The Value of Junk

By: Louise Francis

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Abstract:

The recent insolvency of Executive Life Insurance Company has motivated increased concern about the quality of insurance company assets and the risks associated with those assets. Junk bonds are an asset which was believed by some to provide a relatively high return with minimal risk. Since junk bonds played a significant role in the insolvency of Executive Life, the risks of owning junk bonds currently are of greater concern to regulators.

In this paper, some of the prior research on junk bonds is reviewed. Then statistical models are developed which can be used to predict junk bond default rates. Finally, procedures for incorporating the statistical models into a comprehensive simulation of insurance company performance are described.

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Overview

The recent demise of Executive Life Insurance Company illustrates the crucial role that high yield bonds, also known as junk bonds, can play in the solvency of insurance companies. This insolvency has generated new interest among regulators in the asset side of the insurance company balance sheet.

Not very many years ago high yield bonds were proclaimed the ideal investment with low risk and high return. It was claimed that because of a relatively modest annual default rate of between 1% and 2% per year the losses from defaulted junk bonds were far less than the excess return for this investment. Thus, for an investor who held a diverse portfolio of high yield bonds, the risk adjusted rate of return should exceed that of other investment categories. This anomaly, it was claimed, was due to inefficiencies in the market place and a misperception of the true risk to investors by bond rating agencies. Although some analysts were skeptical about these assertions, the claims about high yield bonds were supported by much of the research which appeared in the 80's.

The experience of more recent years has shown that junk bonds can be very risky investments. Since before the start of the current recession a number of highly publicized defaults of junk bonds have occurred.

Publications have appeared more recently which have pointed out some flaws in the earlier research and presented methodologies which have addressed those flaws. In this paper some of the more recent studies on junk bond default risks will be reviewed. Two of the most widely recognized of these studies are research on corporate bond mortality rates by Altman[1] and an investigation by Asquith, Mullins and Wolff[2] of age as a factor in junk bond defaults. The data contained in these papers will be used to develop statistical procedures for predicting default rates.

A method which has been successfully applied to modelling the variability of insurance losses is simulation. This technique can also be used to study the variability of assets as well as losses. Using simulation the interaction of many factors impacting the solvency of insurance companies can be studied. Data from prior research on junk bonds will be used to develop models which can be used to study the risks associated with investments in high yield bonds. Models will be described which can be incorporated into a simulation. Thus, the risks associated with bond ownership can be used as part of a more comprehensive model of insurance company performance.

Altman's Study of Corporate Bond Mortality Rates

In his paper "Measuring Corporate Bond Mortality and Performance", Edward Altman[3] shows that following the default experience of a group of bonds as they age results in higher default rates than observed in prior studies where the age of bonds is ignored.

The practice in calculating annual bond default rates has been to divide the number of bonds defaulting during a given year by the number of bonds outstanding at the beginning of that year. Similarly, the value of bonds defaulting is computed as the dollar amount of losses to investors from defaulting bonds divided by the value of bonds outstanding at the beginning of the year. In calculating an overall bond default rate, an average of the default rates over a number of years is taken. If the default rate for bonds increases as bonds age or if the population of bonds changes over time, then the default rate estimated by the traditional procedure may be below the true default rate.

Altman[4] suggested that an "actuarial" approach analogous to that used by life actuaries in the evaluation of population mortality be applied to the estimation of bond default rates. That is, the cohort of bonds issued in the same year is followed over time and the mortality of the bonds at each age is computed. Altman computed the mortality rate for each year (marginal mortality rate) as:

$$MMR(t)^{5} = \frac{\text{total value of defaulted debt in year (t)}}{\text{total value of the population of bonds at the start of year (t)}}$$

where t represents the age since issuance of the bond.

The population of bonds at the start of a year was adjusted for defaults, exchanges, sinking funds and calls. Thus, the marginal mortality rate is greater than the rate computed by dividing the value of defaulting debt in year (t) by the value of the population of bonds at the start of year (0). An average is computed of the mortality rates for each age using the data from a number of issue years.

The cumulative mortality rate is then computed as follows:

$$SR(t)^{6} = \text{Survival rate for period } t = 1 - MMR(t)$$

$$CMR(t) = \text{Cumulative mortality rate at time } t = 1 - \prod_{i=1}^{t} SR(i)$$

Altman measured yearly mortality and cumulative mortality by bond rating category.

Exhibit 1 summarizes the results of Altman's research. As expected, the default rate for investment grade bonds (AAA through BBB) is much lower than the default rate for non-investment grade bonds (BB through CCC).

Altman's results suggest that cumulative default rates are relatively low for the first few years after a bond is issued. These rates increase as the bonds age. The 5-year cumulative default rates for B and CCC rated bonds were 11.53% and 31.17% respectively. The default rates for these classes of bonds at one year were 1.98% and 2.99%.

Exhibit 1

Adjusting Mortality Rates by Original S&P Bond Rating Covering Defaults and Issues from 1971 to 1987

Mortality rates are adjusted for defaults and redemption.

	Years After Issuance									
S&P Bond Rating	1	2	3	4	5	6	7	8	9	10
AAA										
Yearly	0.00%	0.00%	0.00%	0.00%	0.00%	0.13%	0.00%	0.00%	0.00%	0.00%
Cumulative	0.00%	0.00%	0.00%	0.00%	0.00%	0.13%	0.13%	0.13%	0.13%	0.13%
AA										
Yearly	0.00%	0.00%	1.81%	0.39%	0.14%	0.00%	0.00%	0.00%	0.13%	0.00%
Cumulative	0.00%	0.00%	1.81%	2.20%	2.33%	2.33%	2.33%	2.33%	2.46%	2.46%
A										
Yearly	0.00%	0.31%	0.39%	0.00%	0.00%	0.06%	0.12%	0.00%	0.04%	0.00%
Cumulative	0.00%	0.31%	0.71%	0.71%	0.71%	0.77%	0.89%	0.89%	0.93%	0.93%
BBB										
Yearly	0.04%	0.25%	0.17%	0.00%	0.45%	0.00%	0.17%	0.00%	0.23%	0.84%
Cumulative	0.04%	0.29%	0.46%	0.46%	0.91%	0.91%	1.07%	1.07%	1.30%	2.12%
BB										
Yearly	0.00%	0.62%	0.64%	0.31%	0.29%	4.88%	0.00%	0.00%	0.00%	0.00%
Cumulative	0.00%	0.62%	1.25%	1.56%	1.84%	6.64%	6.64%	6.64%	6.64%	6.64%
B										
Yearly	1.98%	0.92%	0.74%	4.24%	4.16%	4.98%	3.62%	4.03%	8.47%	4.33%
Cumulative	1.98%	2.88%	3.60%	7.69%	11.53%	15.94%	18.98%	22.24%	28.83%	31.91%
CCC										
Yearly	2.99%	2.88%	3.97%	22.87%	1.37%	NA	NA	NA	NA	NA
Cumulative	2.99%	5.78%	9.52%	30.22%	31.17%	NA	NA	NA	NA	NA

