SURPLUS ALLOCATION: AN OXYMORON

by

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BIOGRAPHIES

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ABSTRACT

In this paper we explore the concept of surplus allocation and demonstrate that it is an insurance oxymoron, a contradiction in terms. After the issue is defined, it is viewed and explored from four different points of view: technical, regulatory, investor-owner, and insurer management. In the course of this process, the fallacy of surplus allocation is demonstrated generally. We then discuss the implications of surplus allocation (knowing full well that however impossible the process may be, someone, somewhere, sometime, somehow will still do it) in a reductio ad absurdum vein. Concluding remarks are presented to provide additional perspective on the subject.

THE ISSUE

The issue of surplus allocation may be illustrated through the construction of a few questions, arranged in increasing order of complexity:

- Can an insurer who writes two lines of business in a single jurisdiction meaningfully allocate its surplus to line of business?

- Can an insurer who writes a single line of business in each of two jurisdictions meaningfully allocate its surplus to each jurisdiction?

- Can an insurer who writes two lines of business in each of two jurisdictions meaningfully allocate its surplus to line of business within each jurisdiction?

- Can an insurer who writes n lines of business in each of k jurisdictions meaningfully allocate its surplus to line of business within each jurisdiction?

The issue we propose to explore in this paper may be generalized as follows: Can an insurer meaningfully allocate its surplus on any basis whether it be line of business, jurisdiction, classification, loss reserves, limit of liability, size of risk, exposure units, or any other objective criterion?

A key term here is "meaningful." We know that surplus can be allocated arithmetically by any one or more of the criteria noted above. The question then is whether surplus can be allocated so that the result of the allocation may serve some useful (i.e., meaningful) purpose without, at the same time, violating its use in other constructs. Thus, the constraint of the term "meaningful" is introduced here in order to assure that the focus of the issue remains on the purpose of an allocation exercise rather than on the mechanical means for performing the allocation. One of the most conspicuous and general attempts at using "allocated surplus" has been by certain regulatory authorities in testing proposed premium rates against target rates of return.

BACKGROUND & PERSPECTIVE

Surplus is a buffer against many types of contingencies.

A useful way to begin this discussion is to consider the reasons why an insurer needs surplus in the first place.

From a purely statutory standpoint, regulatory authorities require some minimum amount of surplus (and/or capital) in order for an insurer to obtain a license to conduct an insurance business. As a practical matter, such requirements tend to be minimal and may be ignored in the context of the issues discussed in this paper, since the surplus of the typical insurer usually far exceeds the minimum statutory requirements.

Aside from meeting the minimum statutory requirement, the key function of surplus is to provide a buffer fund that would enable an insurer to do **all** of the following:

- Withstand the inherent fluctuations in claim experience it can expect from time to time. Some years, claim experience may be much better than expected while in other years, claim experience may be much worse than expected. The insurer cannot be sure what result each year's experience will bring, yet it must be prepared to absorb such contingencies in its claim experience in order to be able to honor its contractual obligations to its policyholders.

- Withstand the inherent fluctuation in investment experience. Some years investment experience may be much better than expected while in other years investment experience may be much worse than expected. The insurer cannot be sure just what kind of an investment environment it is going to have to face in the course of fulfilling its contractual claim obligations and must be prepared to absorb such contingencies of investment fluctuation. In life insurance, an attempt has been made by regulators to identify a specific segment of surplus for that special purpose: the so-called Market Security Valuation Reserve.

- Withstand general business risk. For insurance, as for other businesses, this risk is rooted in many different sources. For

example, an insurer may be unable (and in some cases estopped by a rate freeze) to implement rate increases needed to balance its premium revenue stream with its projected claim and expense experience; court interpretations can summarily invalidate many of the coverage assumptions that the insurer contemplated at the time it set its premiums or its loss reserves; a "voter revolt" can introduce totally different concepts into the way the insurance business is conducted, the most prominent recent example being California's Proposition 103 in 1988; changes in governing statutes can quickly change the rules of the game, such as the introduction of a "take all comers" requirement or the introduction "excess profits" laws, etc.

