THE SPIRAL IN THE CATASTROPHE RETROCESSION MARKET

BY

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ABSTRACT

The risk of a major hurricane or earthquake is spread throughout the world by the property catastophe reinsurance market. This forms a complicated web of contracts with many reinsurers reinsuring little pieces of each other's catastrophe covers. Following Hurricane Alicia in 1983, the market began to notice an anomalous effect called the "London market spiral". This paper will examine:

(1) An overview of the catastrophe market.

(2) A simple example of the operation of the spiral.

(3) Under what conditions the spiral would stop, and how a loss gets distributed among market participants.

THE SPIRAL IN THE CATASTROPHE RETROCESSIONAL MARKET

I INTRODUCTION

The risk of a major hurricane or earthquake is spread throughout the world by the property catastrophe reinsurance market. Primary insurers buy catastrophe covers from reinsurers. These reinsurers in turn buy catastrophe covers from the retrocessional market. This all forms a complicated web of contracts with many reinsurers reinsuring little pieces of each other's catastrophe covers. Following Hurricane Alicia in 1983, the market began to notice an anomalous effect called the "London market spiral". Although the direct insured loss from Alicia settled down to its ultimate value fairly quickly, the gross reinsured amount of loss continued to grow year after year. This happened because market participants kept receiving additional claims and in turn submitting additional claims to their own cat covers, thereby generating more reinsurance claims (which generated more recoveries etc.).

This paper will examine:

(1) An overview of the catastrophe market.

(2) A simple example of the operation of the spiral.

(3) Under what conditions the spiral would stop, and how a loss gets distributed among market participants.

The existence of the spiral has been fairly well understood in London for several years. Nevertheless, the authors are aware of only five published articles dealing with it ([2], [3], [4], [5], [6]). Exhibit 1 shows major industry losses since 1983 that have triggered the spiral. These figures are very crude guesses, but can give the reader a feel for the magnitude of the losses

EXHIBIT 1

ESTIMATED TOTAL INSURED LOSSES FROM SELECTED MAJOR CATASTROPHES

1983 - 1990

Loss Event	Year	Affected Area	Estimated Direct Insured Loss
Hurricane Alicia	1983	U.S.	$1.2 billion
U.K. Storm 87J	1987	U.K.	£1.4 billion
Piper Alpha Oil Platform	1988	Marine Mkt	$1.4 billion
Hurricane Hugo	1989	U.S./Caribbean	$6.5 billion*
European Storm 90A	1990	U.K./Europe	£2.7 billion
European Storm 90D	1990	Europe/U.K.	£1.0 billion
European Storm 90G	1990	Europe/U.K.	£1.4 billion

* $4 billion Continental U.S.
  $2.5 billion non-U.S. and Marine.

the reinsurance market has had to digest. The most interesting example is the Piper Alpha loss where a $1 billion direct loss to the London Market is currently a gross loss of at least $10 billion and projected by some to reach $30 billion.

II PROPERTY CATASTROPHE TREATIES

Property catastrophe reinsurance treaties (also known as "cat covers") are typically simple excess contracts that can be defined with three variables:[1]

(1) The excess retention (denoted R). This is the total amount of loss from one occurrence (such as one storm) that the reinsured retains before the cat cover will respond.

(2) The limit of the cat cover (denoted L).

(3) The pro rata share of the cat cover limit retained by the primary company (denoted p). This can be zero, but is typically 5% or 10%.[2]

If G is the amount of ultimate gross loss subject to such a contract, the loss amount ultimately ceded to (i.e., recovered from) the reinsurers is given by:

$$C = \begin{cases} 0, & \text{if } G \le R \\ (1-p)\cdot(G-R), & \text{if } R < G < R+L \\ (1-p)\cdot L, & \text{if } R+L \le G \end{cases}$$

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1 Other terms commonly found such as a limitation to one reinstatement, the 72 hour clause etc., are not of importance to the spiral and, therefore, will not be discussed.

2 p can result from either a contractually agreed retained amount or from a failure to find enough reinsurers to place 100% of the treaty.

While the reinsurance purchased by reinsurers sometimes incorporates additional features, the definitions introduced above describe the essential structure of catastrophe reinsurance for insurers and reinsurers alike.

Cat covers are usually purchased in excess layers. A typical program for a large U.S. insurance company might begin with a retention of $25 million and have seven successive excess layers, giving a total coverage of $200 million. There is one international programme (covering New Zealand earthquake) that gives coverage of $600 million.

III CATASTROPHE MARKET PARTICIPANTS

As shown in Exhibit 2, catastrophe market participants can be divided into three groups based on their position in the chain of reinsurance buyers and sellers. Levels of companies are designated by Roman numerals; levels of reinsurance contracts and losses associated with those contracts are designated by Arabic numbers.

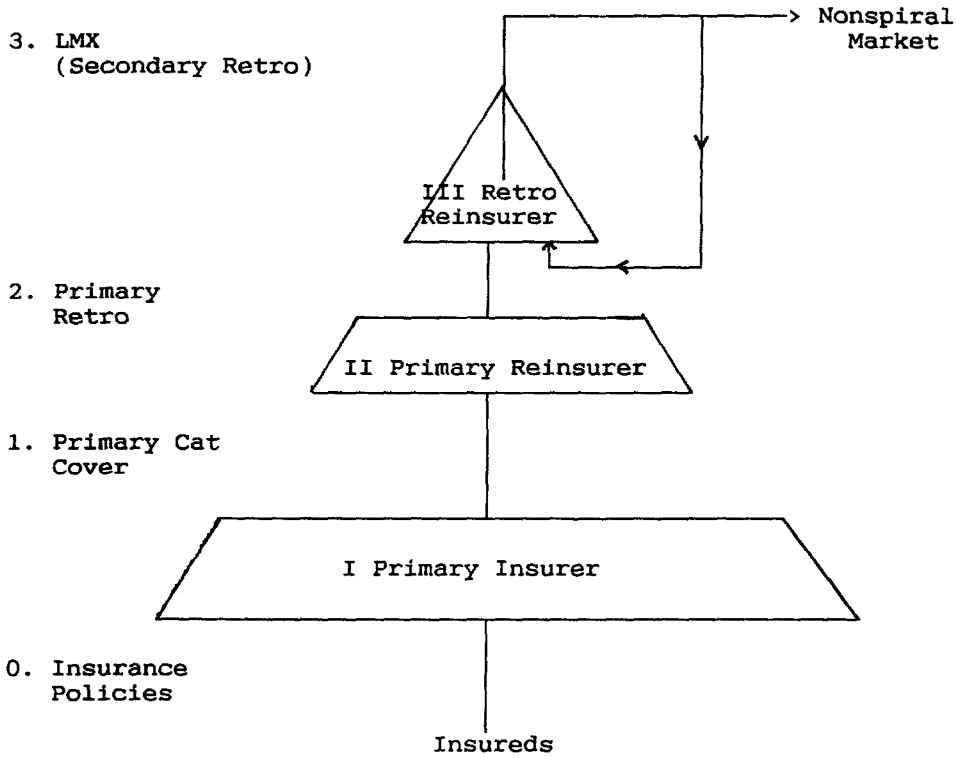
On Level I are the primary insurance companies that issue homeowners policies, commercial fire policies, etc.. They purchase reinsurance contracts known as cat covers from "primary reinsurers", on Level II.

