Estimating and Incorporating Correlation in Reserve Variability

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ESTIMATING AND INCORPORATING CORRELATION IN RESERVE VARIABILITY

By

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Abstract

An actuarial analysis of a book of reserves usually focuses on ultimate loss estimates by exposure period (accident year, report year, etc.) and by business segment. If an actuary is interested in the distribution of total reserves for all segments and exposure periods combined, then the actuary must find a way to combine the distribution estimates from the various parts analyzed. In this paper we present a method that can be used to estimate the correlation of reserve estimates among the various components analyzed, allowing the actuary to combine the resulting distributions in a meaningful way.

Biography

Roger Hayne is a Fellow of the Casualty Actuarial Society, a Member of the American Academy of Actuaries and a Consulting Actuary in the Pasadena, California office of Milliman USA. He holds a Ph.D. in mathematics from the University of California and joined Milliman in 1977. Roger has been involved in reserve estimation for a wide range of property and liability coverages with emphasis on exposures with longer tails and in situations where full data may not be readily available. The winner of the 1995 Dorweiler Prize he long has had interest in reserve variability and has authored several papers on the topic that have appeared in both the *Proceedings* and in the *Forum*.

ESTIMATING AND INCORPORATING CORRELATION IN RESERVE VARIABILITY

1. Introduction

The traditional collective risk model that is sometimes used in estimating reserve variability usually depends on the assumption that the various claim count and severity variables are all independent of one another. Though the assumption makes estimating the resulting aggregate distributions rather tractable, it often may not be realistic in practice.

Heckman and Meyers[1] recognized this in their 1985 paper and incorporated dependency among the various distributions by recognizing the uncertainty that affects the estimates of the parameters of the underlying distributions.

The Casualty Actuarial Society (CAS) recognized the importance of this issue and commissioned research into estimating aggregate distributions when the underlying random variables were correlated. This resulted in Shaun Wang's 1998 paper[2].

Meyers et al.[3] have taken up this issue again recently and considered correlation of risks within the framework of the Heckman and Meyers model. There they build on Shaun Wang's work to use the parameter uncertainty variables as a means of incorporating correlation among variables in the collective risk model. We will take a similar tack here, first measuring correlation, and then using its effects on the variance of the aggregate distribution to select the "mixing" parameter for the Heckman and Meyers version of the collective risk model.

In reserve problems we are faced with potential correlation of distributions for various accident years since inflation, court decisions, and other factors could induce correlation in reserves among various accident years. In addition, such factors could also cause correlation among lines of insurance.

In this paper we consider a hindsight method, similar to the bootstrap approach, to measure correlation among various distributions going into the collective risk model. Such correlations could be between accident years in a single line of business or more broadly across accident years and lines of business. With this measure we modify the "mixing parameter" in the Heckman and Meyers algorithm to incorporate the effects of correlation in the estimate of the aggregate distribution of reserves.

This paper builds on and refines the approach discussed in "Measurement of Reserve Uncertainty"[4] and explores an approach that provides insight into the correlation of reserve estimates, both across accident years and across lines of business. We will assume that the reader has access to that paper for detailed background. We will, however, briefly cover the major items relevant to this discussion.

2. A Brief Digression

One of the primary conclusions in "Measurement of Reserve Uncertainty" is that current stochastic methods of reserve estimation lack much of the robustness of the traditional reserving approach. That traditional approach recognizes its limitations by incorporating several different forecasting methods to assist the actuary in estimating reserves.

That paper used the Heckman and Meyers modification of the collective risk model to estimate the aggregate distribution of reserves. There reserves for an accident year were first looked on as the aggregate of an unknown number N of independent open and IBNR claims all drawn from the same distribution, with N and each of those variables all independent. This is simply the formulation of the classic collective risk model.

Heckman and Meyers introduced additional random variables χ_i and β that are independent from the claim count random variables N_i and claim size random variables X_i with

$$
\begin{aligned}
&E(\chi_i) = 1, \operatorname{Var}(\chi_i) = c_i \\
&E(1/\beta) = 1, \operatorname{Var}(1/\beta) = b.
\end{aligned} \tag{2.1}
$$

They then used Fourier transforms to calculate the aggregate distribution resulting from the following algorithm repeated several times:

1. Randomly select values for each χ_i,

2. Randomly select the number of claims N_i from a Poisson distribution with expected value $\chi_i\lambda_i$,

3. Randomly select N_i claims from the i[th] claim size distribution,

4. Sum all claims from all distributions,

5. Randomly select a value for β, and divide the sum of claims by β.

In this case λ_i is the expected number of claims for the i[th] accident year. The key in this algorithm is that the variable β affects all claims. Heckman and Meyers called the parameters c_i the "contagion parameters" and the parameter b the "mixing parameter." The contagion parameters affect the distributions of reserves for each accident year separately, while the mixing parameter affects the distribution of all years combined.

The approach set forth in "Measurement of Reserve Uncertainty" suggests considering the range of accident year results from various traditional methods to estimate the c_i parameters and then to consider variation in severities to estimate the b parameter. Here we maintain the first concept but follow Meyers, Klinker, and LaLonde and incorporate correlation among accident years (or across lines of business) to assist in the estimate of the *b* parameter. For this analysis, we will use the same data set as that used in "Measurement of Reserve Uncertainty."

