

**Residual Market Pricing**  
*Richard B. Amundson, F.C.A.S.*

## RESIDUAL MARKET PRICING

Richard Amundson

### Abstract

Residual market plans often review their rates based on the experience of the plans themselves. The typical result is an indication for a large increase, which the regulator then judgmentally reduces. To the extent that equilibrium exists between voluntary and residual markets, it results from ignoring the indications. Plans' experience can call for rate decreases as well as increases, especially with no allowance for profit. *Indications for decreases are politically harder to ignore, and could destroy the voluntary market if followed.*

Break-even residual market pricing, if truly followed, has unpredictable consequences on prices and market shares for the residual and the voluntary markets. This paper proposes an alternative to break-even pricing. With input from all concerned, a state should first establish specific goals for the residual market plan in terms of market share, burden on insureds in the voluntary market and maximum surcharge for insureds in the plan. Regulators can then set plan prices at a consistent level above voluntary prices to meet the established goals.

### Biography

Richard Amundson is Actuarial Director with the Minnesota Department of Commerce. Among other responsibilities, he directs and participates in the actuarial review of loss reserves and rate filings of property-liability companies licensed in Minnesota. He holds a bachelors degree in mathematics from Antioch College and a masters degree in mathematics from the University of Minnesota. He became a Fellow of the CAS in 1986 and is also a member of the American Academy of Actuaries and the International Actuarial Association.

Mr. Amundson served for four years on the CAS Examination Committee. He has made presentations before numerous groups, including the Twin Cities Actuarial Club, committees of the Minnesota Legislature, and working groups of the National Association of Insurance Commissioners. In addition he has spoken to student groups, both at the high school and college level, on careers in actuarial science.

## RESIDUAL MARKET PRICING

### Abstract

Residual market plans often review their rates based on the experience of the plans themselves. The typical result is an indication for a large increase, which the regulator then judgmentally reduces. To the extent that equilibrium exists between voluntary and residual markets, it results from ignoring the indications. Plans' experience can call for rate decreases as well as increases, especially with no allowance for profit. Indications for decreases are politically harder to ignore, and could destroy the voluntary market if followed.

Break-even residual market pricing, if truly followed, has unpredictable consequences on prices and market shares for the residual and the voluntary markets. This paper proposes an alternative to break-even pricing. With input from all concerned, a state should first establish specific goals for the residual market plan in terms of market share, burden on insureds in the voluntary market and maximum surcharge for insureds in the plan. Regulators can then set plan prices at a consistent level above voluntary prices to meet the established goals.

## RESIDUAL MARKET PRICING

### 1. INTRODUCTION

#### 1.1. *The First Paradox*

A state insurance commissioner once decided to freeze the auto insurance rates for the drivers in the state's assigned risk plan (ARP). For several years thereafter the rates for those drivers did not budge. During that time the ranks of ARP steadily grew. The kinder the state was toward the drivers insured by ARP, the bigger ARP got.

Time brought a new commissioner, one who believed that the ARP drivers should pay a price more directly related to their cost. For several years thereafter the rates for those drivers rose sharply. During that time, the ranks of ARP steadily dwindled.

At the end of the rise and fall of ARP, everything was more or less as it had been in the beginning. But the years in between were curious. During the time that the rates were frozen, despite continuous inflation in the general economy, the loss ratios of those drivers were not really deteriorating. This seemed to be evidence that commissioner number one was doing the right thing.

Commissioner number two had rather the opposite experience. After each big rate increase, ARP's results weren't much better than they had been the year before. ARP was smaller, but the goal that it be self-supporting seemed as far away as ever.

#### 1.2. *The Actuary Explains All*

None of this was too hard to explain. There are degrees of undesirability. In the beginning insurers rejected only the very most unwanted drivers—the worst of the worst. They were happy to write a certain borderline driver for \$1000. But when the voluntary market price reached \$1100, ARP, whose rates hadn't budged, snapped this driver up at \$1050. These borderline drivers moving into ARP were the best of the worst, and they

improved the quality of ARP's book of business as it grew. Exactly the opposite occurred when ARP shrank.

After years of increases, when things were back to the original balance between voluntary and assigned risk, the indications for ARP were as high as ever. An actuary wrote a memo explaining why this was, and what one might have to do in the future to keep everything in balance. To continue following indications blindly seemed sure to lead to the disappearance of ARP — not a bad idea in the eyes of some, but not politically viable in this case. The presence of a contingency factor in the analysis posed a problem. It added to the price of each policy, not unlike a profit margin, even though this was non-profit business. ARP rates tended to rise mercilessly, and the contingency factor only exacerbated the tendency, pushing rates for the dwindling number of policyholders to truly unaffordable levels. It seemed a good idea to get rid of the contingency factor.

### 1.3. *Out Of The Frying Pan And Into The Fire: Paradox Regained*

Some time after this, the workers compensation assigned risk plan submitted an analysis of its experience. An amazing thing was happening: this ARP, whose rates were already low, needed a rate *decrease*. The actuary was puzzled by this anomaly. Whether this was just random noise or a true reflection of the risks in ARP, it seemed unwise for the rates to get too close to those of the voluntary market. The voluntary market charges for the same coverages as ARP but in addition charges for profit because of the risk of writing business. The ARP analysis had no charge for risk, even though, of course, the ARP business is just as risky as the voluntary business. This gave ARP a rate advantage. It could pick up market share and constantly improve its book. The voluntary market could eventually disappear. Maybe the analysis should include a contingency factor.

All this seemed strangely familiar. The same actuary (who happens to be the author of this paper) had argued, not so long before, against a contingency factor in the case of auto assigned risk. What was wrong? What was the truth?