On Level II are companies such as large professional reinsurers, many Syndicates at Lloyd's as well as large and small broker market reinsurers worldwide. Some reinsurers specialize in this business. Those companies would be "leads" who would quote terms on contracts which other companies would then "follow" (i.e., sign on to).

On Level III are companies who reinsure the primary reinsurers. They provide cat covers referred to as "primary retrocessional contracts" for the primary reinsurers. Although many of the primary reinsurers will write a handful of these primary retro

EXHIBIT 2

STRUCTURE OF THE CATASTROPHE REINSURANCE MARKET



contracts, the number of companies that specialize in and write a significant volume of this business is a small sub-set of the universe of reinsurers. Certain Syndicates at Lloyd's are specialists in this type of business.

These companies on Level III themselves buy secondary retrocessional cat covers referred to as LMX (or Level 3). There is not a distinct fourth level of companies writing these, but they are written by a subset of Level III companies themselves.

It is interesting to note that if these hierarchies are strictly followed, and if the companies writing Level 3 coverage did not themselves buy reinsurance, but kept all their exposures on a net basis, then there would not be a spiral. The spiral is created when companies on a particular level reinsure other companies on the same or higher levels and feed the exposure from those contracts back into their own retrocessional protections.

A second interesting point is that in terms of number of companies and their total surplus, this hierarchy forms a pyramid with by far the most surplus available at the base on the first level and a relatively small amount of surplus on the third level. Yet in the event of a major catastrophe the companies at the top of the pyramid are likely to receive a disproportionately large share of the loss.

IV A SIMPLE TWO COMPANY MODEL OF THE SPIRAL

Let's start with a simple two company example with the following assumptions:

(1) There are two companies in the market, A and B.

(2) A and B are Level III companies that each issue total policy limits of $1 million of primary retro covers written for primary reinsurers. (Level 2 coverage)

(3) In addition, A reinsures B, and similarly, B reinsures A for $1 million excess of $500,000 per occurrence with respect to all catastrophe losses. (Level 3 coverage)

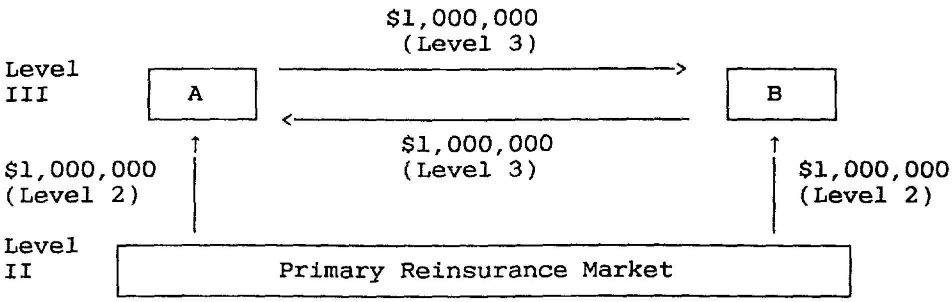


Exhibit 3 goes through the sequence of events triggered by a total loss of $1 million for each A and B from their "outside" (Level 2) business. Focus on company A for a moment. Initially things seem to work well because it can recover $500,000 of the initial loss from company B, keeping a net loss of $500,000. At time two, it receives a loss notice from company B for $500,000. It can turn around and recover this loss from company B using the remaining $500,000 of limit left under the cat cover. However, exactly the same thing is happening from company B, and at time three company B puts in a recovery for an additional $500,000 under its cat cover (which exhausts the limit). Company A has run out of limit under its own cat cover so it must retain this $500,000 net out the top. And, of course, the same thing happens to company B.

What just happened? First of all, the initial loss of $2 million ($1 million each) has become a gross loss of $4 million. However, it is still a net loss of $2 million. Secondly, each company has exhausted their reinsurance programme out the top. It is easy to see that the same thing would have happened no matter how much coverage companies A and B purchase from each other. If the layer of coverage was $10 million excess of $500,000 instead of $1 million, they would have continued to

EXHIBIT 3

LOSS FLOWS IN THE SIMPLE TWO COMPANY MODEL

	Company A Losses			Company B Losses		
Time	Gross	Ceded to B	Net	Gross	Ceded to A	Net
1	1,000	500	500	1,000	500	500
2	500	500	0	500	500	0
3	500		500	500		500
Total:	2,000	1,000	1,000	2,000	1,000	1,000

trade losses back and forth twenty times until each retained a $500,000 loss out the top.

This example illustrates a common enough phenomenon in the real market. While at first blush it appears that companies A and B each had reduced their net retention from $1,000,000 to $500,000, in fact they had not. In the process, company A's total policy limits issued increased from $1,000,000 to $2,000,000. Since it had purchased only $1 million of reinsurance coverage, company A still had $1 million of net exposure after the transaction. Company B is in the same position.

This should not be too surprising. No new reinsurance capacity had been injected into the company A-B system. So the reciprocal reinsurance between companies A and B could not serve to reduce their collective net exposure.

For a more general description of how the spiral affects an individual company, see Appendix A.