Thus, when the volume of new issues is growing as in the 1980's, the average age of the outstanding debt will be low and the default rate calculated from the immature bonds may not be indicative of the rate that will be experienced later. In addition, if the volume of debt issued in the lowest non-investment grade categories (like CCC or B) is growing relative to the volume of debt in higher non-investment grade categories, historical default rates based on a different mix of bond classes will underestimate the default rate of new issues.

A Statistical Model of Mortality Rates Fit to Altman's Data

The data presented in Altman's analysis can be used to model the default probabilities of both investment grade and non-investment grade corporate bond debt. In this paper, statistical models will be developed for studying bond default risks. Because of the small sample of bonds used in his study, and the immaturity of those bonds, mortality probabilities are not available for CCC bonds after 5 years of maturity. By fitting a curve to Altman's data, mortality probabilities can be projected beyond five years. Also, the analyst may wish to smooth the observed mortality probabilities for the other bond categories.

The Weibull distribution has frequently been applied to modelling mortality data. This distribution is also convenient for such applications because its parameters can be estimated using ordinary linear regression. If it is believed that cumulative mortality probabilities for CCC bonds follow the Weibull distribution, one could estimate the parameters of the Weibull distribution using the following procedure:

Let $F(x) = 1 - e^{-cx^t}$ denote the Weibull distribution where c, t > 0 are the Weibull distribution parameters and X is the age of the bonds.

Then regress ln (-ln (1-F(x))) on ln (x). The parameter c is the exponential of the fitted constant from the regression and the parameter t is the fitted coefficient.

Due to the small number of observations in the sample and the extreme variability of the observations, any curve fit to the observed data will exhibit large deviations between observed and fitted values. This is illustrated on Exhibit 2. In addition, extrapolation of the curve beyond the range of the observations involves considerable uncertainty, not only because of uncertainty about the estimated parameters, but because the actual cumulative survival distribution may not be Weibull.

In addition to estimating the parameters of an incomplete mortality distribution, curve fitting can be used to develop a parametric model for the mortality rates for all categories of bonds. The parameterized model can then be used in simulation.

In the example above, a Weibull distribution was fit to cumulative survival probabilities. Because of Altman's definition of annual mortality rates as conditional rates, adjusted for exits from the population, annual rates cannot be derived by differencing the cumulative mortality rates. Therefore it may be preferable to fit a curve to annual, rather than cumulative mortality rates.

Exhibit 2

Bond Cumulative Mortality Distribution for CCC Bonds

Age In Years	Actual Mortality Distribution	Estimated Mortality Distribution	Error
1	2.99%	2.40%	-0.59%
2	5.78%	7.41%	1.63%
3	9.52%	14.02%	4.50%
4	30.22%	21.63%	-8.59%
5	31.17%	29.76%	-1.41%
6		38.02%	
7		46.10%	
8		53.78%	
9		60.89%	
10		67.33%	

To illustrate the statistical modeling of mortality rates, the double lognormal distribution is used because of its tractability. The double lognormal distribution is defined for X, where X is a random variable with values between 0 and 1. It has the following form[7]

$$f(x) = \frac{1}{\sqrt{(2\pi)}\sigma\, x \ln(x)} \exp\left(-\frac{1}{2\sigma^2}(\ln(-\ln(x))-\mu)^2\right.$$

X therefore equals $e^{-e^{(\mu+\sigma z)}}$, where z is Normal (0,1) random variable. X has median $e^{(-e(\mu))}$ This distribution can have a variety or shapes and thus can be used to model a variety of data which are defined on the interval (0,1). The properties of the double lognormal distribution are described in more detail by Meinhold and Singpurwalla[8] and Holland and Ahsanullah[9].

If the random variable p is defined as the annual mortality rate for corporate bonds, then define

$$Y = \ln(-\ln(p))$$

If Y is regressed on X, where X is a vector of independent variables (such as age and bond rating category), $\hat{Y}$ will be normally distributed with mean $\mu = E(Y/X)$ and variance σ^2 equal to the standard error of the regression. That is:

$$\sigma^2 - \sum_{1}^{N} \frac{(Y-\hat{Y})^2}{n-k-1}$$

where k is the number of independent variables in the regression. In simulation this error must be adjusted for the variance of the estimated parameters using the formula for the standard error of the forecast or σ_f:

$$\sigma_f^2 = \sigma^2 (1 + x_0 X'X))^{-1} x_0$$

where X is a matrix of independent variables, X' denotes the transpose of X, X^{-1} denotes the matrix inverse of X and x_0 is a vector of actual values for the independent variables.

The ability to use linear regression makes the double lognormal distribution particularly convenient for modeling dependent variables with values between 0 and 1.

To model the mortality rates, a curve could be fit to the data from each bond rating category separately. However, for some of the categories, only a small number of bonds were included in the sample, thus the observed default rates are not fully credible. In addition, some of the observed results appear to be counterintuitive. For instance, mortality rates for the AA category exceed mortality rates for the lower A and BBB categories. There may in fact be no statistically significant difference between mortality rates by age for the categories which include investment grade bonds. Regression analysis can be used to assess the statistical significance of the relationship between bond rating class and bond mortality.

