

MLE for Claims with Several Retentions

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In the *Loss Models* readings, CAS students learn how to fit severity distributions by MLE, including the case of fitting a ground-up distribution where only losses above a deductible are available. In that case the MLE looks for the ground-up distribution parameters that provide the best fit to the known excess losses. This procedure falls apart, however, when different deductibles are used and there are different degrees of exposure to each. This note derives the likelihood function for that situation.

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A not atypical fitting problem for reinsurance losses is trying to find the severity distribution that is generating claims, where the data is provided for groups of excess policies, each with its own retention and limit. The case considered here is where information is also presented about how much exposure is included in each group. Losses are of course truncated from below at their retentions and censored from above at their limits.

What is the likelihood function for severity in this situation? It turns out that that question is intrinsically linked with the likelihood function for frequency, as the exposure information comes in through frequency. First some notation. To simplify the typing and also to increase the visibility of the sub-variables, subscripts will not be used.

So suppose you have k groups of claims, and the j th group has retention R_j , upper limit or plafond (i.e., retention plus policy limit) U_j , and E_j exposures. The data for the group consists of M_j claims at the policy limit, (some of which are probably censored by the limit, so would have been larger without the limit) and N_j claims less than the limit. All the ground-up claim sizes are assumed to come from the same distribution, with severity distribution function F and density f . The exposures for all the groups are assumed to have the same ground-up Poisson loss frequency h per unit of exposure, so the observed frequency for the j th group is $hE_j(1-F(R_j))$, which will be denoted by h_j , and is still Poisson. Estimating h is part of the problem to be addressed.

With this setup, what is the likelihood function for the set of losses observed in the j th group? This is the product of the frequency and severity probabilities of observing that many claims of those sizes. Let \mathbf{a} denote the severity parameters, considered to be a vector, and X_{ji} the ground-up amount of the i th loss in the j th group. Then the likelihood function at h and \mathbf{a} for the j th group is:

$$L_j(h, \mathbf{a}) = h_j^{N_j + M_j} \exp(-h_j) \left[\prod_{i=1}^{N_j} f(X_{ji} | \mathbf{a}) \right] [1 - F(U_j | \mathbf{a})]^{M_j}$$

The log-likelihood for all the groups combined is the log of the product of these:

$$(1) \quad LL(h, \mathbf{a}) = \sum_{j=1}^k \{ \ln(h_j) (N_j + M_j) - h_j + \sum_{i=1}^{N_j} \ln[f(X_{ji} | \mathbf{a})] + M_j \ln[1 - F(U_j | \mathbf{a})] \}$$

Since h_j is a function of F , this cannot be separated into frequency and severity sections.

Formula (1) is the answer, in the sense that this is the function that has to be maximized to estimate h and \mathbf{a} . Some insight can be gained by considering its partials, however. First, wrt h :

$$\partial LL / \partial h = \sum_{j=1}^k \{ (N_j + M_j) / h - E_j(1 - F(R_j | \mathbf{a})) \}, \text{ which setting to zero gives:}$$

$$h = \sum_{j=1}^k (N_j + M_j) / \sum_{j=1}^k (E_j(1 - F(R_j | \mathbf{a})))$$

This gives the ground up frequency in terms of the severity parameters.

The partial of the LL wrt \mathbf{a} can be seen to have two components – the partial of the first two terms is a frequency component, and the partial of the second two is the usual severity component that does not consider exposures. If these separately become zero at the maximum, then the exposure information is not affecting the parameters. But it can be shown that for this to happen, a different value of h would result. Thus the exposure information does make a difference.

A large number of exposures with high retentions would be expected to produce several large ground-up claims, but no small ones. But if retentions are small, the same sample would suggest that the severity distribution tends to produce larger claims. Thus including the exposure information should make a difference of this kind. Some practical testing of MLE with this LL could discern if this is the case.