3. A Hindsight Measure of Correlation

We will use hindsight results from traditional actuarial methods in an attempt to measure correlation among accident years. We begin with the traditional approach that calls upon a variety of different methods to assist the actuary in deriving ultimate loss and reserve estimates. Exhibit 1 shows the data underlying the examples in this paper as well as in "Measurement of Reserve Uncertainty." In Exhibit 2, we applied a range of methods to arrive at our final estimates of ultimate losses. The range of estimates from the various methods may provide some information regarding the uncertainty inherent in the reserve estimates. In fact in that earlier paper, the parameter *c* for a particular accident year is selected using the variance in reserve estimates implied by the various methods along with the following relationship derived in Heckman and Meyers[5]

(3.1) $$\mathrm{Var}(R) = \lambda\left(\mu^2 + \sigma^2\right) + c\lambda^2\mu^2 .$$

Here μ and σ represent the mean and standard deviation of the claim size distribution for a single accident year and λ the expected number of claims for that year.

These forecast methods and final selection can also be used to assess the behavior of the individual forecast methods. For example, the development factor (link ratio, chain ladder) method assumes that there is a variable d_j such that an estimate of the ultimate losses for an accident year at age j is given by:

(3.2) $$U_i = d_j C_{ij} .$$

Here C_{ij} is the amount paid at age j for accident year i. Thus given our set of ultimate loss selections from Exhibit 2 that take into account the information provided by all the forecast methods the factors in (3.2) give us a hindsight view of what development factors would have resulted in the final selections. These hindsight historical factors are then:

$$(3.3) \qquad d_{ij} = \frac{U_i}{C_{ij}}.$$

We can then get a set of "what if" alternate ultimate loss estimates for a particular accident year, implied by our final selections and the historical development data by simply applying these hindsight factors at the appropriate age to the amount paid to date for that accident year. For example, paid amounts to date for accident year 1991 times the 12-to-ultimate factor implied by the 1978 forecasts and the 1978 losses at 12 months gives an "alternate" forecast for the 1991 losses. These alternative "what if" reserve estimates for the i^{th} accident year based on the development of the k^{th} accident year would then be given as:

$$(3.4) \qquad R_i^k = C_{iN-i}\left(d_{kN-i} - 1\right), k = 1, 2, \ldots, i.$$

Here N represents the number of years of experience in the triangles which, for ease in notation, are assumed to have as many years of development as accident years. Using the sample data and the forecasts from Exhibit 2, Exhibit 3 shows the resulting hindsight factors and resulting alternate reserve loss estimates for the development factor method.

We can take the same type of approach for the incremental severity method projections. These are similar to the forecast methods presented by Berquist and Sherman[6] where the ultimate loss forecast is the sum of the amount paid to date and the incremental average payment in each future development period times an exposure base, very often estimated ultimate claims or exposures earned. In this case, rather than taking the age-to-ultimate development factor implied by the loss emergence for a previous accident year, we take the future average incremental average implied by that year, adjust for expected trend, and multiply by the expected exposure units. The following formula shows this, assuming an annual trend of τ and exposure units for year j of e_j.

$$R_i^k = e_i\left(\frac{U_k - C_{kN-i}}{e_k}\right)(1+\tau)^{i-k},\ k = 1, 2, \ldots, i. \tag{3.5}$$

One can obtain similar formulations for other forecasting methods brought to bear on a particular reserve review.

In this manner, we can construct triangles of hindsight alternative reserve estimates for each of the methods used in the analysis. At this time, one could review these triangles to assess the correlation between the various forecast methods. However, since we have taken the final selections on Exhibit 2 as weighted averages of the forecasts of the various methods, we focus on the corresponding weighted averages of the alternate estimates for each accident year in order to measure correlation among accident years. We note that this same approach can be used to measure correlation across both accident years and lines of business. Exhibit 4 shows the hindsight alternate reserve estimates taken as the weighted averages of the alternates for each accident year, using the selected weights shown in Exhibit 2.

Just as the bootstrap method uses historical data to estimate distributions, one can look at a row of the triangle in Exhibit 4 as one potential realization of future losses for subsequent accident years. With this view we can calculate the correlation coefficient between the alternates in pairs of accident years. For example, to estimate the correlation coefficient between accident years 1976 and 1991, we would calculate the correlation coefficient between the first three amounts in the 1991 column and those in the 1976 column. The top portion of Exhibit 5 shows the results of these calculations.

We note at this point that there is no restriction to applying this approach only to measuring correlations among accident years for a single line of business. The same approach could be just as easily used to measure correlation both across accident years and lines of business. In that case, the rows and columns of the correlation and covariance matrices would represent combinations of lines of business and accident years. For example, one entry may be Accident Year #1 for Line #1 and another may be Accident Year #5 for Line #3, and so forth.

From here we would suggest reference to Shaun Wang's paper.[7] This paper, the product of a CAS research project, does an excellent job at discussing issues that arise when dealing with correlated distributions and presents several approaches that can be used in modeling and estimating such distributions. In all cases, however, there is the need to assume some sort of structure on the correlation of the distributions. One approach would be to assume some joint distribution as a model, such as a joint lognormal. Wang gives an algorithm to model such a distribution if the covariance matrix of the corresponding joint normal distribution is positive definite. It happens that the matrix associated with our correlation structure is not positive definite and Wang's algorithm breaks down.