#### 1.4. *The Scales Fall From Our Eyes*

The truth is all of the above. Both of these scenarios can happen, even though they are complete opposites. The problem lies in assuming that one can base the rates of a residual market plan on the plan's own experience and thereby achieve an acceptable equilibrium with respect to size and price of the plan and the voluntary market. This paper shows that a residual market plan whose prices are based on its own experience has no mathematical certainty of reaching any acceptable equilibrium. To achieve the goals normally desired for an assigned risk plan, the state should base the plan's rates on the voluntary market rates and not on the plan's own experience.

## 2. A MODEL OF RESIDUAL MARKET PRICING

### 2.1. *Some Assumptions*

We will look at residual market plans that set prices to break even based on their own experience. Of course, with break-even pricing a plan may still realize profits or losses. The plan design may or may not give the profit to insurers, but it will virtually always give insurers the loss. The examples in this paper assume that insurers get the profit as well as the loss. The conclusions of the paper are still valid if insurers don't get the profit, but the examples are a bit more complex.

We'll ignore self-insurance for the moment. Assume that all employers, drivers, etc. must buy insurance and that they have two options: an insurance company in the voluntary market or ARP (our surrogate for all residual markets). Assume further that there is a full spectrum of quality among insureds in the sense that within each classification there is a continuum of expected losses per exposure: there are insureds with very few losses expected for each exposure unit, and there are others with very high expected losses, and there is everything in between.

Let's look at a simplified financial model that illustrates some important relationships between the residual and voluntary markets. Suppose there is no ARP. Put yourself in the shoes of an insurer who needs a \$100 investment in surplus to take on \$200 of expected loss at the end of the coming year. Suppose that there are no

expenses. Suppose that you can earn 5% risk-free on invested assets and that given the uncertainty in the expected losses you need a 15% return on the venture. If you collect \$200 in premium up front and invest it along with your \$100 of surplus, you will earn \$15 during the year. If losses materialize as expected, you will pay off \$200 at year-end and will be left with your original \$100 plus \$15 of investment income. The expected return is exactly what you need.

Now let's bring ARP into the picture. In fact let's go to the opposite extreme and assume that ARP collects *all* the premium and pays the entire \$200 of loss from the prior paragraph. You are now an insurer with no voluntary premium, but with continued responsibility for potential bottom line losses of ARP. Do you still need the \$100 in surplus that you needed when you were the one collecting premium and paying claims? Yes, you do. That \$100 was needed to provide some level of protection against insolvency, and all the risks that it protected against are still around. Not only are they all still around, but they are all on your back: ARP carries no surplus and assesses you for any losses at the end of the day whether they arose from excessive claims or investments or no matter what.

This is a game with little appeal. Remember that you have the alternative of getting a return of \$5 with no risk. You might want to add some risk in exchange for an increased return. In this venture, though, unless you share in ARP's profit, you add risk in exchange for a *decreased* return, moving yourself squarely off the efficient frontier. You'd be interested only if you got the profit in addition to the loss. In order to realize the full profit, ARP must charge the full \$200 of premium. The key points are that, *no matter what market share ARP has*, the full \$100 of surplus is still needed and the full \$200 of premium is still needed.

This would be true even if the state took over all the risk and there were no insurers in the equation. The risk takers, whether taxpayers or policyholders, would put up the surplus and reap the rewards explicitly or implicitly. One might argue that a state with its larger spread of risk could get by with less surplus. Not necessarily. Not all risk is diversifiable, and a lot of states aren't much bigger than Orange County. They are not invulnerable. The efficiencies that result in theory from combining many operations under one control don't always pan out in reality whether the control is governmental or private

monopoly. Diversity in the marketplace is still important. It is to foster marketplace diversity that residual market plans have a legitimate function in the first place; it would be rather ironic if the residual markets destroyed the diversity of the insurance market in the process. But this is a digression; let's return to the typical scenario.

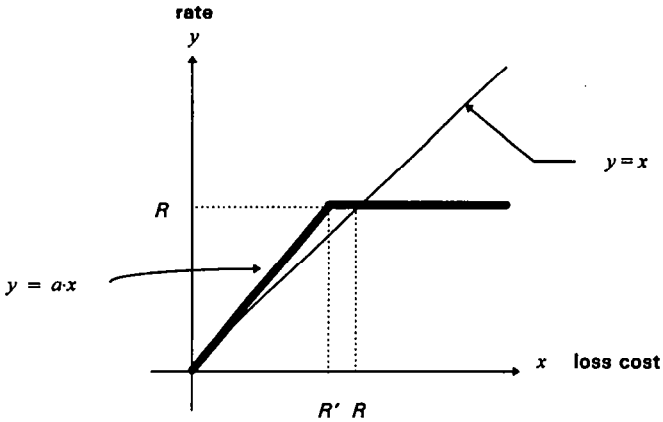
Typically ARP will have part of the market and insurers will have the rest. Let's look at a single class. Suppose ARP charges a rate of  $R$  per exposure. ARP may vary its rate somewhat due to merit rating, but, unlike the voluntary market, it doesn't do any underwriting, so it won't charge the variety of rates of the voluntary market. Assume that ARP charges the same rate to all insureds in the class. The voluntary market by contrast, through the forces of underwriting and competition, charges a rate proportional to expected losses. (This will result from a combination of schedule rating, experience rating, retro rating, and underwriting by companies with differing rates and differing niches.) Remember that there is a whole spectrum of expected losses. For the moment let's assume the underwriting cost is negligible; it won't change the result to assume it is significant, but it clutters the argument. Let's say the market price is  $a \cdot x$ , where  $x$  is the expected loss. In order to attract any business, the market must charge less than ARP.

