SOLVENCY MEASUREMENT FOR PROPERTY-LIABILITY RISK-BASED CAPITAL APPLICATIONS

By Robert P. Butsic

Biography

Mr. Butsic is an Assistant Actuary at Fireman's Fund Insurance Company, responsible for results forecasting, actuarial applications of finance, and measurement of total profit. Previously he worked at CNA Insurance. He currently is a member of the Industry Advisory Committee to the NAIC Property & Casualty Risk-Based Capital Working Group. He has a B.A. in Mathematics and an M.B.A. in Finance, both from the University of Chicago. He is an Associate in the Society of Actuaries, a Member of the American Academy of Actuaries and has written papers for several previous Discussion Paper programs.

Abstract

In 1990 the NAIC began a project to establish risk-based capital formulas. This paper shows how risk can be quantified for setting RBC for property-liability insurers. From an understanding of the general process, rules and methods are formed for practical use, either in regulation or to an insurer's in-house capital management.

The valuation of policyholders' security forms the economic basis for the development. Capital and risk are defined, leading to the *expected deficit to policyholders* as the relevant solvency risk measure. A balance sheet model relates capital and expected deficit amounts, giving results for the normal and lognormal distributions.

The paper shows how insurance risk is time-dependent, concluding that market valuation is needed to remove measurement bias and that a proper time horizon is the period between risk-based capital evaluations, even though the realization of assets and liabilities may extend beyond one period. The present value of the policyholder deficit is shown to be equivalent to a *financial option* implicitly given by the policyholders.

Finally, covariance of risk elements is discussed, indicating that the degree of correlation is a critical factor in properly setting capital levels. A linear approximation gives a simple square root rule to treat covariance.

INTRODUCTION

The recent failure of several large life insurers, following the disastrous experience of the savings & loan industry, has pushed solvency oversight to the top of the regulatory agenda. In late 1990 the National Association of Insurance Commissioners began a mission to establish risk-based capital formulas for both life and property-liability insurance, as well as model laws to institute the capital requirements.

Formula-driven capital requirements are not new to insurance. For about 40 years, European authorities have used various formulas to set solvency margins.[1] In the U.S., detailed risk-based capital formulas for other financial institutions (banks and savings & loans) have recently been adopted and are now undergoing a phase-in period.

The purpose of the paper is to show how risk can be quantified in establishing risk-based capital for property-liability insurers. From an understanding of the general process, we establish rules and methods for practical use, either in regulation or an insurer's in-house capital management.

The valuation of policyholders' security forms the economic perspective for developing results. We initially discuss the roles of parties to the insurance contract, establishing why the risk-based capital concept is economically useful. Capital and risk are defined, leading to the *expected deficit to policyholders* as the relevant risk measure for solvency analysis. A balance sheet model relates capital levels to the size of the expected deficit, providing results for the commonly-used normal and lognormal distributions.

Next, the time dimension is introduced with a discussion of *bias*. Market valuation, for both assets and liabilities, is used to remove bias in risk measurement. We describe diffusion processes to show how insurance risk is time-dependent and then determine that a proper *time horizon* for solvency determination is the period between risk-based capital evaluations. Even though the duration of assets and liabilities may extend beyond one period, the risk-based capital recalibration assures a continual minimum protection level to policyholders.

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[1]Finland has had a specific formula for a required solvency margin since 1952. See Byrnes (1986) or Daykin *et al* (1987) for an extensive overview of international approaches to solvency regulation.

We complete the risk measurement model by taking the *present value* of the policyholder deficit and showing that this measure is equivalent to a *financial option* implicitly given by the policyholders. Next, covariance and independence of risk elements are analyzed, indicating that the degree of correlation is a critical factor in properly setting capital levels. By using a linear approximation, we are able to develop a simple *square root rule* to incorporate independence. The covariance problem is illustrated with a hypothetical balance sheet application. The paper concludes with a discussion of applications and implications of the results.

ECONOMIC BASIS FOR RISK-BASED CAPITAL

The purpose of insurance is to spread the costs of unforeseen economic loss over a wide base of policyholders. In turn, the main purpose of solvency regulation is to ensure that the promised insurance protection is available to an acceptable degree of certainty.

The solvency of an insurer is intimately linked to the condition of its balance sheet. As indicated below, capital is the excess of assets over obligations. It represents the owners' stake, or equity in the firm.

Assets		*Obligations*
[Primarily Investments and Receivables]	=	[Primarily Unpaid Losses & LAE and Unearned Premium] +
		Capital

Under statutory accounting (SAP), capital is called surplus; GAAP capital is called equity. An insolvency occurs when obligations (primarily to policyholders) exceed assets[2]. In this event, the capital providers lose their entire stake in the insurer and the holders of the obligations, mostly policyholders, take over the assets (usually through regulatory intermediation).

In general, risk-based capital is the theoretical amount of capital needed to absorb the risks of conducting a business. More specifically, it is the amount of capital necessary to assure the major parties to an insolvency that the danger of failure is acceptably low. The standard for this low expectation will be addressed later.

For assessing the consequences of insolvency, the primary parties to the insurance contract are the policyholder and the providers of capital (equityholders) to the insurance firm (the

[2]This condition is called a *technical insolvency*. Usually at this point regulators will have intervened to place the company in conservatorship or will have severely curtailed its operations. Theoretically, however, an insurer could operate beyond the point of technical insolvency if payment of losses and expenses sufficiently lagged cash inflows.

policyholder can also be a capital supplier in the case of a mutual insurer). Both the insurer and the regulator are intermediaries. Third-party claimants also have a stake in the success of the insurer.

In a perfectly efficient market, solvency regulation would not be necessary. Consumers would know the likelihood of their insurer's going insolvent, with the price of the policy being adjusted to reflect the expectation that not all claims would be fully paid. Also, the insurers could adjust their capital levels to reflect their customers' preferences for more or less protection at higher and lower prices. The result would likely be a range of capitalization from high to low leverage, with increasing degrees of policyholder security.

In the real world, however, few policyholders have ready access to the information needed to assess the insolvency potential of specific insurers. Nor would they normally have the ability or desire to process the information into their insurance-buying decision. Accordingly, regulation must determine how best to compensate for this deficiency.

It is reasonable for the regulator to assume that the public would require at least a *minimum* level of protection from the adverse effects of insurer insolvency. And it would be the proper role of regulation to provide that minimum level through its legal authority. The insurance market would provide additional security through competitive means.[3]

Desirable Features of a Risk-Based Capital Method

In order to ensure equity for all parties to the insurance contract (policyholders, claimants, capital providers and insurers), the risk-based capital method should satisfy several criteria. First, the solvency standard should be the same for all classes of the above parties[4] (e.g., personal vs. commercial insureds, second- vs. third-party claimants and primary insurers vs. reinsurers).

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[3]There currently exists a wide range of capitalization in the insurance industry, with many companies publicizing their strength through their advertising.

[4]There is an issue as to whether the policyholder, in view of the price paid for the policy, actually anticipated potential non-payment of claims. Some would argue that a customer accepting the contract for a low price has implicitly "paid" the expected value of the shortage. This objection could be partially met by establishing different insolvency standards for the different classes. However, since an insurer's insolvency affects *all* of its policyholders, this scheme would require companies to insure only one class.

Second, the risk-based capital (RBC) should be objectively determined. This means that two insurers with the same risk measures will have identical RBC. Also, a single insurer will obtain the same results under different regulatory jurisdictions using the same RBC method. The objectivity criterion dictates that the risk-based capital method can be expressed as a mathematical formula incorporating financial data from insurers.

Third, the method must be able to discriminate between quantifiable items that differ materially in their riskiness. For example, if stocks are significantly riskier than bonds, and the amounts of these two assets are known for each insurer, then the RBC method should incorporate the distinction. We define each such distinct item as a *risk element*. As shown later, when we discuss the effect of time, a risk element must be a *balance sheet* quantity.

These features will be useful in the development of appropriate solvency measures for a risk-based capital program. Our goal is to determine how much capital is needed for the entire insurer. This is done by evaluating each risk element singly and then combining the capital amounts of all risk elements.

EXPECTED DEFICIT AS MEASURE OF INSOLVENCY RISK

In general, risk is the possibility of experiencing harm or loss. In the context of solvency analysis, the harm occurs when obligations (primarily reserves) exceed assets, both items being balance sheet quantities. For a balance sheet item, risk is present when the future realization of the item can be one of several values, but the particular outcome is currently unknown. Stated differently, there is a *spread* of possible future outcomes. Loss reserves may develop either upwards or down, for instance, and stock values may also fluctuate in either direction due to changing market conditions. Generally, the greater the spread of possible realizable values subsequent to the current valuation, the greater the risk.

To clarify the discussion we will use a simplified model along with a parallel numerical example, both of which we will extend to incorporate additional features.