We will follow the approach in Meyers, Klinker, and LaLonde and use the Heckman and Meyers algorithm with the covariance structure guiding our choice of mixing parameter *b*. Given the covariance matrix in Exhibit 5, it is a simple matter to obtain the standard deviation of total reserves using the standard formula

$$\text{(3.6)}\qquad \begin{aligned}\operatorname{Var}\left(\sum_{i=1}^{n}X_i\right) &= \sum_{i=1}^{n}\operatorname{Var}(X_i)+\sum_{i=1}^{n}\sum_{j=1}^{i-1}2\operatorname{Cov}(X_i,X_j)\\ &= \sum_{i=1}^{n}\operatorname{Cov}(X_i,X_i)+\sum_{i=1}^{n}\sum_{\substack{j=1\\ i\neq j}}^{n}\operatorname{Cov}(X_i,X_j)\\ &= \sum_{i=1}^{n}\sum_{j=1}^{n}\operatorname{Cov}(X_i,X_j).\end{aligned}$$

The calculations on the bottom of Exhibit 5 show that correlation of reserves among accident years does add substantially to the standard deviation of reserves.

We can then use this information to estimate the mixing parameter b in the Heckman and Meyers algorithm. To this end we note that the variance for the total distribution is given by:

$$\text{(3.7)}\qquad \begin{aligned}\operatorname{Var}\left(\sum_{i=1}^{n}R_i\right) &= \mathrm{E}_\beta\left(\operatorname{Var}\left(\sum_{i=1}^{n}R_i\,|\beta\right)\right)+\operatorname{Var}_\beta\left(\mathrm{E}\left(\sum_{i=1}^{n}R_i\,|\beta\right)\right)\\ &= \mathrm{E}_\beta\left(\left(\sum_{i=1}^{n}\lambda_i\left(\mu_i^2+\sigma_i^2\right)+c_i\lambda_i^2\mu_i^2\right)\Big/\beta^2\right)+\operatorname{Var}_\beta\left(\left(\sum_{i=1}^{n}\lambda_i\mu_i\right)\Big/\beta\right)\\ &= \left(\sum_{i=1}^{n}\lambda_i\left(\mu_i^2+\sigma_i^2\right)+c_i\lambda_i^2\mu_i^2\right)\mathrm{E}_\beta\left(1/\beta^2\right)+\left(\sum_{i=1}^{n}\lambda_i\mu_i\right)^2\operatorname{Var}_\beta\left(1/\beta\right)\\ &= \left(\sum_{i=1}^{n}\lambda_i\left(\mu_i^2+\sigma_i^2\right)+c_i\lambda_i^2\mu_i^2\right)(1+b)+\left(\sum_{i=1}^{n}\lambda_i\mu_i\right)^2 b\\ &= \sum_{i=1}^{n}\operatorname{Var}(R_i)+b\left(\sum_{i=1}^{n}\operatorname{Var}(R_i)+\left(\sum_{i=1}^{n}\mathrm{E}(R_i)\right)^2\right).\end{aligned}$$

Solving for b we have

$$\text{(3.8)}\qquad b=\frac{\operatorname{Var}\left(\sum_{i=1}^{n}R_i\right)-\sum_{i=1}^{n}\operatorname{Var}(R_i)}{\sum_{i=1}^{n}\operatorname{Var}(R_i)+\left(\sum_{i=1}^{n}\mathrm{E}(R_i)\right)^2}$$

Using the results from Exhibits 2 and 5 we derive a value of the mixing parameter *b* of 0.025748. We can now use this *b* value with the other parameter estimates derived in "Measurement of Reserve Uncertainty" to derive an estimate of the distribution of total reserves in this example. Using the program CRIMCALC written by Glenn Meyers to implement the algorithm set out in his paper with Phil Heckman, we derived the estimates shown graphically in Exhibit 6.

As can be seen in that exhibit, parameter uncertainty is by far the most significant contributor to overall reserve uncertainty in this case. The correlation among accident years also has a marked contribution to overall uncertainty, as evidenced by the difference between the "With Parameter Uncertainty" and the "Independent Accident Year" distributions. The only difference between these two is that the "Independent Accident Year" distribution assumed that the mixing parameter *b* was 0 instead of the estimated 0.025748.

4. Conclusion

We believe this approach, though somewhat ad-hoc in nature, can provide very useful information with regards to the correlation structure of reserve estimates, both across years and across lines of business. Doubtlessly, there remains much more to be done.

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[1] Heckman, P.E., and Meyers, G.G., "The Calculation of Aggregate Loss Distriubtions from Claim Severity and Claim Count Distributions," *Proceedings of the Casualty Actuarial Society*, LXX, 1983, pp. 22-61, addendum in LXXI, 1984, pp. 49-66.

[2] Wang, S. "Aggregation of Correlated Risk Portfolios: Models and Algorithms," *Proceedings of the Casualty Actuarial Society*, LXXXV, 1998, pp. 848-939.

[3] Meyers, G.G., Klinker, F.L., and LaLonde, D.A., "The Aggregation and Correlation of Reinsurance Exposure," *Casualty Actuarial Society Forum*, Spring 2003, pp. 69-152.