V A CLOSER LOOK AT THE SPIRAL MARKET

Imagine the following idealized model of the spiral market. Assume there are "n" Level III (with "n" roughly at least 100) companies similar to A and B of the preceding section, each of which write proportionate shares in the same Level 1 and 2 covers. Each purchases retro protections (Level 3) having the same limits and retentions and each assumes a proportionate share of the n-1 Level 3 retro covers for the other market participants. These companies differ only by a scale factor, i.e., if company C assumes twice as much business as company D, then it will purchase a retention and limit twice as large, etc.

These companies are referred to as the spiral market, and the variables defined below are the sum over the n companies of their losses, retentions, etc. All losses assumed from or ceded to companies outside of this group will be referred to as the nonspiral market.

We will examine what happens to the Level III spiral and nonspiral markets when an initial loss is ceded to the spiral market. Define the following variables:

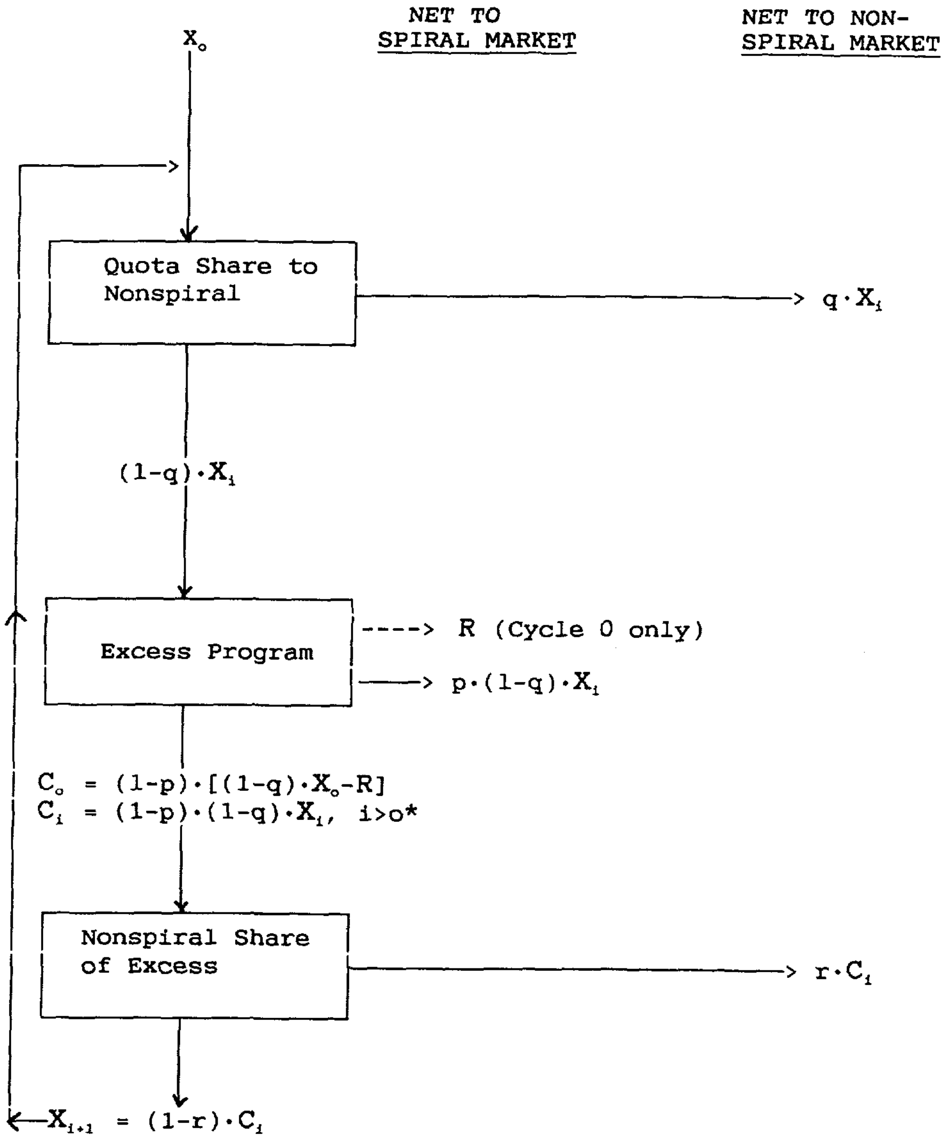
L	=	Total limit of spiral market's excess cat retro program
	=	Sum of limits of spiral market companies' excess retros
R	=	Net retention of spiral market's excess cat retro program = sum of spiral market companies' excess retentions
p	=	Average percentage of L retained net by spiral market companies under excess cat retros
r	=	Average percentage of $(1-p)\cdot L$ ceded under excess cat retros to nonspiral market = average percentage of excess cat retros placed with nonspiral market companies
q	=	Average percentage of assumed level 2 cat loss ceded to nonspiral market companies under <u>quota share</u> retros
X_o	=	Initial level 2 loss to the spiral market
X_i	=	The level 3 loss assumed by the spiral market during cycle i $(i>0)$
C_i	=	The loss ceded by the spiral market's excess program on cycle i $(i\geq 0)$
G	=	Ultimate gross loss to the spiral market = $\sum_{i=0}^{\infty} X_i$

LMX/retro quota share treaties have been a common feature in the market. Their effect on the spiral, which is different from the effect of the excess retros, has been included here in the form of the factor, q, in order to provide as comprehensive and realistic a description of the spiral as possible. However, if the reader wishes to focus on the more central role of the excess cat retros in the spiral process, q may be set to 0.

Exhibit 4A is a flowchart of the course of losses through the spiral market and the ways that losses can be removed from the spiral. It describes what happens after the spiral has been triggered (i.e., where $(1-q)\cdot X_o>R$), and until retro coverage has

EXHIBIT 4A

FLOW CHART OF LOSSES IN SPIRAL



* Subject to remaining retro coverage. If limit, L, has been exhausted, any excess will be retained net within the spiral market.

been exhausted (i.e. through cycle n, provided that $\sum_{i=0}^{n}(1-q)\cdot X_i<R+L$).