Hence the need for a surplus fund that would enable an insurer to meet its contractual obligations.

Different aspects of the surplus allocation question

The functions of surplus listed above can be viewed from at least four separate and distinct viewpoints:

(1) Technician. The issue in this context is one of feasibility: can surplus in fact be allocated in a manner that is technically sound?

(2) Regulator. The regulatory viewpoint exerts itself in at least two ways. One is in regulating for solvency. In this

context, regulators perform functions without regard to line of business or jurisdiction, treating surplus as an indivisible quantity relating to the whole of the insurer's business. The second is the issue of regulating rates, to see that rates (premiums) are not inadequate and not excessive. In this context, regulators perform their function by line of business and by jurisdiction, sometimes attempting to treat surplus also on this basis in order to measure rates of return.

(3) Investor-owner. This is an issue of protecting the investment (capital plus surplus) in an attempt to obtain a reasonable return.

(4) Manager of the insurance enterprise. The issue here is that of assisting in the management of the business (e.g., estimating profitability by line, by branch office, etc.).

The remainder of this paper discusses the subject from these four perspectives.

TECHNICALLY SPEAKING

For purposes of discussion, we make two temporary simplifying assumptions:

(1) We remove the investment risk by assuming that all insurers can invest their funds solely in risk-free instruments, such as U.S. Government obligations. [Even U.S. Government obligations are not totally risk-free; however, let the authors allow themselves that last leap of faith and simply accept U. S. Government obligations as truly "risk-free" investments.]

(2) We remove the general business risk. For the moment, assume that insurance is not subject to the various risks that simply being in business brings. We understand that this assumption is neither realistic nor practical; we make it solely to simplify the analysis and focus the issue.

The purpose of these assumptions is to temporarily narrow the issue to its statistical aspect so that there can be no mistaking the conclusions that follow. Extending the treatment to include all contingencies would complicate the treatment of the issue without fundamentally changing its character. Once the demonstration is made, we shall go back and remove these constraints.

The discussion, then, is temporarily concerned with the remaining single category of contingencies that surplus is intended to cover: **statistical fluctuations in claims experience.** The technical treatment is included in the Appendix. In the text we sketch the outline and describe the conclusions.

First we construct an insurance Utopia, with profound apologies to Sir Thomas More[1], in which all risks are identical; all insurers are of the same size, have the same appetite for risk, invest their funds identically, settle claims with identical skill, provide the same coverage, etc. The only variation is that arising out of fluctuation in claim experience, all of which is process driven. All insurers are equivalent.

In this environment, the true means of the pure premium of each of the various insurers are identical, μ, and the true variances of the pure premium of each of the various insurers are identical, σ^2. We also note that, even though the true expected means of the pure premium of the various insurers are identical and equal to μ, the pure premium of each insurer is a random variable that is independent of all the others.

The annual experience of every insurer can be considered as a sample of n observations (for example, n can be 365 daily observations of the pure premium, 52 weekly observations, annual observations over a number of policy years, individual policy experience repeated for each insured during one policy year, etc.) of the pure premium. Whatever the unit of observation finally chosen, the mean of these observations will be μ and the variance will be σ^2/n, where n is the number of units in the sample. Should two insurers

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[1] Sir Thomas More, author of Utopia, meaning literally "no place." Among other characteristics is that Utopia is a place which functions, and well, without having to resort to the use of money or any of its equivalents.

decide to merge, the expected pure premium of the resulting merged insurer will continue to be μ however the variance of the combined sample (of the pure premiums) will be $\sigma^2/2n$. Since surplus is intended to serve as a buffer against the risk of adverse claim fluctuation (only process risk in this case), surplus is a function of the variance of the **observed** pure premium. The merger of the two insurers means that the sample size is greater (in this case by an order of two) and thus the variance of the observed pure premium is smaller.

