TITLE: INSURANCE PROFITS: KEEPING SCORE

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ABSTRACT: This paper describes an accounting system designed to take investment income from insurance float into account when evaluating insurance operations. It makes use of data contained in the annual statement to estimate premium, loss, and expense cash flows. It discusses the use of a risk free asset portfolio for estimating the investment income that can be earned from insurance float. This investment income is used in evaluating the performance of insurance operations. The paper suggests that the difference between earnings from risk free assets and actual assets is the reward for the financial intermediary role of insurance companies. Thus this difference should be the basis for evaluating the performance of insurance company investment operations.

The paper also presents data on the development of industry paid losses in past years. The data indicates that these payment patterns have been quite stable over the years. Also presented is the difference in loss payment patterns and average payment dates between incurred losses and for loss reserves. A method is developed that can be used in many circumstances for estimating the investment income that can be earned from loss payment patterns where the only information available is incurred losses and loss reserves. Such a situation occurs in statutory data by line and state where only such information is available.

Richard G. Woll

INSURANCE PROFITS: KEEPING SCORE

> "Every man did that which was right in his own eyes"
> Judges 21:25

Almost everybody agrees that it is impossible to evaluate the performance of Property/Casualty insurance companies without integrating underwriting and investment results. Almost everyone has their own way of doing this.

A situation of anarchy reigns in the whole area of analyzing insurance profits. Ancient Israel was in a similar fix in the days before it had its first king. Everyone did what they thought was right and chaos reigned.

A major part of the problem is that current insurance accounting statements have not been designed to address the needs of profit evaluation[1].

This paper describes an accounting system that integrates underwriting earnings and earnings from funds supplied by customers. It also takes into account earnings from capital funds. Such a system would serve as a reference source which would allow everyone to keep score in the same way. This would improve everyone's understanding of insurance company performance. Company management, insurance regulators, consumer advocates, and Wall Street analysts would all benefit.

We begin by considering the logic of insurance company profitability. It is convenient to divide the profits earned by insurance companies into three sources. The first source is the profit earned by investing capital funds. The second is underwriting profit. The third is the profit earned by investing policyholder supplied funds. The second and third sources come from being in the insurance business. Thus their sum is called the insurance profit in this paper. The first source would exist whether capital funds were employed in the insurance business or not. Each source of profit is evaluated on an after-tax basis.

Historically, insurance profits have consisted of underwriting losses and investment gains[2]. The sum of the two has usually been positive. This indicates that underwriting operations have primarily served as a source of funds for investment purposes.

1. Current statutory accounting statements were designed to serve the purpose of measuring insurance company solidity. This purpose would be better served by the inclusion of all information relevant to evaluating insurance company performance.

2. Between 1926 and 1985, the industry paid out, on average, $100.60 for losses, expenses, and policyholder dividends for every $100.00 of premiums it earned. From 1981 through 1985, these payments exceeded premiums earned by more than $50 billion.

Economic theory teaches that profitability depends, in the long run, on the amount of risk that is undertaken by a business enterprise. Insurance companies have the opportunity to take risk in two distinct areas: Underwriting, and Investments. The total profit from insurance operations of the company depends on how much risk is taken in each area. We can state this as follows:

INSURANCE PROFIT = MARKET RISK PREMIUM ON INSURANCE RISK (1)
+ MARKET RISK PREMIUM ON INVESTMENT RISK

But insurance profit is also the sum of the underwriting profit and the profit earned by investing policyholder supplied funds. Thus:

INSURANCE PROFIT = UNDERWRITING PROFIT (2)
+ INVESTMENT PROFIT AT RISK FREE RATES
+ MARKET RISK PREMIUM ON INVESTMENT RISK

Therefore underwriting profits are a function of the market risk premium on insurance risk and the investment profit at risk free rates. That is:

UNDERWRITING PROFIT = MARKET RISK PREMIUM ON INSURANCE RISK (3)
- INVESTMENT PROFIT AT RISK FREE RATES

The market reward for bearing insurance risk is unknown. If it were zero, however, the insurance profit for an insurer investing in risk free assets would also be zero. This insurance company takes no risk (since neither underwriting or investment risk exists) and cannot expect any profit. In this case, the underwriting profit will be equal to the negative of the investment profit at risk free rates.

If this insurer were to invest in risky assets, however, it could expect a higher return from its investments than it would from risk free assets and would earn a profit. This profit would exist because the company secured the use of funds by writing insurance. It would make money on the spread between the investment return it could get by taking risk and the return available at risk free rates.

When <u>insurance risk</u> is greater than zero, the insurance profit for an insurer investing in risk free assets will also be greater than zero. This additional profit will reflect the reward for insurance risk. Insurance companies, therefore, are in a position to bear two types of risk; Insurance Risk and Investment Risk, and to earn two types of reward.

In terms of their investment risk, insurance companies are basically financial intermediaries like banks. They obtain funds from policyholders and invest them. Their profits are earned by the spread between the cost of funds and the return on funds.

An interesting way to think of insurance operations in terms of the types of risk is to recognize that the investment department of an insurance company functions like a bank. It takes funds provided by the underwriting operation and pays for them at risk free rates. It invests the funds in risky assets and expects to earn a profit from the risk it is taking. The difference between the amount of investment income it earns and its cost of funds is the profit it earns. The amount of investment income earned by the investment department includes interest, dividends, and rent as well as all capital gains and losses as well as interest and other investment income. The underwriting operation gets credited for the funds it provides to the investment department at risk free rates. Thus the underwriting operation earns a profit which depends on the insurance risk it takes on, as shown in formula (3).

We see that knowledge of investment income from policyholder supplied funds is an essential part of any analysis of insurance operations. We also see, however, that only risk free rates are relevant to understanding profits from underwriting operations. The investment risk premium earned by the investment department should make little difference to the profit earned by the insurance operation.

This paper will focus on the problems of measuring investment income in a way that provides as accurate a picture as possible of the insurance profit being earned by the insurer. We are interested in putting together a framework for keeping score. Determining the market reward for bearing insurance risk is beyond the scope of this paper. We are convinced, however, that we cannot even begin to study the latter subject until we have better measures of insurance company operations.

Current accounting statements allow for the determination of underwriting profits. They also allow for the determination of total investment profits. They do not provide direct information about insurance profits. Thus it is very difficult to use these statements to determine insurance profits. It is currently difficult, therefore, to measure the performance of insurance companies with regard to either their underwriting risk or their investment risk.

Accounting statements should provide integrated information about insurance underwriting results and the associated investment earnings from policyholder supplied funds. They should present underwriting profit margins, investment earnings expected from assets underlying policyholder supplied funds, and earnings from funds provided by owners.

Companies will have to provide more information in their financial statements than they currently do if they or others wish to gain more understanding of their operations. A lot, however, can be done with information already available. Annual statement data is now supplied in a computer readable form. It is now possible to design additional or replacement pages using this data that will aid the analysis of the financial intermediary aspects of property/casualty insurance operations. The approach described in this paper will be called the Expected Value Accounting (EVA) approach.

This paper will proceed by discussing some principles to use in designing the EVA system. It will discuss how much data is now available for this purpose. Much of this data can only be used by making certain assumptions. Many of these are quite reasonable and can be used with some confidence that they will provide the needed insight. Others are stronger (ie. less reasonable) and cannot be used with the same degree of confidence. In these areas, additional information would provide more insight.

I. INSURANCE FUNDS AND FLOWS

The first step in the EVA method is to reorganize the Balance Sheet.

TABLE I

TOTAL: ALL COMPANIES (1985)

FUND ORIENTED BALANCE SHEET

(000,000 Omitted)

INVESTED ASSETS			LIABILITIES		
	$	%		$	%
Bonds	170,645	65.9	Unearned Prem Reserve	56,850	22.0
Stocks	52,754	20.4	Less Uncollected Prems.	-27,191	-10.5
Other Inv. Assets	35,409	13.7	Net Unearned Prem	29,660	11.5
			Loss & LAE Reserves	155,597	60.1
Total	258,808	100.0	Unearned Expenses	-10,063	-3.9
			Less Unpaid Expenses	4,347	1.7
			Net Unearned Expenses	-5,716	-2.2
			Net Insurance Funds	179,540	69.4
			Net Other Liabilities	-1,960	-0.8
			Equity Funds (Surplus)	81,228	31.4
			Total	258,808	100.0

The percentages show that about two thirds of the invested assets are supplied by customers and one third by the owners. The percent of bonds (65.9%) is quite close to the percent of customer supplied assets. This may be coincidental but matching these two items has long been an investment rule of thumb[3].

The groupings shown on this balance sheet can all be derived from statutory balance sheets, with the exception of prepaid expenses[4]. The value of this approach is that it places invested assets together in one place and the suppliers of these assets in another.

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[3] Herron, S.D., "Insurance Company Investments," PCAS LII, 238-243

[4] Prepaid Expenses are not available from statutory sources.

An income statement will be constructed by adding changes in balance sheet items to the amount paid on that item during the year. Earned premiums are shown as being equal to collected premiums plus the change in net unearned premiums over the year. Incurred losses are shown as being equal to paid losses plus the change in loss reserves during the year. Incurred expenses are shown as being equal to paid expenses plus the change in the prepaid expense reserve during the year.

The next step in developing the EVA method is to show the flow of insurance funds. Table II shows this flow for the industry in 1985:

TABLE II
TOTAL: ALL COMPANIES (1985)
INSURANCE FUNDS FLOWS

	1985
Written Premiums	144,347
Change in Uncollected Prem.	-3,565
Premiums Collected	140,796
Paid Loss & LAE	-97,950
Expenses Paid	-39,519
Total Insurance Cash Flow	3,327

Premiums collected are backed into by subtracting the change in premium receivables from written premium. This item is now included directly in the annual statement but has not been in prior years.

Next we calculate the change in nominal insurance reserves.