Assets	A	cash (realizable value is certain)
Loss Reserve	$\mathbf{L}$	unpaid loss (realizable value is a random variable[5])
Capital	$\mathbf{C}$	assets – loss reserve (realizable value is a random variable)

For simplicity, we initially assume that the following conditions hold:

a) The passage of time does not affect value (i.e., interest is zero).
b) Other assets and liabilities are zero (e.g., receivables and tax liability).
c) There are no other transactions (taxes, expenses, etc.)
d) Losses include loss adjustment expenses.
e) Losses are valued at the beginning of the year and paid at the end of the year.

Numerical Example: Beginning Balance Sheet

Assets	\$13,000	Unpaid Loss	\$10,000
		Capital	\$3,000

[5]Boldface type denotes random variables; their expectations are denoted by plain type. Notice that since the loss is a random variable, the capital is also a random variable.

A key issue is defining what is meant by the "level of protection" suggested by our earlier minimum security standard. The usual measure of risk with respect to insurance solvency is the *probability of ruin.* Although this measure may appear reasonable from the internal perspective of insurance management[6] (whose employment opportunities are correlated with solvency), it is inadequate for public policy.

To illustrate, suppose that two insurers each have the above beginning balance sheets. However, their unpaid losses have different probability distributions, producing the following three possible end-of-year results for each insurer:

Insurer A	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
Scenario 1	13,000	6,900	.2	6,900	0
Scenario 2	13,000	10,000	.6	10,000	0
Scenario 3	13,000	13,100	.2	13,000	100
Expectation:	13,000	10,000		9,980	**20**

Insurer B	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
Scenario 1	13,000	2,000	.2	2,000	0
Scenario 2	13,000	10,000	.6	10,000	0
Scenario 3	13,000	18,000	.2	13,000	5,000
Expectation:	13,000	10,000		9,000	**1,000**

The payoff to policyholders is limited to the insurer's assets of $13,000. Both insurers have a 20% chance of becoming insolvent under Scenario 3, but the policyholders from Insurer B are clearly worse off. They will on average forfeit .2(18,000 - 13,000) = $1,000 of their claim payments. The policyholders from Insurer A, on the other hand, will forego an expected .2(13,100 - 13,000) = $20 of their claim payments. Clearly, the probability-of-ruin criterion is inadequate to express the policyholders' exposure to loss. It is not sufficient merely to consider the probability of ruin—its *severity* must also be appreciated.

[6]Classical risk theory, which has guided the development of European solvency margins (e.g., Beard *et al* [1984]) seems to have ignored the severity of ruin. Even the extensive simulation modeling by Daykin *et al* (1987), which provides an excellent individual insurer approach to risk-based capital, casts its results in terms of ruin probabilities.

This example suggests that a reasonable measure of insolvency risk is the *expected value* of the difference between the amount the insurer is obligated to pay the claimant and the actual amount paid by the insurer[7]. We will call this difference the *policyholder deficit.*

Using the expected policyholder deficit (EPD) risk measure, we can consistently measure insolvency risk in such a way that a standard minimum level of protection is applied to all classes of policyholders and insurers. The EPD measure can apply equally to all risk elements, whether assets or liabilities. To adjust to the scale of different risk element sizes, we will use the ratio of the expected policyholder deficit to expected loss, or the *EPD ratio*, as the basic measure of policyholder security. We denote the EPD ratio by d. The respective EPD ratios for insurers A and B are 0.2% and 10%.

We now extend the preceding numerical example to *assets*. Insurer C has a *known* loss of $5,000 about to be paid, but it has $6,300 of assets whose year-end value is uncertain (for this example we have assumed that the expected future value of the assets equals their current value):

Insurer C	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
Scenario 1	12,000	5,000	.1	5,000	0
Scenario 2	6,000	5,000	.8	5,000	0
Scenario 3	3,000	5,000	.1	3,000	2,000
Expectation	6,300	5,000		4,800	**200**

Capital:	1,300
EPD / Expected Loss:	.040
Capital / Assets:	.206

Here the policyholders will come up short 10% of the time, when assets turn out to be worth $3,000. The deficit in this case is $2000, giving an EPD of $200 and an EPD ratio of 4%. Here the ratio of capital to assets needed to provide the 4% EPD/expected loss is 20.6%.

Now suppose that the regulator wishes to set a capital standard so that the EPD is the same for all insurers, at 5% of the expected losses. This amounts to $500 for Insurer B and $250

[7]Here we ignore guaranty fund or other external sources of recoupment such as another insurer or the federal government. However, the value of insolvency insurance can be determined by methods outlined in this paper. Cummins (1988) develops risk-based guaranty fund premiums using similar principles.

for C (for brevity, we omit Insurer A from the comparison). In order to satisfy this requirement, we assume that the amount of the expected loss is given for each insurer and so we adjust the level of beginning assets to reach the desired capital. The added or subtracted assets are the same type as the original assets, so that the probability distribution of ending asset values stays constant.

The capital for Insurer B must be increased in order to meet the 5% EPD mark, while Insurer C's capital is reduced to bring down its EPD to the 5% standard:

Insurer B **5% EPD**	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
Scenario 1	15,500	2,000	.2	2,000	0
Scenario 2	15,500	10,000	.6	10,000	0
Scenario 3	15,500	18,000	.2	15,500	2,500
Expectation	15,500	10,000		9,500	**500**

Capital:	5,500
EPD / Expected Loss:	.050
Capital / Expected Loss:	.550

Insurer C **5% EPD**	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
Scenario 1	10,000	5,000	.1	5,000	0
Scenario 2	5,000	5,000	.8	5,000	0
Scenario 3	2,500	5,000	.1	2,500	2,500
Expectation	5,250	5,000		4,750	**250**

Capital:	250
EPD / Expected Loss:	.050
Capital / Assets:	.048

Determining the EPD From Probability Distributions

The preceding numerical examples are not very realistic. We need more general probability models to represent the distribution of actual realized balance sheet items. Assume that assets are certain and liabilities (losses) are uncertain. From our earlier notation we have $A = L + C$. Let $c = C/L$ be the capital per unit of expected loss. The expected loss is

$$L = \int_0^{\infty} xp(x)dx, \text{ where } p(\bullet) \text{ is the probability density for the losses } (0 \leq x < \infty).$$

The expected policyholder deficit is the expectation of losses exceeding assets, or

$$D_L = \int_A^{\infty} (x - A)p(x)dx. \tag{1a}$$

For certain losses and uncertain assets, the EPD is similar. It is the expectation of assets being less than losses:

$$D_A = \int_0^{L} (L - y)q(y)dy, \tag{1b}$$

where $q(\bullet)$ is the probability density for the assets ($0 \le y < \infty$). From the balance sheet definition, the expected value of assets is $A = (1{+}c)L$. The reader should notice the similarity of the respective expressions for D_L and D_A to the familiar costs of excess loss (with retention A) and primary (with limit L) coverages.

The Appendix applies these formulas to derive the EPD ratios for the important case of *normally distributed* risk elements. The results are expressed in terms of two important parameters denoting the levels of capital and of risk:

(1) the ratio of capital to the expected value of the risk element, and
(2) the ratio of the standard deviation of the risk element to its mean, or the coefficient of variation (CV).

$$d_L = \frac{D_L}{L} = k\,\varphi\!\left(\frac{-c}{k}\right) - c\,\Phi\!\left(\frac{-c}{k}\right) \tag{2a}$$

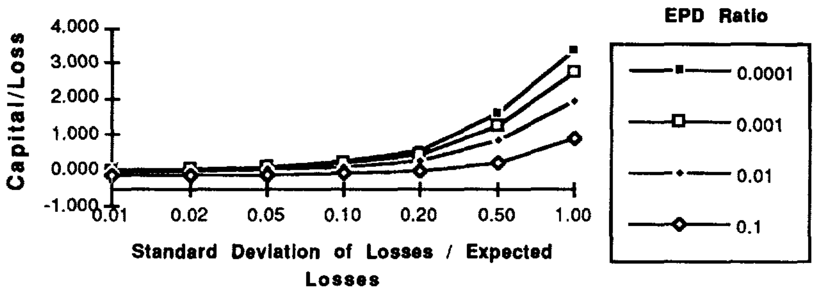
$$d_A = \frac{D_A}{L} = k_A\left(\frac{c}{c_A}\right)\varphi\!\left(\frac{-c_A}{k_A}\right) - c\Phi\!\left(\frac{-c_A}{k_A}\right) \tag{2b}$$

Here k is the ratio of the standard deviation of losses to the mean, k_A is the CV of assets, c_A is the capital to assets, $\Phi(\bullet)$ is the cumulative standard normal distribution and $\varphi(\bullet)$ is the standard normal density function.