The model used to fit mortality rates will relate the dependent variable, Y = ln (-ln (mortality rate)) to the age of the bond. Examination of the data suggests that the relationship between Y and age may be nonlinear. That is, as age increases, the value of Y appears to increase, then level off, and even decrease. This effect is more evident for the non investment grade bonds than for investment grade bonds. Therefore, Y will be modeled as a quadratic function of age. The regression will be

$$Y = a + b1\ age + b2\ age^2 + b3\ Dummy1 + b4\ Dummy1\ age + b3\ Dummmy1\ age^2.$$

where Dummy1 is a indicator variable which is 0 if the bond is investment grade and one if the bond is below investment grade. The use of an indicator variables in the equation shown above is equivalent to fitting a separate regression to the investment grade and noninvestment grade data. The results of fitting a regression to Altman's data are presented in Exhibit 3. The coefficients of age and age^2 were not significant therefore those variables were eliminated from the regression. The coefficient of the dummy variable was significant indicating that the bond's status as investment grade or non investment grade significantly impacts the mean probability of default. The results of the regression also suggest that age may be related to the mortality rate for junk bonds but not for investment grade bonds.

The regression analysis described above can be extended to investigate additional parameters which can be used to predict bonds mortality rates. If all bond rating categories are significant in predicting probabilities of default the following regression would be fit to the data:

$$Y=a+b1\ age+b2\ age^2+b3\ Dummy1+b4\ Dummy1\ age+b5\ Dummy1\ age^2 +\ldots b15\ dummy6+b16\ Dummy6\ age+b17\ Dummy6\ age_2$$

where a dummy indicator variable, Dummyi is used to denote a bond in rating group i+1. For each rating group an interaction terms, Dummyi*age and Dummyi*age^2 are created to model the effect of the bond rating group on the coefficients of age and age^2. No dummy variable or interaction term is created for bonds in the AAA category as one category of bonds becomes a reference group against which the other categories are measured. Although the reference category need not be the AAA group, there must be one category for which a dummy variable is not created.

The model with dummy variables and interaction terms for all bond rating categories has 18 parameters. Since the model is fit to data with only 65 observations, it is unlikely that all the parameters will be significant. Therefore stepwise regression was used to search for a subset of significant variables for predicting bond mortality rates. A stepwise procedure uses a statistical algorithm to select the variables one at a time to enter the regression. At each step the algorithm selects the independent variable which has the strongest relationship to the dependant variable.

The stepwise regression procedure resulted in the following equation for predicting default probabilities:

$$Y = 2.1 + .28 \text{ Dummy1} + 1.22 \text{ Dummy2} - .91 \text{ Dummy3} - .32 \text{ Dummy2 age} + .04 \text{ Dummy2 age}^2$$

where Dummy1 = 1 if the bond is rated AAA

Dummy2 = 1 if the bond is rated BB

Dummy3 = 1 if the bond is a junk bond (BB or lower)

The statistics for this regression are shown on Exhibit 4. Note that variables which are associated with lower default probabilities have positive coefficient and variables which are associated with higher default probabilities have negative coefficients. This is caused by the relationship between Y and p. When Y decreases, p increases.

The double lognormal distribution and the regression procedure used to model bond default rates is one of a number of possible procedures which can be used to model default probabilities. Other possibilities include logit regression and nonlinear regression. Logit regression will be described in more detail below where it is used to model the frequency of junk bond defaults. Nonlinear regression which is available with most of the widely used statistical software packages, can be used to model mortality probability functions which cannot be transformed into a form suitable for estimating linear regression parameters.

Exhibit 4

Regression Results
Mortality Rate Analysis

Variable	Coefficient	T
Constant	2.1043	36.048
Dummy1	0.2842	2.434
Dummy2	1.2185	3.165
Dummy3	-0.9122	-9.022
Dummy2*age	-0.3203	-2.039
Dummy2*age^2	0.0353	2.538

R^2 =	0.8189
Adjusted R^2	0.6427
Standard Error	0.3197

Notes:

Dummy1 = 1	if bond is rated AAA
Dummy2 = 1	if bond is rated BB
Dummy3 = 1	if bond is non investment grade

Simulation of Bond Mortality Rates

Once a model has been fit to bond mortality data, it can be incorporated into a simulation which models the default experience of an insurance company's bond assets. In order to perform this simulation, information about the composition of the company's bond portfolio is needed. The distribution of bonds by bond rating category and age within category is needed. Schedule D of the statutory annual statement is a useful source for this information although the bond quality classes used by the NAIC do not correspond exactly to the bond ratings given by the rating agencies. The simulation is performed as follows:

1) Generate a Normal (0,1) random variable, z.

2) Compute $\hat{Y}_{tca}$, the estimate of Y at time t for bonds of age a in class c, using the fitted regression parameters.

3) Compute p_{tca}, the probability of default for bonds of age a in class c at time t as

   $p_{tca} = 1/\exp(\exp(\hat{Y} + z\sigma_f))$

   where σ_f is the forecast standard error of the regression.

4) Use the following formula to compute the value of bonds defaulting at time t, D_{tca}, and the amount of the company's assets invested at time t in bonds of age a and class c, A_{tca}.

$$D_{tca} = p_{tca} \; A_{t-1,c,a}$$

$$A_{tca} = A_{t-1,c,a}(1-p_{tca}) + CF_{t-1} ri_{tca} - A_{t-1} re_{tca}$$

where CF_t is the net cash flow at time t for the company, ri_{tca} is the proportion of cash flow invested in bonds of class c and age a and re_{tca} is the proportion of bonds of class c and age a which are sold, mature, or are called. This procedure is repeated for T future years from the time of origin, where T is judgementally selected. It may be of little interest to the analyst to follow mortality experience into the distant future, therefore a T of 10 years or less may be reasonable. However, if the analyst is modelling the probability that assets supporting an insurance company liabilities will be sufficient to pay for the liabilities and the company writes very long tail business, a longer period of observation may be desirable.

To illustrate the use of the regression model in simulation, a simple scenario in which all future cash flows are 0 and no bonds are called was performed. The distribution of bonds by age and bond class are shown on Exhibits 5 and 6. For the first scenario only a small percentage of bond assets are invested in below investment grade bonds. This bond distribution is a typical one for the property casualty industry. According to the 1990 Bests Aggregates and Averages[10], less than 2.5% of industry assets invested in bonds were invested in below investment grade bonds in 1990. Note that the assumed average age for non investment grade bonds is lower than the average age for investment grade bonds since the low grade bonds are a more recent financial instrument. A probability distribution of default rates derived from the simulation is shown on Exhibit 8. This distribution indicates that if there is no net cash flow into or out of the company there is a 10% probability that assets will decline by more than 11.87% in a 10 year period as a result of defaults.