## 2.2. *A Natural Limit: Assigned Risk Must Charge Strictly More Than Market Average*

The graph in Figure 1 illustrates the market in equilibrium. The  $x$ -axis represents expected losses per exposure, and the  $y$ -axis represents premium per exposure (i.e., rate). Keeping with the simplified assumption of the earlier example, we ignore expenses and assume that premium equal to expected losses will suffice to pay claims and leave appropriate profit (from investments) for insurers. In an unfettered market the line  $y = x$  represents the appropriate relationship between premium and loss. With ARP charging a rate of  $R$  and the voluntary market charging a rate of  $a \cdot x$ , the actual rates charged are represented by the bold line on the graph. If the expected loss is greater than  $R'$ , where  $R' = R/a$ , the risk will be written by ARP at a rate of  $R$ . If the expected loss is less than  $R'$ , the risk will be written in the voluntary market.



FIGURE 1



Insureds whose expected losses are less than  $R$  pay more than they would in a completely free market. Insureds whose expected losses are greater than  $R$  pay less.

We will show that if  $R \leq L$ , where  $L$  is the average expected loss, there is no solution to the pricing problem of insurers. That is, there is no premium they can charge which would attract customers and which would give them enough to pay claims and adequately reward them for the risks they would be taking.

If  $R > L$ , there is a solution, but it is not necessarily robust. If  $R$  is allowed to increase or decrease depending on ARP's own experience, ARP will most likely not be in equilibrium: it will grow or shrink depending on the distribution of expected losses.

For the case  $R \leq L$ , it's really almost self-evident that insurers can't compete. If there are a total of  $n$  insureds, the total premium needed is  $nL$ . If ARP has  $m$  insureds, it collects  $mR$ . The voluntary market must then collect a total of  $nL - mR$  from the  $n - m$  policies that it insures. If  $R \leq L$ , then  $(nL - mR)/(n - m) \geq R$ ; that is, the voluntary market would have to charge on average *at least* as much as ARP.

More generally, if  $F$  is the distribution function of the expected losses per exposure, we have

$$L = \int_0^{\infty} x dF = \int_0^{R/a} ax dF + \int_{R/a}^{\infty} R dF. \quad (2.1)$$

Equation (2.1) merely says that the expected losses must equal the premium collected by the voluntary market plus the premium collected by ARP. The insurers' pricing problem is to solve for  $a$ . Set

$$g(a) = \int_0^\infty x dF - \int_0^{R/a} ax dF - \int_{R/a}^\infty R dF. \quad (2.2)$$

Solving equation (2.1) for  $a$  is equivalent to finding a zero of the function  $g$  defined by equation (2.2).  $g$  is a continuous, monotone decreasing function on the interval  $(0, \infty)$ , so it has at most one zero. If it ever changes sign, it has exactly one zero.

$$g(1) = \int_0^\infty x dF - \int_0^R x dF - \int_R^\infty R dF = \int_R^\infty (x - R) dF > 0.$$

$$g(N) = \int_0^\infty x dF - \int_0^{R/N} Nx dF - \int_{R/N}^\infty R dF.$$

$$\lim_{N \rightarrow \infty} \int_0^{R/N} Nx dF = 0, \quad \text{and} \quad \lim_{N \rightarrow \infty} \int_{R/N}^\infty R dF = R, \quad \text{so}$$

$$\lim_{N \rightarrow \infty} g(N) = L - R,$$

which is negative if and only if  $L < R$ . Thus if  $L \geq R$ , there is no  $a$  for which  $g(a) = 0$ , and equation (2.1) has no solution. If  $L < R$ , there is a unique solution.

(If we removed the requirement that there exist insureds with arbitrarily large expected losses, our conclusion wouldn't change. For values of  $R$  greater than the largest expected loss the solution would be  $a = 1$ , and all the business would be in the voluntary market. If we removed the requirement that there exist insureds with arbitrarily small expected losses, there might be some degenerate solutions. In that case  $g$  would no longer be monotone decreasing on the entire interval  $(0, \infty)$ , but only on  $(0, R/b)$ , where  $b$  is the smallest possible expected loss — more precisely,  $b = \inf\{x : F(x) > 0\}$ . For all  $a > R/b$ , we'd have  $g(a) = L - R$ , so for  $R = L$  there would be infinitely many solutions of the equation  $g(a) = 0$ . These solutions are rather trivial; they are simply all multipliers,  $a$ , large enough to charge the tiniest risk more than  $R$ , so that ARP writes all the business.)

What this has demonstrated so far is that, however ARP sets its rates, it should not simply gear them to the average risk. They must be higher, otherwise the voluntary market will deconstruct. Is there any danger that the rates will be geared to the average risk? There certainly is, especially for segments of the market, such as small employers. Not only may the ARP rates be pegged to the average risk, but they may be pegged to it with no allowance for profit, which leaves the voluntary market no alternative but to abandon the segment in question.

### 3. THE ELUSIVE SEARCH FOR EQUILIBRIUM

#### 3.1. *The Rate Review*

Let us suppose that  $R > L$ , and that the market has spent some time in equilibrium in the sense that the relative prices and market shares of ARP and the voluntary market have remained stable. Now the time has arrived for ARP to review its rates. What happens? Look back to the graph in Figure 1. ARP has been overcharging insureds with expected losses between  $R'$  and  $R$  and undercharging those with expected losses greater than  $R$ . The net effect is an undercharge, which has been made up by the voluntary market overcharging all its insureds.