TABLE III
TOTAL: ALL COMPANIES (1985)
CHANGE IN NOMINAL INSURANCE FUNDS

	1984	1985	Change
Unearned Prem Reserves	45,832	56,850	11,019
Uncollected Premiums	-23,626	-27,191	-3,565
Net Unearned Prem	22,206	29,660	7,454
Loss & LAE Reserves	131,872	155,597	23,724
Net Prepaid Expenses	-5,152	-5,716	-564
Net Insurance Funds	148,926	179,541	30,615

Statutory Insurance funds increased by $30.6 billion during 1985. Prepaid expenses reduced funds otherwise available funds by nearly half a billion. Net unearned premiums increased by $7.5 billion. Loss Reserves increased by $23.7 billion.

If we add the change in the insurance funds to the amount paid during the year we have:

TABLE IV

TOTAL: ALL COMPANIES (1985)

INCURRED INSURANCE FLOWS

	Payments	Reserve Changes	Incurred
Premiums Earned	140,796	-7,454	133,342
Incurred Loss & LAE	-97,950	-23,724	-121,673
Incurred Expenses	-39,519	564	-38,956
GAAP Underwriting Profit[5]	3,327	-30,615	-27,287

We see that the GAAP underwriting profit is the sum of the various payments (the flow of funds) and the changes in reserves (the stock of funds). Premiums flow into the net unearned fund when they are collected. They flow out into the loss and LAE reserve fund, the expense fund, and surplus when exposures are earned.

Incurred losses are eventually paid from the loss and LAE reserve fund. That is, losses are paid from reserves. Incurred expenses flow from the unearned premium reserve into the expense reserve and then out as expenses are paid. While this treatment is the same as that used for losses, it is more confusing because most expense payments are made before exposures are earned. Losses are always paid after exposures are earned. Loss reserves represent the obligation to make future payments on losses which have already occurred.

In general, the prepaid expense reserve represents payments on business which has not yet been earned. It represents, on balance, an advance of funds. Thus it represents a liability which is negative because it usually represents past payments. Expense payments made after exposures are earned represent a future liability in the same sense that loss liabilities do. Policyholder dividends represent this type of expense since they are paid after premiums are earned. They represent a future expected payment in the same sense that losses do.

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5. The term "GAAP" Underwriting Profit is being used here to recognize the inclusion of prepaid expenses. Other GAAP adjustments have not been made.

The insurance funds flow process can be modeled in terms of these accounting entries. Exhibit I shows a diagram of this process. Funds flow into the company as collected premiums. They flow out as stockholder dividends, paid loss and loss adjustment expenses, and expenses (including policyholder dividends). The flows rarely take place at the exact time that exposures are earned. Thus they form either positive pools of money (such as loss and LAE reserves) or negative pools (such as prepaid expenses). The negative pools represent funds flowing from the company before the premium is earned.

Interestingly enough, even the net unearned premium fund has been negative in some situations. This happens when the delay in collecting premiums is *longer, on average, than the time it takes to earn the exposure. If, for* instance, the company collects its premiums an average of one year after it writes its policies then the net unearned premium fund will be negative. At the height of the cash flow crisis of 1983, there is evidence that several commercial lines companies were collecting premiums after policies were earned.

Underwriting profit is what is left over from earned premiums after the proper contributions have been made to the loss and expense reserve funds. This can be seen from Exhibit I. Surplus is credited with the <u>expected</u> underwriting profit or loss at the moment losses occur and exposures are earned. Premiums are matched to losses and expenses to record the underwriting profit.

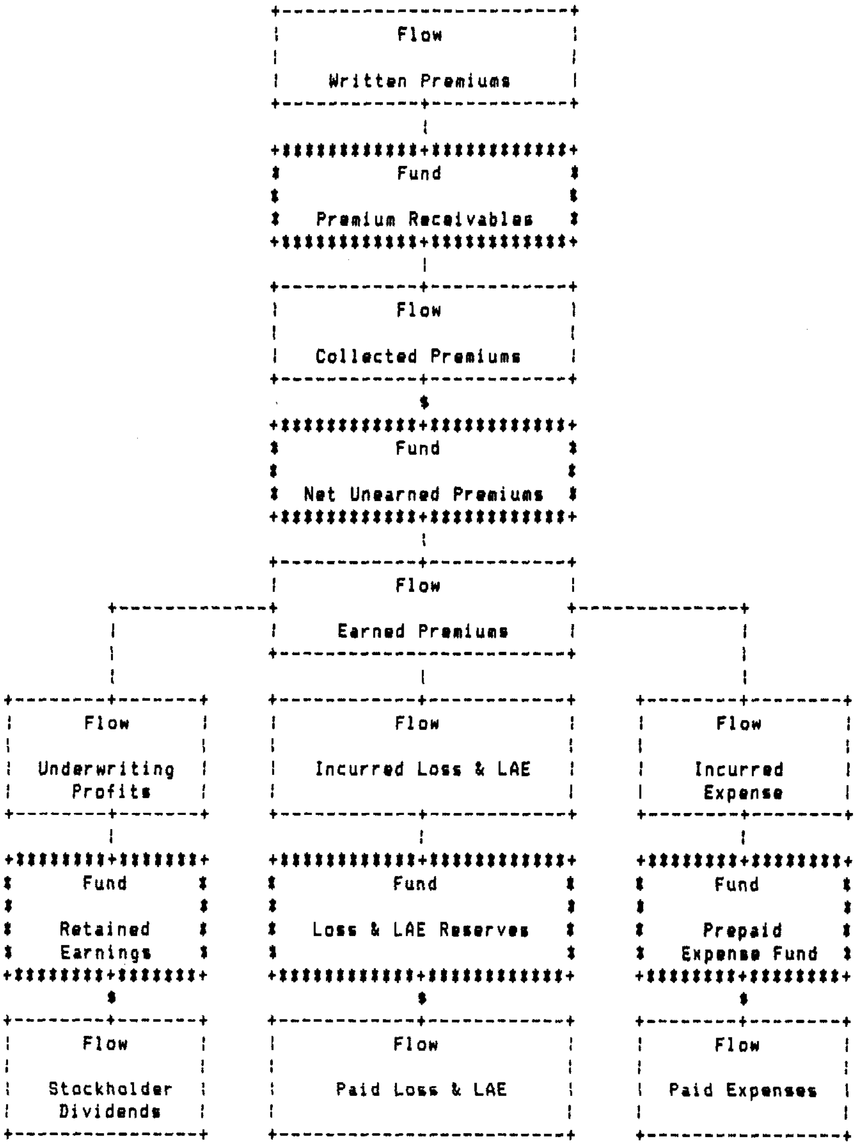
This concept of recognizing underwriting profits or losses at the time that applicable exposures are earned is intended to provide an accurate picture of the company's underwriting profit performance at the time it takes place. *If losses and premiums were perfectly recorded* at the time *when applicable* premiums were earned, this procedure would provide an accurate picture of the company's underwriting profit performance at that time. Neither premiums or losses are perfectly known at the moment of exposure, however, and must be estimated.

Eventually, losses and premiums become known and must be taken into account. Both GAAP and statutory accounting practice is to have misestimates flow into profits when they are corrected. Thus the actual underwriting profit recorded in any given year is the estimated profit on the current accident year (the exposures earned during the year) plus any changes in estimated *underwriting profits from prior year's exposures. That is:*

$$
\begin{aligned}
\text{UND PROFIT}_{\text{current cal yr}} &= \text{ESTIMATED UND PROFIT}_{\text{current acc yr}} \\
&\quad + \Delta\text{ESTIMATED UND PROFIT}_{\text{prior yrs}}
\end{aligned}
$$

Exhibit I

DIAGRAM OF INSURANCE FUNDS AND FLOWS



Actuaries call this the "calendar-year" method because all corrections in loss estimates affect the underwriting profit for the year in which the correction is made. An alternate method is to reestimate prior years underwriting profits whenever corrections are made. Thus, if losses occurring in 1980 increase by $100 million in 1983, the estimated underwriting profit for 1980 decreases by $100 million. This is called the "accident-year" method of accounting.

There are advantages and shortcomings to both approaches. The advantage of the accident year approach is that the true underwriting profit for any year eventually becomes known. The disadvantage is that it may become known only when it is a matter of purely historical interest. The calendar year method records misestimates at the time they become known. This makes management responsible for them at that time.

The calendar year approach seems to be a necessity for accounting documents intended to inform users of everything that has happened to a company during a particular accounting period. This includes the impact of corrections for prior year's misestimates where adherence to accrual principles of accounting is impossible. At the same time, however, calendar year accounting can provide a misleading picture of the company's current underwriting performance.

Accounting statements can provide estimates of both current year's underwriting performance and the impact on profits of corrections for prior years misestimates. This can be done by showing reserve changes and payments applicable to both the current year and prior years separately.

Shown below is an exhibit which separates loss information into the current year and all prior years:

TABLE V

TOTAL: ALL COMPANIES (1985)

ANALYSIS OF UNDERWRITING PROFIT

(000,000,000 Omitted)

RESERVES	12/84	12/85	Change	PAYMENTS	INCURRED
Unearned Premiums				Premiums Paid	Premiums Earned
Unearned	-46	-57			
Uncollected	24	27			
Net	-22	-30	-7	141	133
Loss & LAE Reserves				Paid Losses	Incurred Losses
<1985	-132	-90	42	-51	-9
1983	XXX	-65	-65	-47	-113
Unearned Expenses				Paid Expenses	Incurred Expenses
Unearned	9	10	1		
Unpaid	-4	-4	0		
Net	5	6	1	-40	-39
Net Insurance Funds				Net Ins. Payments	
	-149	-179	-30	3	-27

At the beginning of 1985 there were $149 billion dollars which had been paid to the industry and had not yet flowed out as losses, expenses, or underwriting profits. Of this amount, $132 billion had flowed through the unearned premium fund and on into the loss reserve fund. Another $22 billion had flowed into the unearned premium fund and had yet to leave as earned premium. In addition, the industry paid $5 billion in expenses before 1985 on behalf of policies earned after the end of 1984. This reduced the funds otherwise available for investment at the time by that amount.