Notice that $d_A = d_L$ if $c/k = c_A/k_A$. In other words, the EPD ratios for assets and liabilities are equal if their capital to CV ratios are also equal. The graph below shows d_L values for a range of c and k values; the relationships are identical for asset risk. Notice that for values of k less than about .20, the EPD ratio could be approximated by a linear function[8] of c and k.

Figure 1

Capital/Loss vs. EPD Ratio
and Loss Volatility
Under Normal Distribution



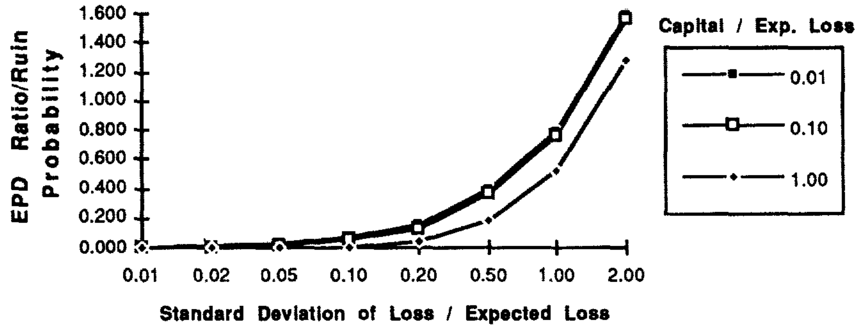
Notice that the capital ratio is *negative* for high values of d_L and low values of k. When k equals zero, there is *no risk* to policyholders—in order for there to be a positive policyholder deficit, the amount of (riskless) assets must necessarily be less than that of the certain loss. This situation, not likely occur in practice, would guarantee a deficit equal to D_L.

It is informative to compare the EPD ratio solvency criterion with the ruin probability notion. The probabilities of ruin for the respective loss and asset risk elements are simply $\Phi(-c/k)$ and $\Phi(-c_A/k_A)$. As shown below, the relationship between the EPD ratio and the probability of ruin is not constant, even for the well-known normal distribution.

[8]The linearity is stronger than appears in the graph, which has a log scale for k.

Figure 2

EPD Ratio/Ruin Probability
vs. Capital/Loss and Loss Volatility
Under Normal Distribution



The normal distribution might be a reasonable approximation for the variation of aggregate incurred loss amounts arising from a population having a known mean, where individual losses occur independently of each other. An example is non-catastrophe property insurance. For correlated events, and where the mean is unknown, a popular assumption is the *lognormal* distribution[9]. This has the desirable property that negative values are impossible, and the skewness of outcomes appears to accord with observed results. However, the sum of two lognormal variables is only approximately lognormally distributed (the *product* is a lognormal variable).

For the lognormal distribution, the Appendix also derives analogous formulas for d_L and d_A. The capital ratio c is determined by solving

$$d_L = \Phi(a_1) - (1+c)\Phi(a_2), \tag{3}$$

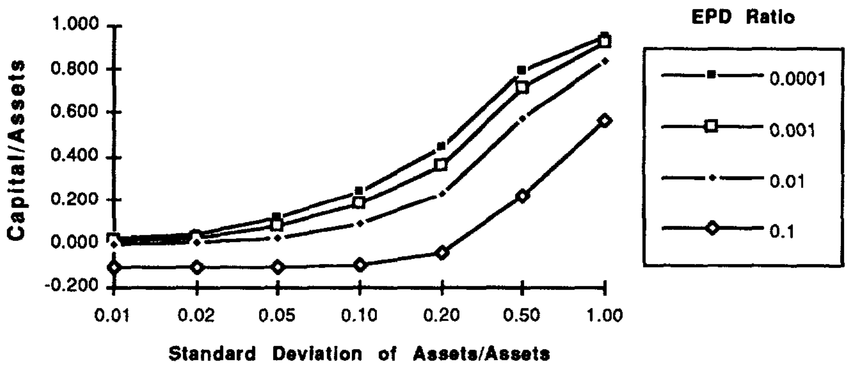
[9]Aitchison and Brown (1969) present a thorough explanation of the lognormal distribution and its economic applications.

where $a_1 = \frac{k}{2} - \frac{\ln(1+c)}{k}$, $a_2 = a_1 - k$ and $\Phi(\bullet)$ is the cumulative normal distribution. The formula for d_A is similar, with k_A and c_A replacing k and c.

For the lognormal distribution, the chart below shows the capital/assets ratio as a function of the standard deviation of year-end assets to the value of beginning assets:

Figure 3

Capital/Assets vs. EPD Ratio and Volatility of Assets Under Lognormal Distribution



Here, for a coefficient of variation less than about .20, the capital requirement is nearly the same as in the normal case (Figure 1) and again is approximately linear with the standard deviation. Above .20, less capital is required to provide the same EPD ratio as with the normal distribution.

RISK MEASUREMENT AND TIME

The development of consistent solvency risk measurement has been fairly straightforward so far. We now extend our model to incorporate the important dimension of time.

Accounting Conventions and the Bias Problem

Valuation distortions often appear for financial statement items subject to accounting measurement at any time prior to realization. Assets or liabilities may change in realizable value even if the accounting convention keeps their financial statement values constant through time. For example, bonds and real estate may vary in market value, (based on purchase price rather than current realizable value) but their statutory or GAAP values may remain constant[10] until sold. Conversely, change in an accounting value *per se* does not connote risk; rather, it is the uncertainty in the actual realized value itself (represented by the accounting value) that conveys risk. To illustrate, the ultimate value of a discounted unpaid loss may be known with certainty, but although its accounting measure will change (increase) through time, there is no risk present. On the other hand, an unpaid loss with a 50% chance of either a $1,000 payment or no payment might carry a constant $500 reserve for several years until the uncertainty is resolved.

For solvency risk measurement we need an accounting treatment that directly reveals realizable value variations. An appropriate accounting system sets all balance sheet items at current realizable value—in other words, *market-value accounting*. This valuation standard is particularly suitable for solvency assessment, since an insurer's failure usually results in liquidation of the balance sheet or purchase of the company, both in market transactions.[11]

Using the market-value approach, capital is now defined as the excess of the market value of assets over the market value of liabilities. In other words, it is the net liquidation (also

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[10]A major reason why bonds are valued at historical cost, rather than market value, is that by doing so an insurer's equity (surplus) will remain stable. However, the market value of an insurer's stock (the real economic equity of the firm) readily reflects changes in the market value of its bond portfolio caused by interest rate variation.

[11]A good example is the recent sale of Executive Life in California. Several groups have bid on various combinations of the insurer's assets and liabilities.

called break-up or winding-up) value of the company.[12] As discussed next, defining capital as the accounting book value (e.g., statutory surplus or GAAP equity) severely limits the usefulness of a risk-based capital methodology.

A major problem with measuring risk from financial statement items is *bias*. This occurs when the current recorded value differs from the current realizable value. Two insurers may carry an identical financial statement element at different amounts. For example, one insurer may record its loss reserves with a margin for adverse deviation, while another may discount its loss reserves to reflect present value. Also, identical bonds purchased at different times by two insurers may be carried at different amounts.

The difference between the carried and market value of a risk element is a measure of its *bias*. In general, bias does *not affect the risk* of a financial item, because the *spread* of potential realizable values does not depend on the valuation basis for the original estimate.

Bias may exist because the valuation standard is conservative or liberal (e.g., it ignores salvage or income tax liability). It may also be present where the estimation process consistently overstates or understates the realizable value (e.g., reserves are set using a faulty method). In the latter case the bias may be either deliberate or unintended.

Removing bias is essential in assessing the risk of insolvency. As an example, suppose two insurers have the following simplified balance sheets:

	Insurer A	Insurer B
Assets	$13,000	$13,000
Indicated Reserve	$11,000	$9,000
Indicated Capital	$2,000	$4,000

The assets are cash (with no interest). Further, both companies have identical unpaid loss obligations: $5,000 with 50% probability and $15,000 with 50% probability; the expected loss value is $10,000. Thus, their ability to pay the loss would also be identical. In each case, the expected policyholder deficit is .5(15,000 - 13,000) = $1,000.

[12]An ongoing, profitable firm normally has value exceeding the liquidation value. The excess is called goodwill or franchise value and equals the present value of profits (net cash flow) from future business. However, in the event of insolvency, the goodwill is often worthless.

However, Insurer A carries the reserve at $11,000 and Insurer B at $9,000. This gives the *appearance* that B has $2,000 more capital to withstand the adverse development, but of course it does not. Further assume that a regulator has determined a $3,000 risk-based capital standard from a universe of companies having identical assets and expected losses (but different carried reserves). Insurer A would need to increase its capital by $1,000 while Insurer B could decrease its by the same amount. In neither case would the policyholder have the same financial security.

Consequently, when establishing risk-based capital, the financial statement should first be adjusted to remove bias. Then application of the RBC technique could start from an equitable foundation.