From this flowchart it should be evident that there are five places that an initial loss, X_0, to the spiral market can end up, three within the spiral market and two outside of it:

Net to the spiral market:

1. Within the net retention (up to a maximum of R)
2. Within the retained portion of cat program (up to a maximum of $p\cdot L$)
3. Out the top of the cat program

Ceded outside the spiral market:

1. To quota share reinsurance ($q\cdot G$)
2. To participants on the retro cat program (up to a maximum of $r\cdot(1-p)\cdot L$)

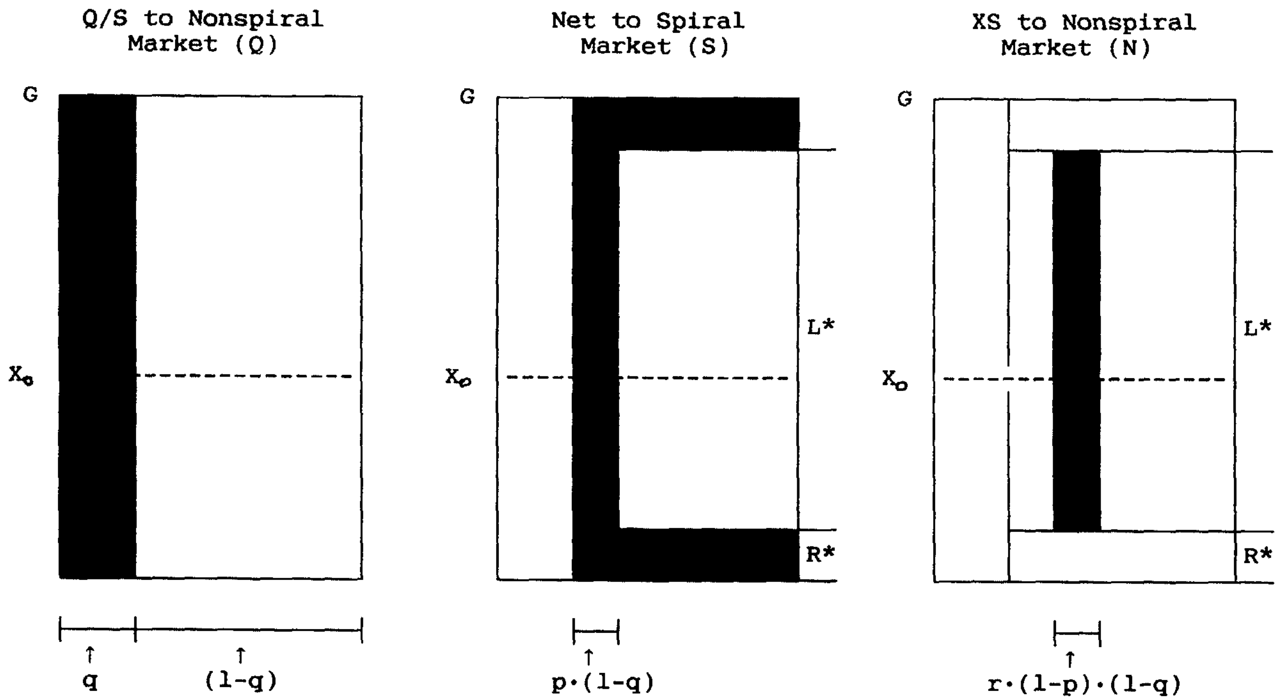
X_0 must end up being paid one of these five ways. We will call this simple principle "conservation of net losses". Exhibit 4B shows graphically how X_0 is ultimately distributed by the spiral process.

Note the difference in how the spiral and nonspiral markets participate in the spiral process. By ceding some of its losses back into the spiral retro market, the spiral market actively feeds the process. The nonspiral market, on the other hand, is passive. It assumes losses from the spiral market but does not feed them back.

The process of recycling assumed losses into the market where they become new cycles of losses to be assumed <u>is</u> "the spiral". It will continue indefinitely unless the spiral market's retro limits are reached. Whether or not losses converge at a level within the spiral market's retro coverage depends on whether a

EXHIBIT 4B

Ultimate Distribution of Initial Spiral Loss, X_o



$L^* = L/(1-q)$
$R^* = R/(1-q)$
$X_o = Q + S + N$ (Note that diagram is not to scale)

sufficient proportion of them "leak" out the spiral and avoid the recycling process.

There are three potential sources of such leakage: 1) the pro rata retention of spiral market companies' excess cat retros; 2) quota share catastrophe treaties placed in the non-spiral market, and 3) excess cat treaties placed in the non-spiral market. The first type is internal to the spiral market. The last two types represent external leakage from the spiral market, since the losses leave not only the spiral, but the spiral market, too.

Define the "leakage factor" as

$$w = 1 - (1-r)\cdot(1-p)\cdot(1-q)$$

From Exhibit 4A we can see that for losses within the range of excess retro coverage, i.e. $(1-q)\cdot X_o>R$ and $(1-q)\cdot\sum_{k=o}^{i+1} X_k<R+L$,

$$X_{i+1} = (1-r)\cdot(1-p)\cdot(1-q)\cdot X_i = (1-w)\cdot X_i, \text{ for all } i \geq 0$$

and the total spiral market gross loss is

$$\begin{aligned} G &= X_o + X_1 + X_2 + \ldots. \\ &= X_o + X_1\cdot(1+(1-w) + (1-w)^2+\ldots.) \\ &= X_o + X_1/w \quad \text{for } 0<w\leq 1 \end{aligned}$$

Let us look at what share each market segment ultimately pays of initial loss X_o. Looking at Exhibit 4A we can see that:

Spiral Market Net (denoted S)

= excess retention + proportional retention first time through + proportional retention subsequent times through

$$= R + p\cdot[(1-q)\cdot X_o-R] + \sum_{i=1}^{\infty} p\cdot(1-q)\cdot X_i$$

$$= R + p\cdot[(1-q)\cdot X_o-R] + p\cdot(1-q)\cdot X_1/w$$

$$= R + p\cdot[(1-q)\cdot G-R]$$

Quota share to nonspiral (denoted Q)

$$= \sum_{i=0}^{\infty} q \cdot X_i = q \cdot G$$

Nonspiral share of excess (denoted N)

$$= \sum_{i=0}^{\infty} r \cdot C_i$$

$$= r \cdot (1-p) \cdot [(1-q) \cdot X_0 - R) + \sum_{i=1}^{\infty} (1-q) \cdot X_i]$$

$$= r \cdot (1-p) \cdot [(1-q) \cdot G - R]$$

The distribution of losses is illustrated graphically on Exhibit 4B. The conservation of net losses principle requires that the sum of the above three must equal X_0. A proof that this is so is included in Appendix B.

