These figures were calculated by taking the projected premium times the expected loss ratio and divided by the limited expected value at the corresponding policy limit. The expected number of claims in the layer were calculated using equation (3.11) with the curves in 2005 and 2000 values respectively. The frequency trend is then the ratio of the expected number of claims in the layer using 2005 severity distribution and the expected number of claims in the layer using 2000 severity distribution. It can be seen from Table 4 that although in this example we assumed no ground up frequency trend (hence the frequency trend for primary layers is zero) the majority of the increase in loss cost in excess layers is expected from additional claims trending into the layer. Table 5 shows a summary of the trend factors by layer as well as the frequency and severity trend factors. As discussed above most of the expected increase in loss cost in excess layers is due to an increase in claims in the layer rather than an increase in severity in the layer. Having the excess trend split into frequency and severity has several advantages when pricing excess of loss reinsurance. In practice frequency tends to be more stable than severity, thus often one models frequency from the ceding company's experience and uses severity from the loss distribution and then multiplies the two to estimate the total loss cost. There are several variations of this "mix" method used in practice.

3.3 Adjustment for Changes in Limit Profile

While primary pricing is concerned with total exposure growth, excess of loss reinsurance pricing is concerned with exposure growth by layer. Traditional pricing assumes exposure growth by layer is consistent with total exposure growth: an assumption that frequently fails in a stable operating environment, let alone an environment where limits usage expands or contracts greatly due to significant shifts in underwriting appetite and pricing adequacy. One of the fundamental components of a typical reinsurance submission is the Limits Profile. The Limits Profile will typically show the amount of business the ceding company has written during the latest calendar period segmented by the limits of the primary policies. It may be based on written or earned premium, or it may be based on policy or exposure counts. If there is significant variability in deductibles, or if the book of business is written on an excess basis with varying attachment points, the Limits Profile may also be segmented by deductible or attachment. While the form of the Limits Profile may differ depending on the characteristics of the business, its purpose is the same: to provide the reinsurer with an estimate of the ceding company's limits usage. By comparing Limits Profiles across time periods, the reinsurer can estimate not only current period limits usage, but also the change in limits usage over time. The experience of the ceding company will greatly vary depending on the composition of the book. For example if the maximum limit capacity of a ceding company in year 2000 was \$3MM the maximum loss for any one claim will be \$3MM. If the ceding company has expanded its capacity to a maximum limit of \$5MM for year 2005 and the reinsurer is interested in pricing the layer \$2.5MM xs \$2.5MM then using the experience for 2000 to project the loss cost in the layer for year 2005 will result in an understatement of the the loss cost for the reinsurers as the maximum exposure for the layer in 2000 was \$500,000 whereas for 2005 the layer is fully exposed. In this section we develop a method based on the mathematics of exposure rating that will help us estimate the differences in exposure across layers. A very similar version of the method presented below was presented by Robert Giambo at the CARe seminar in 2004. In his presentation he used the results of the exposure rating method to adjust for changes in limits profile. There are two main differences between the approach presented by Giambo and the approach presented in this paper:

1. Giambo's approach is based on the experience loss cost expressed as a percentage of the historical on-level premium. Hence his adjustment factor did not include an adjustment for the total limits exposure change. In this paper the estimated exposure adjustment factor includes overall exposure change (through the relative change in on-level premium) as well as exposure in the layer due to shifts in limit profile.

2. We take the methodology one step further and we split the exposure adjustment into its frequency and severity component. This split is helpful when one is interested in applying a mixed method where frequency is estimated from the experience and severity is estimated from the exposure method.

We follow the notation outline in Section 3.1.

Steps to estimate exposure adjustment by layer

1. Given rate changes for the ceding company during the experience period calculate on-level factors with standard on-level methodologies. See, for example, McClenahan [8] .

2. Calculate on-level historic limit profile. In absence of additional information we assume that the same rate changes (hence on-level factors) apply to all policy limits.

3. Using the severity distribution with parameters for the <u>prospective period</u> calculate the expected severity by policy limit $E_Y[S_{Y,p}]$.

4. With the results from item 3 above calculate the expected severity in the layer for each policy limit $E_Y[S_{Y,p}(A, L)]$.