What this means is that the surplus of the new, combined entity can be smaller. In other words, if the surplus of each original insurer was S, then the surplus of the new insurer resulting from the merger can be $(2S-\epsilon)$, where ϵ is a number greater than zero which is statistically determined and which is a function of n, μ, σ^2, and a (the fluctuation standard that the insurer feels reflects its appetite for risk -- see Appendix).

Thus, in the course of merging the two insurers, a release of surplus of magnitude ϵ can be realized by the investor-owners of the merged insurers.[2] For purposes of this discussion we make the assumption that the new surplus, $(2S-\epsilon)$, is the surplus that will

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[2] This, of course, assumes that each insurer has the perfect amount of surplus S to avoid ruin to the same degree that each insurer requires. This discussion does not mean that two insurers who merge should necessarily reduce the surplus from the combined amount, all other things being equal, for we do not intend to make a statement about the absolute necessary amount of surplus.

be maintained by the new merged insurer in order to preserve the integrity of the question we are trying to answer.

Let us now suppose that the resultant merged insurer somehow maintained complete operational separation between the component insurers (i.e., the merger is technically a paper transaction). To the regulatory authorities and the rest of the world, the merged company looks like a single organism operating as a collection of two separate "branches." Can the new surplus account ($2S-\epsilon$) be allocated back to the component companies in any way that is meaningful? The answer is "no" and the reason is obvious. No matter how one allocates ($2S-\epsilon$) among the two member insurers, there is no way in which each of the individual branches (insurers) can have a surplus equal to S, its original "perfect" amount of surplus. If one is somehow able to unbundle, not allocate, ($2S-\epsilon$) into its component surpluses, then each of the resultant surplus "unbundlings," producing a surplus of S for each component insurer, is <u>fictitious</u> and cannot be used meaningfully.

One may question the size of ϵ relative to the size of S. In the technical appendix we note that ϵ is not negligible and in fact can be quite substantial.

In the technical appendix, we extend the discussion in concept in order to bring in other functions of surplus and conclude that the same logic operates for those contingencies as well. Thus, from a

strictly technical perspective, we conclude that surplus allocation is a vacuous term.

As a final note, nothing we say here should be construed as challenging the idea of allocating surplus for the purpose of deriving estimates of branch office profitability, deriving estimates of line of business profitability, etc., for purposes of internal management of an insurer's operation. All such calculations are gross approximations at best and have no life of their own outside the particular, narrow application. Whatever results they show, they have no bearing on assessing the instantaneous solvency of an insurer. The section dealing with the insurer manager will focus more on this issue.

REGULATORY ASPECTS

A responsibility of the regulator is to regulate for insurer solvency. Another, related, responsibility is that of regulating rates (premiums) and, at times, rates of return. An extension of the rate regulatory function which has emerged in recent history is that of "excess profits laws" under which all profits in excess of prescribed levels are termed "excess" and required to be refunded to policyholders.

Regulating for solvency

No insurer, as far as we can determine, has ever been found insolvent for a single line of business to the exclusion of all other lines, for a class of business to the exclusion of other classes, for a limit of liability to the exclusion of other limits, or for a single jurisdiction to the exclusion of other jurisdictions. Solvency always has been evaluated in the aggregate. [The manner in which insurer rating services assess insurers, while not directly relevant, is nevertheless interesting. They, too, assess the entire insurer, not a single line of business or a particular jurisdiction.[3]]

The concept of surplus allocation simply does not exist when examining an insurer with a view toward assessing solvency, financial strength, etc. An offshoot of this observation is that sometimes a regulator has been known to order a company to cease writing business in his jurisdiction in recognition of some concern about the solvency of the insurer in the aggregate because the regulator cannot order an insurer to cease writing business anywhere but in his jurisdiction.