Exhibit 5

Bond Age Distribution Assumptions

Years After Issue	Investment Grade Bonds	Junk Bonds
1	10.5%	7.0%
2	9.6%	16.0%
3	8.7%	20.0%
4	7.9%	20.0%
5	7.2%	15.0%
6	6.5%	8.0%
7	5.9%	6.0%
8	5.4%	3.0%
9	4.9%	1.5%
10	4.5%	1.0%
11	4.1%	1.0%
12	3.7%	0.5%
13	3.3%	0.5%
14	3.0%	0.5%
15	2.8%	0.0%
16	2.5%	0.0%
17	2.3%	0.0%
18	2.1%	0.0%
19	1.9%	0.0%
20	1.7%	0.0%
21	1.6%	0.0%
	100.0%	100.0%

Exhibit 6

Simulation Assumptions
Low Junk Bond Percentage Scenario

Class	Percent of Bonds in Class	Maturity in Years: 1	2-5	5-10	10-20	> 20
AAA	40.0%	28.0%	25.0%	22.0%	13.0%	12.0%
AA	20.0%	19.0%	27.0%	31.0%	14.0%	9.0%
A/B	14.0%	9.0%	33.0%	37.0%	9.0%	12.0%
BB	2.0%	4.0%	15.0%	44.0%	31.0%	6.0%
B	0.5%	6.0%	25.0%	46.0%	19.0%	4.0%
CCC	0.5%	29.0%	12.0%	16.0%	5.0%	38.0%
	77.0%	25.2%	25.6%	24.5%	13.5%	11.2%

Note: 23% of the bonds are assumed to be Government bonds with no default risk

Exhibit 7

Simulation Assumptions
High Junk Bond Percentage Scenario

Class	Percent of Bonds in Class	Maturity in Years				
		1	2-5	5-10	10-20	> 20
AAA	20.0%	28.0%	25.0%	22.0%	13.0%	12.0%
AA	12.0%	19.0%	27.0%	31.0%	14.0%	9.0%
A/BBB	50.0%	9.0%	33.0%	37.0%	9.0%	12.0%
BB	4.0%	4.0%	15.0%	44.0%	31.0%	6.0%
B	11.0%	6.0%	25.0%	46.0%	19.0%	4.0%
CCC	3.0%	10.0%	15.0%	30.0%	20.0%	22.0%
	100.0%	25.2%	25.6%	24.5%	13.5%	11.2%

Distrbution of Bond Mortality Values

Exhibit 8

Percentile	Annual Default Rates (Percent of Initial Value) 1	2	3	4	5	6	7	8	9	10	Cumulative Default Rate
5	1.22%	0.42%	0.30%	0.19%	0.11%	0.10%	0.10%	0.10%	0.10%	0.09%	4.48%
10	1.27%	0.46%	0.35%	0.22%	0.13%	0.13%	0.12%	0.12%	0.12%	0.11%	4.82%
15	1.32%	0.51%	0.37%	0.23%	0.15%	0.14%	0.14%	0.13%	0.13%	0.12%	5.20%
20	1.35%	0.54%	0.40%	0.25%	0.16%	0.16%	0.16%	0.15%	0.15%	0.14%	5.45%
25	1.37%	0.57%	0.42%	0.27%	0.18%	0.18%	0.18%	0.16%	0.16%	0.16%	5.74%
30	1.40%	0.60%	0.44%	0.29%	0.20%	0.19%	0.20%	0.18%	0.18%	0.18%	5.96%
35	1.43%	0.63%	0.47%	0.31%	0.22%	0.21%	0.22%	0.20%	0.21%	0.20%	6.25%
40	1.46%	0.66%	0.50%	0.33%	0.24%	0.24%	0.24%	0.22%	0.23%	0.22%	6.45%
45	1.49%	0.69%	0.53%	0.36%	0.26%	0.26%	0.27%	0.25%	0.26%	0.24%	6.72%
50	1.52%	0.73%	0.56%	0.39%	0.29%	0.30%	0.31%	0.29%	0.28%	0.27%	7.05%
55	1.56%	0.76%	0.60%	0.42%	0.34%	0.33%	0.35%	0.33%	0.31%	0.32%	7.34%
60	1.60%	0.81%	0.64%	0.47%	0.38%	0.39%	0.41%	0.37%	0.35%	0.36%	7.59%
65	1.64%	0.86%	0.68%	0.52%	0.42%	0.44%	0.48%	0.45%	0.42%	0.43%	7.97%
70	1.71%	0.93%	0.75%	0.59%	0.48%	0.50%	0.58%	0.52%	0.48%	0.51%	8.33%
75	1.78%	1.01%	0.85%	0.66%	0.57%	0.61%	0.69%	0.64%	0.57%	0.58%	8.80%
80	1.89%	1.13%	1.00%	0.80%	0.66%	0.71%	0.82%	0.75%	0.69%	0.72%	9.47%
85	2.07%	1.30%	1.24%	0.95%	0.89%	0.90%	1.02%	0.95%	0.90%	0.93%	10.38%
90	2.41%	1.60%	1.59%	1.28%	1.13%	1.14%	1.42%	1.31%	1.36%	1.28%	11.87%
95	3.04%	2.15%	2.29%	1.94%	1.87%	1.80%	2.16%	1.96%	2.07%	2.29%	14.55%
98	4.19%	2.93%	3.85%	2.91%	3.18%	3.22%	3.43%	2.82%	3.21%	3.95%	18.82%
99	5.46%	4.57%	6.94%	3.80%	5.17%	4.91%	5.16%	3.73%	4.00%	5.76%	21.87%