During 1985, these funds increased by $30 billion. Premiums paid amounted to $141 billion. Approximately $31 billion was in the unearned premium fund at the end of the year. Almost all of the $23 billion in that fund at the beginning of the year had flowed into losses, expenses, and underwriting profit. Thus there was $7 billion more in the unearned premium fund at the end of the year than there was at the beginning.

At the same time the industry paid $51 billion from the 12/82 loss reserve fund which was reduced by only $41 billion. Thus, during 1985, the impact on underwriting profits from revaluing prior years losses was almost $9 billion. At the same time the industry paid $41 billion and posted $65 billion of reserves on losses occurring during 1985. Loss reserves increased by $23 billion as a result of these activities. This increase was funded by withdrawals from unearned premiums and surplus.

There was an increase of about $.5 billion in unearned expenses during 1985. This reduced funds otherwise available for investment by the same amount. It also meant that $33.9 billion out of the total $39.5 billion of expenses paid in 1985 applied to business earned in 1985 while $5.7 billion applied to business earned after 1985. The total expenses incurred in 1985 were $39.0 billion. This was the sum of the $33.9 billion paid during 1985 on 1985 business plus the $5.1 billion paid before 1985 on 1985 business.

In 1985, the premiums paid to the industry were about $3.3 billion more than the losses and expenses paid by the industry. At the same time, however, loss liabilities and unearned premium liabilities increased by approximately $32 billion.

II. INVESTMENT INCOME

We have shown how to account for underwriting profits in terms of funds flows and in changes in the various stocks of funds. Underwriting profits, however, are statements of nominal funds flows. The timing of these flows is not considered. At the same time, the industry has recorded bonds at values based upon their purchase price plus any changes due to amortization. Thus it has basically treated the largest body of invested assets on a nominal basis as well.

There is a principle which should be observed in financial statements which is clearly violated in current approaches to insurance accounting. This is the principle that accounting items with equal value should be shown as equal in value. Items that have different values should be shown with different values.

In insurance accounting, for instance, a dollar of loss reserves on a homeowners policy is expected to be paid within a year. A dollar of medical malpractice reserves, however, is expected to be paid after about 4.5 years. No distinction is made between these types of reserves on current balance sheets. It would be necessary to invest almost a dollar today to have a dollar available to pay the homeowner's loss tomorrow. Much less would be needed to invest today to pay the medical malpractice loss day after tomorrow. Thus, in terms of funds needed today to pay the loss tomorrow, the dollar of homeowner's reserves is not equal to the dollar of malpractice reserves.

We can address this problem by determining how much we would have to invest today to provide the funds needed tomorrow. We use this information to construct a balance sheet where the value of a liability is the value of the assets that must be provided today to pay the liability tomorrow.

Consider the sale of a policy for $99 which has no expenses and is expected to have a loss of $120 to be paid two years later. For expositional purposes we will have the policy run for a day. This means the premium is earned when it is paid. We are able to buy two year zero coupon bonds paying 10%. The policy is sold at mid-year at a breakeven price.

First we illustrate the way current accounting methods would treat this policy. Exhibit II illustrates the accounting statements for the three years there is activity on the policy.

Exhibit II shows that during year one, the company experienced a $21.00 underwriting loss. At the same time it earned $5.00 of investment income from the accrued interest on its bond. Thus its net income during the year from the single policy it wrote was -$16.00. The company looks as if it took a net loss on its operations during the year.

We now look at the situation during the following year. The company writes no more premiums. Even though the company did no business during year two, it continued to hold the bond purchased in year 01 which was now worth $114.40. Thus it earned a profit of $10.40 during the year.

TOTAL: COMPANY X EXHIBIT II

ANALYSIS OF UNDERWRITING PROFIT (YEAR 1)

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RESERVES                                      PAYMENTS              INCURRED
           12/00     12/01    Change
Unearned Premiums                             Premiums Paid         Premiums Earned
            0.0       0.0       0.0                         99.0         99.0
Loss & LAE Reserves                           Paid Losses           Incurred Losses
  <01       0.0       0.0       0.0                          0.0          0.0
   01       X.X    -120.0    -120.0                          0.0       -120.0
Net Insurance Funds
            0.0    -120.0    -120.0                         99.0        -21.0
Invested Assets                               Cash Flows            Net Inv. Income
                                                Net Sales -99.0
                                                Interest    0.0
            0.0     104.0     104.0             Total     -99.0           5.0
```

(YEAR 2)

```
RESERVES                                      PAYMENTS              INCURRED
           12/01     12/02     Change
Unearned Premiums                             Premiums Paid         Premiums Earned
            0.0       0.0       0.0                          0.0          0.0
Loss & LAE Reserves                           Paid Losses           Incurred Losses
  <02     120.0     120.0     120.0                          0.0          0.0
   02       X.X       0.0       0.0                          0.0          0.0
Net Insurance Funds
         -120.0    -120.0       0.0                          0.0          0.0
Invested Assets                               Cash Flows            Net Inv. Income
                                               Net Sales    0.0
                                               Interest     0.0
          104.0     114.4      10.4            Total        0.0          10.4
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(YEAR 3)