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[3] We recognize that frequently different insurer members of a holding company group file separate financial statements with dedicated surplus accounts to the different insurers. In this setting, each insurer is viewed as a separate free-standing economic unit from the point of view of solvency regulation. In fact, there are documented cases where a member of a holding company group is found insolvent while other parts of the enterprise continued to do business as usual.

An interesting, and not too extreme, scenario can be described if a regulator actually tried to regulate for solvency by line of business -- by way of allocating and monitoring surplus to line of business by jurisdiction. Imagine, for example, what could happen as a result of allocating surplus to California earthquake coverage. For the vast majority of years, results are favorable and the line is seemingly "profitable." Then a major earthquake occurs and one insurer is particularly hard hit. The insurer suddenly stops paying earthquake claims because it just "ran out" of California earthquake surplus although it has plenty of auto physical damage surplus on hand.

Thus, from the point of view of regulating for solvency, surplus allocation can lead to anomalous situations, as illustrated above. We can conclude that the overall surplus of an insurer is relevant only in toto when evaluating solvency. Allocated surplus whether by line, by jurisdiction, or by any other criterion, simply is not relevant as it is, at best, a fiction, since an insurer cannot stop drawing on its other "surplus" once an allocated surplus has been exhausted.

One related issue that we do not treat specifically in this paper, but one which we feel compelled to point out, is the extra noise that reinsurance introduces into the issue of solvency regulation vis-a-vis surplus allocation. Once an insurer has reinsured any part of its book of business, then the premium-to-surplus ratio measure is "contaminated" to some extent, and any consequent allo-

cation of this "contaminated" surplus is in turn suspect, whatever use is made of such allocated surplus amounts.

Rate regulation

Because regulators must make judgments on the reasonableness of rates (premiums) on a line of business and jurisdiction specific basis, they may be tempted to allocate surplus in order to measure the total return on equity underlying the particular ratemaking exercise or to consider the investment income flowing from the invested surplus of an insurer in the calculation of the profit provision. And some may do so without regard for the other function of surplus: solvency protection. Occasionally, regulators have argued that, while surplus is indivisible for solvency regulation purposes, it is divisible for rate regulation purposes and that the two uses of surplus are unrelated. In fact, they are not unrelated.

In protecting against adverse claim fluctuation, surplus, in fact, protects against the chance that premiums have been set at an inadequate level (for whatever reason, both purposeful and accidental). Nonetheless, regulators and insurers alike have offered many means by which to allocate surplus, each of which fails at some extreme, but not implausible, situation.

Those who would allocate surplus in proportion to premium ignore the differences in the likelihood among the different lines of

business for parameter and process error to affect the premium estimation process. Is automobile physical damage insurance the same as excess medical malpractice? They ignore the differences in likelihood among different jurisdictions for parameter and process error to affect the premium estimation process. Is homeowners insurance in Pennsylvania the same as homeowners insurance along the Texas coastline?

Those who would allocate surplus in proportion to premium also ignore the need for surplus in growth or decline situations. If an insurer ceases to write business, can it release all its surplus the day after it has earned the last premium dollar although large liabilities may remain on its books for many years to come? The answer is painfully obvious.

Then there are those who would allocate surplus in proportion to loss and loss expense reserves saying that the reserves are reflective of the riskiness of adverse claims experience and the imprecisions of the ratemaking process. This assigns relatively more surplus to the long-tail lines. Is earthquake insurance, although short-tail in nature, less in need of surplus than workers compensation?

Some would prefer to allocate surplus on the basis of the relative riskiness of the business, thus solving the dilemma of allocating relatively less surplus to earthquake insurance in the case of using loss reserves as the allocation basis. This is an admirable

goal, but there is no universal agreement on the definition of riskiness or how to set about measuring it for the individual insurer. Even if riskiness is defined, then over what time frame is riskiness measured: one year, ten years, or in perpetuity?

Some favor a middle of the road process and would allocate surplus based on a blend of premium and loss reserves. This is akin to the Chinese proverb about the middle of the road. The surplus allocation still fails and now it fails for both sets of reasons, not just one.