Time Horizon for Risk Measurement

Because we have adopted the *market valuation* standard, it is clear that in order for the value of capital to change, time must elapse. For example, if the current market value of assets and loss reserves are $1,200 and $1,000 respectively, the capital will have an unambiguous value of $200. No matter how risky these items are, market valuation provides a single result when the items are measured concurrently. Since the change in value of capital depends on the passage of time, and solvency is directly related to positive capital, it follows that insolvency risk must be measured by weighing the possible capital values at a *future* time. But the future capital values are assets minus obligations—both *balance sheet* quantities. Consequently, the relevant insolvency risk elements must be capable of point-in-time estimation, as opposed to a flow-through-time measurement[13]. The capital ratios (c and c_A, for example) in our model are applied to balance sheet quantities in order to produce the required risk-based capital.

The time span between the current valuation of a financial statement item and a subsequent valuation will greatly affect the measurement of risk. For example, it is more likely for a share of stock to decline 10% in value in one year than in one day; similarly, liability reserves to be paid five years from now are more likely to develop adversely by 10% than

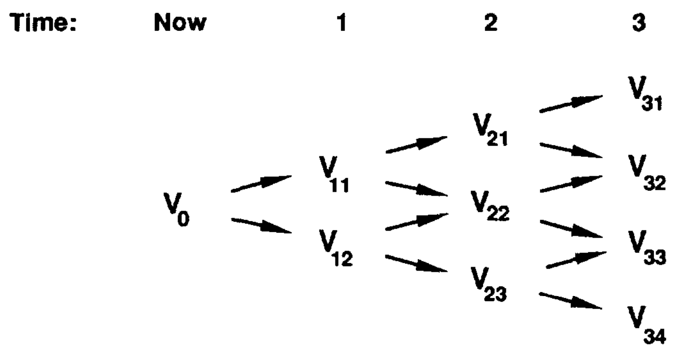
[13] In accounting theory, balance sheet items are known as *stock* quantities, while cash flow and income statement items are called *flow* quantities. Notice that the commonly-used premium-to-surplus solvency measure is the ratio of a flow to a stock quantity. A more proper measure would be the ratio of two stock items, such as unearned premium reserves and surplus.

reserves paid in the next six months. Therefore, the degree of risk depends on the time interval between valuations as well as the intrinsic volatility of the item.

The dispersion of future realizable values for many assets, notably stocks, is characterized by what is called a *diffusion* process[14]. Here the spread of future values becomes greater as the amount of time elapses. Similarly, we know that the variance of unpaid losses increases with the time required to pay claims. Conversely, and for liability claims in particular, the spread of possible values diminishes as the settlement process winds to a conclusion.

The time-dependent nature of risk is illustrated in Figure 4, where the range of possible risk element values increases with time for three periods.

Figure 4



Here the current market value is V_0, and V_{tk} represents the kth possible value at the end of the tth period. The transition of value from one period to the next is governed by a particular probability rule. When the relationship between adjacent nodes is constant (e.g., the probability of moving from, and the rate of change from V_{11} to V_{22} is the same as from V_{23} to V_{34}), the spread of values at any future point will have the same probability distribution, but with a regularly changing mean and variance. The variance will increase

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[14] A diffusion process is a type of continuous stochastic process (wherein the probability structure depends on time). The prototype for diffusion processes is Brownian motion, where changes in position are independent increments. It is commonly assumed that infinitesimal stock price changes are normally distributed, producing lognormally distributed stock returns. See Brockett and Witt (1990) for additional details.

through time (except for trivial cases) but the mean could remain constant or decrease. A critical notion, therefore, is the *variance per unit of time.*

Because the variance of realizable values is time-dependent, in order to measure risk consistently, especially for different types of risk elements, it is necessary to establish a common time horizon. Extending the earlier example, suppose that the value of the stock at the end of *four* years has a standard deviation of 10% of its current value, and the value of the unpaid loss at the end of *one* year also has a standard deviation of 10% of its current value; both risk elements follow a normally distributed diffusion process (meaning that the standard deviation is proportional to the square root of the elapsed time). With a capital level equal to 10% of the risk element value, we get the following results for $1,000 of each item:

	Value	Standard Deviation		Probability of Ruin (10% Adverse Change)		Expected Policyholder Deficit	
Time	Now	1 Year	4 Years	1 Year	4 Years	1 Year	4 Years
Stock	1,000	50	100	.023	.159	0.42	8.33
Unpaid Loss	1,000	100	200	.159	.309	8.33	39.60

The two items do not have the same risk: the reserve is far more likely to vary by 10% over the same time frame than the stock. The EPD difference for the risk elements is even more pronounced.

Continual Recapitalization With a Fixed Time Horizon

Although the parties to the insurance contract are concerned about insolvency of the insurer over a very long time horizon, not just one year, we show in this section that a sufficient and consistent solvency protection can be achieved simply by assessing risk-based capital periodically with a short time horizon.

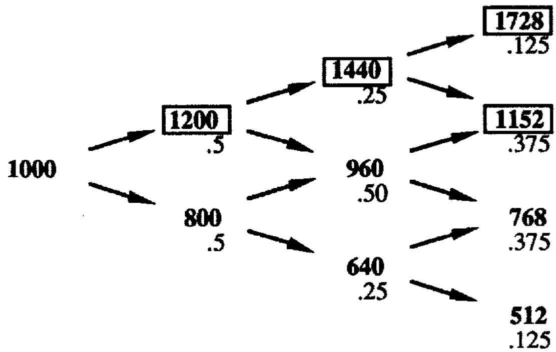
Since our basis for measuring capital is market, or liquidation value, in establishing RBC we will assume that the insurer can be liquidated at the end of each period (even though it rarely would need to do so). In this event, the unpaid losses would be valued as in a loss reserve portfolio transfer.[15] Here we assume that the reinsurer will have sufficient capital to pay the actual loss when due (otherwise, the transfer value would be less than the market value by the EPD related to the reinsurer's default prospects). For clarity, we will generally assume that the period between valuations is one year.

In the pro-forma sale of the insurer, if the market value of the assets net of losses is positive, then the reinsurer will return the excess to the owners of the insurer and pay the *entire* ultimate loss, regardless of its size. If the net market value is negative, the reinsurer will pay a *portion* of the ultimate loss equaling the ratio of the assets to the expected unpaid loss. As illustrated in the following example, this procedure guarantees that the reinsurer gets a fair deal.

Assume that an insurer has an initial unbiased loss reserve of $1,000. The actual loss payment will occur in three years; to simplify the example, there is no time value of money. Initial assets are $1,100. During each year we gather information enabling us to reevaluate our expectation of ultimate loss. The nature of this information is limited, such that the reserve can change, with equal probability, each year up or down by by 20% of the previous reserve. Thus, the reserve sequence through time is a simple binomial stochastic process[16]. Also assume that we can, at the end of each year, cede the reserve to a reinsurer for its expected value. The chart below shows the progression of possible reserve values, along with the associated probabilities. The liquidation EPD values at each stage are also given. Notice that the EPD increases each year as the spread of possible reserve values increases.

[15] The market value of loss reserves would in theory include a provision for both the present value of expected payments and the risk that the stream of actual payments will be more adverse than expected. The market value would also implicitly assume that there is no insurer default; otherwise, it would depend on the capital levels of individual insurers and thus could not be a market value. In the absence of a ready market for the trading of loss reserves, market values can be approximated by discounting using a risk-adjusted interest rate. See Butsic (1988) and D'Arcy (1988) for additional details.

[16] By using sufficiently small time intervals, a simple binomial structure can replicate a continuous diffusion process. In particular, if the relationship between adjacent nodes is multiplicative (as in the example here), the result will tend toward a lognormal distribution. Cox, Ross and Rubinstein (1979) and Trigeorgis (1991) show how the binomial method can be applied to evaluate time-dependent contingencies. Most of the numerical results for normal and lognormal distributions in this paper were obtained using this technique.



	Now	One Year	Two Years	Three Years
Expected Loss	1000	1000	1000	1000
Assets	1100	1100	1100	1100
EPD if Liquidated		50	85	98
EPD Ratio		.050	.085	.098

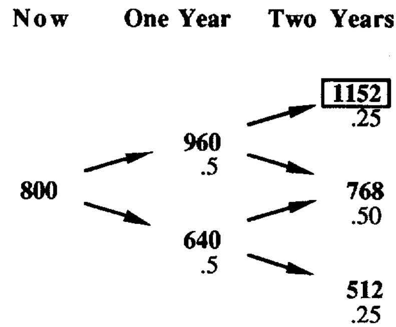
The reserve values in boxes represent a policyholder deficit if the insurer is liquidated at that point (or at the third year, when the actual claim must be paid). If the insurer dissolves at the end of the first year, there are two possibilities:

(1) The reserve is $800. Here the reinsurer accepts $800 of the insurer's assets, returning $300 to the owners, and agrees to pay the entire ultimate loss (either $1152, $768 or $512; a $1728 amount is no longer possible). Thus, the EPD is zero. The deal is fair because the value of the ceded reserve equals that of the assets transferred.