Because ARP has been undercharging, shouldn't its experience indicate that it needs an increase? Not necessarily. ARP has been undercharging when one considers the need for profit, but ARP doesn't include a profit margin in its rate analysis. It is possible that ARP has in fact charged enough to pay claims and that its analysis on a non-profit basis will actually show a need for a rate reduction. This is not the normal course of events with residual market plans, but it is possible — and not merely theoretically possible — especially for individual segments of the market. Whether ARP's analysis will show the need for an increase or for a decrease is a function of the distribution of expected losses. One can construct distributions which go both ways.

If ARP uses a market-level profit margin in its analysis, it will of course see the need for an increase. Often Plans do include a "contingency" allowance, which serves somewhat the same purpose and does increase the probability that the analysis will

indicate the need for a rate increase. For just the right distribution and just the right value of  $R$  and just the right contingency factor one can arrive at situations where equilibrium appears to exist, but this will be a very precarious equilibrium. The tendency is rather for continual indications for rate increases, or continual indications for decreases. In the first case, if the indications are followed, ARP will eventually price itself out of existence. In the second case it is the voluntary market which will disappear if the indications are followed. The more common scenario is the first. To the extent that equilibrium is reached, it is generally reached by ignoring the indications: ARP takes lesser increases, amidst grim predictions for the future. The predictions are often a touch exaggerated, but given that this is an inherently unpredictable road to equilibrium, it certainly does open the door to many problems.

The more serious scenario, and fortunately the more rare so far, is the one in which ARP sees a need for a decrease. It is more serious because if ARP follows its indications under this scenario, the voluntary market may well disappear. As in the case where increases are indicated, the only sure way to remain in equilibrium is to ignore the indications, but that's not easy in the face of political pressures to lower rates. Let's look at some simple, finite examples which show the two possibilities.

### 3.2. *Assigned Risk Plans That Follow Their Own Experience May Grow*

First, imagine a distribution of expected losses with ten equally likely possible outcomes: the integers ranging from 20 to 29. Continue with our earlier assumptions. The voluntary market with its diversity of players and underwriting capabilities distinguishes among policies with different expectations and charges accordingly; ARP takes all comers at the same price. The voluntary market sets its prices for a break-even underwriting return, getting its profit from investment income; ARP prices at a 5% discount in order to break even *after* investment income (i.e., ARP is non-profit). Table 1 below summarizes this situation.

$X$  is the random variable representing a policy's expected losses, with its 10 possible outcomes in column 1; each outcome has probability 0.10 (column 2). The data in columns 3, 4, 5 and 6 assume that ARP writes all risks with expected losses greater

than the value of  $x$  in column 1. If ARP writes all the risks with expected losses greater than 20, for example, it will have to charge 23.81 per risk in order to break even (column 4 first row). With investment income it will have  $25.00 = 23.81 \times 1.05$  to pay claims; 25.00 is the average value of expected losses for policies whose expected losses are greater than 20.

**TABLE 1**

(1)	(2)	(3)	(4)	(5)	(6)
If ARP writes all risks with losses greater than $x$					
$x$	$P[X = x]$	voluntary market rate for $x$	ARP rate ( for all )	voluntary market rate for $x + 1$	1: ARP gains $x$ -1: ARP loses $x$ 0: equilibrium
20	0.10	30.71	23.81	32.25	1
21	0.10	25.98	24.29	27.21	1
22	0.10	25.03	24.76	26.16	1
23	0.10	25.02	25.24	26.11	0
24	0.10	25.40	25.71	26.46	0
25	0.10	25.97	26.19	27.01	0
26	0.10	26.65	26.67	27.67	0
27	0.10	27.39	27.14	28.40	1
28	0.10	28.18	27.62	29.19	1
29	0.10	29.00			

24.5 = average expected loss =  $E[X]$

column (4): ARP rate =  $A(x) = E[X | X > x] / 1.05$

column (3): vol mkt rate for  $x = V(x) = ax$ ,

where  $a = (E[X] - A(x) \cdot P[X > x]) / (E[X | X \leq x] \cdot P[X \leq x])$

column (5): vol mkt rate for  $x+1 = V(x) \cdot (x+1) / x = a \cdot (x+1)$

The first entry in the third column, 30.71, is what the voluntary market would have to charge for a risk with expected losses of 20 given that ARP writes everything with greater expected losses. The voluntary market must collect not only the 20 needed to pay the claims and provide for profit for the risk it writes, but it must also collect enough to provide for the profit on all the risks that ARP writes, since it is the voluntary market, not

ARP, that is taking the risk. The premium that the voluntary market and ARP combined collect on average would then be  $0.1 \times 30.71 + 0.9 \times 23.81 = 24.5$ , which is the overall expected loss and what is needed to keep the voluntary market in the game. That forces the voluntary market to charge more than ARP ( $30.71 > 23.81$ ), so the voluntary market would lose the risks with expected losses of 20 to ARP in this situation. The 1 in the sixth column of the first row is a flag to indicate that ARP would capture this risk, too, once it had all the larger risks.