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RESERVES                                      PAYMENTS              INCURRED
           12/02     12/03     Change
Unearned Premiums                             Premiums Paid         Premiums Earned
            0.0       0.0       0.0                          0.0          0.0
Loss & LAE Reserves                           Paid Losses           Incurred Losses
  <03    -120.0       0.0     120.0                       -120.0          0.0
   03       X.X       0.0       0.0                          0.0          0.0
Net Insurance Funds
         -120.0       0.0     120.0                          0.0          0.0
Invested Assets                               Cash Flows            Net Inv. Income
                                               Net Sales  120.0
                                               Interest     0.0
          114.4       0.0    -114.4            Total      120.0           5.6
```

During year three the bond accrued another $5.60 in value and the company then sold it for $120. The company used this amount to pay the loss of $120. Thus the company earned $5.60 from its investment operations.

The story these statements tell us is that the company sold a policy on which it took a loss of $21.00. At the same time, with no other assets than those provided by that policy, it earned an investment profit of $21.00. It is clear, in retrospect, that there was no loss or gain on the policy. The *investment income profits earned during years two and three were really part* and parcel of the results expected when the policy was sold.

If the expected investment earnings had been recognized at the time the *premiums were earned, the underwriting loss for this policy would have been* equal to the investment income expected. This is the result shown in formula (2). Thus the expected profit would be zero. Earlier in this paper it was pointed out that in calendar year accounting, profits or losses that are recognized after the year in which premiums are earned flow through to profits when they are recognized. At the same time expected profits are recognized, as well as they can be, by posting reserves representing losses expected to be paid in later years.

Current insurance accounting approaches recognize losses that are expected to be paid in the future. They do not recognize the investment income expected *from the assets underlying these losses until that income is earned. Neverthe*less, these investment earnings are just as predictable as losses are. When one is recognized and the other is not, there is a mismatch between earned revenue (the investment income) and the expenses incurred in producing that revenue (the losses). Since investment income is reasonably predictable, it should be recognized when exposures are earned.

Table VI is similar to Exhibit II, but adds a receivable item which does recognize the expected investment earnings. Only the first year is shown. During the second year the investment income expected from loss reserve funds is reduced by $10.4. This is matched by $10.4 of interest accrued on the bond. The interest accrued is shown as a positive paid loss. During the third year the investment income receivable from loss reserve funds is reduced to zero. The bond is sold as in the prior example. All transactions cancel out leaving no assets and having no effect on profits. For convenience, bond accruals are treated as a payment even though no actual funds flow takes place.

TABLE VI

TOTAL: COMPANY X

ANALYSIS OF UNDERWRITING PROFIT (YEAR 1)

RESERVES				PAYMENTS	INCURRED
	12/00	12/01	Change		
Unearned Premiums				Premiums Paid	Premiums Earned
	0.0	0.0	0.0	99.0	99.0
Loss & LAE Reserves				Paid Losses	Incurred Losses
<01					
Reserve	0.0	0.0	0.0	0.0	0.0
Receivable	0.0	0.0	0.0	0.0	0.0
Net Reserve	0.0	0.0	0.0	0.0	0.0
01					
Reserve	X.X	-120.0	-120.0	0.0	-120.0
Receivable	X.X	16.0	16.0	5.0	21.0
Net Reserve	X.X	-104.0	-104.0	5.0	-99.0
Net Insurance Funds				Net Cash Flow	Insurance Profit
	0.0	-104.0	-104.0	104.0	0.0

In Table VI the insurance profit booked on this policy at the time it is earned is zero. This is equal to the profit expected.

Exhibit III is similar to Table VI but is a little more complicated. The general approach developed in this paper will be illustrated from this example. We begin by determining the current market value of the assets required today to pay all of the losses expected tomorrow. The difference between the nominal value of the reserves (the actual dollar amount of losses expected) and the market value of the assets supporting these reserves is shown as an investment receivable. This investment receivable is the amount of investment income expected from supporting funds by the time losses are paid.

Since both loss reserves and investment receivables are estimates, reality will eventually replace expectation. Errors in estimates will create positive or negative profit flows. Errors in estimates of *expected investment earnings* from assets supporting insurance funds should be recognized in accounting statements in the same way that errors in *loss estimates* are recognized. The investment receivable entry should be thought of as a reserve which recognizes future investment flows just as loss reserves recognize future payment flows. As time passes, actual losses will gradually replace expected losses. At the same time a more accurate picture of the remaining losses will also emerge. Both of these will create differences between the *investment income expected* and that *actually earned*. *These changes and the changes resulting from loss payments and changes in reserves should flow through to the balance sheet and income statements in the same way*.

The investment receivable items represent the amount of interest expected from the various assets supporting the insurance funds. Thus it is necessary to take into account the rate of interest expected from these assets. For loss reserves, this means forecasting future interest rates. In practical terms, forecasting these future interest rates has proven to be a fruitless task. Some economists have argued that the best predictor of future interest rates is the present rate.

This suggests that the best estimate of investment income that will be earned from assets supporting insurance funds is the amount of interest they would earn if employed at the rates of interest prevailing at any given moment. The market value of assets themselves depends upon the same rates of interest.

The procedure proposed in this paper records assets at market values. Insurance funds are recorded as nominal liabilities together with the investment income expected from those funds. This expected investment income will be called an investment receivable. The sum of the nominal liability and its associated receivable will be called a net liability. Changes in interest rates from one year to the next only flow through to profits to the extent that changes in assets and net liabilities do not offset each other. The degree to which these changes offset each other reflects what is called immunization. Complete immunization means that changes in interest rates will have no effect on profits. Interest rate risk is eliminated.

The company purchased a zero coupon bond with the same cash flow as that for losses in the example used in Table VI. Suppose interest rates had changed from 10% at the time the policy went from 8% by the end of the year and then to 7% by the end of the second year. Under this procedure, no difference in profits would be recognized as a result of these interest rate changes. The loss investment receivable at the end of the first year, expressed in terms of the current interest rate of 8%, is $13.10. This is $2.90 less than the receivable would have been if the interest rate had not changed. At the same time, however, the market value of the zero coupon bond goes from $104 at 10% interest to $106.90 at 8% interest. The fall in interest rates makes the 10% rate built into the bond more attractive and so the price rises to the point where it has an 8% yield. Thus we have a loss on the receivable of $2.90 and a gain on the market value of the bond of $2.90. These two changes offset each other and there is no effect on profits. The same thing happens during the third year. This is a special case of immunization, since the flows connected with both the assets and liabilities are the same.

TOTAL: COMPANY X

EXHIBIT III

ANALYSIS OF UNDERWRITING PROFIT (YEAR 1)

RESERVES				PAYMENTS	INCURRED
	12/01	12/02	Change		
Unearned Premiums				Premiums Paid	Premiums Earned
	0.0	0.0	0.0	99.0	99.0
Loss & LAE Reserves				Paid Losses	Incurred Losses
<01					
Reserve	0.0	0.0	0.0	0.0	0.0
Receivable	0.0	0.0	0.0	0.0	0.0
Net Reserve	0.0	0.0	0.0	0.0	0.0
01					
Reserve	XXX.X	-120.0	-120.0	0.0	-120.0
Receivable	XXX.X	13.1	13.1	5.0	18.1
Net Reserve	XXX.X	-106.9	-106.9	5.0	-101.9
Net Insurance Funds				Net Cash Flow	Insurance Profit
	0.0	-106.9	-106.9	104.0	-2.9

YEAR 2

RESERVES				PAYMENTS	INCURRED
	12/01	12/02	Change		
Unearned Premiums				Premiums Paid	Premiums Earned
	0.0	0.0	0.0	0.0	0.0
Loss & LAE Reserves				Paid Losses	Incurred Losses
<02					
Reserve	-120.0	-120.0	0.0	0.0	0.0
Receivable	13.1	6.6	-6.5	8.6	2.1
Net Reserve	-106.9	-113.4	-6.5	8.6	2.0
02					
Reserve	0.0	0.0	0.0	0.0	0.0
Receivable	0.0	0.0	0.0	0.0	0.0
Net Reserve	0.0	0.0	0.0	0.0	0.0
Net Insurance Funds				Net Cash Flow	Insurance Profit
	-106.9	-113.4	-6.5	8.6	2.1

YEAR 3

RESERVES				PAYMENTS	INCURRED
	12/02	12/03	Change		
Unearned Premiums				Premiums Paid	Premiums Earned
	0.0	0.0	0.0	0.0	0.0
Loss & LAE Reserves				Paid Losses	Incurred Losses
<03					
Reserve	-120.0	0.0	120.0	-120.0	0.0
Receivable	6.6	0.0	-6.6	6.6	0.0
Net Reserve	-113.4	0.0	113.4	-113.4	0.0
03					
Reserve	0.0	0.0	0.0	0.0	0.0
Receivable	0.0	0.0	0.0	0.0	0.0
Net Reserve	0.0	0.0	0.0	0.0	0.0
Net Insurance Funds				Net Cash Flow	Insurance Profit
	-113.0	0.0	113.4	113.4	0.0

Exhibit III shows what can happen as a result of interest rate changes when an immunized situation does not exist. We will assume that instead of buying matched zero coupon bonds to support the funds the company buys a six month bond yielding 10% at the end of the year. The company purchases the bond for $99 at the time the policy is sold (midyear) and sells it for $104 at the end of the year. By this time interest rates have fallen to 8%. If interest rates remained at 8% for the next year and a half it would not be possible to earn the $16 originally expected. It would require assets of $106.9 invested at 8% to have $120 available at the time the loss payment was due. Thus the amount of interest the company will expect to earn by the time the loss is paid is $13.1 rather than $16. This new expectation produces the financial statement shown in Exhibit III, year one.