(2) The reserve is $1200. Here the reinsurer takes the entire $1100 of the insurer's assets, agreeing to pay 1100/1200 of the ultimate loss (i.e., 11/12 of either $1728, $1152 or $768). The expected value of the payoff to the policyholders is (11/12)[.25(1728) + .50(1152) + .25(768)] = $1100, thus providing a fair exchange for the reinsurer. However, the expected policyholder deficit is $100, equal to the difference between the $1200 expected loss and the $1100 expected payoff.

The unconditional EPD for the first year equals the sum of the year-end EPD's times the probability of reaching each state, or $50 = .5(0) + .5(100). The EPD ratio is 5%.

Now suppose that we recalibrate each year, adjusting the capital so that the EPD/expected loss remains at the initial one-year forward value of 5%. Then if the one-year reserve value turns out to be \$800, the capital will be \$300. We can shed most of it, leaving only \$80 of capital and then face the remaining two years:



	Now	One Year	Two Years
Expected Loss	800	800	800
Assets	880	880	880
EPD if Liquidated		40	68
EPD Ratio		.050	.085

Here, if we liquidate at the end of the year, the EPD will be \$40, or 5% of the original reserve. We can repeat the procedure for the third year in this example. As long as the insurer remains solvent, the capital can be recalibrated to maintain the same one-year forward EPD ratio. And if the insurer becomes insolvent, the procedure terminates (if it could not, then the EPD would necessarily be zero).

The process can be stated symbolically in concise fashion by extending our earlier notation. Let A_t, L_t and C_t be the random variables denoting respective assets, liabilities and capital at time t. The amounts of these variables are known at present: A_0, L_0 and C_0, with $A_0 = (1+c)L_0$. Further let $A_1 = (1+r)A_0$ and $L_1 = (1+g)L_0$, where r and g are random variables denoting the annual return on assets and the annual rate of change in value of the liabilities (i.e., the expected value of g is a risk-adjusted discount rate).

An important variable is C_1, the amount of capital at the end of one period. Define $c_1 = C_1/L_0$ as the amount of capital relative to the original expected loss. Then $c_1 = c + (1+c)r - g$ and we have the one-period EPD ratio

$$d_1 = \int_{-\infty}^{0} -zp(z)dz\,, \tag{4}$$

where $p(\bullet)$ is the density of c_1. For a given value of d_1, then, we solve the formula for the beginning capital/loss ratio c. The Appendix applies this formula to derive EPD ratios for the normal distribution when assets and liabilities are both random variables (the negative sign in the integral converts negative capital amounts to positive policyholder deficits).

To summarize, the recalibration process guarantees that the policyholders will maintain or exceed the same EPD ratio[17] each year. Therefore, by choosing a common time horizon, we can provide a consistent level of policyholder safety without regard to the actual duration of the risk element. A similar example will demonstrate that this procedure will work equally well with assets. The key requirements are that we know the time-dependent nature of the future realizable values, that the insurer can liquidate its assets and liabilities at each evaluation point (although it does not have to do so) and that the recalibration interval is long enough to allow insurers to add capital when needed. Notice that it is not necessary for the time-based probability structure to be uniform, as in the preceding example—we only need to know the current market value and the probability distribution of future market values at the *next* valuation date.[18]

The Insurer As a Going Concern

The preceding discussion treated the *runoff* of an insurer; in other words, we did not consider the risk of policies (both new and renewal) becoming effective in the future. This contingency is considered a major risk element, with rapid growth in business a primary cause of property-liability insolvencies.[19] Nevertheless, the periodic recalibration to assure a minimum EPD ratio will also work for an insurer as a going concern, continually writing new business.

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[17]Recalibration does not, of course, guarantee that the insurer will remain solvent over each successive control period. It merely limits the expected policyholder deficit to a minimum threshold.

[18]The current market value will embody the market's knowledge of all possible future values. Thus, knowing (or being able to estimate) the market value will in effect provide information concerning the forward probabilities.

[19]A study by A.M. Best (1991) showed that over the period from 1969 to 1990, 21% of Property-liability insolvencies had rapid premium growth as the major cause.

To incorporate the future business into the recalibration procedure, we still assume the capability of liquidating the insurer at the end of each successive period. However, now the year-end capital C_1 will be affected by both the runoff of the initial balance sheet (assets and liabilities A_0, L_0) and by the period-ending value of the business added during the interval. Let P be the premium (net of expenses), assumed to be collected just after the beginning of the period and let L_P be the loss from the added premium, assumed to be incurred at the end of the period and paid at the end of a subsequent period.[20] Then the end-of-period assets and liabilities are $A_1 = (A_0 + P)(1+r)$ and $L_1 = L_0(1+g) + L_P$, where r and g are as previously defined (we assume that the premiums are invested in the same assets as A_0; also the premium is known in advance[21] and thus is not a random variable).

By choosing a constant p we relate the premium to the initial capital: $P = pC_0 = pcL_0$. We also define a random variable b equal to the incurred loss ratio: $L_P = bP = bpcL_0$. Using the previous definition of the period-ending capital to the initial liabilities, we get

$$c_1 = c(1+p) + [1+c(1+p)]r + pcb - g. \tag{5}$$

Finally, we solve equation (5) for the value of c needed to determine d_1.

Equation (5) a linear function of three important random variables comprising the bulk of the risk facing the typical property-liability insurer. We have already discussed the role of the asset and loss reserve risk, represented by r and g. These can be modeled as diffusion processes. The incurred losses (represented by b), as well as a portion of the unearned premium reserve, have components that are qualitatively different than the other risks. In particular, property coverages are subject to catastrophes, which are highly unpredictable, and being paid quickly, cannot be modeled as diffusion processes. We will return to the three risk categories later when we discuss correlation of risk elements.

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[20]The beginning and end-of period assumptions are not necessary, but help to clarify the analysis. A more refined approach would place the premiums and incurred losses in the middle of the period.

[21]This assumption is reasonable, since we can predict the amount of premium for the next year fairly well from the insurer's business plans and knowledge of the current market. However, it would be increasingly difficult to accurately forecast premiums longer into the future. Even though there is considerable risk to the long-range premium forecast, the insurer does not need extra capital *now* to offset the future uncertainty. The annual RBC recalibration process places the capital at the time when it is needed.

We stated earlier that risk elements are balance sheet quantities. Although premiums and incurred losses above are *income statement* items occurring between evaluations, their *present value* is a balance sheet quantity. A true market valuation would include the present values of future premiums collected and losses and expenses paid arising from business not yet written (see footnote 12). But since assessing the worth of an insurer's future business would be a formidable task for a regulator using only public financial statements, we will ignore this item (except possibly for business added in the upcoming year) as a risk element for practical RBC applications.

Presuming that we want to incorporate future writings, the preceding numerical illustration can be extended accordingly. Assume that $200 of both premiums and expected incurred loss are added during Year 1, with no business written thereafter. The premiums are added to the $1100 of other assets, giving $1300 in initial assets. Suppose that the joint probability distribution of the incurred loss and the existing reserve amounts gives the following distribution of liabilities:

Now		**One Year**
	↗	**1500** .4
1200		
	↘	**1000** .6

The EPD ratio is .067 = [.4(1500 − 1300)]/1200. To achieve the 5% standard we need to add $50 of capital, raising assets to $1350: .05 = [.4(1500 − 1350)]/1200.

Again, the recalibration process guarantees that the policyholders will maintain at least the minimum EPD ratio each year, even if more exposures occur between evaluation points.

Insolvency Cost As a Financial Option

The model for the EPD ratio as developed up to now is nearly complete. However, we need to also consider the *present value* of the policyholder deficit: a dollar of forfeited claim a year from now is worth less than a dollar lost now. Since we are evaluating the expected policyholder deficit occurring one year hence, its value is reduced by $1/(1+i)$, where i is a default-free interest rate with a one-year duration (i.e., a one-year treasury note). From now on, we will assume that the EPD is measured at present value.

With the addition of the present value concept, the EPD is now completely analogous to a *financial option* having a one-year duration. Return to the example with a 50/50 chance of a \$1200 or \$800 loss reserve value at the end of one year, with \$1100 in assets at the end of the year. The EPD valued at that point is \$50. Now suppose that i is 8%. Then the present value of the EPD is \$50/1.08 = \$46.30.

Now suppose that a share of stock has a current price of \$1000, but will be worth either \$1200 or \$800 in one year with equal probability. An option to buy (a *call* option) one share a year from now[22] for \$1100 (the exercise price) is available. If the stock turns out to be worth \$1200, the option is worth \$100. If its price is \$800, the option would not be exercised (if so, the holder would lose \$300) and thus its value would be zero. The expected option value at the exercise date is therefore \$50 = .5(100) + .5(0). Its present value at the same 8% interest rate is \$46.30, which is identical to the value of the policyholder deficit.