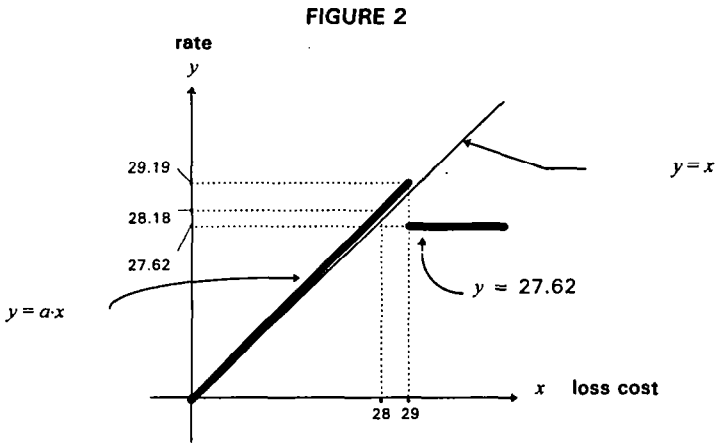
We continue to assume that the voluntary market uniformly loads its expected ARP assessment as a multiple,  $a$ , times the rate it would otherwise charge. The value in the fifth column, 32.25, is what the voluntary market would charge for a risk with expected losses of 21, again given that ARP writes everything with expected losses greater than 20. If the voluntary market rate in column 5 were less than the ARP rate in column 4, then ARP would lose the risks with expected losses of 21 to the voluntary market; in that case the flag in column 6 would be set to -1. A zero in column 6 indicates equilibrium. That occurs when the voluntary market rate for  $x$  is less than the ARP rate which is in turn less than the voluntary market rate for  $x + 1$ ; i.e., column 3 < column 4 < column 5.

Each row represents a distinct rating scenario: the columns of voluntary market rates for  $x$  and  $x + 1$  are not lists of rates all of which would be available at the same time. For example, the table contains two voluntary market rates for risks with expected losses of 21: 32.25 in row 1, column 5, and 25.98 in row 2, column 3. 32.25 is the voluntary market rate if ARP writes everything greater than 20; 25.98 is the voluntary market rate if ARP writes everything greater than 21. The full schedule of voluntary market rates is not displayed for every ARP rate. The table displays only the two rates (in columns 3 and 5) which lie at the boundary of ARP's book of business for the row in question. To know if ARP will grow or shrink or remain in equilibrium, we need only look at the boundary.

For each row of Table 1 one could construct a graph similar to that in Figure 1. Figure 2, for example, corresponds to the row  $x = 28$  of Table 1. As in Figure 1, the bold line segment through the origin represents the rates that the voluntary market charges, while the bold horizontal segment represents ARP's rate. The rates represented by the bold line segments generate an average premium of  $L = E[X]$ , just as in the case of the graph in Figure 1. The obvious difference is that the graph in Figure 2 is discontinuous.

For Figure 1 we required the two segments to join at  $(R', R)$  and we varied  $R'$  to obtain an adequate total premium, without regard for the adequacy of ARP by itself. We showed that for  $R > L$ , there is always an  $R'$  that solves this problem.

For Figure 2 we fixed the left end point of the horizontal ARP segment at 29 on the  $x$ -axis and allowed the segment to move up or down until ARP's premium balanced its own discounted expected losses. The voluntary market segment was then permitted to pivot at the origin to attain the desired *total* premium. The discontinuity in the graph represents a state of disequilibrium between ARP and the voluntary market. ARP is momentarily in balance but the system is not: ARP sets its rates for one group of insureds, but the rates themselves will cause that group to change.



If ARP starts out writing only risks with expected losses greater than 28, it will charge 27.62 ( $29.00 / 1.05$ ). Because the voluntary market must then charge 28.18 for a risk with expected losses = 28, ARP, with its lower price, will take over this level as well. ARP's price (based on its own new experience for the risks with expected losses of 28 and 29) will drop to 27.14. (Row  $x = 27$  of Table 1.) The voluntary market then needs to charge 27.39 for a risk with expected losses of 27, but that still exceeds ARP's rate, so ARP will capture the risks with expected losses of 27, too. Now, based on the experience of risks with expected losses of 27, 28 and 29, ARP will again lower its rate, this time to

26.67. (Row  $x = 26$  of Table 1.) This time, however, because the voluntary market will need only 26.65 for risks with expected losses of 26, it will keep risks with that level of expectation or better, and equilibrium will be reached.

There is nothing robust or inevitable about this equilibrium. Table 2 presents the same scenario as Table 1 except that the probabilities have changed. The overall expected loss is still 24.5, but the distribution is more concentrated. In this case if ARP starts with risks whose expected losses are greater than 28 and bases its future rates on its own experience, it will capture the entire market before reaching equilibrium. ARP will underprice the voluntary market at the high-priced end of the voluntary market's book, causing the highest-priced business to move to ARP. This will improve ARP's experience. ARP will lower its price. The voluntary market will have a higher risk load, which will increase the voluntary market's price. After these price adjustments, ARP will undercut the voluntary market at the next highest level, and business will again flow to ARP. With the loss distribution shown in Table 2, the cycle will continue until ARP has all the business.

**TABLE 2**

(1)	(2)	(3)	(4)	(5)	(6)
If ARP writes all risks with losses greater than $x$					
$x$	$P[X = x]$	voluntary market rate for $x$	ARP rate ( for all )	voluntary market rate for $x + 1$	1: ARP gains $x$ -1: ARP loses $x$ 0: equilibrium
20	0.0028	429.57	23.35	451.05	1
21	0.0095	115.79	23.38	121.30	1
22	0.0316	47.96	23.46	50.14	1
23	0.1053	29.86	23.65	31.16	1
24	0.3508	25.23	24.21	26.28	1
25	0.3508	25.23	25.14	26.24	1
26	0.1053	26.06	26.04	27.07	1
27	0.0316	27.02	26.89	28.02	1
28	0.0095	28.01	27.62	29.00	1
29	0.0028	29.00			

-----  
24.5 = average expected loss



One needs to take care with the conclusions that one draws from these examples. It's true that as a distribution becomes more dispersed ARP is less likely to take over, but not all uniform distributions result in a balanced equilibrium between ARP and the voluntary market. One can construct examples where just about anything happens. The important conclusion is that the evolution of an assigned risk plan is sensitive to the distribution of expected losses among insureds, and there is no mathematical certainty of equilibrium nor even the direction that the evolution will take.