Whether by premium or by liabilities, the allocation of surplus fails to measure the differences among the various insurers and among the various risks they write. Are all writers of general liability alike, including those who insure flower shops and those who insure manufacturers of three wheeled vehicles?

Whether allocating surplus by premium or by liabilities, the purpose of such an allocation in the context of rate regulation is usually to regulate the rate of return on equity (or surplus). This, in turn, can have its own grave consequences, but such discussion is beyond the scope of this paper.

No matter how unscientific the idea of regulating rates by measuring rates of return to allocated surplus may be, there is one case where we can envision a bit of latitude. That case involves a regulator who is trying to set a benchmark rate for the whole

industry for a single line of business where this benchmark rate represents a base line from which each insurer can apply for deviations, both upward and downward. The latitude on the issue of surplus allocation is generated by the fact that any individual insurer can opt out of the benchmark rate if, in fact, the individual insurer determines that the benchmark rate is not appropriate for its circumstances. In these cases, the regulator has to use some attributed surplus to a line of business throughout the jurisdiction in order to be able to erect the benchmark rate if he wishes to rely on a total rate of return approach to ratemaking. This acknowledgment should not be viewed as a blessing of surplus allocation, even under these circumstances. It is merely and acknowledgment of the possibility of using surplus allocation in order to create a benchmark which is **not mandatory** on any insurer.

INVESTOR-OWNER ASPECTS

The investor-owner's interest in the insurer is generally focused on the return on investment [in the business] as a totality which speaks to his consideration of surplus as a totality. After all, a person who chooses to invest in Insurer A cannot invest in its Iowa business only or in its fire insurance business only. He invests in all of Insurer A's business or he places his money elsewhere.

Further, it would be nonsensical to think that an investor-owner is concerned with the rate of return on a single line of business or

jurisdiction to the exclusion or diminution of another line of business or jurisdiction. This is akin to the owner of a grocery store who makes a greater profit from selling coffee than from selling doughnuts. Surely he strives to make a profit from every commodity that he sells, but in the end, if doughnuts continue to bring less profit than coffee but they are instrumental to bringing the coffee customers to the store, he would be foolish to discontinue the sale of doughnuts.

Among the more practical views of this issue is the view that various financial gurus have of the insurance industry. They slice, dice, mince, and rearrange industry data in many ways. The results have been viewed by line -- in which case profitability is always measured against revenue [loss and expense ratios against premiums]. But once the profitability analysis shifts to a return on investment, the results are never stated by line of business. The reason, we are certain, is as noted above: the investor-owner invests in all of Insurer A and not in a single line of business.

Illustrating this idea yet one more way, imagine a person wanting to invest only in IBM's printer business, or its mainframe manufacturing operations, or its telecommunications business, etc. The idea is just not rooted in reality -- on any level of consideration of the question. And the same conclusion extends to an investor-owner viewing his investment in an insurer at the line of business level (or by any other subdivision).

INSURER MANAGEMENT VIEW

It is in this arena that surplus allocation seems to have its best case. We are aware of a few insurers who view the profitability of their business on an allocated surplus basis -- and then only at the line of business level, branch office level, or some other broad subdivision of the operation. In these cases, surplus is allocated on any one or a combination of bases (such as premiums, loss reserves, riskiness, etc.) and profit or loss is converted into a rate of return calculation.

However sound or flawed the methodologies supporting such an exercise may be, if management derives a certain level of comfort from such calculations, whether it assists in setting its marketing strategies and objectives, making other business decisions, etc., then that is fine for that management -- as the market forces are very efficient, and over time, the value of such an exercise is borne out by the overall return on investment of this insurer, since that is how the investor-owner will view the results of the enterprise. In other words, surplus allocation may be useful as a management tool to help improve the overall return on equity that is of interest to investor-owners.