We see, from this Exhibit, that the change in interest rates has changed the expected profit from $0 to a loss of $2.90. This is funded by a negative profit flow of $2.90.

The company buys an annual bond bearing an 8% coupon at the end of year 01 for $106.9 and earns $8.6 during year 02. Thus it has $115.50 on hand. Interest rates now go from 8% to 12% by the end of year 02. It is now possible to buy a six month bond yielding 12% for $113.40 at that time which will earn $6.60. At the end of the final six month period the company will have $120 on hand which will be the exact amount needed to pay the loss. The interest expected at the end of year 02 is $6.6 as shown in Exhibit III, year 2. This second change in interest rates has improved the situation. The reduction of two percentage points in the interest rate during the first year produced a loss of $2.9. The increase of two percentage points during the second year produced a profit of $2.10. While the company had $115.50 on hand at the end of year 02, it only needed $113.40 to buy a bond that would provide the funds needed to pay the $120 loss. The difference flows back to surplus just as the effect of the first interest rate change caused a flow from surplus. The net profit is now $-.80.

During the third year the bond asset is sold for the amount that will pay the loss. No contribution from capital or to capital takes place.

In the above example, all assets were sold at the end of each year. Thus the value shown for them was exactly the value the market placed on them.

In this example, interest rate changes produced profits and losses. In the example from Table VI, interest rate changes did not have an effect on profits and losses. The reason for this difference is that in the first example assets and liabilities were immunized. In the second they were not. In this example interest rate changes almost canceled out over the period loss reserve liabilities were held. Thus the net difference between the profit expected at the time the policy was sold in year 00 and that actually gained was quite small. Had interest rates increased or decreased throughout the period, the difference in profits would have been significantly larger.

III. THE EXPECTED VALUE ACCOUNTING (EVA) APPROACH

We propose to measure insurance profit by first determining nominal funds and flows and then the investment funds and flows associated with them. The different funds and flows we consider are shown in Exhibit I.

We see in Exhibit I that a company writes premiums, collects them, and then earns them. Premiums are in the Uncollected Premiums Fund from the time the company writes premiums until it collects them . Premiums are in the Net Unearned Premium Fund from the time the company collects the premiums until it earns them. Thus Net Unearned Premiums are equal to Unearned Premiums minus Uncollected Premiums.

When premiums are earned, they flow into loss and LAE reserves, expense reserves and into surplus as a profit or loss. Eventually the company pays losses and expense and books a profit. Premiums are available for investment from the time the company collects premiums until it pays them out as profits, losses, or expenses.

GAAP accounting recognizes the expected benefit of nominal flows at the time the earned premium flow takes place. The Expected Value Accounting (EVA) Approach recognizes the expected benefit of investment flows at the same time.

We will evaluate the expected benefit of investing premiums before we earn them. We make these investments while premiums are in the Net Unearned Premium Fund. Thus we look at the interest earned on premium funds from the time a company collects the premiums until it earns them. We then look at the interest expected on assets supporting loss and LAE reserves. We also look at the interest lost or gained by timing differences in the payment of expenses. That is, we look at the interest lost or gained when the company pays its expenses. The company loses interest income when it pays expenses early (ie. when it pays expenses before it earns the associated premiums). It gains interest income when the company pays expenses late.

We call the sum of after-tax flows the investment credit. We call the sum of the underwriting profit and the investment credit (where we evaluate both on an after-tax basis) the insurance profit. This name comes from the recognition that an insurance company's total profit (or loss) can be looked at as the sum of three elements:

$$\begin{aligned}\text{TOTAL PROFIT} = \; & \text{UNDERWRITING PROFIT} \\ & + \text{INVESTMENT CREDIT} \\ & + \text{INVESTMENT INCOME FROM CAPITAL FUNDS}\end{aligned} \qquad (4)$$

The third element is one the capital provider would earn if he were not in the insurance business at all. We call this equity profits. We treat equity profits on an aftertax basis. In Massachusetts, Myers and Cohn[5] recognized that the capital provider could avoid paying federal income taxes on these earnings if he were not in insurance. Thus a firm's insurance profit should be at least equal to the taxes incurred on the investment of capital funds. If it is *not*, the capital provider will have a higher after-tax return outside the insurance business than within it.

The first two items, underwriting profit and investment credit, result from entering the insurance business. Thus it makes sense to call their sum the insurance profit.

The insurance profit is the sum of nominal and investment flows as follows:

$$\begin{aligned} \text{INSURANCE PROFIT} = {} & \text{PREMIUM EARNED} \times (1 + \text{INTEREST EARNED}_{\text{premium}}) \quad (5) \\ & - \text{LOSSES INCURRED} \times (1 + \text{INTEREST EARNED}_{\text{losses}}) \\ & - \text{EXPENSES INCURRED} \times (1 + \text{INTEREST EARNED}_{\text{expenses}}) \\ & - \text{FEDERAL TAXES INCURRED} \times (1 + \text{INTEREST EARNED}_{\text{fit}}) \end{aligned}$$

The EVA approach shows the insurance profit expected from premiums earned during a particular year. It also shows the adjustments made to prior years' expected profits as a better picture of these profits emerges over time. The EVA Balance sheet shows the nominal value of the insurance funds, together with the interest from investing the assets underlying the funds. The EVA Income Statement shows nominal changes in funds plus payment flows. It also shows changes in expected investment income plus the actual interest earned during the year.

Exhibit IV presents EVA balance sheets and income statements for the total P/C insurance industry from 1983 through 1985. The figures in these statements come from annual statement data provided by A.M. Best and Co. We will discuss the details of how these figures were derived in following sections of this paper.

It is necessary to know the amount and timing of nominal insurance funds flows to determine the insurance profit. These are the premium, loss, expense, and federal tax flows. It is also necessary to know what interest rate to use in estimating the investment income which will be earned from these flows.

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5. Myers and Cohn, "Insurance Rate of Return Regulation and the Capital Asset Pricing Model", Massachusetts Private Passenger Automobile filing for 1982 Rates.

EXHIBIT IV
Page 1

EVA METHOD: INSURANCE LIABILITIES

PREMIUMS	1983	Change	1984	Change	1985
1. Unearned Premiums	42,478	3,354	45,832	11,019	56,850
2. Uncollected Premiums	-20,755	-2,871	-23,626	-3,565	-27,191
3. Net Unearned Premiums	21,723	483	22,206	7,454	29,660
4. Interest Earned	567	45	612	1	613
5. EVA Unearned Premiums	22,290	527	22,818	7,455	30,272
EXPENSES					
6. Unearned Expenses	-7,654	-1,125	-8,779	-1,284	-10,063
7. Exp. Booked but Unpaid	3,137	489	3,626	720	4,347
8. Net Unearned Expenses	-4,517	-636	-5,152	-564	-5,716
9. Interest Lost	-157	-16	-173	9	-164
10. EVA Unearned Expenses	-4,674	-652	-5,326	-554	-5,880
LOSSES					
PRIOR YEARS					
11. Loss Reserve Carried	69,455	7,885	77,340	12,869	90,208
14. Inv. Income Expected	-11,040	-1,037	-12,077	962	-11,115
15. EVA Loss Reserve	58,415	6,848	65,263	13,831	79,093
CURRENT YEAR					
16. Loss Reserve Carried	49,298	5,235	54,533	10,855	65,388
19. Inv. Income Expected	-5,799	-743	-6,542	164	-6,378
20. EVA Loss Reserve	43,499	4,492	47,991	11,019	59,010
ALL YEARS					
21. Loss Reserve Carried	118,752	13,120	131,872	23,724	155,597
24. Inv. Income Expected	-16,839	-1,780	-18,619	1,126	-17,493
25. EVA Loss Reserve	101,913	11,340	113,255	24,850	138,103
NET INSURANCE FUNDS					
26. Nominal Ins. Funds	135,958	12,967	148,926	30,615	179,541
27. Net Interest	-16,529	-1,751	-18,181	1,136	-17,044
28. EVA Funds	119,530	11,216	130,746	31,750	162,496

EXHIBIT IV
Page 2

EVA METHOD: INSURANCE PAYMENTS

PREMIUMS	1984	1985
1. Collected Premiums	115,664	140,796
2. Interest Earned	1,385	1,386
3. EVA Collected Prem.	117,048	142,181
EXPENSES		
4. Paid Expenses	-35,087	-39,519
5. Interest Lost	-308	-291
6. EVA Paid Expenses	-35,395	-39,810
LOSSES		
PRIOR YEAR		
7. Paid Loss & LAE	-44,857	-50,516
8. Interest Earned	4,897	5,733
9. EVA Paid Losses	-39,960	-44,782
CURRENT YEAR		
10. Paid Loss & LAE	-43,630	-47,434
11. Interest Earned	1,489	1,496
12. EVA Paid Losses	-42,141	-45,937
ALL YEARS		
13. Paid Loss & LAE	-88,487	-97,950
14. Interest Earned	6,386	7,230
15. EVA Paid Losses	-82,100	-90,720
NET INSURANCE PAYMENTS		
16. Nominal Payments	-7,910	3,327
17. Interest Earned	7,463	8,325
18. EVA Ins. Payments	-447	11,652

EXHIBIT IV
Page 3

EVA METHOD: INCOME STATEMENT

	1984			1985		
	Paid	Change	Incurred	Paid	Change	Incurred
PREMIUMS						
1. Earned Premiums	115,664	-483	115,181	140,796	-7,454	133,342
2. Interest Earned	1,385	-45	1,340	1,386	-1	1,385
3. EVA Earned Prem.	117,048	-527	116,521	142,181	-7,455	134,727
EXPENSES						
4. Earned Expenses	-35,087	636	-34,452	-39,519	564	-38,956
5. Interest Lost	-308	16	-292	-291	-9	-300
6. EVA Earned Exp.	-35,395	652	-34,743	-39,810	554	-39,256
LOSSES						
PRIOR YEAR						
7. Incurred Loss Stat.	-44,857	41,413	-3,444	-50,516	41,664	-8,852
9. Inv. Receivable	0	218	218	0	563	563
10. Net Incurred Loss	-44,857	41,631	-3,226	-50,516	42,227	-8,289
11. Interest Earned	4,897	-4,762	136	5,733	-7,504	-1,770
12. EVA Incurred Loss	-39,960	36,651	-3,309	-44,782	34,160	-10,622
CURRENT YEAR						
13. Incurred Loss Stat.	-43,630	-54,533	-98,163	-47,434	-65,388	-112,822
