The Seventh Game—
An Example of Pricing Arbitrage

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Two friends of mine happen to each have a pair of tickets to the seventh game of the World Series between the Chicago Cubs and the Boston Red Sox. For those who don't know, the World Series is the annual championship of Major League Baseball, and it (currently) is a best of seven series[1]. What that means is that the first team to win four games wins the Series and play stops. In baseball, there can be no ties[2], so if the seventh game is played it will be the final game, but the Series could end before the seventh game.

I happen to know that if there is a Game 7, a pair of tickets will be worth $1000. If there is no Game 7, a pair of tickets can be returned for a refund of $300. How much are the tickets worth before Game 1 is played?

What is the value of something? For our purposes, we will assume that the value of a ticket is the amount that a willing buyer and willing seller would agree upon as a price. This is the "fair market price." In the problem posed, my friends would be the sellers. They know that if there is a Game 7, they could sell their pair of tickets for $1000, but would they want to? I don't know, perhaps they would not be willing sellers. To get around this problem, let's assume that we know that we could either sell <u>or buy</u> the tickets immediately before Game 7 (if there is one) for $1000.

One of my friends, Steve, works in an office where there are fans of both teams. Before any game he can, if he chooses to, place a bet on that game on either team for any amount of money at even odds.

In Steve's office, the tickets must sell for exactly $518.75 for the pair[3]. Let's see why.

Suppose that five games have been played and that the Cubs have won three and the Red Sox have won two (the case when the Red Sox lead, is similar). Now if the Cubs win again, Steve's tickets are worth $300 (because the series is over) while if the Red Sox tie the series, Game 7 will be played and his tickets will be worth $1000. Steve is $700 better off if the Red Sox win. Such a big swing in value makes Steve nervous.

Steve has an idea. He approaches one of the Red Sox fans in his office and asks, "Who's going to win Game 6?" The Red Sox supporter answers, "Why the Red Sox, of course!" Steve says, "Here's $350, if the Red Sox win you keep it, if they don't you pay me $700." The Red Sox supporter answers, "Done!"

This looks like a good deal to Steve, if the Cubs win he gets the $300 refund and he collects $700 from his colleague, $1000 total. If the Red Sox win, he has a pair of tickets

[1] The 1903 Series was best-of-nine and ended in eight. The Series was also best-of-nine from 1919-21.
[2] Well, okay, maybe there can be in the All-Star Game.
[3] Since this amount has an odd number of cents, a transaction for one ticket at the fair price isn't possible.

worth $1000 total. Steve can put himself in this happy position for a cost of $350. If he does this, Steve won't care what happens in Game 6. He pays $350 and his tickets and bet together are worth $1000, so each pair of tickets must be worth $650 at the start of Game 6.

We are making progress. We now know the value of the tickets at the start of Game 6. What about at the start of Game 5? Well, if the series is tied, 2 games each, we know that no matter what happens, it will be 3 games to 2 next and that our tickets are worth $650 in that case, so they must be worth $650 at the start of Game 5 (if it's 2-2). What if it's three games to one? Again, there are two possible outcomes, either the series will end and Steve's tickets will be worth $300 or the team that has won one game will win and we will have a 3-2 situation, which is the case we just valued (the tickets are worth $650).

Just as before, Steve can place an even-money bet to make himself ambivalent as to which team wins. This time he will bet $175 on the team that is ahead. If they win, he gets $350 and has a pair of tickets worth $300 for a total of $650; if they lose, he has tickets worth $650. The cost of this insurance is $175, so the value of the tickets before Game 5 is played (if the series is 3-1) is $475.

The reader should continue this process to see that the price before Game 1 for the pair of tickets is $518.75.

Steve's bookies think that he is odd. He had no interest in betting on Game 1, but suddenly wanted to bet on Game 2 for a very specific amount on the team that was ahead. As it happened, he lost that bet. The series was now tied and again, he had no interest in betting on Game 3, but when Game 4 came along, he was suddenly interested in betting even more than before (and on the other team as it happened, since they were now ahead in the series). Again, he lost. What upsets his bookies the most is that he doesn't care whether he wins or loses these bets!

I said that I have two friends with pairs of tickets, the other is Glenn. Glenn works in a different office from Steve and, as it happens, his coworkers also like to bet. Glenn, too, can buy or sell a pair of tickets for $1000 if there is a Game 7 and can get the $300 refund if Game 7 is not held. Glenn's coworkers have reviewed the history of the Series and believe that there is a 50/50 chance that Game 7 will be held. In Glenn's office, before Game 1 is held a pair of Game 7 tickets must sell for $650. This is because Glenn, like Steve before, could bet $350 against there being a Game 7. If there is a Game 7, he has tickets worth $1000 and if there is no Game 7, he has tickets with a refund value of $300 and he wins his side bet and collects $700 --- again $1000. The cost of this insurance is $350, so the tickets must be worth $650.

I have lunch with Steve and Glenn (before Game 1) and they each tell me about their tickets and how much they are worth. In Steve's office, the tickets are worth $518.75; in Glenn's office they are worth $650. In order to pay for lunch, I will buy tickets from Steve for $518.75 a pair and sell them to Glenn at $650 a pair. This will pay for lunch (and after rescaling, much more). We've all heard the expression that there are no free lunches, but here we have one; we have found an inter-market arbitrage.

The people in Steve's office offer him even money bets on each game. We do not know the actual probabilities for the Cubs winning a given game --- the bookies in Steve's office may not either. It doesn't matter. So long as Steve knows that he can place his bets at those odds before each game, he can guarantee a payoff of $1000 if there is a Game 7 and $300 if there is no Game 7 for an initial cost of exactly $518.75.

Some questions to consider:

1) Suppose the bookies in Steve's office offer two to one odds on the visiting team and one to two odds on the home team. How does this change the price of his tickets? The home teams are scheduled: Red Sox, Red Sox, Cubs, Cubs, Cubs, Red Sox. Does the order matter?
2) In the original problem, as I keep buying from Steve and selling to Glenn what eventually happens? Why?
3) Suppose that in the original problem the bookies in Glenn's office change their odds and decide to offer bets at two to one against there being a game 7, now what should Steve, Glenn, and I do to get our free lunches?