Valuing An Insurance Enterprise

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by
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Abstract

The valuation methodology described in this paper follows from minimum sufficiency levels for reserves. The valuation is risk adjusted both for uncertainty in claims payments and uncertainty in investments. Attribution of capital is inherent in the method of determining minimum sufficiency levels. Value of an enterprise is seen as consisting of two parts: (1) current asset levels beyond what is required for minimum reserve sufficiency; and (2) capital release that is expected by virtue of the chance-constrained properties of the conservative minimum sufficiency levels. The valuation of an insurance enterprise in a runoff mode seeks to know the capital required to support the runoff of the enterprise and the probability distribution of the release of excess capital back to shareholders for each of the forecast periods. Because the approach relies on bootstrap methods, there is no explicit measurement of either process or parameter risk that ordinarily appears in dynamic financial analysis.

Introduction

The *Minimum Sufficiency Level* is defined to be the level of assets necessary to fund future claims payments with a specified level of confidence [Scheel, 2001]. This level is risk adjusted both with respect to uncertainty in the stream of future claims payments and uncertainty in the returns on assets needed to fund those claims payments. Risk adjustment is in the form of chance-constrained confidence; confidence that investments will grow to a target minimum sufficiency level and that the target will be sufficient to immunize future claims payments.

This paper describes a valuation procedure based on minimum sufficiency levels. It seeks to establish:

1. The valuation of the insurance enterprise.
2. Capital requirements for a runoff of the enterprise.
3. The probability distribution of capital release for each of the forecast periods.

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The difference between the market value of assets and the minimum sufficiency level is excess value beyond what is required for claims against the enterprise. The minimum sufficiency level of assets contains capital attribution; it is the risk-adjusted amount that will fund future claims obligations.[2] We refer to this difference between market value and minimum sufficiency level as the *current excess value*. Remaining value lies in the difference between asset accrual and claims payments over time; this value is referred to in this paper as capital release. Future capital release is a random variable and only can be measured with a probability distribution. We can speak of chance-constrained values—values that confidently lie under a threshold or within a range.

Both the nature of the distribution of capital release and how it may change over time are the foundations of enterprise valuation.

Aquisition Value

Table 1: Current Excess Value describes the initial valuation of the enterprise.[3]

Table 1: Current Excess Value

Source of Value	*Amount*
Current market value	5,534,719
Less:	
Current min sufficiency level	1,591,549
Current ultimate loss for lines not analyzed	2,565
Net Current excess value	3,940,605
PV E(capital release) (@.05)	297,109

[2] The concept of capital attribution used in this paper avoids the accounting distinction between liabilities and earmarked surplus. The author rejects the concept that equity that has been segregated is still equity; rather, it is a liability in the sense that the real liability has been misstated during the course of accounting ministrations. Whether there are different legal attributes afforded liabilities and segregated surplus is irrelevant to this paper. This paper is concerned with risk-adjusted measures of future obligations. The author refers to them as liabilities even if an accountant does not.
[3] The data used in this paper were provided to authors participating in the 2001 DFA Seminar Call Papers contest held by the Casualty Actuarial Society. They include Schedule P information for a hypothetical insurance company and other financial statements.

The table indicates that a large proportion of current value is excess; it could be distributed to stockholders as a dividend without impairment to the enterprise. The minimum sufficiency level is risk-adjusted and will provide for future obligations. We now turn to a discussion about what this minimum sufficiency level is and how it is determined.

Distribution of Minimum Sufficiency Levels

The minimum sufficiency level shown in Table 1: Current Excess Value is the amount, which with a confidence level of 0.9, will grow through investments and be sufficient to cover future claims payments.[4] This study assumes that investment returns are described by a multinormal distribution.[5] The investment data are those used in one of the author's prior articles [Scheel, et al, 2000]; they are summarized in Appendix A: Review of Data Sources. The proxy investment choices cover a broad range, including fixed obligations, collateralized mortgage obligations, foreign and domestic bonds and equities.