### 3.3. *Assigned Risk Plans That Follow Their Own Experience May Shrink*

Let's look at some examples where ARP's experience will lead to a rate increase. The distribution in Table 3 is essentially Poisson with  $\lambda = 2.74$ , but it is shifted so that it starts at 1 instead of at zero, and it is truncated at 10 so that the probabilities of the tail are all concentrated at 10. Now we see negative flags in column 6 meaning that ARP will be increasing rates and losing business to the voluntary market if it follows its own indications — even with non-profit pricing. If it starts out writing everything with expected losses greater than 2, it will have a beginning rate of 4.17, but the voluntary market will undercut it with a rate of 4.13 for risks with expected losses of 3. ARP's market share will drop; ARP's rate will increase; the voluntary market will then beat ARP's price for risks with expected losses of 4. The cycle will continue until equilibrium is reached with ARP writing only risks with expected losses of 9 and 10, at a rate of 8.85.

This is an interesting example not just because it illustrates that ARP's experience can cause it to lose, as well as gain, market share. It also illustrates that equilibrium, even within a single distribution, can occur at rather extremely different points. ARP and the voluntary market can be in equilibrium if ARP writes all risks with expected losses larger than 1, at a rate of 3.74, or if ARP writes all risks with expected losses larger than 8, with a rate of 8.85. In the first case ARP will have a market share of 93.6%; in the second, 1.7%. ARP and the voluntary market won't be in equilibrium anywhere in between these two extremes.

TABLE 3

(1)	(2)	(3)	(4)	(5)	(6)
If ARP writes all risks with losses greater than $x$					
$x$	$P[X = x]$	voluntary market rate for $x$	ARP rate ( for all )	voluntary market rate for $x + 1$	1: ARP gains $x$ -1: ARP loses $x$ 0: equilibrium
1	0.0646	3.71	3.74	7.42	0
2	0.1769	2.76	4.17	4.13	-1
3	0.2424	3.32	4.79	4.43	-1
4	0.2214	4.16	5.52	5.20	-1
5	0.1516	5.08	6.32	6.10	-1
6	0.0831	6.04	7.16	7.05	-1
7	0.0379	7.02	8.02	8.02	-1
8	0.0149	8.01	8.85	9.01	0
9	0.0051	9.00	9.52	10.00	0
10	0.0021	10.00			

3.74 = average expected loss

A market share of 1.7% for ARP is certainly not extreme, but there is no guarantee that ARP will stop at 1.7%. Look at one last example. Table 4 is a truncated geometric distribution. For  $x$  less than 10,  $P[X = x] = 0.54 \times 0.46^{x-1}$ ; the balance of the distribution is concentrated at  $x = 10$ . In this case there is no equilibrium for the voluntary market at the small end of the market. Either ARP has all of the market or nearly none of it. Equilibrium can occur with ARP writing risks with expected losses of 10, at a rate of 9.52, and a market share of 0.5%. Even this equilibrium occurs only because the distribution was truncated. If it had not been truncated, equilibrium would not have been reached until ARP's market share was less than 0.01% and its rate nearly 17, more than 9 times the average market rate. By tweaking the parameters a little, this equilibrium market share can be pushed to any extreme.

TABLE 4

(1)	(2)	(3)	(4)	(5)	(6)
If ARP writes all risks with losses greater than $x$					
$x$	$P[X = x]$	voluntary market rate for $x$	ARP rate ( for all )	voluntary market rate for $x + 1$	1: ARP gains $x$ -1: ARP loses $x$ 0: equilibrium
1	0.5400	1.12	2.71	2.23	-1
2	0.2484	2.07	3.66	3.11	-1
3	0.1143	3.05	4.61	4.07	-1
4	0.0526	4.03	5.56	5.04	-1
5	0.0242	5.02	6.49	6.02	-1
6	0.0111	6.01	7.40	7.01	-1
7	0.0051	7.01	8.26	8.01	-1
8	0.0024	8.00	9.01	9.00	-1
9	0.0011	9.00	9.52	10.00	0
10	0.0009	10.00			

1.85 = average expected loss

The above examples assume that the voluntary market operates freely. Companies do indeed find ways to compete against one another even when regulation constrains them. If that constraint becomes severe, however, none of these examples will bear much resemblance to the real behavior of the market. They are still relevant, though — just as the force of gravity is relevant to an engineer — because they show the natural forces at work against the barriers thrown up by regulation. What will actually happen in the case of overly stringent regulation will depend on the nature of the regulation. Typically, profits will decrease, which will drive capital from the market, which in turn will decrease the efficiency of the market, which finally will push insureds into self-insurance or no insurance. For insureds who need the risk reduction that insurance provides, this is not a desirable result. The interaction among residual market, voluntary market and self-insurance is important but complex. It is beyond the scope of this paper, whose focus is the interplay between the residual and voluntary markets.