A much more common, and nearly universal, view of profitability by business subdivision (line of business, jurisdiction, classification, branch office, etc.) is measuring performance against target (or budgeted or planned or expected) loss and expense

ratios. In this sense, the manager of the particular business segment has a number of objectives to achieve (financial and otherwise) and the financial criteria are most commonly given in terms of loss and expense ratios for the particular segment of business.

The intersection of the practice of measuring performance in terms of loss and expense ratios and the practice of surplus allocation (measuring performance in terms of rates of return on equity or surplus) is where most of the discussion occurs. Such discussion normally occurs between the insurer and the regulator, and that conflict takes us back to the section on the regulatory view. The problem arises when an insurer views its results in terms of return on surplus (equity), as an internal management tool, among many, and the regulatory authorities wish to extend the idea to make a universal ratemaking science out of it, as witnessed during the recent Proposition 103 proceedings. Any attempt to view the allocation of surplus as a science is fallacious from premise to result.

Thus while an insurer's management is certainly free to make use of the idea of allocating surplus to assist it in the management of various segments of the enterprise, care needs to be taken to make sure that such practice is properly viewed (by the insurer) as just one of many tools that an insurer's management has available to it. Furthermore, even greater care must be taken to make sure that any such attempts, aimed at internal management of the enterprise, are

not inadvertently extended beyond its natural limited application, to the arena or rate regulation as the sole measure of the soundness of rates.

One of the major pitfalls of the use rate of return, when the subject is taken out of the confines of internal management, is that an insurer may be compared to other insurers in an area where comparisons simply are not possible. For example, take two insurers, A and B, identical in all respects except that A has a greater appetite for risk than B and is willing to do business at a premium-to-surplus ratio of x-to-1 while B is willing to do business at a ratio of y-to-1, with $x > y$. Now the allocations of surplus may be performed on any number of bases, all technically flawed, but producing answers, nevertheless. The regulator is in a position to view two very different rates of return, since the two insurers are identical in all other respects. What is the regulator to do? Any attempt at normalizing these results (projected rates of actual, after the fact results) to a common surplus standard are fatally flawed as are all conclusions that are drawn therefrom -- for all the reasons cited earlier in this paper.

CONCLUSIONS

In this paper we have demonstrated that meaningful allocation of surplus is a mathematical impossibility -- even in the most rudimentary sense of surplus as a buffer against fluctuation due to statistical process risk. Even though it is impossible, there are

those who would try to do it any way. We have illustrated some of the anomalies and contradictions that flow from such practices, whether they are invoked by the technician, regulator, investor-owner, or insurer management. We also discussed the idea of the use of surplus allocation as a means of assisting company management in measuring performance and cautioned against extending this idea beyond the confines of the individual insurer as the results then have no standing since that individual insurer is not comparable to any other insurer, at the most fundamental level.

EPILOGUE

This discussion would not be complete if we did not introduce one natural extension of the ideas discussed in this paper. Given that the merger of two identical insurers, all other things remaining equal, yields a release of surplus, then we can logically extend this idea to reach the conclusion that the most efficient deployment of capital in the insurance business occurs when all of the business, in all jurisdictions, is written in a single entity. It is at this point that the maximum release of [excess] surplus occurs.

We should also point out that we are aware that the size of this most efficiently capitalized insurer, can bring about its own problems of operational (in)efficiency as well as a host of related issues. Our only wish is to point out the logical extension of the idea that the sum of two insurers will release excess surplus out

of the insurance system -- that a single monolithic insurer, all other things remaining equal, will require the least amount of surplus when compared to any other configuration or distribution of business among insurers.

TECHNICAL APPENDIX

Elements of Risk

In the main body of the paper, we enumerated the three primary classes of risk against which surplus serves as a buffer: statistical fluctuation risk (both process and parameter risks), investment risk, and general business risk. This technical note deals primarily with the statistical fluctuation risk and then extends the discussion to include the other elements of risk on an intuitive level.