[4] Hayne, R.M., "Measurement of Reserve Variability," *Casualty Actuarial Society Forum,"* Fall 2003, pp. 141-172.

[5] Heckman, P.E., and Meyers, G.G., op.cit.

[6] Berquist, J.R., and Sherman, R.E., "Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach," *Proceedings of the Casualty Actuarial Society*, LXXIV, 1977, pp. 123-184.

[7] Ibid.

EXAMPLE PRIVATE PASSENGER AUTO BODILY INJURY LIABILITY DATA

Cumulative Paid Losses

ccident	Months of Development																	
Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	$267	$1,975	$4,587	$7,375	$10,661	$15,232	$17,888	$18,541	$18,937	$19,130	$19,189	$19,209	$19,234	$19,234	$19,246	$19,246	$19,246	$19,246
1975	310	2,809	5,686	9,386	14,884	20,654	22,017	22,529	22,772	22,821	23,042	23,060	23,127	23,127	23,127	23,127	23,159	
1976	370	2,744	7,281	13,287	19,773	23,888	25,174	25,819	26,049	26,180	26,268	26,364	26,371	26,379	26,397	26,397		
1977	577	3,877	9,612	16,962	23,764	26,712	28,393	29,656	29,839	29,944	29,997	29,999	29,999	30,049	30,049			
1978	509	4,518	12,067	21,218	27,194	29,617	30,854	31,240	31,598	31,889	32,002	31,947	31,965	31,986				
1979	630	5,763	16,372	24,105	29,091	32,531	33,878	34,185	34,290	34,420	34,479	34,498	34,524					
1980	1,078	8,066	17,518	26,091	31,807	33,883	34,820	35,482	35,607	35,937	35,957	35,962						
1981	1,646	9,378	18,034	26,652	31,253	33,376	34,287	34,985	35,122	35,161	35,172							
1982	1,754	11,256	20,624	27,857	31,360	33,331	34,061	34,227	34,317	34,378								
1983	1,997	10,628	21,015	29,014	33,788	36,329	37,446	37,571	37,681									
1984	2,164	11,538	21,549	29,167	34,440	36,528	36,950	37,099										
1985	1,922	10,939	21,357	28,488	32,982	35,330	36,059											
1986	1,962	13,053	27,869	38,560	44,461	45,988												
1987	2,329	18,086	38,099	51,953	58,029													
1988	3,343	24,806	52,054	66,203														
1989	3,847	34,171	59,232															
1990	6,090	33,392																
1991	5,451																	

Claims Closed with Payment

ccident	Months of Development																	
Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	268	607	858	1,090	1,333	1,743	2,000	2,076	2,113	2,129	2,137	2,141	2,143	2,143	2,145	2,145	2,145	2,145
1975	294	691	913	1,195	1,620	2,076	2,234	2,293	2,320	2,331	2,339	2,341	2,343	2,343	2,343	2,343	2,344	
1976	283	642	961	1,407	1,994	2,375	2,504	2,549	2,580	2,590	2,596	2,600	2,602	2,603	2,603	2,603		
1977	274	707	1,176	1,688	2,295	2,545	2,689	2,777	2,809	2,817	2,824	2,825	2,825	2,826	2,826			
1978	269	658	1,228	1,819	2,217	2,475	2,613	2,671	2,691	2,706	2,710	2,711	2,714	2,717				
1979	249	771	1,581	2,101	2,528	2,816	2,930	2,961	2,973	2,979	2,986	2,988	2,992					
1980	305	1,107	1,713	2,316	2,748	2,942	3,025	3,049	3,063	3,077	3,079	3,080						
1981	343	1,042	1,608	2,260	2,596	2,734	2,801	2,835	2,854	2,859	2,860							
1982	350	1,242	1,922	2,407	2,661	2,834	2,887	2,902	2,911	2,915								
1983	428	1,257	1,841	2,345	2,683	2,853	2,908	2,920	2,925									
1984	291	1,004	1,577	2,054	2,406	2,583	2,622	2,636										
1985	303	1,001	1,575	2,080	2,444	2,586	2,617											
1986	318	1,055	1,906	2,524	2,874	2,958												
1987	343	1,438	2,384	3,172	3,559													
1988	391	1,671	3,082	3,771														
1989	433	1,941	3,241															
1990	533	1,923																
1991	339																	