14. Interest Earned	1,489	6,542	8,031	1,496	6,378	7,875
15. EVA Incurred Loss	-42,141	-47,991	-90,132	-45,937	-59,010	-104,947
ALL YEARS						
16. Incurred Loss Stat.	-88,487	-13,120	-101,607	-97,950	-23,723	-121,673
17. Interest Earned	6,386	1,780	8,166	7,230	-1,126	6,104
18. EVA Incurred Loss	-82,100	-11,340	-93,440	-90,720	-24,850	-115,570
INVESTMENT INCOME						
23. Interest	7,463		7,463	8,325		8,325
24. Capital Gains		82	82		2,333	2,493
25. Total	7,463	82	7,545	8,325	2,333	10,658
NET INSURANCE PAYMENTS						
26. Underwriting Profit	-7,910	-12,967	-20,877	3,327	-30,614	-27,287
27. Investment Credit	7,463	1,833	9,296	8,325	1,197	9,522
28. Net Federal Taxes	701	5,146	5,847	-4,856	13,527	8,671
29. Insurance Profit	254	-6,026	-5,772	6,796	-15,890	-9,094

EXHIBIT IV
Page 4

EVA METHOD: BALANCE SHEET

ASSETS

	1983	Change	1984	Change	1985
1. Market Value of T-Bills	119,530		130,746		162,496
2. Asset changes					
3. Funding Required					
4. EVA Premiums		-116,521		-134,727	
5. EVA Expenses		34,743		39,256	
6. Prior Yr. EVA Loss		3,226		8,289	
7. Curr. Yr. EVA Loss		90,132		104,947	
8. Total		11,580		17,765	
9. Cash Flow					
10. Collected Prems.		115,664		140,796	
11. Expenses Paid		-79,944		-90,035	
12. Losses Paid		-43,630		-47,434	
13. Total		-7,910		3,327	
14. Interest Earned		7,463		8,325	
15. Capital Gains		82		2,333	
16. Total Asset Change		11,215		31,750	

LIABILITIES

	1983	Change	1984	Change	1985
17. EVA Unearned Premiums	22,290	527	22,818	7,455	30,272
18. EVA Unearned Expenses	-4,674	-652	-5,326	-554	-5,880
19. EVA Loss Reserves	101,913	11,340	113,255	24,850	138,103
20. Total EVA Ins. Liabs.	119,530	11,215	130,746	31,750	162,496

The EVA approach expresses the insurance profit in the following terms:

$$\begin{aligned}\text{INSURANCE PROFIT} = {} & \text{PREMIUMS PAID} + \Delta\text{UNEARNED PREMIUMS} \qquad (6)\\ & + \text{INTEREST PAID} + \Delta\text{INTEREST EARNED ON UNEARNED}\\ & - \text{LOSSES PAID} - \Delta\text{LOSS RESERVES}\\ & + \text{INTEREST PAID} + \Delta\text{INTEREST EARNED ON LOSS RESERVES}\\ & - \text{EXPENSES PAID} - \Delta\text{PREPAID EXPENSE RESERVES}\\ & - \text{INTEREST LOST} - \Delta\text{INTEREST LOST ON PREPAID EXPENSES}\end{aligned}$$

The interest paid and earned in formula (6) depends on the time the related flows of funds take place. It also depends on the interest earned while the company holds the funds. Current annual statement data does not provide this information but it does provide the basis for reasonable estimates of it.

The next section of this paper will examine how to determine the timing of insurance flows. Another section will deal with interest rates.

IV. THE TIMING OF INSURANCE FLOWS

In this section, we will see what we can learn about the timing of insurance flows using annual statement data. We will find that we can learn quite a lot. We will also find that the areas with the biggest difficulties are those which make the least difference.

We begin by examining what we can learn about premium flows from annual statement data. We continue by examining expense flows and finally we will look at loss and LAE flows.

We will find that we cannot determine the exact timing of premium and expense flows from annual statement data. We can, however, make reasonable estimates of the average time that flows take place. We will show that knowledge of average flow times provides estimates of the interest we can earn from Net Unearned Premium and Expense funds. These answers are very similar to the answers gained by an exact knowledge of the flows.

The timing of loss and LAE payments is more important than the timing of premium and expense payments. Schedules O and P of the annual statement, however, provide information about the timing of these payments.

We will examine how much we learn about these flows by line. We will see that we can learn almost as much about flows by line as we can about company-wide flows. Nevertheless there are limits on what we can learn about cash flows by line. The annual statement, for example, makes no distinction between personal and commercial lines. Thus we must estimate personal and commercial cash flows from data that aggregates the two. In addition, annual statement data does not separate premium receivables into different lines. There is reason to believe that premium receivables for personal lines are quite different from those for commercial lines.

PREMIUMS

The flow of insurance funds begins when a company collects premiums. It continues when the company earns them. At that time funds flow into loss and expense funds and into retained earnings. In this section we will learn how to estimate the length of the period between the time premiums are collected and the time they are earned. We want to see how long premiums remain in the Net Unearned Premium Reserve.

At any given time, the company will be earning premium from policies written and still active. The premiums on these policies are called "premiums in force," or PIF. Premiums earned over any period of time are equal to the PIF times the ratio of the length of the period to the term of the policies.

Exhibit V diagrams the relationship between the process of writing premiums and the process of earning them. It consists of three triangles which are partly shaded and partly clear. The shaded areas represent time during which the company has collected premiums but has not yet earned them. The two lower triangles form a trapezoid representing the flow of premiums written in year -01. The two rightmost triangles represent premiums earned during year 00. The middle triangle represents premiums unearned at time zero.

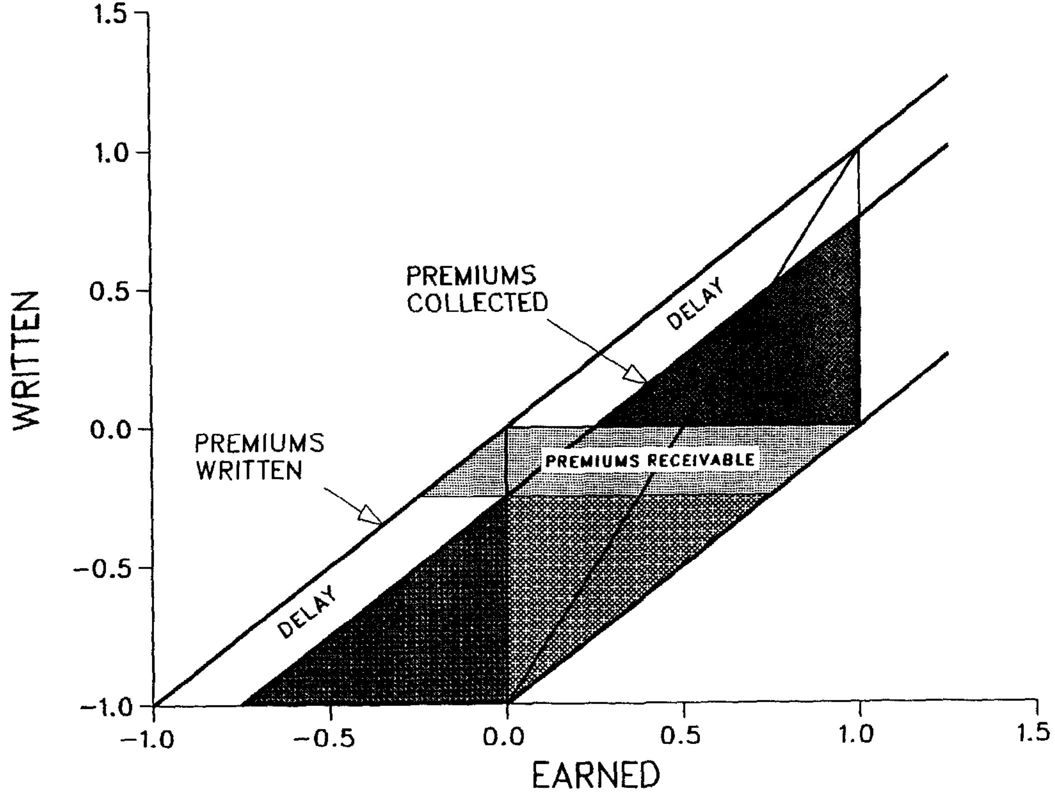
The white area within each triangle represents premiums written but not yet received. In this exhibit, the delay in collecting premiums is .25 years. At time zero, all the premiums earned in the Uncollected Premiums area are still uncollected. The total premiums written but uncollected are equal to 25% of the total premiums written during year -01.

The area of the unearned premium triangle is one half the area of the total premiums written during year zero. Thus the ratio of unearned premiums to written premiums is one half. If the policy term is shorter, say six months, only premiums written within the past six months will be partly unearned. We can imagine this situation by moving the right hand line to the left by half a year. With six month policies, the value of the Unearned Premium Reserve is one half of the premiums written during the past six months. Under steady state conditions this will equal one quarter of the premiums written over the past year. Thus the ratio of unearned premiums to written premiums in this case is .25. The average time lapse between the time the company writes its premiums on six month policies to the time it earns them is .25 years. <u>Thus we estimate the amount of time premiums spend in the Unearned Premium Reserve by dividing unearned premiums by written premiums.</u>

$$\text{AVG WRITTEN LAG} = \frac{\text{UNEARNED PREMIUM}}{\text{WRITTEN PREMIUM}} \qquad (7)$$

Exhibit V

THE PREMIUM PROCESS



Net Unearned Premiums are equal to Unearned Premiums less Uncollected Premiums. Uncollected Premiums are Agents Balances plus Bills Receivable taken for Premiums, lines 8.1, 8.2, and 10 from page 2 of the annual statement. Thus:

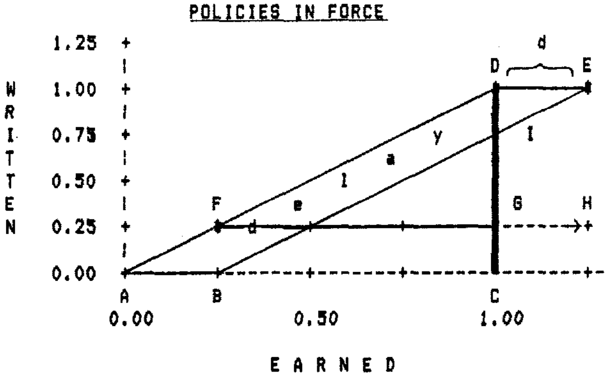
NET UNEARNED PREMIUMS = UNEARNED PREMIUMS - UNCOLLECTED PREMIUMS (8)

The ratio of Net Unearned Premiums to Premiums Written provides an estimate of the amount of time that passes from the moment the company collects premiums to the moment it earns them. Thus we estimate the time premiums spend in the Net Unearned Premium Reserve by dividing Net Unearned Premiums by Written Premiums.

$$\text{AVG INVESTMENT LAG} = \frac{\text{NET UNEARNED PREMIUM}}{\text{WRITTEN PREMIUM}} \quad (9)$$

Fig. 1 is taken from Exhibit V and shows the premium flow process as it applies to premiums in force at an instant in time.

Fig. 1



Line AD represents the time the premiums in force at time 1.0, represented by line CD, were written. Line BE represents the time they were collected. Triangles BIC and DIE represent the time these premiums are in the Net Unearned fund. (Triangle DIE represents premiums written but uncollected at the time *they are earned.) Line FG represents the process of collecting and earning* premiums written at time .25 and earned at point G. The distance d represents the delay in collecting premiums. Thus we see that these premiums are in the fund from time (.25+d) till time 1.0, a length of time equal to .75-d. It has earned interest i for this time. Thus the value of this premium at the time it is earned is $(1+i)^{1-(.25+d)}$.

The value of the premiums in force at time 1.00, including the investments earned while these premiums have been in the Net Unearned Premium Reserve, is:

$$PIF_{pv} = PIF \int_0^1 (1+i_{(p)})^{1-t-d} \, dt$$

Where p is the policy term
t is the time premiums are written
d is the ratio of the time that premiums are uncollected to the policy term.

The ratio of PIF at present value compared to nominal PIF is:

$$ER = PIF_{pv}/PIF$$

Where ER = Ratio of Premiums in Force at Present Value to Premiums in Force at nominal value

This is equal to:

$$ER = \int_0^1 (1+i_{(p)})^{1-t-d} \, dt$$

$$= \frac{i_{(p)}}{(1+i_{(p)})^d \ln(1+i_{(p)})} \qquad (10)$$

Since ER is the present value of the premium flow at the time it is earned, (ER - 1) represents the interest earned by the company from existing premium funds. We use equation (10) to estimate how much a company earns between the time it collects premiums and the time it earns them. Shown below are premiums earned on annual policies with zero, three month, and six month delays in collecting premiums. Also shown are earnings from six month policies with zero and three month collection delays.

TABLE VI

INVESTMENT EARNINGS FROM UNEARNED PREMIUMS PER DOLLAR OF PREMIUMS!

	Annual Policies			Six Month Policies	
	Delay			Delay	
	0	3	6	0	3
5%	.025	.012	.000	.012	.000
10%	.049	.025	.000	.024	.000
15%	.073	.036	.001	.036	.000

The amount earned from six month policies with no delay is about equal to that earned from annual policies with a six month delay. The average time the company invests premium funds for annual policies with three month delays is three months. The average time premiums are in the fund for six month policies with no delay is also three months. The amount earned in these two cases is about the same even though the policy terms are different. There are almost no earnings on either annual policies with six month delays or six month policies with three month delays. In both cases, the average time premiums are in the fund is almost zero.