Thus, for a liability risk element paired with riskless assets, we have the following call option equivalents:

	Stocks			*Insurance*
S_0:	Current Stock Price	$\leftrightarrow$	L_0:	Current Liability Value
S_1:	Stock Price in One Year	$\leftrightarrow$	L_1:	Liability Value in One Year
E_1:	Exercise Price	$\leftrightarrow$	A_1:	Asset Value in One Year
E_0:	PV of Exercise Price	$\leftrightarrow$	A_0:	Current Asset Value
	$E_0 - S_0$	$\leftrightarrow$	C_0:	Current Capital Value
	$\text{Max}[0, S_1 - E_1]$: Option Value When Exercised	$\leftrightarrow$		$\text{Max}[0, L_1 - A_1]$: Policyholder Deficit

[22] An option exercisable only at the expiration date, as we have assumed here, is called a *European* option. An option exercisable at any time until the expiration date is called an *American* option. Technically, policyholders have implicitly written American options against their claims, since the insurer's owners can "exercise" during the year, rather than at the evaluation dates. However, the difference in option value (present value of the EPD) would not be significant.

In effect, because liabilities may exceed the insurer's assets, its policyholders have given the insurer's owners the *option* to abandon full payment of claims. The legal concept of corporate limited liability (non-assessment for mutual policyholders/owners) creates this option.

For an asset risk element, paired with a riskless liability, we have similar equivalents, but to a *put* option[23]:

Stocks				*Insurance*
S_0:	Current Stock Price	↔	A_0:	Current Asset Value
S_1:	Stock Price in One Year	↔	A_1:	Asset Value in One Year
E_1:	Exercise Price	↔	L_1:	Liability Value in One Year
E_0:	PV of Exercise Price	↔	L_0:	Current Liability Value
	$S_0 - E_0$	↔	C_0:	Current Capital Value
	$\text{Max}[0, E_1 - S_1]$: Option Value When Exercised	↔		$\text{Max}[0, L_1 - A_1]$: Policyholder Deficit

Here, if the asset value (stock price) in one year is less than the liability value (exercise price) in one year, the difference is *put* to the policyholders (the option seller).

For both a risky asset and a risky liability, again we have a *put* option:

Stocks				*Insurance*
S_0:	Current Stock Price	↔	C_0:	Current Capital Value
S_1:	Stock Price in One Year	↔	C_1:	Capital Value in One Year
E_1:	Exercise Price	↔		Zero
E_0:	PV of Exercise Price	↔		Zero
	$S_0 - E_0$	↔	C_0:	Current Capital Value
	$\text{Max}[0, E_1 - S_1]$: Option Value When Exercised	↔		$\text{Max}[0, -C_1]$: Policyholder Deficit

[23]Brealy and Myers (1988) gives a good basic discussion of option relationships. A more thorough treatment is Cox and Rubinstein (1985).

The idea of insurer solvency cost being a financial option is a fairly recent development, trailing the rapid growth of stock option trading in the 1970's. For a more thorough treatment of the topic, see Doherty and Garven (1986) and Cummins (1988). In particular, Cummins shows that the value of the risky asset-liability put option (our EPD) is the fair risk-based guaranty fund premium.

We now have a fairly complete capital-setting model for individual risk elements: determine how much capital per unit of risk element satisfies a standard value of the one-year discounted[24] EPD ratio. For a liability risk element, we assume that the related asset is riskless, with annual return $r = i$. In parallel fashion, a risky asset is paired with a riskless liability, whose market value grows at an annual return of $g = i$. The next section extends our results to the more likely case where both assets and liabilities are risky.

[24]Due to the annual horizon of a practical risk-based capital program, taking the present value of the EPD will not change the *relative* capital ratios needed for one risk element versus another. This is because the same riskless interest rate should be used for all risk elements. Thus, for example, if the EPD ratio standard is set by requiring a specified percentile of insurers failing to reach the standard, then taking the present value of the EPD is not necessary.

CORRELATION AND INDEPENDENCE OF RISK ELEMENTS

We have demonstrated how risk-based capital for each risk element can be calculated separately by treating each element as a mini-insurer. Now we need a way to *combine* the risk capital for the separate elements. As shown next, we cannot simply add their required capital amounts together unless the risk elements are highly correlated with the proper sign.

A Numerical Illustration

For example, suppose that we have a line of business with riskless assets and risky losses, which can have only two possible realizable values. The values and their probabilities are given below. The desired EPD ratio is 1%. The risk-based capital needed for this degree of protection is easily calculated at $2,900:

Single Line	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
	6,900	2,000	.6	2,000	0
	6,900	7,000	.4	6,900	100
Expected Value	6,900	4,000		3,960	**40**
Capital:		2,900			
Capital / Loss:		.725			
EPD Ratio:		**.01**			

Now suppose that we have another line of business with an identical loss distribution, but directly correlated with the first: if a $2,000 loss amount occurs for the first line, the same amount occurs for the second line; similarly, a $7,000 amount will occur concurrently for both lines. The effect of combining the two lines is the same as if we now had a single line twice as large as the original single line:

Two Correlated Lines	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
	13,800	4,000	.6	4,000	0
	13,800	14,000	.4	13,800	200
Expected Value	13,800	8,000		7,920	**80**
Capital:		5,800			
Capital / Loss:		.725			
EPD Ratio:		**.01**			

Now suppose that the two lines are statistically *independent*: the value of the loss for one line does not depend on the value for the other. Then we have the following possible total losses with their associated probabilities:

Amount		Probability	
4,000	= 2,000 + 2,000	**.36**	= (.6)(.6)
9,000	= 2,000 + 7,000 or 7,000 + 2,000	**.48**	= (.6)(.4) + (.4)(.6)
14,000	= 7,000 + 7,000	**.16**	= (.4)(.4)

Adding the two $2,900 risk-based capital amounts and using the above combined losses and probabilities, we can determine the EPD for the total of the two lines:

Two Independent Lines	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
	13,800	4,000	0.36	4,000	0
	13,800	9,000	0.48	9,000	0
	13,800	14,000	0.16	13,800	200
Expected Value	13,800	8,000		7,968	**32**
Capital:		5,800			
Capital/Loss:		.725			
EPD Ratio:		**.004**			

Notice that the $32 expected deficit for the combined lines is less than the sum of the individual expected deficits ($80). This produces a 0.4% protection level, compared to the 1% value for the separate pieces. To reach the same 1% level as before, we need *less* capital than obtained by adding the separate amounts of risk-based capital:

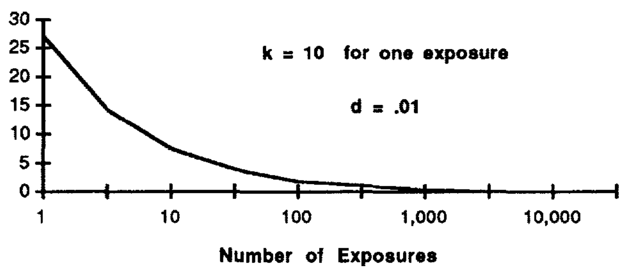
Two Independent Lines	Asset Amount	Loss Amount	Probability	Claim Payment	Deficit
	13,500	4,000	0.36	4,000	0
	13,500	9,000	0.48	9,000	0
	13,500	14,000	0.16	13,500	500
Expected Value	13,500	8,000		7,920	**80**
Capital		5,500			
Capital/Loss		**.687**			
EPD/Loss		**.01**			

As shown here, we only need $5,500 in capital, which is $480 less than the $5,980 needed when the losses are correlated. The capital ratio to loss drops from .725 to .687.

The reason for the reduced capital requirement through independence of risk elements is the *law of large numbers*. The spread of realizable values (relative to their mean) is reduced when independent elements are combined. The following graph depicts the diminishing capital needed to provide a 1% protection level for losses arising from independent normal exposures (having a standard deviation to mean ratio of 10 for a single exposure):

Figure 5

Capital / Loss For Independent Normal Exposures



This illustrates that if losses are truly independent of each other, a small line of business will need a relatively large amount of capital, while a larger one requires much less capital. In reality, however, there is a limit to the risk reduction allowed by the law of large numbers. The mean or other parameters of the loss distribution are rarely known with certainty, introducing *systematic*, or parameter risk affecting all exposures. Thus, an insurer with a very large homogeneous book of business will still be subject to considerable uncertainty, and consequent capital needs.

Correlation Under the Normal Distribution

Although the preceding numerical example illustrates the capital reduction due to independence of risk elements, one must be careful not to generalize regarding the degree of reduction.[25] More robust conclusions can be reached by analyzing a continuous probability model, such as the normal distribution.