EXAMPLE PRIVATE PASSENGER AUTO BODILY INJURY LIABILITY DATA

Cumulative Reported Claims

ccident	Months of Development																	
Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	1,912	2,854	3,350	3,945	4,057	4,104	4,149	4,155	4,164	4,167	4,169	4,169	4,169	4,170	4,170	4,170	4,170	4,170
1975	2,219	3,302	3,915	4,462	4,618	4,673	4,696	4,704	4,708	4,711	4,712	4,716	4,716	4,716	4,716	4,716	4,717	
1976	2,347	3,702	4,278	4,768	4,915	4,983	5,003	5,007	5,012	5,012	5,013	5,014	5,015	5,015	5,015	5,015		
1977	2,983	4,346	5,055	5,696	5,818	5,861	5,884	5,892	5,896	5,897	5,900	5,900	5,900	5,900	5,900			
1978	2,538	3,906	4,633	5,123	5,242	5,275	5,286	5,292	5,298	5,302	5,304	5,304	5,306	5,306				
1979	3,548	5,190	5,779	6,206	6,313	6,329	6,339	6,343	6,347	6,347	6,348	6,348	6,348					
1980	4,583	6,106	6,656	7,032	7,128	7,139	7,147	7,150	7,151	7,153	7,154	7,154						
1981	4,430	5,967	6,510	6,775	6,854	6,873	6,883	6,889	6,892	6,894	6,895							
1982	4,408	5,849	6,264	6,526	6,571	6,589	6,594	6,596	6,600	6,602								
1983	4,861	6,437	6,869	7,134	7,196	7,205	7,211	7,212	7,214									
1984	4,229	5,645	6,053	6,419	6,506	6,523	6,529	6,531										
1985	3,727	4,830	5,321	5,717	5,777	5,798	5,802											
1986	3,561	5,045	5,656	6,040	6,096	6,111												
1987	4,259	6,049	6,767	7,206	7,282													
1988	4,424	6,700	7,548	8,105														
1989	5,005	7,407	8,287															
1990	4,889	7,314																
1991	4,044																	

Outstanding Claims

ccident	Months of Development																	
Year	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	1,381	1,336	1,462	1,660	1,406	772	406	191	98	57	23	13	3	4	0	0	0	0
1975	1,289	1,727	1,730	1,913	1,310	649	358	167	73	30	9	6	4	2	2	1	1	
1976	1,605	1,977	1,947	1,709	1,006	540	268	166	79	48	32	18	14	10	10	7		
1977	2,101	2,159	2,050	1,735	988	582	332	139	66	38	27	21	21	8	3			
1978	1,955	1,943	1,817	1,384	830	460	193	93	56	31	15	9	7	2				
1979	2,259	2,025	1,548	1,273	752	340	150	68	36	24	18	13	4					
1980	2,815	1,991	1,558	1,107	540	228	88	55	28	14	8	6						
1981	2,408	1,973	1,605	954	480	228	115	52	27	15	11							
1982	2,388	1,835	1,280	819	354	163	67	44	21	10								
1983	2,641	1,765	1,082	663	335	134	62	34	18									
1984	2,417	1,654	896	677	284	90	42	15										
1985	1,924	1,202	941	610	268	98	55											
1986	1,810	1,591	956	648	202	94												
1987	2,273	1,792	1,059	626	242													
1988	2,403	1,966	1,166	693														
1989	2,471	2,009	1,142															
1990	2,642	2,007																
1991	2,366																	

EXAMPLE PRIVATE PASSENGER AUTO BODILY INJURY LIABILITY DATA

Outstanding Losses

ccident Year	Months of Development 12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	$5,275	$8,867	$12,476	$11,919	$8,966	$5,367	$3,281	$1,524	$667	$348	$123	$82	$18	$40	$0	$0	$0	$0
1975	6,617	11,306	13,773	14,386	10,593	4,234	2,110	1,051	436	353	93	101	10	5	5	3	3	
1976	7,658	11,064	13,655	13,352	7,592	4,064	1,895	1,003	683	384	216	102	93	57	50	33		
1977	8,735	14,318	14,897	12,978	7,741	4,355	2,132	910	498	323	176	99	101	32	14			
1978	8,722	15,070	15,257	11,189	5,959	3,473	1,531	942	547	286	177	61	67	7				
1979	9,349	16,470	14,320	10,574	6,561	2,864	1,328	784	424	212	146	113	38					
1980	11,145	16,351	14,636	11,273	5,159	2,588	1,290	573	405	134	81	54						
1981	10,933	15,012	14,728	9,067	5,107	2,456	1,400	584	269	120	93							
1982	13,323	16,218	12,676	6,290	3,355	1,407	613	398	192	111								
1983	13,899	16,958	12,414	7,700	4,112	1,637	576	426	331									
1984	14,272	15,806	10,156	8,005	3,604	791	379	159										
1985	13,901	15,384	12,539	7,911	3,809	1,404	827											
1986	15,952	22,799	16,016	8,964	2,929	1,321												
1987	22,772	24,146	18,397	8,376	3,373													
1988	25,216	26,947	17,950	8,610														
1989	24,981	30,574	19,621															
1990	30,389	34,128																
1991	28,194																	

ccident Year	Earned Exposures
1974	11,000
1975	11,000
1976	11,000
1977	12,000
1978	12,000
1979	12,000
1980	12,000
1981	12,000
1982	11,000
1983	11,000
1984	11,000
1985	11,000
1986	12,000
1987	13,000
1988	14,000
1989	14,000
1990	14,000
1991	13,000