5. Using the projected limit profile with premium distribution by policy given by β_p, for $p = 1, \ldots, P$, calculate the exposure rate as follows:

$$(\text{Loss Cost})_{projected} = (SP)(ELR) \sum_{p=1}^{P} \beta_p \frac{E_Y[S_{Y,p}(A, L)]}{E_Y[S_{Y,p}]}. \tag{3.13}$$

6. Using the historic on-level limit profile with premium distribution given by α_p for $p = 1, \ldots, P$ calculate the exposure rate as follows:

$$(\text{Loss Cost})_{historic} = (OLP)(ELR) \sum_{p=1}^{P} \alpha_p \frac{E_Y[S_{Y,p}(A, L)]}{E_Y[S_{Y,p}]}. \tag{3.14}$$

7. The exposure adjustment in the layer $L\ xs\ A$ is given by the ratio of equations (3.13) and (3.14):

$$
\begin{aligned}
\text{Layer Exposure Adjustment} &= \frac{(\text{Loss Cost})_{projected}}{(\text{Loss Cost})_{historic}} \\
&= \frac{(SP)(ELR)}{(OLP)(ELR)} \frac{\sum_{p=1}^{P} \beta_p \frac{E_X[S_{X,p}(A,L)]}{E_X[S_{X,p}]}}{\sum_{p=1}^{P} \alpha_p \frac{E_X[S_{X,p}(A,L)]}{E_X[S_{X,p}]}}.
\end{aligned} \tag{3.15}
$$

Note from equation (3.15) the term $(SP)/(OLP)$ represents the total limits exposure adjustment. This term is then multiplied by the change in exposure in the layer given by changes in limit profile distribution across policies. If the ceding company's premium or exposure distribution by policy type has remained constant the right hand side term of equation (3.15) would equate $(SP)/(OLP)$, i.e. the total limit exposure adjustment. The method presented in this section only takes into account changes in limit profile, it does not reflect changes in average loss cost as we have assumed the severity distribution is in future value terms. The exposure adjustment as presented in equation (3.15) is applied to ultimate trended losses in the layer, where trended losses can either be calculated using the standard per claim trending method or the aggregate excess trend method as presented in Section 3.2.1. The exposure adjustment and the trend factor are multiplicative.

3.3.1 Frequency and Severity Exposure Adjustment

In this section we extend the method of adjusting for changes in limit profile to frequency and severity. In other words we estimate the increase in exposure due to increase in frequency and severity separately. The following steps provide the methodology to estimate the frequency exposure adjustment for a generic layer $L\ xs\ A$.

1. Calculate the expected number of claims by policy type for the prospective period as follows:

$$E[N_p] = \frac{(SP)(ELR)\beta_p}{E_Y[S_{Y,p}]}, \tag{3.16}$$

   i.e. dividing the projected total expected loss cost by policy type by the expected severity for that policy type.

2. Calculate the "as if" expected number of claims by policy type for the experience period as follows:
$$E[M_p] = \frac{(OLP)(ELR)\alpha_p}{E_Y[S_{Y,p}]}, \tag{3.17}$$
i.e. the expected loss cost for the experience period in current rate level by the expected severity by policy type using the severity distribution for the prospective period.

3. Calculate the expected claim count in the layer by multiplying the expected claim count by policy type times the probability of claims penetrating the layer as follows:
$$\begin{aligned} E[N_p(A,L)] = &\quad E[N_p](1 - F_Y(A+D)) \\ E[M_p(A,L)] = &\quad E[M_p](1 - F_Y(A+D)) \end{aligned} \tag{3.18}$$

4. Add the expected claim count in the layer across policies for both the prospective and the experience period:
$$\begin{aligned} (\text{Claim Count})_{projected} = &\quad \textstyle\sum_{p=1}^{P} E[N_p(A,L)] \\ (\text{Claim Count})_{historic} = &\quad \textstyle\sum_{p=1}^{P} E[M_p(A,L)] \end{aligned} \tag{3.19}$$

5. Calculate the frequency exposure adjustment as:
$$\text{Frequency Exposure Adjustment} = \frac{(\text{Claim Count})_{projected}}{(\text{Claim Count})_{historic}}$$

The severity exposure adjustment is then calculated as the ratio between the layer exposure adjustment and the frequency adjustment. The following section presents a detailed worked example showing the exposure adjustment for various layers of loss.

3.3.2 Worked Example

We continue to use the same assumptions as in the example shown in Section 3.2.3. Table 6 shows the ceding company's Limits Profile for business written during underwriting year 2000 and the projected Limits Profile for underwriting year 2005.

The Limits Profile and rate increase information are typically provided by the ceding company as a part of the reinsurance submission. Assume we know that rates for business written in underwriting year 2000 will have increased a cumulative 50% by underwriting year 2005.