Under what conditions will the spiral loss stop within the coverage limits, L?

Let

f = the percentage of L consumed by the ultimate loss subject to the excess cat retro.

$= [(1-q) \cdot G - R]/L$

Appendix C shows that

$f = (1/w) \cdot [(1-q) \cdot X_0 - R]/L$

Then $f < 1$ implies $[(1-q) \cdot X_0 - R]/w < L$ and $(1-q) \cdot X_0 - R < w \cdot L$. In other words in order for the ultimate loss to converge within the excess retro coverage, the initial loss in excess of the retention must be less than the leakage factor times the limit. This makes sense. The loss has to end up somewhere, and if it cannot be contained by leaking out the "side" of L, it will be forced out the top.

How will a loss be divided between the spiral and nonspiral markets?

Let us take as our basis for comparison the spiral market net loss for the special case where $w = p$, the "no external leakage" situation, and contrast it with the case where $w > p$.

External Leakage	Spiral Market Net
NO $(w = p)$	$R + p\cdot(G - R)$
YES $(w > p)$	$R + p\cdot[(1-q)\cdot G^{*}-R]$

Since a larger w results in a smaller gross loss, the gross loss for the external leakage case G^{*} is always less than G, so it is easy to see that $w > 0$ results in savings to the spiral market, sometimes in large amounts. This is illustrated on Exhibits 5A and 5B, where we have tabulated the spiral market's gross and net losses resulting from an entry loss, X_o = \$1,000, for a range of values of q and r. Exhibit 5A uses p = 10% and 5B uses p = 1%. The excess retention, R, is \$100 on both exhibits.

For example, when q = r = 0, the spiral market ultimately keeps the entire \$1,000 net. When p = 1%, the height of the spiral is about ten times higher than when p = 10%, but that just means it takes longer to distribute the \$1,000 - it does not change the outcome. Of course, the higher the spiral the more likely it is that losses will go out the top of the market's protections. But that also does not change the outcome - the spiral market pays 100% of X_o.

Now contrast this with a situation involving a very low rate of external leakage: q = r = 1%. If p = 10%, the spiral market ultimately retains \$885 net, a 12% reduction. If p = 1%, the spiral market retains only \$400 net, a 60% reduction, unless it exhausts its protections before the spiral converges! It is remarkable that such a low rate of external leakage results in a disproportionately large amount of leakage. However, this

EXHIBIT 5A

Spiral Market Gross and Net Loss*

Initial Loss: X_0=$1,000
Excess Retention: R=$100
Prorata Retention: p=10%

q	r	w	Gross Loss (G)	Net Loss (S)
0%	0%	10.00%	9,100	1,000
0%	1%	10.90%	8,357	926
0%	5%	14.50%	6,307	721
0%	10%	19.00%	4,837	574
0%	0%	10.90%	8,349	917
1%	1%	11.79%	7,725	855
1%	5%	15.36%	5,956	680
1%	10%	19.81%	4,639	549
5%	0%	14.50%	6,276	686
5%	1%	15.36%	5,932	654
5%	5%	18.78%	4,871	553
5%	10%	23.05%	3,987	469
10%	0%	19.00%	4,789	521
10%	1%	19.81%	4,598	504
10%	5%	23.05%	3,967	447
10%	10%	27.10%	3,391	395

*Assuming no truncation by retro coverage limits.

EXHIBIT 5B

Spiral Market Gross and Net Loss*

Initial Loss: X_0=$1,000
Excess Retention: R=$100
Prorata Retention: p=1%

q	r	w	Gross Loss (G)	Net Loss (S)
0%	0%	1.00%	90,100	1,000
0%	1%	1.99%	45,326	552
0%	5%	5.95%	15,226	251
0%	10%	10.90%	8,357	183
1%	0%	1.99%	45,276	547
1%	1%	2.97%	30,369	400
1%	5%	6.89%	13,148	229
1%	10%	11.79%	7,725	175
5%	0%	5.95%	15,143	243
5%	1%	6.89%	13,090	223
5%	5%	10.65%	8,505	180
5%	10%	16.36%	5,932	155
10%	0%	10.90%	8,266	173
10%	1%	11.79%	7,650	168
10%	5%	15.36%	5,900	152
10%	10%	19.81%	4,598	140

*Assuming no truncation by retro coverage limits.

would require a very large L to avoid going out the top.

VI WHO FINALLY PAYS THE LOSS - A NUMERICAL EXAMPLE

Suppose the total market's cat covers have the following approximate coverage features applicable to an event in the U.S. Amounts are in millions. Note that this hypothetical structure more closely approximates the non-marine market than the marine market.

	Description	Excess Retention	Excess Limit	Prorata Retention
I	Insurance	$1,000	$ 9,000	5%
II	Reinsurance	$ 500	$ 7,000	10%
	Spiral Market			
III	Retro/LMX	$ 100	$15,000	10%

Note that the nonspiral market ultimately keeps its losses net. For the sake of this illustration, let us further assume that the spiral market writes 75% of the total market's primary retro coverage, leaving 25% for the nonspiral market. Finally, assume that q=r=5%.

Even without the LMX spiral, the total loss processed by insurers and reinsurers will be greater than the direct insured loss whenever the event is large enough to result in cessions to reinsurers. The total market gross loss is the sum of the incremental gross losses experienced by market participants at all levels. These participants are insurers, primary insurers, primary retrocessionaires as well as both spiral and non-spiral LMX writers.

Suppose the insurance market suffers a $5.0 billion direct loss. Then we have the direct and ceded loss amounts shown on Exhibit 6 in millions.

This gives a multiplier of 4.4. Note that of the $2,228 that entered the spiral market at Level 2, only $1,174, or 53%, is

EXHIBIT 6

WHO PAYS THE LOSS?