Distrbution of Bond Mortality Values

Exhibit 9

Percentile	Annual Mortality Rates (Percent of Initial Bond Value)										Cumulative Mortality Rate
	1	2	3	4	5	6	7	8	9	10	
5	2.90%	1.24%	0.94%	0.72%	0.52%	0.51%	0.48%	0.43%	0.43%	0.39%	12.80%
10	3.01%	1.39%	1.07%	0.82%	0.62%	0.58%	0.56%	0.50%	0.50%	0.46%	13.48%
15	3.10%	1.48%	1.16%	0.87%	0.68%	0.64%	0.62%	0.56%	0.55%	0.51%	14.04%
20	3.16%	1.55%	1.22%	0.92%	0.73%	0.68%	0.67%	0.61%	0.59%	0.56%	14.64%
25	3.25%	1.63%	1.29%	0.97%	0.78%	0.73%	0.71%	0.66%	0.64%	0.60%	15.12%
30	3.32%	1.69%	1.35%	1.01%	0.81%	0.76%	0.76%	0.70%	0.68%	0.64%	15.63%
35	3.38%	1.75%	1.40%	1.07%	0.86%	0.81%	0.80%	0.75%	0.72%	0.70%	16.10%
40	3.44%	1.82%	1.46%	1.11%	0.90%	0.87%	0.86%	0.80%	0.78%	0.74%	16.65%
45	3.52%	1.88%	1.52%	1.16%	0.94%	0.92%	0.90%	0.85%	0.83%	0.80%	17.16%
50	3.58%	1.96%	1.59%	1.22%	1.00%	0.98%	0.95%	0.90%	0.87%	0.86%	17.84%
55	3.64%	2.04%	1.66%	1.28%	1.06%	1.03%	1.03%	0.97%	0.93%	0.92%	18.45%
60	3.71%	2.14%	1.73%	1.37%	1.13%	1.10%	1.10%	1.05%	1.01%	1.00%	19.19%
65	3.80%	2.23%	1.80%	1.46%	1.22%	1.19%	1.23%	1.17%	1.10%	1.11%	19.77%
70	3.93%	2.36%	1.93%	1.56%	1.32%	1.32%	1.37%	1.29%	1.21%	1.23%	20.82%
75	4.12%	2.51%	2.08%	1.70%	1.44%	1.49%	1.56%	1.47%	1.35%	1.40%	22.09%
80	4.35%	2.68%	2.34%	1.86%	1.66%	1.70%	1.82%	1.68%	1.60%	1.63%	23.39%
85	4.65%	3.11%	2.66%	2.24%	1.98%	2.20%	2.37%	2.05%	1.89%	1.98%	25.33%
90	5.06%	3.69%	3.64%	2.78%	2.49%	3.10%	3.15%	2.67%	2.76%	2.65%	29.19%
95	6.97%	5.34%	5.90%	4.47%	4.24%	4.24%	4.78%	4.45%	4.32%	4.05%	38.23%
98	11.27%	7.93%	10.67%	8.58%	7.58%	7.15%	7.75%	7.40%	7.44%	8.47%	48.06%
99	15.37%	10.68%	16.38%	10.78%	12.10%	12.12%	11.06%	10.19%	11.42%	16.10%	56.66%

Although most property and casualty companies are not heavily invested in junk bonds some companies have a much higher percentage of their assets in low grade bonds than the average insurance company. Exhibit 9 presents the results of a simulation in which 18% of a company's assets are invested in below investment grade bonds. For this scenario there is a 10% probability that assets will decline by more than 29.19% in a 10 year period as a result of defaults.

Asquith's Study: Aging Analysis of High Yield Bonds

In his 1989 paper, Altman calculated mortality rates for a universe of corporate bonds which included both investment grade and noninvestment grade bonds. Much of the research on default rates which is relevant to modelling default risk used data for low grade bonds only.

In their paper "Original Issue High Yield Bonds: Aging Analysis of Defaults, Exchanges and Calls", Asquith, Mullins and Wolff[1] studied the relationship between bond default rates and bond age for high yield bonds. They note that the default rates estimated in many prior studies were biased downwards because of failure to properly account for exchanges as well as the bond's age. An exchange involves swapping the high yield bond for some other security such as equity shares or bonds with lower coupon payments. Many researchers eliminated exchanged bonds from their sample when calculating default rates. However, if an exchange occurs because a company is in a distressed situation, the default rate for the exchanged security may be greater than the default rate for nonexchanged securities. Thus, default rate studies need to follow the history of exchanged securities subsequent to the exchange.

The study by Asquith et. al. included a sample of 731 bonds, beginning with bonds issued in 1977. Their sample was larger than Altman's sample and contained sufficient data to estimate default rates up to 12 years after bond issuance. The default or mortality probabilities were not conditional probabilities. That is, the value of bonds defaulting was divided by the value of bonds issued. Asquith et. al. observed long term default rates in excess of 30% for the earliest years in their sample. The 12 year rate for bonds issued in 1977 was 33.92% and the 11 year rate for bonds issued in 1978 was 34.26[12], indicating that the investor bears the risk that a large percentage of high yield bonds will default. Their study indicated that default rates increase as bonds age. Thus, default rates of between 0% and 3% were observed for bonds of less than one year of maturity.

The hypothesis that age is an important, if not the most important factor in predicting default rates has not gone unchallenged. In a recent paper Blume, Keim and Patel[13] suggest that economic factors, not age are the most important determinants of default rates. They noted that inspection of the data used by Asquith et. al. shows that high default rates are associated with particular calendar years. That is, bonds of all ages seem to have a higher propensity to default in some years. Blume et. al. note that after adjustment of Asquith, Mullins and Wolff's data for economic effects the rank correlation between age and default rate decreases from .49 to .22[14].

Analysis of Asquith, Mullins and Wolff's Data

In order to assess the effect of both economic variables and age in determining bond default rates, we applied a technique known as logistic regression to Asquith, Mullins and Wolff's data. Logistic regression can be utilized when a dependent variable can have only two possible values. This is the case for bond default rates since a bond either defaults or survives. When using logistic regression, the probability that an event occurs is defined as

$$Prob(event) = \frac{\exp(a + b\,x)}{(1+\exp(a + b\,x))}$$

where x is an independent variable, such as age (x can also be a vector of variables). It is convenient to think of logistic regression as a procedure which relates the logit of a variable to one or more independent variables. The logit of a variable is defined as the log of the odds ratio for that event. The odds ratio is the probability that an event will occur divided by the probability that the event will not occur. If p_{ta} is defined as the probability that a bond of age a will default in period t then the logit at age a of p_{ta} is

$$Y_{ta} = \text{logit(default rate)} = \ln(p_{ta}/(1-p_{ta})) = a+bx$$

Logistic regression, like linear regression is a multivariate technique which can be used to investigate the simultaneous influence of a number of independent variables on the dependent variable. Most commonly used statistical packages include maximum likelihood routines for estimating the parameters of a logistic regression. For the analysis presented in this paper, the logistic regression procedure of SPSS was applied to the default rate data.