A full description of how the distribution of minimum sufficiency levels was determined appears in Scheel [2001]. A brief summary is given here. Non-linear optimization methods were used to evaluate portfolio weights and determine a reserve that is risk-adjusted both for uncertainty in claims amounts and uncertainty in investments; this reserve is the minimum sufficiency level. Managerial decision-making established an acceptable level of confidence in a probabilistic sense. Within these levels of confidence, the minimum sufficiency level of assets will grow to a target amount that will both fund claims for the period and immunize (within a specified confidence level) the company both from fluctuations in investment return and remaining claims. The minimum sufficiency level is a risk-adjusted reserve that contains capital attribution. Additional capital is needed only to assure margins beyond those already built into the minimum sufficiency level,or for other risk-bearing purposes.[6]

Targets for Sufficiency

Targets for the required growth levels were obtained from simulations of correlated link ratios for ultimate losses and payment patterns.[7] They were applied to current loss triangle diagonals to provide estimates of: (a) changes in ultimate loss, and (b) the relationship between paid and ultimate loss for each accident period during the forecast development periods. The statistical foundation for the simulation was the use of bootstrap methods applied to loss triangles.

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[4] Detailed descriptions of how sufficiency levels were calculated appear in the companion paper [Scheel, 2001].

[5] The hypothetical insurance company has invested assets but their efficacy was not examined in this paper. Rather, the analysis considers current assets to have been rebalanced into the proxy portfolio. The paper describes in detail how an optimal portfolio was constructed and rebalanced over time using the rich set of securities in the proxy portfolio.

[6] This study did not include all joint costs associated with claims and, therefore, overstates net current excess value. Only claims costs included in Schedule P paid losses have been considered.

[7] Correlation among lines of business was considered for the determination of each link. However, correlation among different development periods either within a line of business or among lines of business was not considered.

The initial step involved multivariate bootstrapping using the link ratios for ultimate and paid loss triangles. This bootstrap was done in multivariate fashion to measure correlation among lines of business in ultimate links. The next step used the estimates of means and covariances obtained from the bootstrapping in a multinormal simulation. This simulation produced ultimate link ratios for forecasting changes in ultimate loss. Then, a secondary simulation elicited the speed of claims payment. These simulations were the source of cash flow during the forecast period.[8]

The forecast period for paid loss cash flow extended ten years; these paid losses were discounted at a risk-free rate. Many scenarios were derived for loss payment cash flow. These were discounted and the result was a probability distribution of end-of-period target sufficiency levels. A chance-constrained target was measured with this probability distribution; it is referred to as the *target sufficiency level.*

The target sufficiency level was the upper confidence level associated with the .8 percentile of the present value of future claims cash flow. In this case, the upper .9 confidence point of the percentile was used.[9] Other risk tolerances would lead to different levels; but the fundamental approach taken to firm valuation would be unchanged.

The target sufficiency level is similar to a conventional GAAP reserve calculation because cash flow is discounted.[10] A conservative interest rate was used in the discounting of future claims obligations. Cash flow measurement for paid losses followed from simulations of paid-loss/ultimate ratios. First, link ratios were simulated for transition in ultimate loss estimates across calendar periods. Then, payments were generated based on the simulated ultimate loss.

Sufficiency Levels for Investments

The second phase of the reserve determination is the translation of the target (end-of-period) sufficiency level into a beginning-of-period sum required for investment. This sum is risk-adjusted for investment uncertainty. It is the *minimum sufficiency level*, and non-linear optimization methods are used to calculate it. It is an invested amount that grows with income to the target sufficiency level within a managerial-selected confidence level.

[8] This study did not deal with unearned premiums or any other accrual items. The only source of cash flow was assumed to be claims payments as they are reflected in estimates of ultimate loss.

[9] The percentile is binomially distributed. Its confidence band is a function of sample size. Simulations used in this paper were always at least 2,500 iterations so the normal approximation could be used to evaluate the confidence band for percentiles. See John C. Freund, *Mathematical Statistics*, 1971, Prentice-Hall, Inc., p. 276.

[10] Reserves are not always discounted for GAAP. In fact, the GAAP rules can be interpreted as either allowing or not allowing discounting. But, were discounting to be demanded, the target sufficiency levels are an abstraction from the probability distribution of GAAP reserves.