An Insurance Utopia

First we describe an insurance Utopia. We assume that all insurers are identical in every respect: they are of the same size (units insured and premium volume), they write identical distributions of coverage, the business is spread geographically in an identical manner, they use the same classification plans, they use the same rates, they have the same appetite for risk, etc. In other words, all Utopian insurers are equivalent. And the Utopian investor-owners, at the end of every policy year, true up the surplus accounts so that excess surplus is removed and surplus reductions are made so that the ideal surplus is present at the conclusion of the policy year (and thus at the beginning of the new policy year). In Utopia, at the beginning of any given policy year, we can expect

each insurer to (1) carry a surplus of size S, (2) to have a true, known, expected mean pure premium μ and (3) the expected pure premium to have a true, known, variance σ^2.

Simplifying Assumptions -- Peeling the Onion

In this Utopia, therefore, it would not be unreasonable to make two broad simplifying assumptions:

1. Investment risk is zero. In other words, the only investments made by Utopian insurers are made in Utopian Service (U.S.) Government obligations. While we recognize that even U.S. Government obligations are not risk-free, we propose to take that last leap of faith and assume that U.S. Government obligations are risk-free.

2. General business risk is zero. In other words, we temporarily suspend this aspect of the problem in order to keep the presentation and discussion simple.

Thus the issue is temporarily confined to the risk due to statistical fluctuation in claim experience. Surplus is to serve the sole purpose of buffer against statistical fluctuation in claim experience.

Merger of two insurers

For good and sufficient reason, two of these Utopian insurers decided to merge. The owners of the new, merged insurer were faced with the problem of how much surplus the new entity needed to carry. After only a brief consideration of the problem, they realized they needed to know how the surplus S of each of the two companies was derived if they hoped to reach a scientific answer to the problem. Since S, the mathematically correct surplus for each of the two insurers simply evolved over time, they didn't know the formula and hence could not answer the question systematically. Then they began to discuss the question in terms of "what if" in order to get a fix on the correct answer.

For example, if the pure premium, μ, is assumed to be normally distributed (which they knew to be a bad assumption but which they accepted temporarily to see what the process would yield), with a variance of σ^2, and they wanted to meet the conditions of their appetite for risk (for example: to be 99% certain that, in any one year, the observed pure premium μ is within 5% of the true mean), they found that surplus would have to be a multiple of the standard deviation that was expressed by a formula as follows:

$$f(n,\mu,\sigma,a) = S = k*n*\sigma$$

where:

- f is a mathematical function,
- n is the number of risks insured by the insurer,
- μ is the true, known, expected mean pure premium,
- σ is the true, known, expected standard deviation of the pure premium,
- a is the parameter describing the appetite for risk of the insurer,
- S is the resulting surplus, and
- k is the multiple that is produced by the formula, generally a function of a. Note that k is the same for all Utopian insurers.

Thus when the two insurers wanted to merge, they applied this logic to the merged entity and they came up with the following:

$$f(2n,\mu,\sigma,a) = k*2n*(\sigma/\sqrt{2}),$$

since they found in their book of Utopian statistics the theorem that stated that when the sample size is doubled, for a normally distributed random variable, then the standard deviation of the larger sample is equal to $\sigma/\sqrt{2}$.[4]

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[4] Theorem: If X is normally distributed with mean μ and variance σ^2 and a random sample of size n is taken, then the sample mean X' will be normally distributed with mean μ and variance σ^2/n. (Hoel, Paul G. Introduction to Mathematical Statistics (fourth edition), 1971, John Wiley & Sons.)

Thus the Utopian investor-owners decided that, for this illustration, the surplus of the merged insurer, when compared to the sum of the surpluses of the two insurers, would be in the ratio of

$$[k*2n*(\sigma/\sqrt{2})] \;/\; [2*(k*n*\sigma)] = 1/\sqrt{2},$$

therefore releasing to the investor-owners of the original insurers an amount equal to $[1-(1/\sqrt{2})]$, or about 29% of the original combined surpluses.