In this example, we have assumed that the 50% cumulative rate increase has equally impacted policies written at each limit. On-level written premium for underwriting year 2000 is 50% higher in Table 7 (Historic On-Level Limits Profile) than its counterpart in Table 3.6 (Historic Limits Profile). If rate changes have varied by limits due to a change in Increased Limits Factors, an appropriate differential increase should be applied separately to each limit.

We then calculate the expected losses in the layer using the standard exposure rating method. Table 8 shows the % of loss by layer by policy limit using the severity distribution with 2005 parameter. As in the example in Section 3.2.3 we are using a lognormal with $\mu = 9.31$ and $\sigma = 2.29$. The % of loss in the layer is calculated as the ratio of the expected severity in the layer $E_Y[S_{Y,p}(A, L)]$ and the expected severity for the underlying policy $E_Y[S_{Y,p}]$. Using this percent of loss in layer as a proxy for the exposure in each layer, we then multiply Table 8 by our 2000 limit profile (on-level) and the assumed 60% expected loss ratio to estimate historic exposure by layer. This yields Table 9.

Then we calculate the projected exposure by multiplying the results from Table 8 by the ELR of 60% and by the 2005 Limit Profile from Table 7. The results are shown in Table 10.

Taking the relativity between Table 9 and Table 10 yields the exposure adjustment to be applied to the trended ultimate losses in the layer for experience year 2000. These results are shown in Table 11. Note that the exposure in 2005 for the \$5MM *xs* \$0 layer is 1.09 times the exposure in 2000 for the \$5MM *xs* \$0 layer. As this layer represents total limits (there are no policies in this example greater than \$5 million), this is our total limits change in exposure. It is simply the ratio of on-level total limits premium in 2005 relative to that in 2000, or \$25,875,000 divided by \$23,737,500. This is the total limits change in exposure yielded by standard on-leveling procedures.

It is interesting to note how this 9% increase in total limits exposure differs by layer. Exposure in the \$250,000 *xs* \$0 layer is actually down slightly, while exposure in the \$4MM *xs* \$1MM layer has doubled. Table 12 shows the expected frequency by policy for underwriting years 2000 and 2005 respectively. These results were calculated by taking the total expected loss cost by policy type and dividing it by the expected severity for such policy. Table 13 then shows the expected frequency in the layer. These results are calculated by multiplying the total expected frequency by the probability of a loss penetrating the layer given that the underlying policy exposes the layer.

The severity adjustment is calculated as the ratio of the layer adjustment and the frequency adjustment. Table 14 shows a summary of the exposure adjustment by layer of loss. As we can see from Table 14 there is a significant difference in exposure changes across layers. Standard experience rating method would assume that the total limit exposure change apply consistently across layers. In our example above the total limit exposure changes is 1.09, however we can see that although the total exposure has increased by 9% the exposure in the layer \$4MM *xs* \$1MM has doubled. Hence, if we had adjusted the experience in the \$4MM *xs* \$1MM layer by a factor of 1.09 we would have significantly understated the experience rate. It is also interesting to note from Table 14 the contribution to the exposure change of frequency and severity separately. As we can see most of the exposure change for total limits is due to an increase in severity since frequency has reduced slightly.

4 CONCLUSIONS

We have presented in this paper an improved method for experience rating excess of loss treaties based on the mathematics of exposure rating. The methodology presented in Section 3.2 to estimate a trend factor by layer based on the unlimited trend factor presents several advantages over the standard per claim trending procedures widely used in practice. First, the aggregate trend method is useful when individual claim information does not contain policy details such as limit and deductible. In this case we simply calculate the excess trend factor and apply it to nominal losses in the layer. Second, when reinsurers receive individual losses greater than a certain amount the aggregate trend method is useful as it helps us overcome the problem of having to eliminate all data that falls below a selected threshold. Hence, by using an aggregate trend instead of the per claim trending method we can experience rate lower layers that would have been otherwise impossible to do. Third, the aggregate excess trend methodology brings consistency between unlimited trend factors and trend factors by layer of loss. Finally, the aggregate trend method helps us to quantify the impact of frequency and severity trend in excess layers which is useful for measuring the impact of increased frequency in excess layers even when ground up frequency is stable. The methodology presented in Section 3.3 helps us to quantify the exposure adjustment by layer of loss due to shifts in limit profile for the ceding company. Traditional experience rating methods quantify the exposure change for the underlying book of business through changes in on-level premium or other measure of exposure. However, this method assumes that the total limits exposure change is consistent across layers. Through the example presented in Section 3.3.2 we have seen that using the total limit exposure change across layers can result in a significant misestimation of the expected loss cost. The exposure adjustment method is based on the mathematics of exposure rating, which are widely used in reinsurance pricing, and data that is typically available in a standard reinsurance submission, making the method relatively easy to implement in practice.