During the second phase, investment scenarios were generated using an asset model.[11] During a trial solution, a profile of weights is tested by the optimizer. The trial profile is applied to every simulated scenario to ascertain a portfolio return for it. The asset weights in the profile were constrained by the optimizer to eliminate the possibility of short sales.[12]

The simulated portfolio return is a growth factor for the invested sum, the minimum sufficiency level. In a converse manner, it can be used as a discount rate to convert the target sufficiency level into the required minimum sufficiency level. The present value is the minimum sufficiency amount that is a required; it is a beginning-of-period amount. It leads to the end-of-period target level within prescribed confidence levels. The distribution of these present values is obtained for all asset scenarios using the trial profile provided by the optimizer. A chance-constrained limit of this distribution was returned to the optimizer as an objective value.

The optimizer repeats the process with different sets of proxy investment weights until the objective function is found to be a minimum. In summary, the optimizer minimized the invested sum need to provide risk-adjusted growth to a target sufficiency level. The objective function for the optimizer is a confidence level of the probability distribution of the discounted value of the target sufficiency level. There is a simulated set of returns for all asset categories, and apportionment among them is given by the optimizer. Because (a) we know the end-of-period target and (b) we have a simulated set of portfolio returns, the discounted beginning-of-period invested amount can be ascertained for any desired confidence level.[13] We refer to this risk-adjusted reserve as the *minimum sufficiency level.*

Capital Attribution

The minimum sufficiency levels contain attribution of capital. The release of that capital is of interest because it can be a source of future stockholder dividends.

Capital may be released when minimum sufficiency levels grow in an ***expected*** fashion that leads to sums greater than the target sufficiency level. There is an additional expected capital release because expected claims are less than the conservative target sufficiency level that immunizes the company.

Capital release during period t is defined by (1.1).

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[11] The choice of an asset model was not particularly important for this paper; any asset model that produces investment scenarios for a broad spectrum of securities could work. The one used here was a multi-variate normal simulation. Of course, different models for investment and claims scenario generation would produce different valuations.

[12] Investments were not constrained to limits imposed by regulation. For example, no constraints were set on the proportion of assets invested in equities. These and other similar limitations could be added to the constraints used by the optimizer.

[13] It was assumed that claims were incident at the end of each period. In this analysis, periods were calendar years.

$$SR_t = MSL_t(1 + p_t) - (C_t + MSL_{t+1}) \qquad (1.1)$$

where:

SR_t = capital released at the end of period t,

MSL_t = minimum sufficiency level at the beginning of period t,

p_t = portfolio return during period t,

C_t = claims payments during period t.

Capital release, SR_t, is a random variable. We now turn to procedures used to estimate the probability distribution for capital release each period.

Distribution of Capital Release

The capital release random variable is a function of two other random variables that are independent: investment growth, p_t, and paid losses, C_t. The distribution of each of them is found through modeling. The asset model provides scenarios for invested assets. The liability model provides scenarios for losses. The minimum sufficiency levels are determined using both the investment scenario and the paid loss generators.

The steps are:

1. Randomly generate an investment scenario for the period. Assume the investment portfolio backing the minimum sufficiency level is apportioned according to the optimized profile used to measure it. Determine the period's return, p_t, for the weighted portfolio.
2. The result of step 1 is the growth rate of minimum sufficiency assets. This end-of-period value, $MSL_t(1 + p_t)$, is used to pay the period's claims[14] and fund next period's minimum sufficiency level.
3. Paid claims for the period, C_t, are obtained from the liability simulator. A payments scenario is randomly generated.[15]
4. The results of steps (1)-(3) are used in equation (1.1) to calculate SR_t, a simulated observation of capital release.
5. Steps (1)-(4) are repeated many times to build up the distribution of capital release for the t^{th} period.
6. This method is extended through remaining periods of the analysis.

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[14] This study simplifies the analysis by assuming claim payments occur at the end of the period.

[15] The loss generator could be any of the conventional ones deployed in popular dynamic financial analysis models. But, it should the same one used to evaluated the minimum sufficiency levels. The one used in this study relies on bootstrapping and involves no other data than published information. See Scheel [2000] Because it relies on bootstrap methods, there is no explicit measurement of either process or parameter risk that ordinarily appears in dynamic financial analysis

The results are shown in Table 2: Distribution of Capital Release. The discounted present value of the expected capital release is one of the items shown in Table 1: Current Excess Value.