The Utopian investor-owners tried out a number of formulas and they always concluded that, as long as the distribution of μ was normal, then they can expect a fairly substantial release of their original investment upon merger of the two entities. This was good news indeed and confirmed their intuitive feeling about the situation, that if you have a larger "bank", so to speak, then the stability of results should improve -- all other things remaining equal. So far, so good. But the investor-owners knew that pure premiums are not normally distributed. What is an investor-owner to do?

Process risk for variables that are not normally distributed

The Utopian investor-owners further studied the question and discovered that their conclusions based on normality of the pure premium distribution held as well even if the pure premium is not normally distributed. They discovered a theorem called the Central

Limit Theorem[5] that, in effect, transformed random variables of the type they were dealing with (where the means and variances existed) to a normally distributed variable. Thus they found that the conclusions they had reached before hold regardless of the type of the distribution so long as the mean and the variance existed. They did realize, however, that the amount of surplus release may vary from the normal case. There was great jubilation in Utopia since the problem was finally solved and, in fact, surplus could be released to the investor-owners of the original companies and the merger was consummated.

Unpeeling the Onion

The simplifying assumptions that were made to confine the problem to the process risk only can now be relaxed. In the next few paragraphs, we discuss this "unpeeling of the onion" only in a very general, intuitive way since a full discussion of this extension is well beyond the scope of this paper.

So far we have dealt with process risk. Now consider parameter risk. The question is really one of asking whether the merged insurer is in any better shape (with respect to selecting the parameters of the underlying distributions) than each of the two insurers who wished to merge. The answer is intuitively "yes".

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[5] Let X be a random variable with mean μ and variance σ^2, then the random variable $Z = (X' - \mu)\sqrt{n}/\sigma$ has a distribution that approaches the standard normal distribution as n grows large without bound.

Put in the obverse, would the investor-owners of the two companies prefer to set their own individual statistical distribution parameters or would they rather collaborate in a joint effort in selecting the parameters for the combined books of business? Experience to date, McCarran-Ferguson protection notwithstanding, suggests that it is definitely less risky to select the parameters of the underlying distributions in a collective effort than individually. We should point out that parameter risk will reduce more slowly, as a function of size, than process risk.

Moving on to investment risk, it has been demonstrated in the investment literature that hedging of investments can be carried out more effectively as the size of the portfolio increases. Without straying too far afield, it is intuitively clear, if not factually, that the larger the portfolio, the greater the opportunity to subject its management to effective scientific methods of portfolio allocation, immunization, hedging, etc.

As to general business risk, the evidence is not as clear that a merger of two insurers actually reduces general business risk. If one accepted the idea of diversification as a means of hedging general business risk, then one could argue that the overall business risk is reduced by way of merger. However, offsetting such benefits is the fact that the larger insurers tend to be targets for all sorts of private and public campaigns to make a point. So, on balance, we would like to offer the thought that

business risk is not any greater because of the merger of two insurers, all other things remaining equal.

Each of the items discussed above represent a fertile field for future study in statistics, investment, and economics, and we by no means suggest that we have the last word on the subject of compressing surplus needs. We simply wish to point to the statistical case in terms of process risk, which can be demonstrated, and suggest that the other size issues only strengthen the case for the reduced need for proportional surplus growth as companies grow or merge.

The Final Generalization

For the limited example involving process risk that is discussed above, the size of the combined surpluses diminishes as the number of merged companies grows in approximately the following orders of magnitude:

Number of Merged Companies	Ratio of needed to combined surplus
1	1.00
2	0.71
3	0.58
.	...
.	...
.	...
9	0.33
10	0.32
.	...
.	...
.	...
j	$1/\sqrt{j}$

We may then conclude that the most efficient deployment of capital for an insurance business, when viewed as a business to absorb process risk of the underlying statistical risk, occurs when there is exactly one insurer writing all the business.