The following economic variables were considered as possibly significant for predicting default rates: GNP (Percent Change in Gross National Product), S&P (the percent change in the S&P 500 index), UEP (the percent change in the unemployment rate), and MANUCAP (manufacturing capacity).

Using the logistic regression procedure of SPSS, and a stepwise method for selecting variables the following model was fit to Asquith, Mullins and Wolff's data:

$$Y_t = -4.4 + .5 \ln(age_t) + 2.47 \, S\&P_t - 2.3 \, UEP_t$$

where Y_t is the log of the odds ratio at time t. This model indicates that an increase in age and in the S&P index are associated with increased default rates and an increase in the unemployment rate is associated with a decreased probability of default. The signs on the S&P and UEP variables are the opposite of what one would expect. However interactions among the independent variables can have a significant impact on the fitted coefficients. Interaction terms are frequently included in the logistic regression when the effect of an independent variable is believed to depend on the value of other independent variables. An interaction term is generally expressed as the product of the two independent variables. A logistic regression was performed which included interaction terms for all the independent variables. The results are presented in Exhibit 10. The ln(age) variable was dropped from

Exhibit 10

Logistic Regression Results
Default Rate Analysis

Variable	Coefficient	Significance
Constant	-3.5037	0.000
S&P	-2.3400	0.049
UEP	3.2207	0.130
ln(age)*S&P	2.9200	0.001
ln(age)*UEP	-2.6332	0.052
S&P*UEP	-33.1674	0.007

the regression because after inclusion of the interaction terms the probability that it was significant was only 34%. The coefficients of all variables now have the expected sign. An increase in the stock market index is associated with a decreased probability of default and an increase in the unemployment rate is associated with an increased probability of default. In addition, age impacts the probability of default through its interaction with other variables. The signs on the coefficients of the intraction terms indicate that as age increases the impact of S&P and UEP decreases.[15] That is, as the log of age increases the magnitude of the coefficient for UEP decreases and the magnitude of the coefficient for S&P increases.

Simulation of Bond Default Rates

According to Hosmer and Lemeshow[16] $\hat{Y}_t$, the logit of the predicted default rate can be assumed to be normally distributed. Thus, the fitted model can be used to generate random default rates in a simulation and used to model the risks associated with junk bond ownership. One additional parameter is needed for the model, the variance of $\hat{Y}_t$. For a regression of the form

$$\hat{Y}_t = a + b_1 X1_t + b_2 X2_t$$

The variance of $\hat{Y}_t$ is equal to[17]

$$var(a) + var(b_1)X1_t^2 + var(b_2)\, X2_t^2 + 2\, cov\, (b_1, b_2)\, X1_t\, X2_t$$

Thus, the variance of $\hat{Y}_t$ is equal to the sum of the variances plus twice the covariances of the terms in the regression equation above. The variances and covariances of the regression coefficients can be obtained as part of the standard logistic regression output.

To simulate default rates using the logistic regression above, values are needed for the economic variables S&P and UEP. These variables can be simulated using simple statistical models.

The change in the stock market index can be approximated using a white noise model. That is:

$$S\&P_t = \mu_s + \sigma_s Z$$

where μ_s is the mean of the economic series and σ_s is the standard deviation of the residual of the series and Z is a normally distributed random variable. Since it is reasonable to assume that stock market performance and the unemployment rate are related to each other, the following function can be used to model the unemployment rate:

$$UEP_t = \mu_u + \phi(S\&P_t - \mu_s) + \sigma_u Z$$

where μ_u is the mean unemployment rate and σ_u is the standard error of the regression and Z is a standard normal random variable. The parameter ϕ can be determined by regressing UEP_t on $S\&P_t$.

Default rates can be simulated as follows:

1) Simulate values for the independent variables $S\&P_t$ and UEP_t

2) Compute $\hat{Y}_{ta}$, the estimated value of the logit of the default rate, for bonds of age a

3) Simulate a Normal (0,1) random variable, Z

4) The simulated logit default rate $S_{ta} = \hat{Y}_{ta} + Z$ sd where sd is the square root of the variance of $\hat{Y}_{ta}$

5) Compute p_{ta} the default probability for bonds aged a at time t as $p_{ta} = 1/(1+\exp(-S_{ta}))$

6) Compute the number of bonds defaulting at time as $N_{ta}p_{ta}$, where N_{ta} is the number of bonds of age a at time t. If $N_{ta}p_{ta}$ is less than one do the following:

   a) generate a uniform (0,1) random variable, U

   b) If U is less then $N_{ta}p_{ta}$, one bond defaults, otherwise no bond defaults.

On Exhibit 11, the results of a simulation of bond default rates is presented. The default rates shown are for a simple scenario in which no new bonds are added to the portfolio after the initial time period. The initial distribution of bonds by age used for this simulation was presented on Exhibit 5.

The simulation results for the particular set of assumptions incorporated into the model indicate that there is a significant risk of default as the cohort of bonds ages. At the 90th percentile, the cumulative default rate after 5 years is 23% and at the 99th percentile it is 29%.

Probability Distribution for Cumultive Junk Bond Default Rates

	Number of Future Years Simulated				
Percentile	1	2	3	4	5
5%	2.3%	5.2%	8.2%	11.1%	13.9%
10%	2.5%	5.5%	8.6%	11.7%	14.6%
15%	2.6%	5.7%	9.0%	12.1%	15.1%
20%	2.7%	6.0%	9.3%	12.4%	15.6%
25%	2.8%	6.2%	9.5%	12.8%	16.0%
30%	3.0%	6.3%	9.8%	13.1%	16.3%
35%	3.1%	6.6%	10.0%	13.3%	16.6%
40%	3.2%	6.8%	10.2%	13.7%	16.9%
45%	3.3%	7.0%	10.5%	14.0%	17.3%
50%	3.4%	7.2%	10.8%	14.3%	17.6%
55%	3.6%	7.4%	11.0%	14.6%	17.9%
60%	3.7%	7.6%	11.4%	15.0%	18.4%
65%	3.9%	7.9%	11.7%	15.3%	18.8%
70%	4.1%	8.2%	12.1%	15.9%	19.3%
75%	4.3%	8.4%	12.6%	16.4%	19.9%
80%	4.6%	9.0%	13.1%	17.0%	20.6%
85%	5.0%	9.7%	13.8%	17.7%	21.5%
90%	5.6%	10.5%	14.8%	19.0%	23.0%
95%	6.9%	12.4%	17.3%	21.2%	25.4%
99%	11.2%	16.2%	20.5%	25.3%	28.7%

In the simulation described above it was assumed that the defaulting bonds have no value subsequent to the default. However this would be an unnecessarily conservative assumption. It was noted by Altman and Asquith et. al. that when bonds default they frequently do not lose their full market value. Thus we may wish to model the severity as well as the frequency of bond default rates. The data in Asquith's paper cannot be used to develop a model for the severity of defaults as the distribution of bond losses once a default occurred was not studied by Asquith.