The normal distribution has the important property that sums of normal random variables are themselves normal random variables with additive means and easily-computed variances. For two assets (A_1 and A_2), two liabilities (L_1 and L_2), or an asset and a liability (A and L), we have

	Mean	Variance
Two Assets	$A = A_1 + A_2$	$\sigma^2 = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2$
Two Liabilities	$L = L_1 + L_2$	$\sigma^2 = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2$
Asset and Liability	$C = A - L$	$\sigma^2 = \sigma_A^2 + \sigma_L^2 - 2\rho\sigma_A\sigma_L$

Here σ_1 and σ_2 denote the standard deviations of risk elements 1 and 2 (either assets or liabilities) and σ the total SD of combined risk elements (for assets minus liabilities, the SD of the capital). For the asset and liability combination, σ_A is the total asset SD and σ_L the total liability SD. The correlation coefficient between risk elements is ρ.

With perfect positive correlation ($\rho = 1$), we have $\sigma = \sigma_1 + \sigma_2$ for risk elements on the same side of the balance sheet or $\sigma = \sigma_A - \sigma_L$ for assets and liabilities. With perfect

[25]For example, using a 10% EPD Ratio, the capital requirement drops to $2,000 for the single line of business. The *combined* capital need drops to $1,000 for the two independent lines—less capital than for a single line. This effect is due to using a discrete probability distribution with a limited range of outcomes.

negative correlation ($\rho = -1$), $\sigma = \sigma_1 - \sigma_2$ and $\sigma = \sigma_A + \sigma_L$. When the elements are independent, $\rho = 0$, and thus $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$ and $\sigma = \sqrt{\sigma_A^2 + \sigma_L^2}$ for the two cases.

The formula for the EPD ratio with normally distributed combined risk elements is identical to that for individual elements (see the Appendix):

$$d = \frac{D}{L} = k\,\varphi\!\left(\frac{-c}{k}\right) - c\,\Phi\!\left(\frac{-c}{k}\right). \tag{6}$$

Here c is the capital to loss, k is the total standard deviation divided by the total expected loss L and D is the total expected policyholder deficit. The lognormal EPD ratio for combined risk elements is identical to Equation (3) and is not repeated here.

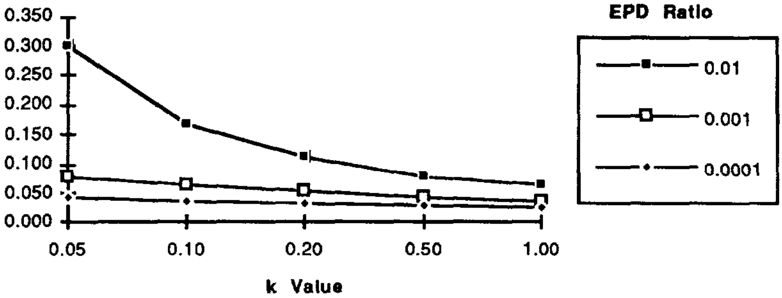
As indicated earlier, for the normal and lognormal distributions the relationship between c and k is approximately linear for a fixed EPD ratio d. Since $c = -d$ when $k = 0$ (no risk), we have $c \cong ak - d$ for some constant a. Under the assumption that we desire a high level of protection (d less than 1% or so), we can further simplify the relationship to $c \cong ak$.

Since the total capital C equals cL and the total SD σ equals kL, it follows that if $c = ak$, then $C = akL = a\sigma$. Therefore, the risk-based capital for the total of separate risk elements is proportional to their combined standard deviation. Risk capital for perfectly correlated items can be added (or subtracted, depending on whether the correlation is positive or negative or whether the items are on the same side of the balance sheet). Risk capital for independent (and partially correlated items) can be combined according to the square root of the sum of the squares of their standard deviations, plus twice the product of their SD's and the correlation coefficient.We will refer to this as the *square root* rule.

The graph below shows the relative error in using the square root rule, for two independent risk elements of the same size and standard deviation:

Figure 6

Relative Error Using Square Root Rule for Equal Independent Normal Risk Elements



This graph shows that the error decreases as the EPD ratio decreases and as the risk increases. For a reasonable (i.e., .001) protection level, the error is less than 10%. To illustrate, suppose that we have two independent lines of business each with a $1,000 expected loss and $200 SD. For a .001 EPD ratio, each requires $438 of capital in isolation. When the lines are combined, Equation (6) produces a capital ratio of .292, or $584 in capital when applied to the $2,000 expected total losses. The square root rule produces $619 = 438$\sqrt{2}$, which is about 6% more than the exact calculation[26] yields.

A parallel calculation using the lognormal distribution shows a 15% error: the true required capital is $694, compared to $800 indicated by the square root rule.[27]

The square root rule can be extended to incorporate more than two risk elements. The total capital C is a function of the individual element risk capital amounts C_i and the separate correlation coefficients between each pair of n risk elements (note that the sign of the correlation coefficient depends on which side of the balance sheet the two items reside):

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[26]Because the error in using the square root for the normal and lognormal distributions overstates the combined amount of capital needed, a closer fit could be had by using a root higher than two. For instance, in the normal example given, using a 2.4th root (.42 power) gives an exact result.

[27]The higher capital amounts are a consequence of thicker tail of this distribution, compared to the normal distribution. For the lognormal model, the error increases with increasing risk (k).

$$C = \left[\sum_{i=1}^{n} C_i^2 + \sum_{i \neq j}^{n} \rho_{ij} C_i C_j \right]^{\frac{1}{2}}. \tag{7}$$

Practical Application of Correlated and Independent Risk Elements

The preceding analysis has shown the effect of correlation between risk elements. Some examples of balance sheet items having varying degrees of correlation are presented in the table below:

Correlation	**Asset/ Asset**	**Liability/ Liability**	**Asset/ Liability**
Positive	Common Stock/ Preferred Stock Common Stock/ Bonds	Loss Reserve/ LAE Reserve	Bonds/ Loss Reserve
Zero	Cash/ Real Estate	Liability Loss Reserve/ Property Unearned Premium Reserve	Common Stock/ Unearned Premium Reserve
Negative	Common Stock/ Put Options	Loss Reserve/ Income Tax Liability Loss Reserve/ Dividend Reserve	Property-Liability Stock/ Loss Reserve Reinsurance Recoverable/ Loss Reserve

In general, reinsurance transactions create a high degree of correlation between ceding and assuming parties. Ownership of insurance subsidiaries (affiliates) or stock also produces highly correlated values. Where it is difficult to determine the numerical correlation between items, a practical approach would be to judgmentally peg the correlation at zero, 1 or -1, whichever is closest to the perceived value.

We can demonstrate the effect of independent and correlated risk elements by constructing a numerical example. The table below shows risk elements from a hypothetical insurer's balance sheet at market values. The capital ratios assume a .001 EPD ratio and are based roughly on empirical data.

	Amount	Capital Ratio	RBC
Stock	200	0.30	60
Bonds	1000	0.05	50
Affiliates	100	0.30	30
Loss Reserve	800	0.40	320
Property UPR	100	0.10	10
Total			**470**

The 30% stock capital factor arises from using the 16.6% standard deviation of 1946 to 1989 annual returns from Ibbotson and Associates (1990). Based on the same source, we have used a 6% annual SD for bonds (the corporate bond SD is 9.8% for a 20-year maturity; adjusting for a more typical property-liability insurer's duration gives a lower value), producing an approximate 5% capital ratio. The loss reserve capital ratio is based on a study of loss ratio variation by Derrig[28] (1986). We have assumed that the affiliate stock risk is the same as for general non-insurance stock, that all the risk elements are lognormally distributed and that the EPD's are discounted at an 8% riskless interest rate. In the loss reserve (equal to the present value of the expected payments), we have also included the loss expenses and the liability portion of losses arising from the unearned premiums.

The sum of the separate risk-based capital amounts is $470. This value assumes that *all* items are fully correlated, ignoring any independence or partial covariance between the items. Now assume that only the following pairs of elements are correlated:

		Correlation Coefficient
Stock	Bonds	0.2
Stock	Affiliates	1.0
Bonds	Affiliates	0.2
Bonds	Loss Reserve	0.4
Affiliates	Loss Reserve	-1.0

The property UPR is independent of all other items. Notice that the bonds/reserve correlation coefficient is positive due the parallel change in value from interest rate movements;

[28]Derrig used a sample of Workers' Compensation and Private Passenger Auto loss ratios from 51 insurers over the period 1976-1985 (since calendar-year losses were used, the variance should be similar to that for loss reserves). The combined annual variance was .059, which we have judgmentally reduced to .045 reflecting a greater variance in the unpaid loss tail; the variance is lowered when the loss is brought to present value. This produces a capital ratio (to the discounted loss) of about 0.40. Notice that a further adjustment would be needed to convert the capital factor for application to an *undiscounted* loss reserve: using an 18% reserve discount, the required *statutory* surplus is $(1 + .40)(1 - .18) - 1 = .15$ times the undiscounted reserve.

since these two items are on opposite sides of the balance sheet, this means that their joint movement will *reduce* total risk.[29] Similarly, the negative sign of the affiliates/reserve correlation coefficient indicates that these opposing items will *increase* total risk when combined.