Exhibit 2

EXAMPLE ULTIMATE LOSS FORECASTS

Accident Year	Incurred Development	Paid Development	Paid Severity	Paid Pure Premium	Paid Hindsight	Adjusted Incurred	Paid Adjusted for Claim Closing Changes Development	Paid Adjusted for Claim Closing Changes Severity	Paid Adjusted for Claim Closing Changes Pure Premium	Paid Adjusted for Claim Closing Changes Hindsight	Weighted Average	Weighted Standard Deviation
	Reserve Estimates by Ultimate Forecast Method											
1974	$0	$0	$0	$0		$0	$0	$0	$0		$0	0
1975	3	0	0	0		3	0	0	0		0	0
1976	33	0	0	0		33	21	0	0		11	14
1977	5	0	0	0		8	24	0	0		5	8
1978	-15	10	9	10		7	26	0	0		6	11
1979	-10	35	34	33		-35	28	0	0		11	24
1980	-7	54	55	50		-29	61	33	31		31	30
1981	-37	49	73	75		-20	77	47	49		39	41
1982	-41	107	136	131		-58	100	79	75		66	70
1983	114	275	297	297		-68	200	176	172		156	126
1984	-161	416	394	446		-135	352	318	351		181	258
1985	403	761	713	812		130	692	702	779		567	249
1986	744	2,143	1,760	1,909	$1,687	394	1,936	1,842	1,950	$675	1,357	637
1987	2,335	6,847	5,583	5,128	5,128	2,348	6,000	5,790	5,220	2,301	4,260	1,620
1988	8,371	19,768	16,246	13,451	14,428	10,391	17,352	16,433	13,399	8,001	12,866	3,525
1989	25,787	44,631	36,887	29,232	32,199	26,048	39,241	36,431	28,512	19,174	30,212	6,426
1990	60,211	83,760	73,987	61,846	62,974	55,734	79,667	70,246	57,192	43,286	62,516	10,197
1991	83,093	130,907	95,283	95,185	78,616	79,573	154,268	87,625	84,688	72,157	90,014	19,165

Accident Year	Incurred Development	Paid Development	Paid Severity	Paid Pure Premium	Paid Hindsight	Adjusted Incurred	Paid Adjusted Development	Paid Adjusted Severity	Paid Adjusted Pure Premium	Paid Adjusted Hindsight
	Selected Weights									
1974	1	1	1	1		1	1	1	1	
1975	0	1	1	1		0	1	1	1	
1976	1	1	1	1		1	1	1	1	
1977	1	1	1	1		1	1	1	1	
1978	1	1	1	1		1	1	1	1	
1979	1	1	1	1		1	1	1	1	
1980	1	1	1	1		1	1	1	1	
1981	1	1	1	1		1	1	1	1	
1982	1	1	1	1		1	1	1	1	
1983	3	1	2	2		3	1	2	2	
1984	3	1	2	2		3	1	2	2	
1985	3	1	2	2		3	1	2	2	
1986	3	1	2	2	2	3	1	2	2	2
1987	3	1	2	2	2	3	1	2	2	2
1988	3	1	2	2	2	3	1	2	2	2
1989	3	1	2	2	2	3	1	2	2	2
1990	3	1	2	2	2	3	1	2	2	2
1991	3	1	2	2	2	3	1	2	2	2

Exhibit 3

HINDSIGHT ALTERNATE PAID LOSS DEVELOPMENT ESTIMATES

Accident Year	Age-to-Ultimate Factors Implied by Ultimate Selections at Age 12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216
1974	72.0824	9.7448	4.1958	2.6096	1.8053	1.2635	1.0759	1.0380	1.0163	1.0061	1.0030	1.0019	1.0006	1.0006	1.0000	1.0000	1.0000	1.0000
1975	74.7065	8.2446	4.0730	2.4674	1.5560	1.1213	1.0519	1.0280	1.0170	1.0148	1.0051	1.0043	1.0014	1.0014	1.0014	1.0014	1.0000	
1976	71.3726	9.6239	3.6270	1.9875	1.3356	1.1055	1.0490	1.0228	1.0138	1.0087	1.0053	1.0017	1.0014	1.0011	1.0004	1.0004		
1977	52.0860	7.7518	3.1267	1.7718	1.2647	1.1251	1.0585	1.0134	1.0072	1.0037	1.0019	1.0018	1.0018	1.0002	1.0002			
1978	62.8524	7.0810	2.6512	1.5078	1.1764	1.0802	1.0369	1.0241	1.0125	1.0032	0.9997	1.0014	1.0008	1.0002				
1979	54.8169	5.9925	2.1094	1.4327	1.1871	1.0616	1.0194	1.0102	1.0071	1.0033	1.0016	1.0011	1.0003					
1980	33.3887	4.4623	2.0546	1.3795	1.1316	1.0623	1.0337	1.0144	1.0108	1.0016	1.0010	1.0009						
1981	21.3919	3.7547	1.9525	1.3211	1.1266	1.0550	1.0270	1.0065	1.0025	1.0014	1.0011							
1982	19.6375	3.0601	1.6701	1.2365	1.0983	1.0334	1.0112	1.0063	1.0037	1.0019								
1983	18.9470	3.5601	1.8005	1.3041	1.1198	1.0415	1.0104	1.0071	1.0041									
1984	17.2274	3.2311	1.7300	1.2782	1.0825	1.0206	1.0089	1.0049										
1985	19.0559	3.3482	1.7149	1.2856	1.1105	1.0367	1.0157											
1986	24.1310	3.6271	1.6988	1.2278	1.0649	1.0295												
1987	26.7449	3.4440	1.6349	1.1989	1.0734													
1988	23.6521	3.1875	1.5190	1.1943														
1989	23.2504	2.6176	1.5101															
1990	15.7485	2.8722																
1991	17.5133																	