Description	Gross Amount	Variable Name	Net Amount	Level
Direct	5,000		1,200	I
Ceded to primary reinsurance	3,800		830	II
Spiral Market				
Primary retro	2,228*	X_0	302	III
Spiral LMX	1,182	$G-X_0$	872	III
Nonspiral Market				
Primary retro/ LMX	743		743	
Q/S from spiral	570	Q	570	III
LMX (spiral Market)	483	N	483	III
	-----		-----	
	22,005		5,000	

* Loss entering the spiral market. Note that this is 75% of the total market's primary retro loss.

retained net by that market. Even though the nonspiral market's participation in the spiral was only through a 5% quota share and a 5% share of the spiral market's excess of loss protections, it receives 47% of the net loss![3]

VI CONCLUSION

Several interesting questions have not been addressed by this paper.

1. How do the initial premiums flow through the spiral market? (Note that 10% brokerage is normally deducted on each excess reinsurance treaty - how much remains to pay the losses)?

2. How do reinsurance premiums flow through after a loss? (Most catastrophe covers provide for payment of an additional premium to reinstate coverage.)

3. What is the effect of relaxing the assumption that each market participant has the same reinsurance program - what types of programs are better?

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[3] In 1986 the All Industry Research Advisory Council (AIRAC) studied the effects of two successive $7 billion hurricanes [1]. That study showed that the insured losses would be retained net as follows:

I	Primary Insurers	$ 4.9 billion	35%
II	Primary Reinsurers	$ 3.0 billion	21%
III	Retro Reinsurers	$ 6.1 billion	44%
	Total:	$14.0 billion	100%

The distribution of net loss by level in Exhibit 6 is not unreasonable compared to this.

While this paper was being written, during November 1990, the non-marine spiral market, after a decade of relative health, virtually disappeared. Level 3 retrocessional coverage became nearly unobtainable. The marine market, which has a more severe spiral due to much smaller R and p, and higher L, is showing signs of strain with several major Level 3 writers dropping out.

Will the spiral become a historical curiosity? If it does reappear we expect there will be a much more thorough actuarial analysis and understanding of it than in the past.

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APPENDIX A

A CLOSER LOOK AT ONE COMPANY

Suppose, as in our first example, that Company A buys retro protection for its Level 2 exposure and obtains coverage for Level 3 exposure within the same treaty. Then a single excess retention applies to the aggregate of its Level 2 and 3 losses. Let us consider the implications of this.

In order to emphasize the contrast between the conditions that feed the spiral and those that do not, let us assume that Company A also writes some primary cat covers (Level 1), which are protected by a primary retro that excludes coverage for assumed retrocessions. Exhibit A-1 summarises Company A's gross losses, X_A, its gross losses subject to reinsurance, G_A, and its ceded losses, C_A, at Levels 1,2, and 3. (Level 0, the primary insurance level, is not used in this example but is included on Exhibit A-1 for the sake of completeness.)

Note that X_A, G_A and C_A and related loss variables relate to ultimate direct losses. Case and IBNR development are included in their definition. Accordingly, the increase in the size of G_A that we demonstrate below is due only to the operation of the spiral, and not to case and IBNR development in the direct losses.

At Level 1, Company A's gross losses subject to reinsurance are confined to those from that level, so ${}_1G_A = {}_1X_A$. The amount of Level 1 loss the company cedes to its primary retro covers is given by

$${}_1C_A = \left| \begin{array}{lll} 0 & , \text{ if} & {}_1G_A \le {}_1R_A \\ (1-{}_1p_A)\cdot({}_1G_A-{}_1R_A) & , \text{ if} & {}_1R_A < {}_1G_A < {}_1R_A+{}_1L_A \\ (1-{}_1p_A)\cdot\ {}_1L_A & , \text{ if} & {}_1R_A+{}_1L_A \le {}_1G_A \end{array} \right.$$

Similarly, the Level 2 subject losses are equal to the gross losses arising from only assumed primary retros: ${}_2G_A = {}_2X_A$.

The loss ceded at Level 2 to A's secondary retro (i.e., LMX) covers is given by:

$$
{}_2C_A = \begin{cases} 0 & \text{, if } {}_2G_A \le {}_2R_A \\ (1-{}_2p_A)\cdot({}_2G_A-{}_2R_A) & \text{, if } {}_2R_A < {}_2G_A < {}_2R_A+{}_2L_A \\ (1-{}_2p_A)\cdot\ {}_2L_A & \text{, if } {}_2R_A+{}_2L_A \le {}_2G_A \end{cases}
$$

The cession formula is identical to the one for Level 1 losses. The only differences are that subject losses are at Level 2 and there are new excess and prorata retentions and a new limit.

Now let us look at what happens at Level 3. We have entered the LMX spiral and things change. Losses arising at this level (from assumed LMX covers) are protected along with those from Level 2 by the secondary retro triggered at Level 2. As a result the losses subject to the reinsurance are given by:

$$
{}_3G_A = {}_2G_A + {}_3X_A = {}_2X_A + {}_3X_A.
$$

One of the difficulties posed by the spiral is that the value of ${}_3X_A$ is not a simple function of the loss that first emerges at Level 3. Instead, ${}_3X_A$ is the sum of a series of loss amounts that include this first cycle of losses (denoted ${}_{3,1}X_A$) as well as a stream of secondary ones that are spawned by the first cycle:

$$
{}_3X_A = {}_{3,1}X_A + {}_{3,2}X_A + {}_{3,3}X_A + \ldots
$$

The gross losses subject to reinsurance after the emergence of the first cycle of Level 3 losses are:

$$
{}_{3,1}G_A = {}_2G_A + {}_{3,1}X_A = {}_2X_A + {}_{3,1}X_A
$$

The ceded losses at Level 3 depend not only on the treaty terms, p, R and L, but also on the Level 2 subject losses:

$$
{}_{3,1}C_A = \begin{cases} 0 & \text{, if } {}_{3,1}G_A \le {}_2R_A \\ (1-{}_2p_A)\cdot({}_{3,1}G_A-{}_2R_A) & \text{, if } {}_2G_A \le {}_2R_A < {}_{3,1}G_A < {}_2R_A+{}_2L_A \\ (1-{}_2p_A)\cdot{}_{3,1}X_A & \text{, if } {}_2R_A < {}_2G_A < {}_{3,1}G_A < {}_2R_A+{}_2L_A \\ (1-{}_2p_A)\cdot({}_2R_A+{}_2L_A-{}_2G_A) & \text{, if } {}_2R_A < {}_2G_A < {}_2R_A+{}_2L_A \le {}_{3,1}G_A \\ (1-{}_2p_A)\cdot\ {}_2L_A & \text{, if } {}_2G_A \le {}_2R_A < {}_2R_A+{}_2L_A \le {}_{3,1}G_A \\ 0 & \text{, if } {}_2R_A+{}_2L_A \le {}_2G_A \le {}_{3,1}G_A \end{cases}
$$