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Appendix: Numerical Results

Table 1: Limits Profile and Limited Expected Values

	Underwriting Year	2005	2000
Policy Limit (PL)	Premium (SP)	$E_Y[Y \wedge PL]$	$E_X[X \wedge PL]$
250,000	2,250,000	48,539	38,900
500,000	5,400,000	64,416	50,191
750,000	2,925,000	74,252	56,947
1,000,000	6,300,000	81,301	61,681
5,000,000	9,000,000	117,221	84,401
Total	25,875,000		

[3]The methods set forth in this paper are the opinion of the authors and they do not necessarily represent the views of the ACE Group of Insurance and Reinsurance companies or Carvill and any of its subsidiaries. The worked examples provided in this paper were derived from purely hypothetical assumptions and any similarity between these examples and any existing insurance company are coincidental only.

Table 2: Exposure Rating using 2005 severity distribution

	Layer of Loss				
Policy Limit	\$250k *xs* 0	\$250k *xs* \$250k	\$500k *xs* \$500k	\$4MM *xs* \$1MM	\$5MM *xs* 0
250,000	1,350,000	0	0	0	1,350,000
500,000	2,441,429	798,571	0	0	3,240,000
750,000	1,147,250	375,256	232,495	0	1,755,000
1,000,000	2,256,760	738,167	785,072	0	3,780,000
5,000,000	2,236,034	731,388	777,862	1,654,717	5,400,000
Total	9,431,473	2,643,382	1,795,428	1,654,717	15,525,000

Table 3: Trend Factor per Policy and Total Layer Trend

	Layer of Loss				
Policy Limit	\$250k *xs* 0	\$250k *xs* \$250k	\$500k *xs* \$500k	\$4MM *xs* \$1MM	\$5MM *xs* 0
250,000	1.248				1.248
500,000	1.248	1.406			1.283
750,000	1.248	1.406	1.456		1.304
1,000,000	1.248	1.406	1.470		1.318
5,000,000	1.248	1.406	1.470	1.581	1.389
Total	1.248	1.406	1.468	1.581	1.328
Annualized	4.53%	7.06%	7.98%	9.59%	5.83%

Table 4: Expected number of claims by policy and frequency trend by layer

		\$250k xs 0		\$250k xs \$250k		\$500k xs \$500k		\$4MM xs \$1MM		\$5MM xs 0	
Limit ('000)	$E[N_p]$	2000	2005	2000	2005	2000	2005	2000	2005	2000	2005
250	27.81	27.81	27.81							27.81	27.81
500	50.30	50.30	50.30	3.17	4.35					50.30	50.30
750	23.64	23.64	23.64	1.49	2.05	0.79	1.13			23.64	23.64
1,000	46.49	46.49	46.49	2.93	4.03	1.55	2.23			46.49	46.49
5,000	46.07	46.07	46.07	2.90	3.99	1.54	2.21	0.75	1.13	46.07	46.07
Total	194.31	194.31	194.31	10.49	14.42	3.88	5.57	0.75	1.13	194.31	194.31
Excess Frequency Trend		1.00		1.37		1.44		1.50		1.00	
Annualized Trend		0%		6.57%		7.50%		8.47%		0%	

Table 5: Summary of trend factors by layer of loss

	\$250k *xs* 0	\$250k *xs* \$250k	\$500k *xs* \$500k	\$4MM *xs* \$1MM	\$5MM *xs* 0
Layer Trend Factor	1.248	1.406	1.468	1.581	1.328
Annual Trend Factor	4.53%	7.06%	7.98%	9.59%	5.83%
Frequency Trend Factor	1.00	1.37	1.44	1.50	1.00
Annual Frequency Trend	0.00%	6.57%	7.50%	8.47%	0.00%
Severity Trend Factor	1.248	1.023	1.022	1.053	1.328
Annual Severity Trend	4.53%	0.46%	0.44%	1.04%	5.83%

Table 6: Historic Limits Profile

	Underwriting Year	
Limit	2000	2005
250,000	2,250,000	2,250,000
500,000	4,500,000	5,400,000
750,000	2,925,000	2,925,000
1,000,000	3,150,000	6,300,000
5,000,000	3,000,000	9,000,000
Total	15,825,000	25,875,000
OL Factor	1.50	1.00
OL Written Premium	23,737,500	25,875,000