Table 2: Distribution of Capital Release

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6	Period 7	Period 8	Period 9
Mean	133,441	59,222	51,999	39,575	17,900	15,222	8,490	6,766	4,395
Standard Deviation	113,283	45,983	48,344	36,444	13,694	12,471	7,220	5,709	2,895
Median	127,131	58,012	50,623	37,486	17,596	14,711	8,308	6,558	4,360
5 percentile	-42,909	-15,214	-23,636	-16,731	-4,230	-4,442	-3,078	-1,956	-347
10 percentile	-6,434	1,953	-8,050	-4,987	402	-195	-905	-535	678
25 percentile	54,018	27,826	18,292	13,848	8,234	6,438	3,511	2,681	2,431
75 percentile	208,578	89,464	83,842	63,171	27,064	23,719	13,385	10,634	6,361
90 percentile	280,925	118,269	114,969	88,253	35,390	31,348	17,719	14,450	8,200
95 percentile	325,144	135,291	134,078	101,404	41,021	35,717	20,375	16,522	9,103

What is the Source of Expected Capital Release?

This is an interesting question. If we were to hold a minimum sufficiency level equal to the expected value of the runoff, would there be no expected capital release? What is the foundation for an expectation of capital release?

Ex ante, the minimum sufficiency levels are conservative, chance-constrained values. As defined in this paper, there are two sources of such conservatism: (1) the target sufficiency level is higher than the expected present value of losses and (2) the beginning level assets is higher than the expect level needed to achieve this deferred target. If the first target was based on the expected present value of claims (and, the expected value was a riskless rate of return during the holding period) *and* if the value of assets held were expected to yield this target amount, there would be no *expectation* of capital release.

The expected source of capital release arises from contingency margins both in the target sufficiency levels and in the required assets backing them—the minimum sufficiency levels. Were such levels to be based strictly on expectations, capital release still could occur. But it would arise from fortuitous events—there would be no expectation of capital release. It means that in an expectation sense the minimum sufficiency levels set at fair value (expected) levels have no expectation of being either excessive or deficient. Sufficiency, at least in the context of this paper and the setting of reserves, requires a higher standard. It requires that there be an expectation of capital release. This expectation is the foundation of insurer solidity.

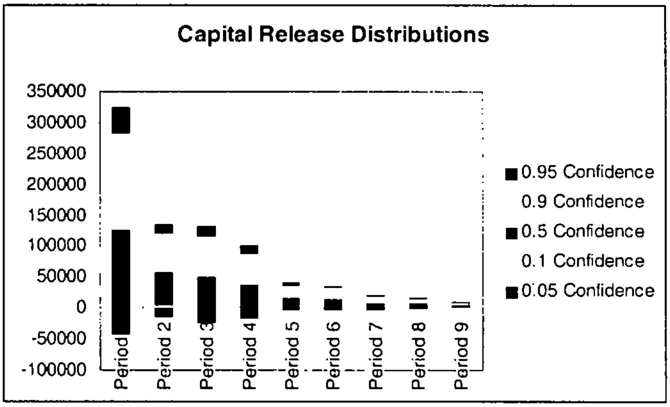
This result becomes clearer if we switch from expected value to median value. Were the distributions to be normally distributed, the mean expectancy and median merge. Under these circumstances, it becomes apparent that either capital release or deficiency has a 50:50 chance of emerging. Neither median-based nor expected value-based estimates seem to be reasonable standards. A regulatory context of sufficiency seeks solidity of the enterprise and the paramount preservation of policyholder interests. This standard imposes conservative chance-constrained levels. It is this conservatism in using high confidence levels that leads to an expectation of capital release.

Value of the Enterprise

The value of the enterprise consists of the current excess value and subsequent capital release. Table 1: Current Excess Value identifies these sources of value. The contribution to enterprise value from future ***expected*** capital release is discounted and added to initial excess value to produce the total net present value. As can be seen in the table, this enterprise value is a high percentage of the current market value of assets.

Capital release will not unfold as expected. Other ways of expressing value are to examine percentiles of the capital release distributions. The capital release percentiles are shown in Figure 1: Value of the Enterprise. Both the Figure and Table show that there is slight positive skewness in the distributions. The median value is less than the expected value for all periods. Because there is a 50:50 chance that capital release will be lower than expected, the valuation of the enterprise might be considered somewhat less. However, risk lovers may see great value in the enterprise if windfall probabilities end up causing some of the high capital release values that lie in the tails of these distributions.