Table VI shows that investment earnings on premiums are almost equal when the average time invested is equal. This shows that differences in payment patterns have little effect. Thus knowledge of policy terms is not necessary to determine investment earnings from premium funds.

We use formula (10) to calcuate the investment income from premium funds shown in Table VI. This formula is a simulation of actual payment flows. Table VI shows only small differences in the amount earned on different policies when the average time invested is the same. This indicates that we can estimate investment income from premium funds using the average time premium funds are invested. This estimate is equal to the discounted value of a single payment taking place at the average time premiums are collected.

In a sense, we are saying that we can treat the premium process as if all premiums were collected t years before they were earned. The value t is the ratio of net unearned premiums to written premiums. Thus we estimate ER as follows:

$$ER = (1+i)^{\text{Net Unearned Premiums/Written Premiums}} \qquad (11)$$

Where ER is the same as in equation (10).

Shown below is a table that shows results corresponding to Table VI.

TABLE VII

INVESTMENT EARNINGS FROM UNEARNED PREMIUMS PER DOLLAR OF PREMIUMS

AVERAGE DELAY FACTOR (ADF) METHOD

Interest Rate	Net Unearned / Written Ratio .50	.25	.00
5%	.025	.012	.000
10%	.049	.024	.000
15%	.072	.036	.000

When there is no delay, the Net Unearned/written ratio will be .50. When there is a three month delay, the ratio will be .25. When there is a six month delay, the ratio will be .00. This method produces results that are quite similar to those shown in Table VI. Thus we can use knowledge of the average time premiums are in the Net Unearned Premium Fund to estimate the amount of interest a company will earn on premium funds.

Different premium growth rates will generate different estimates of the interest earned from unearned premium funds. (Premium growth will not affect the actual interest earned percentage but it will have an effect on the estimate.) The following Table shows the estimated interest from net unearned premium funds under various interest rates and different premium growth rates.

Table VIII

ESTIMATED INTEREST FROM UNEARNED PREMIUM FUND
EFFECT OF DIFFERENT PREMIUM GROWTH RATES

Interest	Premium Growth Rate							
Rate	0%	5%	10%	15%	20%	30%	40%	50%
5%	.025	.025	.025	.025	.025	.026	.026	.026
10%	.049	.049	.050	.050	.050	.051	.052	.052
15%	.072	.073	.074	.074	.075	.076	.077	.077

Even when growth rates are as high as 50% and interest rates are as high as 15%, growth has a minor impact on the estimate of investment earnings from net unearned premium funds. The effect of growth is insignificant if interest rates are 10% or below and growth is 30% or less.

During 1985, industry premiums written grew about 22%. The average time premiums were in the Net Unearned Premium Fund was about .21 years. At the same time, two-year treasury bonds were yielding about 9.4%. Since the corporate tax rate in 1985 was 46%, the after tax yield was 5.1%. Thus we can estimate the interest earnings on premiums earned in 1983 as $106,968 million x (.011 = 1.051^{-21} - 1) = $1,400 million. The industry earned nearly one and a half billion dollars from the prepayment of $133 billion dollars of premiums earned in 1984.

At the end of 1985 the industry had $29,660 million of premiums collected but not earned. We want to estimate how much interest the industry had earned on those premiums by that time. These interest earnings will flow to losses, expenses and surplus when the company earns the premiums. Until that time they are a part of the funds prepaid by policyholders and included in EVA Unearned Premiums.

Figure 1 shows the earned flow of a policy written at time .25. The company collects premiums at time .25+d, and earns interest from that time till time 1. It has earned interest for a period of 1-.25-d. The amount of premiums unearned on this policy at time 1 is .25. Thus the time that premiums have been in the Net Unearned Premium Fund by time 1 is the following weighted average:

$$APD^{\text{net unearned prem}} = \frac{\int_0^{1-d} t(1-t-d)\,dt}{\int_0^{1-d} t\,dt} = \frac{(1-d)\int_0^{1-d} t\,dt - \int_0^{1-d} t^2\,dt}{\int_0^{1-d} t\,dt}$$

$$= \frac{(1-d)}{3}$$

We estimate the the average time that premiums have been in the Net Unearned Premium Fund by dividing Net Unearned Premiums by Written premiums. If there is no delay and all policies are annual policies, the average time premiums will be in the fund will be .5 years. The difference between the ratio of net unearned premiums to written premiums and .5 represents the delay in collecting premiums. Thus we can estimate the value of 1-d by adding .5 to the Net Unearned/Written ratio. That is:

$$1-d = .5 + \text{Net Unearned / Written}$$

Thus:

$$AIP_{\text{unearned}} = \frac{(.5+(\text{Net Unearned / Written}))}{3} \qquad (12)$$

Where AIP_{unearned} is the Average Investment Period for Unearned Premiums.

Since the ratio of Net Unearned Premiums to Written in 1985 was .21, the value of AIP_{unearned} was .71/3 = .237. The investments earned on Unearned Premium Reserves at the end of 1985 were equal to premiums unearned at that time, $56,850 million, times ($.011 = 1.047^{.233} - 1$), or $613 million.

In conclusion, we have shown how to estimate average premium flows. We have shown that the average time companies invest premiums by the time they have earned them is estimated by dividing Net Unearned Premiums by Written Premiums. This is shown in formula (9). We have shown that the average time premiums in the Net Unearned Premium Fund at any time have been invested is estimated by dividing .5 plus the ratio of Net Unearned Premiums to written premiums, by 3. This is shown in formula (11).

We have also shown that these average investment periods can be used to estimate the amount of interest earned on earned premiums and on net unearned premiums. The formula for estimating the average investment income earned on earned premiums is to discount them for the average delay. That is:

$$\text{INT EARNED}_{\text{premiums}} = \text{EARNED PREMIUMS} \times (1+i)^{\text{Net Unearned/Written}} - 1 \qquad (13)$$

The formula for estimating the average investment income earned at any time on premiums collected but unearned is:

$$\text{INT}_{\text{unearned}} = \text{UNEARNED PREMIUM} \times (1+i)^{(.5+(\text{Net Unearned/Written}))/3} - 1 \qquad (14)$$

The amount of premium funds available for investment at any time is equal to the premiums collected but unearned plus any interest earned since they were collected. This is the sum of Net Unearned Premiums and $\text{INT}_{\text{unearned}}$ and is called the EVA Unearned Premium. Thus:

$$\text{EVA UNEARNED PREMIUM} = \text{NET UNEARNED PREMIUM} + \text{INT}_{\text{unearned}} \qquad (15)$$

This is called the EVA Unearned Premium. During the year, this is always the amount available for investment. Thus the total amount of interest earned during the year is the rate of interest being earned at any time multiplied by the EVA Unearned Premium in force at the time.

We assume that the average premium funds available for investment during the year are the average of the beginning and ending EVA Unearned Premium Reserves. That is:

$$\text{AVG PREM FUNDS} = \frac{\text{EVA UNEARNED PREM}_{\text{beginning}} + \text{EVA UNEARNED PREM}_{\text{end}}}{2} \qquad (16)$$

The premiums earned during the year will consist of premiums written earlier but earned during the year plus premiums both written and earned during the year. The interest rate applicable to the first of these is the interest rate underlying the beginning EVA Unearned Premium. This is the interest rate in effect at the end of the prior year. The interest rate applicable to the second is the average interest rate in effect during the year. The weight given to the first interest rate is the ratio of EVA Unearned Premiums at the beginning of the year to premiums earned during the year. That is:

$$\text{WEIGHT}_{\text{unearned prem beginning}} = \frac{\text{EVA UNEARNED PREMIUM}_{\text{beginning}}}{\text{EVA EARNED PREMIUM}}$$

The weight given the remaining premiums is:

$1 - \text{WEIGHT}_{\text{unearned prem beginning}}.$

Thus the interest rate to apply to premium funds during the year is:

$$\text{INT RATE}_{\text{curr yr}} \tag{17}$$

$$= \text{INT RATE}_{\text{prior yr end}} \quad \times \quad \frac{\text{EVA UNEARNED PREMIUM}_{\text{beginning}}}{\text{EVA EARNED PREMIUM}}$$

$$+ \text{INT RATE}_{\text{curr yr avg}} \quad \times \quad 1 - \frac{\text{EVA UNEARNED PREMIUM}_{\text{beginning}}}{\text{EVA EARNED PREMIUM}}$$

The interest earned during the year is:

$$\text{INT EARNED}_{\text{curr yr}} = \text{INT RATE}_{\text{curr yr}} \tag{18}$$

$$\times \frac{\text{EVA UNEARNED PREM}_{\text{beginning}} + \text{EVA UNEARNED PREM}_{\text{end}}}{2}$$

The EVA Paid Premium is equal to the sum of the premiums collected during the year plus the interest earned on those premiums. Thus:

$$\text{EVA PAID PREMIUM} = \text{COLLECTED PREMIUMS} \quad + \quad \text{INT EARNED}_{\text{curr yr}} \tag{19}$$

Finally, the EVA Earned Premium is equal to the EVA Paid Premium plus the change in the EVA Unearned Premium. Thus:

$$\text{EVA EARNED PREMIUM} = \text{EVA PAID PREMIUM} \quad + \quad \Delta\text{EVA UNEARNED PREM} \tag{20}$$

The Balance Sheet on Page 1 of Exhibit IV shows the value of EVA Unearned Premiums for 1983 - 1985. The cash flow statement on page 2 of the Exhibit shows the value of EVA Paid Premiums during the year for 1984 and 1985. The Income Statement on Page 3 of the Exhibit shows the EVA Earned Premium.

In summary, it is possible to estimate investment income earned from the prepayment of premiums using annual statement data. We use the ratio of net unearned premium funds to written premiums to estimate the average time between the time companies collect premiums and the time they earn them. We use formulas (13) - (20 to estimate EVA unearned and earned premiums.

EXPENSES

Statutory accounting documents do not currently provide information about about expense funds or flows. The accounting profession has taken issue with statutory accounting practices in the area of expenses and has demanded recognition of prepaid expenses. Prepaid expenses are an integral part of the flow of insurance funds as can be seen from Exhibit I and should be a part of insurance accounting statements.