Accident Year Paid to Date

1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980	1979	1978	1977	1976	1975	1974
5,451	33,392	59,232	66,203	58,029	45,988	36,059	37,099	37,681	34,378	35,172	35,962	34,524	31,986	30,049	26,397	23,159	19,246

Based on Accident Year	Hindsight Alternate Reserve Estimates for Accident Year 1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980	1979	1978	1977	1976	1975	1974
1974	387,470	292,007	189,292	106,562	46,729	12,119	2,737	1,411	615	208	104	69	22	20	0	0	0	0
1975	401,774	241,911	182,019	97,146	32,262	5,578	1,870	1,037	640	509	179	154	48	44	42	37	0	
1976	383,601	287,968	155,600	65,375	19,472	4,851	1,767	846	519	299	187	60	48	35	12	11		
1977	278,470	225,455	125,967	51,097	15,359	5,753	2,109	497	271	126	66	65	63	5	5			
1978	337,157	203,056	97,803	33,616	10,238	3,688	1,330	893	470	111	-11	51	29	6				
1979	293,356	166,709	65,710	28,644	10,859	2,832	699	379	269	114	57	38	11					
1980	176,551	115,613	62,468	25,125	7,637	2,864	1,215	534	408	54	35	31						
1981	111,156	91,983	56,418	21,261	7,349	2,529	972	240	96	49	39							
1982	101,593	68,790	39,691	15,655	5,707	1,536	406	235	140	66								
1983	97,829	85,488	47,414	20,132	6,954	1,909	377	263	156									
1984	88,456	74,500	43,240	18,415	4,785	947	322	181										
1985	98,423	78,410	42,346	18,911	6,410	1,686	567											
1986	126,087	87,725	41,394	15,083	3,764	1,357												
1987	140,335	81,611	37,608	13,171	4,260													
1988	123,477	73,045	30,740	12,866														
1989	121,287	54,013	30,212															
1990	80,394	62,516																
1991	90,014																	

Exhibit 4

COMPOSITE HINDSIGHT ALTERNATE RESERVE ESTIMATES

Based on Accident Year	Estimates for Accident Year																	
	1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980	1979	1978	1977	1976	1975	1974
1974	119,864	77,831	41,971	22,816	9,430	2,449	60	101	-79	15	50	25	17	6	6	11	0	0
1975	123,373	73,746	43,816	22,471	7,441	1,134	373	314	295	324	170	125	47	41	41	42	0	
1976	115,554	76,766	41,440	19,596	6,356	1,399	677	234	122	147	102	31	11	15	3	11		
1977	97,719	70,593	38,461	17,741	5,412	1,803	736	62	44	55	45	41	10	-1	5			
1978	108,995	70,843	36,227	14,207	4,003	1,282	801	447	199	43	-16	35	13	6				
1979	98,437	65,969	31,990	13,968	4,681	1,253	418	156	151	67	36	15	11					
1980	82,177	62,553	31,790	13,422	4,286	1,776	1,074	374	319	63	47	31						
1981	74,869	56,710	28,954	12,966	4,293	1,433	648	76	79	46	39							
1982	68,912	51,023	26,148	11,271	3,776	1,151	494	141	152	66								
1983	72,409	59,085	31,041	14,104	4,914	1,625	569	203	156									
1984	68,211	53,945	29,544	12,040	3,297	991	492	181										
1985	69,543	55,614	26,719	12,349	4,611	1,490	566											
1986	80,939	58,132	29,839	12,341	3,928	1,274												
1987	82,822	62,227	29,517	12,076	4,009													
1988	85,755	62,235	29,363	12,238														
1989	89,367	56,699	28,871															
1990	82,522	60,382																
1991	88,967																	