Applying Equation (7), we have the sum of the squares of the separate risk capital amounts equal to 109,500. The sum of the cross products (each of the above pairs appears twice) of the capital amounts times their correlation coefficients equals 11,800. Thus the approximate total risk capital is $\$348 = \sqrt{121{,}300}$. If all the risk elements were independent, the total required capital would be only $\$331 = \sqrt{109{,}500}$.

The impact of the bond/reserves covariance can be found by setting the correlation coefficient to zero: here the total risk capital increases to \$366. Thus, the effect of their correlation is to reduce required capital by \$18. Similarly, if the affiliate and reserves values were independent, the required capital would drop by \$28 to \$320.

A more sophisticated RBC calculation would divide the risk elements into additional categories and might include a provision for the value of future business.

[29]The correlation methodology provides a means of allowing for matching of asset and liability durations. If the durations of fixed maturity assets and loss payments were equal, and the movements in value were due solely to interest rate fluctuations, then a (negative) 100% correlation coefficient would be appropriate.

CONCLUSION

We have studied the problem of how to measure insolvency risk for a risk-based capital program. The following points summarize the results:

1) The relevant measure of solvency is the *present value* of the *expected policyholder deficit* as a ratio to the expected loss. This value is equivalent to a *put option* held by the insurer's owners and equals a fair risk-based guaranty fund premium. By requiring sufficient capital to meet or exceed a common EPD ratio standard for each insurer, policyholders are assured a consistent level of protection.

2) To remove measurement *bias* caused by accounting conventions and varying insurer practices, the valuation standard for risk-based capital application should determine a *market value* for each risk element.

3) The major components of insurance risk are *time-dependent*: the longer the time to realization, the greater the risk. This relationship is particularly important for stocks, bonds, loss reserves and loss adjustment expense reserves. In order to properly compare risk between these items, a common time horizon must be used.

4) The EPD ratio is based on expected market values at the *end* of each risk-based capital *valuation interval* (generally one year). When risk capital levels can be set periodically, with sufficient time for insurers to add capital where necessary, there is no need for additional capital to absorb fluctuations in value beyond the valuation interval. Capital is not required *now* for distant contingencies.

5) The risk-based capital for an insurer will always be *less* than sum of the separate RBC amounts for each risk element, to the extent that all the elements are not fully correlated. By assuming a normal distribution, an approximate method for combining risk capital is the square root of the squared individual RBC amounts plus additional terms involving the correlation coefficients. In general, knowing the degree of correlation between risk elements is as important as knowing the risk of individual items.

Although we have looked at the solvency problem from a regulator's viewpoint, the concepts could readily be applied to an insurer's *in-house* capital management. For example, the insurer might want a consistent level of capital higher than the regulatory target RBC. Or, in the absence of a regulatory risk-based capital program, the insurer may wish to set its own standards.

Other applications for the risk measurement concepts presented here include setting risk loadings for reinsurer default, since the relationship between a ceding insurer and an assuming reinsurer is analogous to that of a policyholder and an insurer (the ceding commission for reinsured business should include a provision for the reinsurer's possible insolvency). Another practical use might be establishing solvency ratings for insurers based on the relationship between their recorded capital (adjusted for known bias) and their risk-based capital.

Although we have presented some empirical results in order to explain the application of our methodology, the findings are still rudimentary. It is especially important to determine more accurate distributional assumptions for loss reserve risk and to measure the correlation between risk elements.

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APPENDIX

A. Expected Policyholder Deficit Under the Normal Distribution

The amount of capital is $C = A - L$, where $A = (1+c)L$. A is the value of assets and L is the value of unpaid losses, both random variables. The expected value of C is $cL \equiv \mu$. The variance of C is σ^2. The policyholder deficit is $L - A = -C$ for $L > A$ or $C < 0$.

The expected policyholder deficit is $D = \int_{-\infty}^{0} -zp(z)dz$, where $p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(z-\mu)^2}{2\sigma^2}}$ is the normal probability density function. Let $y = (z - \mu)/\sigma$. Then $dz = \sigma dy$ and we have

$D = \int_{-\infty}^{\frac{-\mu}{\sigma}} \frac{-(\mu + y\sigma)}{\sqrt{2\pi}\sigma} e^{\frac{-y^2}{2}} \sigma dy$. This reduces to $D = \sigma\varphi\left(\frac{-\mu}{\sigma}\right) - \mu\Phi\left(\frac{-\mu}{\sigma}\right)$, where $\Phi(\bullet)$ is the cumulative standard normal distribution and $\varphi(\bullet)$ is the standard normal density. Notice that the probability of ruin ($C < 0$) is $\Phi\left(\frac{-\mu}{\sigma}\right)$.

Define $k_T \equiv \sigma/L$, the ratio of the standard deviation of the capital (total assets minus losses) to the expected loss. Then $\mu/\sigma = (cL)/(k_T L) = c/k_T$. The EPD ratio is

$$d = \frac{D}{L} = k_T\, \varphi\left(\frac{-c}{k_T}\right) - c\Phi\left(\frac{-c}{k_T}\right).$$

Letting the variance of assets be zero, we have the EPD ratio for risky losses:

$d_L = k\varphi\left(\frac{-c}{k}\right) - c\Phi\left(\frac{-c}{k}\right)$ where $k = \sigma_L/L$ with σ_L being the standard deviation of losses.

Let $c_A = C/A = c/(1+c)$ be the ratio of capital to assets and let $k_A = \sigma_A/L(1+c)$ be the ratio of the standard deviation of assets to their expected value. Setting the variance of losses to zero, we get $\sigma = \sigma_A$ and $c_A/k_A = c/k_T$. Then $k_T = k_A\,(c/c_A)$ and we have the EPD ratio for risky assets:

$$d_A = k_A\left(\frac{c}{c_A}\right)\varphi\left(\frac{-c_A}{k_A}\right) - c\Phi\left(\frac{-c_A}{k_A}\right).$$

B. Expected Policyholder Deficit Under the Lognormal Distribution

To determine the lognormal EPD at the end of one period with no time value ($i = 0$), we use the fact that the EPD for risky losses is a call option with exercise price A and current "stock price" L. Since the famous Black-Scholes option pricing model (see Black and Scholes [1973]) assumes that the future stock price is lognormally distributed with instantaneous variance σ^2, we have the option price

$$F = S\Phi(a_1) - Ee^{-it}\Phi(a_2),$$

where $a_1 = \dfrac{ln\left(\frac{S}{E}\right) + (i + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}$, $a_2 = a_1 - \sigma\sqrt{t}$, S is the stock price and E is the exercise price. Substituting $i = 0$, $t = 1$, $A = (1+c)L = E$ and $L = S$, we get the EPD

$$D_L = L\Phi(a_1) - (1+c)L\Phi(a_2),$$

where $a_1 = \dfrac{\sigma}{2} - \dfrac{ln(1+c)}{\sigma}$, $a_2 = a_1 - \sigma$, and $\Phi(\bullet)$ is the cumulative standard normal distribution. The EPD ratio to expected loss is $d_L = \Phi(a_1) - (1+c)\Phi(a_2)$.

For risky assets, the EPD is equivalent to a put option with exercise price L and stock price A. The value of the corresponding call option is

$$D'_L = A\Phi(b_1) - L\Phi(b_2),$$

where (following the preceding derivation) $b_1 = \frac{\sigma}{2} + \frac{ln(1+c)}{\sigma} = -a_2$, and similarly, $b_2 = -a_1$. Here σ denotes the instantaneous standard deviation of the assets. Thus $D'_L = A\Phi(-a_2) - L\Phi(-a_1) = A[1 - \Phi(a_2)] - L[1 - \Phi(a_1)] = D_L + A - L$.

To determine the value of the put option, we use the *put-call parity* relationship $G = F' - S + Ee^{-it}$ where G and F' are the respective values of put and call options with stock price $S = A$ and exercise price $E = L$. Since $i = 0$, the EPD is $G = D_A = D'_L - A + L = D_L$. Thus, if assets and losses have the same variance, they will have the same EPD under the lognormal distribution.

Because σ, the dispersion parameter of the lognormal distribution, is an instantaneous standard deviation, we may want to convert it to an annual rate. Suppose assets, for example, are variable with annual standard deviation to mean ratio $k_A = \sigma_A$. The variance of the lognormal distribution with mean = 1 is $e^{\sigma^2} - 1$, giving the relationship $\sigma_A = \sqrt{e^{\sigma^2} - 1}$ or $\sigma = \sqrt{ln(1 + \sigma_A^2)}$.