The initial gross loss at Level 3, ${}_{3,1}X_A$, receives the benefit of any contribution that the Level 2 subject loss, ${}_2G_A$, made towards satisfying the excess retention, ${}_2R_A$. So the effective excess retention for this loss is less than or equal to ${}_2R_A$, and if ${}_2G_A$ was greater than ${}_2R_A$, it is zero!
As a result when subject losses have already triggered coverage at Level 2, (i.e., ${}_2G_A>{}_2R_A$), the losses emerging in the first cycle at Level 3 are then immediately recoverable, subject only to the coverage limit, ${}_2L_A$, and the prorata retention percentage, ${}_2p_A$. If ${}_2p_A$ = 10%, as is common in the non-marine market, and it has not already exhausted its available coverage, Company A will feed 90% of its assumed Level 3 losses back into the retro market. If ${}_2p_A$ = 0%, as is common in the marine market, the feedback is 100%! In general, the portion of the first cycle loss ceded back into the market is given by ${}_{3,1}C_A = (1-{}_2p_A)\cdot{}_{3,1}X_A$ as long as ${}_2L_A$ has not been exhausted.

If Company A, like its reinsurers, has written LMX contracts (Level 3) that cover assumed LMX (Level 3) as well as assumed primary retros (Level 2), then some of the losses arising from these contracts will actually originate from Company A's own ceded loss, ${}_{3,1}C_A$. By ceding a portion of its initial Level 3 losses, Company A creates a new cycle of losses that will flow back in part to the Level 3 contracts it has written!

The treatment of the new cycle of losses, ${}_{3,2}X_A$, is similar to that of ${}_{3,1}X_A$. The gross losses subject to retro coverage are equal to the subject losses from Level 2 plus the cumulative gross losses from Level 3:

$${}_{3,2}G_A = {}_2G_A + {}_{3,1}X_A + {}_{3,2}X_A$$

Ceded losses from the second cycle are:

$${}_{3,2}C_A = \begin{cases} 0, & \text{if } {}_{3,2}G_A\le{}_2R_A \\ (1-{}_2p_A)\cdot({}_{3,2}G_A-{}_2R_A), & \text{if } {}_{3,1}G_A\le{}_2R_A<{}_{3,2}G_A<{}_2R_A+{}_2L_A \\ (1-{}_2p_A)\cdot{}_{3,2}X_A, & \text{if } {}_2R_A<{}_{3,1}G_A<{}_{3,2}G_A<{}_2R_A+{}_2L_A \\ (1-{}_2p_A)\cdot({}_2R_A+{}_2L_A-{}_{3,1}G_A), & \text{if } {}_2R_A<{}_{3,1}G_A<{}_2R_A+{}_2L_A\le{}_{3,2}G_A \\ (1-{}_2p_A)\cdot{}_2L_A, & \text{if } {}_{3,1}G_A\le{}_2R_A<{}_2R_A+{}_2L_A\le{}_{3,2}G_A \\ 0, & \text{if } {}_2R_A+{}_2L_A\le{}_{3,1}G_A\le{}_{3,2}G_A \end{cases}$$

The incremental gross losses arising from secondary Level 3 exposure, ${}_{3,2}X_A$, benefit from the erosion of the excess retention ${}_2R_A$, performed by the Level 2 and previous Level 3 losses. Subject to remaining limit, $100\cdot(1-{}_2p_A)$% of these losses flow back out through the retrocessional program.

It is now easy to see that the same scenario will repeat itself as Level 3 losses are recycled in successive waves through Company A's LMX coverage attaching at Level 2 and that of its reinsurers.

Of course, if $G_A = \lim {}_{3,n}G_A > {}_2A_A$, Company A's aggregate limits issued at Level 2, the retrocessional coverage limit eventually will be exhausted and Company A will stop feeding the spiral.