ESTIMATES OF CORRELATION AND COVARIANCE AMONG RESERVE PROJECTIONS

Accident Year	Accident Year 1991	1990	1989	1988	1987	1986	1985	1984	1983	1982	1981	1980	1979	1978	1977	1976
	Indicated Correlation Coefficients															
1991	1.0000	0.9432	0.9215	0.8734	0.7696	0.2875	-0.2808	0.2565	-0.1211	0.4603	0.4737	0.4767	0.6662	0.7310	0.5456	0.8307
1990	0.9432	1.0000	0.9422	0.8714	0.7791	0.4501	-0.1859	0.2023	-0.2232	0.2893	0.3644	0.2248	0.2232	0.1849	-0.1977	-0.9699
1989	0.9215	0.9422	1.0000	0.9538	0.8146	0.3558	-0.2467	0.1785	-0.1498	0.5005	0.6072	0.5800	0.6042	0.7182	0.7191	0.9754
1988	0.8734	0.8714	0.9538	1.0000	0.9422	0.4732	-0.4501	-0.0227	-0.2841	0.4906	0.6739	0.5157	0.5869	0.5167	0.5370	0.4059
1987	0.7696	0.7791	0.8146	0.9422	1.0000	0.6381	-0.6052	-0.1570	-0.4458	0.2991	0.5301	0.2915	0.4139	0.2778	0.1618	-0.1753
1986	0.2875	0.4501	0.3558	0.4732	0.6381	1.0000	-0.1700	-0.2617	-0.6127	-0.4809	-0.2305	-0.4074	-0.2994	-0.5792	-0.5933	-0.6619
1985	-0.2808	-0.1859	-0.2467	-0.4501	-0.6052	-0.1700	1.0000	0.4781	0.5570	-0.1380	-0.3085	-0.1644	-0.3493	-0.2700	-0.2493	0.0171
1984	0.2565	0.2023	0.1785	-0.0227	-0.1570	-0.2617	0.4781	1.0000	0.7592	0.3085	-0.0020	0.2528	0.3177	0.3986	0.7332	0.7910
1983	-0.1211	-0.2232	-0.1498	-0.2841	-0.4458	-0.6127	0.5570	0.7592	1.0000	0.5344	0.2792	0.4589	0.5704	0.7256	0.8042	0.8476
1982	0.4603	0.2893	0.5005	0.4906	0.2991	-0.4809	-0.1380	0.3085	0.5344	1.0000	0.9063	0.9074	0.8738	0.9610	0.8801	0.9079
1981	0.4737	0.3644	0.6072	0.6739	0.5301	-0.2305	-0.3085	-0.0020	0.2792	0.9063	1.0000	0.7843	0.7879	0.8632	0.8623	0.9055
1980	0.4767	0.2248	0.5800	0.5157	0.2915	-0.4074	-0.1644	0.2528	0.4589	0.9074	0.7843	1.0000	0.9565	0.9047	0.9829	0.9987
1979	0.6662	0.2232	0.6042	0.5869	0.4139	-0.2994	-0.3493	0.3177	0.5704	0.8738	0.7879	0.9565	1.0000	0.9300	0.9937	0.9891
1978	0.7310	0.1849	0.7182	0.5167	0.2778	-0.5792	-0.2700	0.3986	0.7256	0.9610	0.8632	0.9047	0.9300	1.0000	0.9140	0.9705
1977	0.5456	-0.1977	0.7191	0.5370	0.1618	-0.5933	-0.2493	0.7332	0.8042	0.8801	0.8623	0.9829	0.9937	0.9140	1.0000	0.9957
1976	0.8307	-0.9699	0.9754	0.4059	-0.1753	-0.6619	0.0171	0.7910	0.8476	0.9079	0.9055	0.9987	0.9891	0.9705	0.9957	1.0000
	Estimated Standard Deviation by Year															
	19,165	10,197	6,426	3,525	1,620	637	249	258	126	70	41	30	24	11	8	14
Total Standard Deviation Assuming Independence Among Accident Years																22,983
	Estimated Covariance															
1991	367,314,112	184,342,232	113,490,346	59,002,642	23,900,201	3,511,144	-1,342,580	1,266,255	-291,818	616,198	371,817	278,107	308,697	153,055	82,154	230,220
1990	184,342,232	103,982,827	61,738,485	31,324,197	12,873,454	2,924,467	-472,885	531,256	-286,074	206,038	152,187	69,773	55,026	20,601	-15,844	-143,013
1989	113,490,346	61,738,485	41,296,208	21,605,944	8,483,346	1,456,783	-395,555	295,520	-120,958	224,675	159,802	113,464	93,867	50,421	36,310	90,642
1988	59,002,642	31,324,197	21,605,944	12,425,915	5,381,949	1,062,726	-395,866	-20,606	-125,857	120,794	97,281	55,345	50,020	19,900	14,874	20,688
1987	23,900,201	12,873,454	8,483,346	5,381,949	2,625,950	658,817	-244,652	-65,514	-90,788	33,859	35,177	14,383	16,215	4,918	2,061	-4,108
1986	3,511,144	2,924,467	1,456,783	1,062,726	658,817	405,924	-27,026	-42,945	-49,062	-21,403	-6,014	-7,902	-4,612	-4,032	-2,970	-6,098
1985	-1,342,580	-472,885	-395,555	-395,866	-244,652	-27,026	62,242	30,721	17,464	-2,405	-3,152	-1,248	-2,107	-736	-489	62
1984	1,266,255	531,256	295,520	-20,606	-65,514	-42,945	30,721	66,341	24,576	5,551	-21	1,982	1,979	1,122	1,484	2,946
1983	-291,818	-286,074	-120,958	-125,857	-90,788	-49,062	17,464	24,576	15,797	4,692	1,437	1,756	1,733	996	794	1,541
1982	616,198	206,038	224,675	120,794	33,859	-21,403	-2,405	5,551	4,692	4,880	2,593	1,930	1,476	733	483	917
1981	371,817	152,187	159,802	97,281	35,177	-6,014	-3,152	-21	1,437	2,593	1,677	978	780	386	277	536
1980	278,107	69,773	113,464	55,345	14,383	-7,902	-1,248	1,982	1,756	1,930	978	927	704	301	235	440
1979	308,697	55,026	93,867	50,020	16,215	-4,612	-2,107	1,979	1,733	1,476	780	704	584	246	189	346
1978	153,055	20,601	50,421	19,900	4,918	-4,032	-736	1,122	996	733	386	301	246	119	78	153
1977	82,154	-15,844	36,310	14,874	2,061	-2,970	-489	1,484	794	483	277	235	189	78	62	113
1976	230,220	-143,013	90,642	20,688	-4,108	-6,098	62	2,946	1,541	917	536	440	346	153	113	209

Indicated Total Standard Deviation 39,942

Implied b value:

Exhibit 6

ESTIMATED DISTRIBUTION OF RESERVES