With the information available default severity cannot be modelled without making basic assumptions about the underlying severity distribution. One procedure for deriving severity distribution parameters for use in simulation is suggested below.

A model for the frequency and severity of bond defaults is described in a paper by Spahr, Sunderman, and Amalu[18]. In their paper Spahr et. al. developed a procedure for pricing corporate bond insurance. The authors fit a model to the frequency and severity of default losses. It should be noted that Spahr et. al. did not take into account the age of the bonds when computing the frequency or severity of defaults. However this should not affect their evaluation of severity as Altman's research indicated that default severity does not vary with the age of bonds. Spahr et. al. measured the severity of losses as the decline in market value from three months prior to the default to three months after the default. They estimated the mean loss in value as 89% of the market value three months prior to default and the variance of the loss as 3% of the value of the bonds. It should be noted that other researchers, using a different definition of default severity, estimated a lower average loss size. For instance, Altman notes that defaulting bonds on average sell at about forty percent of par[19] at the end of the defaulting month. Thus, how default severity is defined will affect the mean default rate selected.

In order to use this information within a simulation, a distribution must be assumed for bond severities. In their paper, Spahr et. al. used the normal distribution to model bond default pure premiums. Because the pure premium was an average value based on a large volume of data, they felt justified in making such an assumption. However it is unlikely that the normality assumption is justified for the severity of default losses. The double lognormal distribution was previously introduced for modelling a random variable with values between 0 and 1 and it might be useful for modelling default severity. However, because the moments of the double lognormal distribution cannot be computed analytically, parameters for the double lognormal cannot be derived without significant effort. A simpler method is to use the lognormal distribution to model default severity. If the coefficient of variation for the severity distribution is denoted CV, then the lognormal parameter σ is equal to $\ln(CV^2+1)^{.5}$ and the lognormal parameter μ equals the mean - .5 σ^2, where the mean is the expected severity of loss. Since the default severities are expressed as a percentage of the bond's value, it is also necessary to develop a distribution of bond values. An empirical distribution derived from an insurance company's actual portfolio can be constructed for use in generating bond values in the simulation. Bratley, Fox and Schrage[20] describe procedures for generating random variables from an empirical distribution.

To apply the frequency/severity model to simulating bond default rates use the following procedure.

For each iteration of the simulation do the following for T years

1) For each age category simulate N_a, the number of bonds of age a defaulting at time t, using the procedure already described
2) For each defaulting bond simulate a bond value
3) For each defaulting bond simulate a default severity from the lognormal or other selected distribution
4) Multiply the default severity times the bond's value
5) Accumulate the value of the defaulting bonds for the T year period.

A probability distribution of the amount of defaulting bonds and the ratio of loss to issue value of the bonds can then be constructed.

Call Risk

The procedures described have presented methods for assessing default risk. However other risks to investors in junk bonds exist. One of these risks is call risk. That is, if interest rates decline and a bond is called, the investor will be reinvesting the income from the called bonds at a lower interest rate.

In Asquith, Mullins and Wolff's study, for bonds issued prior to 1983, between 24% and 47%[21] of bonds had been called as of 1988. Data is not available to model the relationship between call risk and the age of bonds or between call risk and economic variables, although call risk is related to both these variables. Because of call protection few bonds under five years of age will be called. In addition, bonds are much more likely to be called when interest rates decline.

It seems reasonable to incorporate provisions for call risk into an analysis of junk bond risk. In the absence of more detailed information, judgmental call rates for bonds over 5 years old can be coded into the simulation. If a variance for the call rate is selected, random call rates can be simulated from a selected distribution such as the normal distribution. Call rates can also be made to vary with interest rates.

Interest Rate and Market Value Risk

Junk bonds, like investment grade corporate bonds are subject to interest rate risk. Interest rates vary over time and the market value of the bonds declines when interest rates rise. The procedures presented in this paper cannot be used to model the variability in the market value of bonds due to interest rate risk. However such risk must be considered.

For a large portfolio of high yield bonds, the risk resulting from the default of bonds and the subsequent decline in market value, as well as the variability due to changes in interest rates and the calling of bonds can be observed in the overall variability of the portfolio's returns.

Using a series of observed returns for high yield bonds, a procedure to model the variability of the returns can be developed.

Using data collected from low-grade mutual bond funds, Cornell and Green[22] investigated several models of junk bond volatility. The data on monthly bond returns collected spanned the period 1960 - 1989. Among the findings of Cornell and Green were

(1) Over the period 1960 - 1989, junk bond funds had a lower variance and higher average returns than investment grade bonds.

(2) For the period 1960 - 1976, junk bond funds had a higher variance than investment grade bond funds. During the period 1977 - 1989, low grade bond funds had a lower variance than investment grade bond funds. Cornell and Green noted that during the period 1976 - 1989 the variability of investment grade bond funds was significantly greater than during the period 1960 - 1976. The increased variability of investment grade bonds partially explained the relative lower variability of junk bonds during the 1976 - 1989 period.

(3) Junk bonds were found to be less sensitive to interest rate changes than investment grade bonds, but more sensitive to changes in the stock market.

Using multiple regression, Cornell and Green modelled the influence of interest rates and stock market returns on junk bond returns. The model they fit for the period 1960 - 1989 was:

$$JB_t = .051 - .009\ TB_{t+1} + .278\ TB_t + .1\ TB_{t-1} + .034\ S\&P_{t+1} + .355\ S\&P_t + .037\ S\&P_t$$

where JB_t is the return for junk bonds at time t

TB_t is the return for treasury bonds at time t

$S\&P_t$ is the percentage change in the S&P 500 index

The adjusted R^2 for this model was .664 suggesting that it explained a significant percentage of the variability in junk bond returns. Cornell and Green also split their data into the periods 1960 - 1976 and 1977 - 1989 and fit separate regressions to the different time periods.