APPENDIX A

EXHIBIT A-1

LOSS FORMULAS FOR COMPANY A

Level	Incremental Gross Losses @ Level	Gross Losses Subject to Reinsurance	Losses Ceded to Reinsurance
0 Insurance	${}_0X_A = \text{direct}$	${}_0G_A = {}_0X_A$	${}_0C_A = \begin{cases} 0, & \text{if } {}_0G_A \le {}_0R_A \\ (1-{}_0p_A)\cdot({}_0G_A-{}_0R_A), & \text{if } {}_0R_A<{}_0G_A<{}_0R_A+{}_0L_A \\ (1-{}_0p_A)\cdot{}_0L_A, & \text{if } {}_0R_A+{}_0L_A<{}_0G_A \end{cases}$
1 Primary Reinsurance	${}_1X_A = \sum_{i}^{i\neq A} {}_1r_{Ai}\cdot{}_0C_i$	${}_1G_A = {}_1X_A$	${}_1C_A = \begin{cases} 0, & \text{if } {}_1G_A \le {}_1R_A \\ (1-{}_1p_A)\cdot({}_1G_A-{}_1R_A), & \text{if } {}_1R_A<{}_1G_A<{}_1R_A+{}_1L_A \\ (1-{}_1p_A)\cdot{}_1L_A, & \text{if } {}_1R_A+{}_1L_A<{}_1G_A \end{cases}$
2 Primary Retrocession	${}_2X_A = \sum_{i}^{i\neq A} {}_2r_{Ai}\cdot{}_1C_i$	${}_2G_A = {}_2X_A$	${}_2C_A = \begin{cases} 0, & \text{if } {}_2G_A \le {}_2R_A \\ (1-{}_2p_A)\cdot({}_2G_A-{}_2R_A), & \text{if } {}_2R_A<{}_2G_A<{}_2R_A+{}_2L_A \\ (1-{}_2p_A)\cdot{}_2L_A, & \text{if } {}_2R_A+{}_2L_A<{}_2G_A \end{cases}$
3 First Cycle	${}_{3,1}X_A = \sum_{i}^{i\neq A} {}_3r_{Ai}\cdot{}_2C_i$	${}_{3,1}G_A = {}_2G_A+{}_{3,1}X_A$	${}_{3,1}C_A = \begin{cases} 0, & \text{if } {}_{3,1}G_A \le {}_2R_A \\ (1-{}_2p_A)\cdot({}_{3,1}G_A-{}_2R_A), & \text{if } {}_2G_A\le{}_2R_A<{}_{3,1}G_A<{}_2R_A+{}_2L_A \\ (1-{}_2p_A)\cdot{}_{3,1}X_A, & \text{if } {}_2R_A<{}_2G_A<{}_{3,1}G_A<{}_2R_A+{}_2L_A \\ (1-{}_2p_A)\cdot({}_2R_A+{}_2L_A-{}_2G_A), & \text{if } {}_2R_A<{}_2G_A<{}_2R_A+{}_2L_A\le{}_{3,1}G_A \\ (1-{}_2p_A)\cdot{}_2L_A, & \text{if } {}_2G_A\le{}_2R_A<{}_2R_A+{}_2L_A\le{}_{3,1}G_A \\ 0, & \text{if } {}_2R_A+{}_2L_A\le{}_2G_A\le{}_{3,1}G_A \end{cases}$
3 (n-th Cycle)	${}_{3,n}X_A = \sum_{i}^{i\neq A} {}_3r_{Ai}\cdot{}_{3,n-1}C_i$	${}_{3,n}G_A = {}_{n-1}G_A+{}_{3,n}X_A$	${}_{3,n}C_A = \begin{cases} 0, & \text{if } {}_{3,n}G_A \le {}_2R_A \\ (1-{}_2p_A)\cdot({}_nG_A-{}_2R_A), & \text{if } {}_{3,n-1}G_A\le{}_2R_A<{}_{3,n}G_A<{}_2R_A+{}_2L_A \\ (1-{}_2p_A)\cdot{}_{3,n}X_A, & \text{if } {}_2R_A<{}_{3,n-1}G_A<{}_{3,n}G_A<{}_2R_A+{}_2L_A \\ (1-{}_2p_A)\cdot({}_2R_A+{}_2L_A-{}_{3,n-1}G_A), & \text{if } {}_2R_A<{}_{3,n-1}G_A<{}_2R_A+{}_2L_A\le{}_{3,n}G_A \\ (1-{}_2p_A)\cdot{}_2L_A, & \text{if } {}_{3,n-1}G_A\le{}_2R_A<{}_2R_A+{}_2L_A\le{}_{3,n}G_A \\ 0, & \text{if } {}_2R_A+{}_2L_A\le{}_{3,n-1}G_A\le{}_{3,n}G_A \end{cases}$

${}_kr_{Ai}$ = Company A's percentage share of Company i's cat cover at Level k.

APPENDIX B

Define: S = Spiral market net loss
Q = Quota share loss (nonspiral market)
N = Excess net loss (nonspiral market)

Identity:

$$S + Q + N = X_0$$

Proof:

$$\begin{aligned} S &= R + p\cdot[(1-q)\cdot G - R] \\ &= R + p\cdot[(1-q)\cdot X_0 - R] + p\cdot(1-q)\cdot X_1/w \end{aligned}$$

$$\begin{aligned} Q &= q\cdot G \\ &= q\cdot X_0 + q\cdot X_1/w \end{aligned}$$

$$N = r\cdot\sum_{i=0}^{\infty} C_i = r/(1-r)\cdot X_1/w$$

$$\begin{aligned} S + Q + N = {} & (X_1/w)\cdot[p\cdot(1-q) + q + r/(1-r)] \\ & + R + p\cdot[(1-q)\cdot X_0 - R] + q\cdot X_0 \end{aligned}$$

Note that the right hand factor of the first term can be rewritten as

$$p\cdot(1-q) - (1-q) + 1 + r/(1-r)$$

$$= 1 - (1-q)\cdot(1-p) + r/(1-r)$$

$$= [1 - (1-q)\cdot(1-p)\cdot(1-r)]/(1-r)$$

$$= w/(1-r)$$

Then $S + Q + N$ simplifies to

$$S + Q + N = X_1/(1-r) + R + p\cdot[(1-q)\cdot X_0 - R] + q\cdot X_0$$

Then

$$X_1/(1-r) = C_0 = (1-p)\cdot[(1-q)\cdot X_0 - R]$$

implies

$$\begin{aligned} S + Q + N &= R + [(1-q)\cdot X_0 - R] + q\cdot X_0 \\ &= X_0 \end{aligned}$$

APPENDIX C

Identify:

$$f = 1/w \cdot [(1-q) \cdot X_o - R]/L$$

Proof:

$$
\begin{aligned}
X_o &= S + Q + N \text{ (From Appendix B)} \\
&= R + p \cdot [(1-q) \cdot G - R] + q \cdot G + r/(1-r) \cdot X_1/w \\
&= R + p \cdot (1-q) \cdot G - p \cdot R + q \cdot G + r/(r-1) \cdot G - r/(1-r) \cdot X_o \\
X_o/(1-r) &= (1-p) \cdot R + [p \cdot (1-q) + q + r/(1-r)] \cdot G \\
&= (1-p) \cdot R + w/(1-r) \cdot G \\
\text{Substitute } G &= (f \cdot L + R)/(1-q), \text{ so} \\
X_o/(1-r) &= (1-p) \cdot R + w/(1-r) \cdot (f \cdot L + R)/(1-q) \\
X_o &= (1-p) \cdot (1-r) \cdot R + w \cdot (f \cdot L + R)/(1-q) \\
(1-q) \cdot X_o &= (1-p) \cdot (1-r)(1-q) \cdot R + w \cdot (f \cdot L + R) \\
(1-q) \cdot X_o &= (1-w) \cdot R + w \cdot R + w \cdot f \cdot L \\
w \cdot f \cdot L &= (1-q) \cdot X_o - R \\
f &= [1/w \cdot [(1-q) \cdot X_o - R]/L
\end{aligned}
$$