Figure 1: Value of the Enterprise



Caveats Regarding this Study

The following shortcuts were made:

1. Ordinarily a DFA analysis would use the existing assets of an enterprise so that rebalancing could trace their disposition. The specific assets held would have to be modeled as part of the investment scenario generation process. This may require modeling other business climate aggregates that are thought to impact on investment returns for these securities. This study assumes that a rich set of investment aggregates serve as a proxy for the real assets. The focus of this study was on the rebalancing that might be required were this set of investment proxies to be used as actual investments. Maintenance of the existing portfolio or how it might be rebalanced was beyond the scope of this study. The implicit assumption is that all assets, valued at market, could be reinvested immediately in the proxy portfolio used in this study.
2. No adjustments for uncertainty respecting inflation were made. In fact, no business scenario generation, other than investment returns, was done in this analysis.
3. The possibility of future inflation differing from expected inflation was not considered. There was no common economic tie binding future loss projections and future asset valuations.

4. Future business writings were not considered.
5. Tax frictions were not analyzed.
6. Financial statement generation was limited to cash flow analysis. Acquisition and integration of an insurance entity would necessitate the modeling of consolidated statements and many aspects of line-of-business integration were the parent company also an insurer. Consideration of these effects on total risk bearing may have a material impact on valuation.
7. The study did not attempt to harvest uncertainty in non-claims accruals or financial accounts other than claims and investments. The total risk of the enterprise might be materially affected were other sources of uncertainty to be considered.
8. Administration and other expenses were ignored.
9. The optimizer constraint set did not limit the extent of portfolio allocations for specific asset classes such as equities or international securities within the proxy portfolio. In general, the mix of assets in the optimal portfolios was under 20 percent equities. Allocations to mortgaged backed securities got as high as 18 percent. These and some allocations to convertible and high yield bonds may not be consistent with statutory limitations on such asset classes. These asset class-specific constraints were deemed to be beyond the scope of this study; however, appropriate changes could be made in the constraints set to limit the asset allocation weight for a class of investments.

Appendix A: Review of Data Sources

This paper uses monthly time series of asset class total returns. A selection of broad asset classes typical of P&C insurance company asset portfolios was chosen for examination. The time series all begin January 1, 1970. However, certain asset classes (e.g. mortgage backed securities) do not have a history that extends back this far. For these classes the time series were backfilled to the January 1, 1970 start date by an investment consultant. The backfill process was based on a consideration of the market conditions of the time (e.g. interest rates, fixed income spreads, inflation expectations) and how the particular sector would have performed given those market conditions. The Start Date in Table 3 refers to the date historical data begins.

Table 3 Asset Components

Class	Code	Source	Start Date
International Equities	EAFEU	MSCI EAFE Index	1/1970
International Fixed Income	INTLHDG	JP Morgan Non-US Traded Index	1/1970
Large Cap Domestic Equities	S&P5	S&P 500 Index	1/1970
Cash	USTB	90 Day US Treasury Bill	1/1970
Mid Cap Domestic Equities	RMID	S&P Mid Cap 400 Index	1/1982
High Yield	HIYLD	CSFB High Yield Bond Index	1/1986
High Yield	HIYLD	CSFB High Yield Bond Index	1/1986
Convertible Securities	CONV	CSFB Convertible Index	1/1982
Convertible Securities	CONV	CSFB Convertible Index	1/1982
Corporate Bonds	LBCORP	Lehman Brothers Corporate Bond Index	1/1973

Class	Code	Source	Start Date
International Equities	EAFEU	MSCI EAFE Index	1/1970
Government Bonds	LBGOVT	Lehman Brothers Government Bond Index	1/1973
Mortgage Backed Securities	LBMBS	Lehman Brothers Mortgage Backed Securities Index	1/1986