No attempt has been made by the author of this paper to construct an alternative model for low grade bond returns. These results are included in this paper for those who wish to model junk bond returns rather than junk bond default rates. Using Cornell and Green's model, junk bond rates of return can be simulated as follows:

1) Generate random treasury bond rates, TB_{t-1}, TB_t and TB_{t+1}. A simple autoregressive model can be used to model the bond rates. The autoregressive model is denoted

$$TB_t = \mu_t + \phi\ (TB_{t-1} - \mu_{tb}) + \sigma_{tb}\ Z$$

where Z is a standard normal variable μ_{tb} is the mean treasury bill rate, σ_{tb} is the standard deviation of the residuals of the model.

2) Generate random stock market returns $S\&P_{t-1}$, $S\&P_t$ and $S\&P_{t+1}$. Stock market returns can be modelled as a white noise process where the deviation of returns from the average stock market returns is a normal random variable.

3) Generate a Normal (0,1) random variable, z

4) Compute $JB_t = \hat{JB}_t + \sigma_{jt} z$

   where $\hat{JB}_t$ is the estimate of JB_t using Cornell and Green's regression and σ_{jt} is the standard error of the regression (which was estimated by Cornell and Green equals to equal 1.44).

5) Save the values of TB_t, TB_{t+1}, $S\&P_t$ and $S\&P_{t+1}$ for use in the future simulated time periods.

Cornell and Green's research indicated that junk bonds are more sensitive to movements in interest rates during recessions. Their research also suggested that junk bonds perform more poorly in recessions than in a nonrecessionary environment. In addition, because the majority of junk bonds were issued in the 80's during a relatively prosperous period, the impact of a severe recession on junk bond values is not known. Thus the behavior of junk bonds during a recession is unlikely to be adequately reflected in the model.

Having described a procedure for modelling junk bond returns, mention must be made about shortcomings of the data used in the study. The data used to model junk bond returns may reflect an inaccurate measurement of actual junk bond returns.

Junk bonds are a very illiquid security. Unlike investment grade corporate bonds or treasury bonds, there is not a large market for trading these bonds. Low grade bonds are not traded daily and frequently only one brokerage firm deals in a particular bond. On days in which a bond is not traded its market value must be estimated by the mutual fund. Various strategies have been used by mutual funds to estimate the market value of thinly traded bonds. According to Lisco[23], many of the estimation techniques used by mutual funds result in an inflated market value for junk bonds. In fact, in a Barrons[24] article titled "Inflated Junk", Lisco stated "According to the former pricing agent, the quotes used for the vast majority of junk bonds have a high degree of fluff in them".

Since the model for junk bond returns was fit to data from low grade bond mutual funds, the results are affected by the pricing procedures used by the funds managers.

Despite its flaws, Cornell and Green's model is a step towards building a model for the volatility of junk bond returns.

Valuation of Junk Bonds

The focus of this paper has been the development of models for analyzing the risks of junk bond ownership. It has been assumed throughout that the market values established by an insurance company for both its investment grade and noninvestment grade corporate bonds are the correct market value. Significant risk exists that the actual value differs from the stated value.

An estimation of the true value of bonds can only be performed by a trained financial analyst. Such an evaluation is based upon an in depth analysis of the company issuing the debt. The factors which must be considered by the analyst are described by Howe[25]. These factors are: (1) competition; (2) asset quality and marketability; (3) leverage; (4) projections of cash flows; (5) corporate structure; (6) management. Most of these factors are related to whether the company issuing the bonds can survive periods of economic stress. In addition, the company must be positioned to profit in good economic times.

It should be noted that property and casualty insurance companies are required to state accurately the market value of their bonds on the statutory annual statement. The Securities Valuation Office of the NAIC provides an evaluation service for both publicly traded and privately placed bonds which can be used by companies to value their assets. However, a valuation of a bond is performed only if it is requested.

Other Limitations of Junk Bond Statistical Models

In their paper on junk bond default rates Asquith et. al. note that simulation can be used to model the return variability or default experience of junk bonds. They also present a long list of the limitations of simulation models. Among the limitations noted are 1) uncertainties associated with inputs to a simulation such as bond calling assumptions 2) the difficulties of realistically modelling asset selling and reinvestment strategies and 3) the use of very limited data for developing models for default rates and return variability. All of these limitations present difficulties in the development of realistic models of bond volatility.

Of the items mentioned by Asquith et. al., the limitations of the data are a matter of serious concern. Since the junk bond market is a relatively new market, defaults rates based on only a few years of historical experience may not be indicative of future default rates. Moreover, in the mid 80's, junk bonds began to be used for mergers and leveraged buyouts. The default experience of this new kind of bond may be different from the default experience of bonds purchased to finance a company's operations.

Conclusion

In this paper, three procedures for modelling junk bond variability have been presented. Using the first procedure, bond rating class and age are used to predict bond mortality rates. The double lognormal distribution is used to generate random mortality rates. The mortality rates represent the percentage of the bonds value lost to investors due to default of the bonds.

A second procedure separately modelled the frequency and severity of losses from junk bond defaults. Logistic regression is used to model the probability of default and the mean and variance of average default amounts are used to derive a model for default severity.

The third procedure presented was a method for modelling the variability of junk bond returns for a portfolio of junk bonds.

Using one or more of these models, the sensitivity of an insurance company's surplus to junk bond variability can be assessed using simulation.

Much of the prior research on junk bonds supported the conclusion that these securities provided investors with very attractive risk adjusted returns. However when economic variables and age are considered, junk bond default rates appear to be much higher than previously documented. Since the research cited in this paper did not include data from the current recession, junk bonds may be even riskier than these models indicate.

The risk that junk bonds pose to insurer solvency is perceived to be greater in the life industry than in the P&C industry. This is because P&C companies in general, pay out their liabilities more quickly than life companies do and tend to invest more conservatively. While many P&C companies do not own junk bonds, some holding companies (of both P&C and Life companies) have pursued a more aggressive investment program and may have invested amounts of between 15% and 20% of their bond portfolio in below investment grade bonds.

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