Table 7: Historic On-Level Limits Profile

	Underwriting Year	
Limit	2000	2005
250,000	3,375,000	2,250,000
500,000	6,750,000	5,400,000
750,000	4,387,500	2,925,000
1,000,000	4,725,000	6,300,000
5,000,000	4,500,000	9,000,000
OL Written Premium	23,737,500	25,875,000

Table 8: Percentage of Loss in Layer using 2005 Severity Distribution

	Layer of Loss				
Limit	\$250k xs 0	\$250k xs \$250k	\$500k xs \$500k	\$4MM xs \$1MM	\$5MM xs 0
250,000	100.0%	0.0%	0.0%	0.0%	100.0%
500,000	75.4%	24.6%	0.0%	0.0%	100.0%
750,000	65.4%	21.4%	13.2%	0.0%	100.0%
1,000,000	59.7%	19.5%	20.8%	0.0%	100.0%
5,000,000	41.4%	13.5%	14.4%	30.6%	100.0%

Table 9: Expected Loss Cost by layer: 2005 Curve 2000 Limit Profile

	Layer of Loss				
Limit	\$250k xs 0	\$250k xs \$250k	\$500k xs \$500k	\$4MM xs \$1MM	\$5MM xs 0
250,000	2,025,000	0	0	0	2,025,000
500,000	3,051,788	998,212	0	0	4,050,000
750,000	1,720,876	562,883	348,741	0	2,632,500
1,000,000	1,692,572	553,625	588,803	0	2,835,000
5,000,000	1,118,015	365,693	388,929	827,363	2,700,000
Total	9,608,250	2,480,413	1,326,474	827,363	14,242,500

Table 10: Expected Loss Cost by layer: 2005 Curve 2005 Limit Profile

	Layer of Loss				
Limit	\$250k *xs* 0	\$250k *xs* \$250k	\$500k *xs* \$500k	\$4MM *xs* \$1MM	\$5MM *xs* 0
250,000	1,350,000	0	0	0	1,350,000
500,000	2,441,430	798,570	0	0	3,240,000
750,000	1,147,250	375,255	232,494	0	1,755,000
1,000,000	2,256,763	738,167	785,071	0	3,780,000
5,000,000	2,236,030	731,385	777,858	1,654,726	5,400,000
Total	9,431,473	2,643,377	1,795,423	1,654,726	15,525,000

Table 11: Exposure Adjustment by Layer of Loss

	Layer of Loss				
Limit	\$250k *xs* 0	\$250k *xs* \$250k	\$500k *xs* \$500k	\$4MM *xs* \$1MM	\$5MM *xs* 0
250,000	0.667				0.667
500,000	0.800	0.800			0.800
750,000	0.667	0.667	0.667		0.667
1,000,000	1.333	1.333	1.333		1.333
5,000,000	2.000	2.000	2.000	2.000	2.000
Total	0.982	1.066	1.354	2.000	1.090

Table 12: Expected Frequency by Policy Limit

Limit	$E[M_p]$	$E[N_p]$
250,000	27.81	41.72
500,000	50.30	62.87
750,000	23.64	35.45
1,000,000	46.49	34.87
5,000,000	46.07	23.03
Total	194.31	197.95

Table 13: Frequency Exposure Adjustment by Layer of Loss

	\$250k xs 0		\$250k xs \$250k		\$500k xs \$500k		\$4MM xs \$1MM		\$5MM xs 0	
$(1 - F_Y(A))$	1.000		0.087		0.048		0.025		1.000	
Limit ('000)	2000	2005	2000	2005	2000	2005	2000	2005	2000	2005
250	41.72	27.81							41.72	27.81
500	62.87	50.30	5.44	4.35					62.87	50.30
750	35.45	23.64	3.07	2.05	1.70	1.13			35.45	23.64
1,000	34.87	46.49	3.02	4.03	1.67	2.23			34.87	46.49
5,000	23.03	46.07	1.99	3.99	1.11	2.21	0.57	1.13	23.03	46.07
Total	197.95	194.31	13.53	14.42	4.48	5.57	0.57	1.13	197.95	194.31
Freq. Adj.	0.98		1.07		1.24		2.00		0.98	

Table 14: Summary of Exposure Adjustment by Layer of Loss

	\$250k *xs* 0	\$250k *xs* \$250k	\$500k *xs* \$500k	\$4MM *xs* \$1MM	\$5MM *xs* 0
Layer	0.982	1.066	1.354	2.000	1.090
Frequency	0.982	1.066	1.245	2.000	0.982
Severity	1.000	1.000	1.087	1.000	1.110