Appendix B: Examples of Capital Release Scenarios

	Minimum Sufficency Level (2)	*Target Sufficiency Level* (3)	*Paid Claims* (4)	*Optimized Portfolio Return* (5)	*Simulated EOP Value* (6)=(2)*[1+(5)]	*Next Period's MSL* (7)	*Capital Release* (8)=(6)-[(4)+(7)]
Period 1							
1	1,591,549	1,683,785	571,004	0.1512	1,832,128	1,064,347	**196,777**
2	1,591,549	1,683,785	550,142	0.1563	1,840,355	1,064,347	**225,866**
3	1,591,549	1,683,785	600,991	0.0353	1,647,748	1,064,347	**-17,590**
4	1,591,549	1,683,785	598,510	0.2693	2,020,096	1,064,347	**357,238**
5	1,591,549	1,683,785	510,979	0.2408	1,974,870	1,064,347	**399,544**
Period 2							
1	1,064,347	1,128,672	448,891	0.0244	1,090,309	680,513	**-39,095**
2	1,064,347	1,128,672	400,138	0.0264	1,092,403	680,513	**11,751**
3	1,064,347	1,128,672	455,988	0.1262	1,198,640	680,513	**62,139**
4	1,064,347	1,128,672	402,754	0.1181	1,190,084	680,513	**106,817**
5	1,064,347	1,128,672	436,252	0.1090	1,180,324	680,513	**63,559**
Period 3							
1	680,513	722,708	225,438	0.0137	689,836	445,919	**18,479**
2	680,513	722,708	233,072	0.0765	732,601	445,919	**53,610**
3	680,513	722,708	281,247	0.0854	738,658	445,919	**11,493**
4	680,513	722,708	273,722	0.0969	746,454	445,919	**26,813**
5	680,513	722,708	292,232	0.1032	750,721	445,919	**12,570**
Period 4							
1	445,919	471,934	156,646	-0.0228	435,732	296,149	**-17,063**
2	445,919	471,934	159,586	0.2777	569,748	296,149	**114,013**
3	445,919	471,934	159,446	0.0511	468,689	296,149	**13,094**
4	445,919	471,934	158,879	0.0589	472,172	296,149	**17,144**
5	445,919	471,934	193,995	0.0627	473,873	296,149	**-16,271**

	Minimum Sufficency Level *(2)*	*Target Sufficiency Level* *(3)*	*Paid Claims* *(4)*	*Optimized Portfolio Return* *(5)*	*Simulated EOP Value* *(6)=(2)*[1+(5)]*	*Next Period's MSL* *(7)*	*Capital Release* *(8)=(6)-[(4)+(7)]*
Period 5							
1	296,149	313,611	109,267	0 0560	312,726	198,432	**5,028**
2	296,149	313,611	111,052	0 0961	324,596	198,432	**15,113**
3	296,149	313,611	110,854	0 1495	340,437	198,432	**31,152**
4	296,149	313,611	112,634	0 1691	346,232	198,432	**35,165**
5	296,149	313,611	106,550	0 1364	336,550	198,432	**31,568**
Period 6							
1	198,432	210,181	69,061	0 1341	225,036	129,609	**26,366**
2	198,432	210,181	71,070	0 1125	220,760	129,609	**20,082**
3	198,432	210,181	69,934	0 1268	223,588	129,609	**24,046**
4	198,432	210,181	77,007	0 2471	247,473	129,609	**40,857**
5	198,432	210,181	69,786	0 0557	209,484	129,609	**10,089**
Period 7							
1	129,609	137,047	51,764	0 0315	133,689	78,977	**2,949**
2	129,609	137,047	59,632	0 1125	144,189	78,977	**5,581**
3	129,609	137,047	51,172	0 1607	150,434	78,977	**20,285**
4	129,609	137,047	55,113	0 1085	143,669	78,977	**9,579**
5	129,609	137,047	59,218	0 1393	147,666	78,977	**9,471**
Period 8							
1	78,977	83,439	47,517	0 2188	96,254	37,711	**11,026**
2	78,977	83,439	42,505	0 1290	89,162	37,711	**8,946**
3	78,977	83,439	45,269	0 1612	91,705	37,711	**8,725**
4	78,977	83,439	45,651	0 0859	85,763	37,711	**2,400**
5	78,977	83,439	43,289	0 0134	80,037	37,711	**-964**
Period 9							
1	37,711	39,879	32,317	0 0686	40,297	0	**7,980**
2	37,711	39,879	37,275	0 0670	40,239	0	**2,963**
3	37,711	39,879	39,955	0 1242	42,394	0	**2,439**
4	37,711	39,879	37,788	0 0741	40,507	0	**2,719**
5	37,711	39,879	32,851	0 1188	42,192	0	**9,341**

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