## 4. HOW TO SET THE RATES

### 4.1. *An Alternative to Break-Even Pricing*

One might be tempted to argue that because the above examples are filled with instances of equilibrium, it is reasonable for assigned risk plans to base their prices on their own experience. Unfortunately, the equilibrium is capricious. One never knows where or whether it will occur. Equilibrium, moreover, desirable as it is, is not an end in itself. An equilibrium that leaves insurers a tiny fraction of the market or an equilibrium that charges assigned risk plan members ten times the voluntary market rates is probably not in the best interests of society. In any case, ARP's pricing strategy should be consistent with public goals. The public may accept letting some residual markets price themselves out of existence and may be well-served by so doing. In those cases break-even pricing with a contingency factor may work well, provided ARP really follows the indications. Where the consensus is in favor of keeping and controlling the residual market, though, the break-even approach is not a good one.

So how should ARP set its rates? If one starts with the assumptions that there should (or in any case *will*) be an assigned risk plan, that it should not be overly burdensome on the insureds in the voluntary market and that it should not have wild swings in market share, there is a reasonable solution to the rate problem. That is to base ARP's rates on total industry experience, but set at a level consistently higher than that which a typical insurer would need to charge in the voluntary market. One can start with industrywide pure premiums, for example, and load them with an expense and profit factor which is 25% above that of the industry average (or whatever percentage seems reasonable in line with studies of the market and the philosophy of a given state). The market will seek its own equilibrium. In the typical case ARP will lose money, but the burden on voluntary insureds will not be excessive. At the same time ARP's rates will be high, but not intolerably high. A start-up employer who truly has a contribution to make to society, for example, will have a chance.

#### 4.2. *Setting Specific Goals*

Words such as *reasonable* and *excessive* are pretty soft. One must define them in order to actually use them in rate setting. Their definitions will vary from state to state and from line to line and probably with the passage of time as well. They will come through compromise and consensus — there is no optimal solution that everyone will accept. The key is to have specific goals and to structure the pricing to accomplish those goals.

The voluntary market attempts to identify true costs underlying whatever it is insuring, and by varying its prices according to those costs it steers production of goods and services toward those which are most efficient. Production processes which are dangerous pay more for insurance and will thus be superseded by safer processes where safer processes exist. Autos or drivers that are prone to cause accidents will cost more to insure, and economic pressure from those higher rates will push people in the direction of safer cars and habits. This feature of insurance is very beneficial to society. When the acceptable boundary is set between voluntary and residual market share, it should be set so as to guarantee the continuation of a large voluntary market so as to give society the benefits just described. The ideal case would be a totally voluntary market.

Critics of governmental intervention in the voluntary market — intervention such as the creation of assigned risk plans — need to realize, however, that governmental intervention has in itself some aspects of a voluntary market. The legislators who pass the laws which the regulators try to enforce are democratically elected by citizens who are making their decisions in a voluntary market of information. One may disagree with some of the legislation that comes out at the end of this process, but eliminating the democratic process hardly promises a better solution.

The free market is analogous to water flowing downhill: at all points it finds, thanks to the force of gravity, the fastest route. If humans design the watercourse, the best they can hope for is to equal what nature would have done without them. More likely they will choose a less efficient course which will require continual future intervention at some cost. But, alas, the free market can't see very far ahead, any more than can a drop of water flowing downhill. If its goal is to reach the sea, too bad if the slope it's on leads to Death Valley. A little guidance may be helpful.

Rightly or wrongly, the government has provided "a little guidance" to the insurance market for a good many decades. Workers compensation statutes are a prime example. Despite the benefits of these statutes, they raise a high hurdle for many small employers. Residual market plans often enable such employers to enter and compete in the marketplace, something which would occur naturally in the absence of the workers compensation statutes. Thus residual markets are part of the "continual intervention" needed because states interfered with the natural flow of the market when they created such statutes in the first place.

The point is that there are strong arguments for the existence of residual markets. They will almost surely continue to have their adherents, and if their prices are unaffordable for virtually everyone, consumers will revolt and probably revolt successfully.

So in determining the parameters of the pricing problem, one must be guided by two somewhat conflicting goals: the bigger the voluntary market the better, and residual market rates shouldn't be unaffordable for all. A third guiding factor is consideration for the voluntary market insureds. The expected assessment of residual market losses on these innocent bystanders should not be punitive. A fourth guiding factor is the status quo. Too abrupt a change can be harmful — partly because it might unleash unexpected and uncontrollable consequences, partly because it would be in some sense a change of the rules under which many people have been operating in good faith.

Reasonable goals for a residual market plan might be a market share of under 1%, a rate of under 150% of the voluntary market, an expected assessment on the voluntary market of under 0.5%, and (during the catch-up period if one is needed) annual adjustments relative to the voluntary market of under 10%. For automobile insurance this is demonstrably achievable. For workers compensation, one might need to start with a higher market share percentage, just given the status quo. Perhaps in some states one can't achieve a 1% market share in the workers compensation residual market without increasing rates excessively. But if one could achieve a 5% maximum market share across the country, it would already be a substantial improvement. (Appendices A and B show the residual market's percent of total premiums by state for workers compensation and automobile.) In any case, this paper is not trying to suggest the exact parameters to use, merely suggesting a way to approach them.

#### 4.3. *Using the Goals to Set Prices*

With a set of specific residual market goals in hand, a state doesn't need to fight the unpredictability of break-even pricing. It can take the more stable path of setting residual market prices as a direct multiple of voluntary market prices, and it can measure its success directly from its goals.

Suppose that a state currently sets ARP rates by looking at ARP's own experience, judgmentally modifying the indication (essentially ignoring it), and finally ending up with rates which average 105% of voluntary. Consider the following alternative. First, the state gathers all the data it needs to monitor the goals it set. What are the market shares of the residual and voluntary markets (separating out companies specializing in non-standard business)? What are the average rates of voluntary writers? What are the average expense ratios? What are the underlying loss costs? Etc. Then the state measures its goals against the data. Are all the goals met? If so, the state leaves the prices at 105% of voluntary (as measured by loss costs and average expense ratios) and the job is done.

Probably, though, 105% of voluntary won't achieve the goals. So the state increases the rates to 110% or 115% of voluntary, depending on the "catch-up" parameter. Next year it looks again at the experience and market data. Gradually the state adjusts the ARP-to-voluntary ratio until it meets its goals — not break-even goals with all their unpredictability, but goals based on ARP's actual impact on society.

### 5. FINAL THOUGHTS

The reader has no doubt questioned a lot of the assumptions in this paper. The temptation to add refinements has been all but irresistible: underwriting costs, other expenses, imprecision of risk selection, competitive strategies, reinsurance pools, stochastic variation all over the place — the list is endless. A potential insured, for example, must fail to find insurance in the voluntary market before getting insurance with ARP, so insureds may stay in the voluntary market even when ARP's rate is lower. But that is a precarious balance that will crumble bit by bit as insureds find ways to move toward the lower rates. A full discussion would entail many detours.

The author considered these potential detours but wanted to look at the forest and not the trees. The paper's perspective is deliberately macro instead of micro and focused on traditional assigned risk plans or their close cousins — plans where a distinct set of rates exists independent of the company which writes the individual policies. Changing that perspective would turn this paper into a book, without really clarifying the underlying relationship between residual market pricing and the voluntary market.

The original impetus for this paper sprang from real-life observation of the outcomes that this simple model predicts; these predictions are not merely theoretical. Of course the most egregious examples of residual markets run amuck are not markets that set rates based on their own experience, but rather markets that suppress rates and ignore the effects. The point of this paper is that what may appear to be the obvious solution to that disaster, namely basing residual market rates directly on residual market experience, is not the real solution.



**APPENDIX A**

**1993 WORKERS COMPENSATION RESIDUAL MARKETS  
PERCENT OF TOTAL PREMIUM IN RESIDUAL MARKET**

<b>STATE</b>	<b>PERCENT</b>	<b>STATE</b>	<b>PERCENT</b>
Massachusetts	63.5%	Nebraska	19.5%
Tennessee	54.8%	District of Columbia	18.8%
New Mexico	53.4%	Iowa	18.4%
Arkansas	52.0%	Delaware	17.9%
Kentucky	44.7%	Alaska	17.0%
Mississippi	44.7%	New Jersey	16.3%
South Carolina	43.4%	Michigan	15.0%
Alabama	41.0%	Indiana	14.9%
Kansas	40.9%	Illinois	13.3%
Vermont	39.7%	Connecticut	11.9%
New Hampshire	38.5%	Wisconsin	9.4%
Missouri	35.6%	Oregon	7.9%
Virginia	33.6%	Arizona	3.2%
Georgia	28.6%	Idaho	2.5%
Minnesota	28.5%	Maryland	0.8%
North Carolina	28.5%	Pennsylvania	0.6%
Florida	28.2%	Colorado	0.1%
South Dakota	23.4%	Oklahoma	0.0%
Texas	20.1%	Utah	0.0%
Hawaii	20.1%		
		<b>Total</b>	<b>21.5%</b>

Twelve states are excluded because they have monopolistic or competitive state funds which provide coverage which might otherwise go to the residual market. The excluded states are Nevada, North Dakota, Ohio, West Virginia, Washington, Wyoming, California, Louisiana, Maine, Montana, New York and Rhode Island.

**SOURCE:** *Residual Markets: Workers Compensation 1993 Experience*, Schaumburg IL, Alliance of American Insurers, 1995, p. 11.

The total is a weighted average using as weights the workers compensation direct earned premium distribution taken from the *Report on Profitability By Line By State 1993*, published in Kansas City by the National Association of Insurance Commissioners in November 1994.

**APPENDIX B**

**1993 AUTOMOBILE RESIDUAL MARKETS  
PERCENT OF TOTAL PREMIUM IN RESIDUAL MARKET**

<b>STATE</b>	<b>PERCENT</b>	<b>STATE</b>	<b>PERCENT</b>
South Carolina	35.2%	Minnesota	0.5%
Massachusetts	21.0%	Mississippi	0.5%
North Carolina	18.2%	Missouri	0.3%
New York	17.2%	New Mexico	0.3%
District of Columbia	14.6%	Tennessee	0.3%
Rhode Island	10.2%	Georgia	0.2%
Delaware	8.1%	Montana	0.2%
Maryland	7.7%	North Dakota	0.2%
Pennsylvania	6.4%	Oklahoma	0.2%
New Jersey	5.7%	Washington	0.2%
Connecticut	5.5%	Wisconsin	0.2%
Michigan	5.0%	Arkansas	0.1%
Texas	4.9%	Colorado	0.1%
Virginia	4.9%	Idaho	0.1%
Vermont	4.4%	Indiana	0.1%
Hawaii	3.9%	Nebraska	0.1%
Alaska	3.0%	Nevada	0.1%
New Hampshire	2.5%	South Dakota	0.1%
Kansas	1.4%	Utah	0.1%
Louisiana	1.4%	Wyoming	0.1%
Maine	1.4%	Alabama	0.0%
California	1.3%	Arizona	0.0%
Florida	1.1%	Iowa	0.0%
West Virginia	0.8%	Ohio	0.0%
Kentucky	0.6%	Oregon	0.0%
Illinois	0.5%		
		<b>Total</b>	<b>4.8%</b>

SOURCE: *AIPSO Facts 1993/94*, Johnston RI, AIPSO, 1994, p. 35.