The tax reform measure of 1986 made 20% of unearned premiums subject to federal income taxes. The purpose of this was to recognize the "equity in the unearned premium reserve," a phrase sometimes used to describe prepaid expenses. The implicit assumption underlying this action is that prepaid expenses are 20% of premiums.

Both GAAP accounting and Federal Tax Law now recognize the mismatch between revenues and expenses created by the omission of prepaid expenses from statutory accounting.

It has been argued over the years that the adoption of prepaid expense reserves in statutory accounting statements might lead to federal income taxation of Prepaid Expenses. Now that this has happened, there is no reason why statutory accounting statements should not join everyone else and show prepaid expenses.

Since this information is not available from the Annual Statement, we must develop our own estimates of prepaid expenses. We begin by recognizing that Expense Reserves are similar to premium reserves. Unearned premiums represent the stock of premiums written but not earned. Unearned expenses are the stock of expenses booked before and after they are earned. Net Unearned Expense Reserves are the stock of expenses paid before and after they are earned. Thus the Unearned Expense Reserve is similar to the Unearned Premium Reserve. The Net Unearned Expense Reserve is similar to the Net Unearned Premium Reserve.

Once these reserves are estimated, they can be used in turn to estimate expense cash flows the same way that premium reserves are used to estimate premium cash flows. We will use Unearned Expense and Net Unearned Expense reserves to make estimates of average expense payment dates. We will use these estimates, in turn, to estimate interest earned on expense flows.

The expense categories that will be covered in this analysis are those available from the Insurance Expense Exhibit:

1) Loss Adjustment Expense

2) Commission and Brokerage

3) Other Acquisition, Etc.

4) General Expenses

5) Taxes, Licenses and Fees

6) Policyholder Dividends

In this paper we will include policyholder dividends in the expense category.

Schedules O and P show the details of the cash flow for loss adjustment expenses. We will discuss them in the next section.

We will discuss the standard expense categories first and then consider the effect of policyholder dividends. These categories are Commissions, Other Acquisition, General Expenses, and Taxes.

COMMISSIONS AND BROKERAGE

Commissions and Brokerage have usually been regarded as expenses which a company pays when it writes a policy. In many companies, however, the agent retains the commission and never sends it to the company. This means, in effect, that the company pays commissions when it collects premiums. Even when a company makes a direct commission payment, it is unlikely that they make the payment before they collect the premium. Thus we will assume that the delay in paying commissions and brokerage is approximately equal to the delay in collecting premiums.

Companies usually record commissions in full at the inception of new policies or on the renewal date of insurance policies being renewed. Thus there is a difference between the time they book commissions and the time they pay them that is similar to the difference between the time they write premiums and the time they collect them.

We estimate prepaid and net prepaid commission reserves by multiplying unearned and net unearned premium reserves by the direct commission percent. That is:

DIRECT COMMISSION RESERVE = -UNEARNED PREM x DIRECT COMMISSION PCT (21)

NET DIR. COMMISSION RESERVE = -NET UNEARNED PREM x DIR. COMMISSION PCT (22)

Since direct commissions are usually prepaid, they are a negative liability.

Not all commissions are paid directly. Companies pay a small portion of commissions (about 5% of all commissions in 1985) on a contingent basis after the end of a year. The delay in payment is half a year, on average, plus the time it takes to determine the bonus and make the payment. The reserve for contingent commissions at the end of a year will be about equal to contingent commissions earned during the year.

The present value of the contingent commission liability is essentially the same as the nominal value. We estimate investment income earned from the delay in paying contingent commissions by multiplying the paid contingent commissions by $(1 - (1+i)^{-.5})$.

Contingent commissions represent a postpaid expense: one which companies pay the year after it earns premiums. Thus contingent commission reserves are a positive liability.

Because of the dependence of contingent commissions on the yearly cycle, the reserve ratio is not an indicator of the average lag. We will handle this situation by showing the full contingent commission reserve as part of the Unearned Expense Reserve and half that amount as part of the Net Unearned Expense Reserve.

The procedure for calculating commission prepaid and net prepaid expense reserves begins by first calculating the contingent commission percent. The contingent commission percent is the contingent commission reserve from line 3, page 3 of the Annual Statement divided by Earned Premiums.

CONTINGENT COMM. PCT = CONTINGENT COMM. RESERVE / EARNED PREMIUMS (23)

We estimate the commission percentage net of contingent commissions by subtracting the contingent commission percent from the ratio of commissions to written premiums. (We are calling this the Direct Commission Percent for convenience since most of it is direct commissions.) Thus:

DIRECT COMMISSION PCT = CONTINGENT COMMISSION PCT (24)
- TOTAL COMMISSIONS / WRITTEN PREMIUMS

We use this result to calculate direct prepaid commissions and net prepaid commission reserves. We multiply the contingent commission percent by earned premiums to calculate the contingent commission reserve. We subtract the direct commission prepaid and net prepaid reserves from the contingent commission reserve to calculate the total commission reserves.

$$\text{COMMISSION RESERVE} = \text{CONTINGENT COMMISSION PCT} \times \text{EARNED PREMIUMS} - \text{DIRECT COMMISSION RESERVE} \quad (25)$$

$$\text{NET COMMISSION RESERVE} = .5 \times \text{CONTINGENT COMMISSION PCT} \times \text{EARNED PREMIUMS} - \text{NET DIRECT COMMISSION RESERVE} \quad (26)$$

ACQUISITION EXPENSES

Acquisition expenses relate to the cost of maintaining a field organization. They do not relate to the costs of specific policies. Under the assumption that most of the work of the field force is connected with sales, we assume that acquisition expenses are all prepaid.

The acquisition expense ratio is the ratio of acquisition expenses to written premiums. We will estimate prepaid acquisition expense reserves by multiplying the acquisition expense percentage by the unearned premium reserve. The net prepaid acquisition reserve is equal to the prepaid acquisition reserve.

$$\text{PREPAID ACQ} = -\text{ACQ EXP PCT} \times \text{UNEARNED PREMIUM} \quad (27)$$

GENERAL EXPENSES

General expenses are all company related expenses not otherwise classified. They consist of salaries, policy handling costs, computer costs, etc. Some proportion of these costs are incurred when a policy is underwritten and processed by the company. We associate other portions of these costs with the maintenance of policies in force. Thus we assume that we prepay part of general expenses and pay the other part when we earn premiums. The traditional rule of thumb for this has been to assume that half of general expenses are prepaid.

We will estimate prepaid general expenses by multiplying one half the general expense percentage by the unearned premium reserve. The net prepaid general expense reserve is equal to the prepaid general expense reserve.

$$\text{PREPAID GEN EXP} = -.5 \times \text{GEN EXP PCT} \times \text{UNEARNED PREMIUM} \quad (28)$$

TAXES, LICENSES, AND FEES

Taxes, Licenses and Fees consist primarily of premium taxes paid on direct written premiums. There is some lag between the time companies write premiums and the time they pay premium taxes since it takes time to report the amount of written premiums to the state. Thus taxes, licenses, and fees are prepaid expenses with a lag of perhaps a third of a year. (This is indicated by the ratio of unpaid taxes, etc. to the incurred taxes, licenses, and fees for the industry in 1985.)

At any time, the percentage of premium taxes payable on premiums written will be equal to the ratio of the taxes, licences and fees liability from line 4 of page 3 of the annual statement to the taxes, licenses and fees expense shown on line 8, column 33 of the Insurance Expense Exhibit. This represents a delay between the time companies write premiums and the time they pay premium taxes. It is conceptually similar to premiums receivable. Thus the prepaid taxes, licenses, and fees reserve will be equal to the ratio of taxes, etc. to written premiums times the unearned premium reserve. The net prepaid taxes, licenses and fees reserve will be the prepaid tax reserve plus the tax ratio times the liability ratio times written premiums. That is:

PREPAID PREM TAX (29)

= -PREM TAX RATIO x UNEARNED PREMIUM

NET PREPAID PREM TAX (30)

= PREPAID PREM TAX

+ PREM TAX RATIO x TAX LIABILITY RATIO x WRITTEN PREMIUM

POLICYHOLDER DIVIDENDS

Policyholder dividends are postpaid expenses. Policyholder dividends declared and paid at any given time are applicable to policies which have expired within a recent period of time (usually a quarter of a year). This means, on average, that companies pay dividends about 3/4 years after they earn premiums. This assumes a one and a half month payment lag after the beginning of the year. Thus we assume that policyholder dividends flow out of unearned expense reserve .25 years after the expiration of the policy. We can diagram this flow as follows.

Fig. 2

DIAGRAM OF POLICYHOLDER DIVIDEND RESERVES

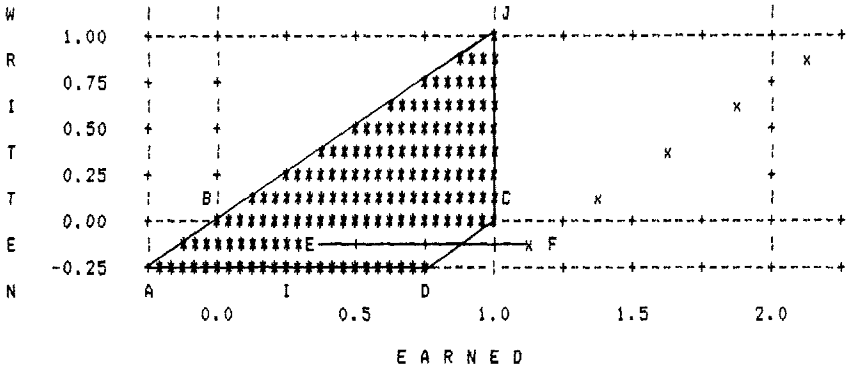


Figure 2 shows the relationships between written and earned premiums. Area ABCD represents the earnings on policies expiring in the fourth quarter. These policies were written from time -0.25 to time 0 (assuming annual policies). Dividends are assumed to have been declared and then paid at time 1.125 as shown by point F. The average time lapse between the time earned and the time paid is shown by line EF which is 3/4 years long. We can see from this diagram that no experience period has been completed for any policy written during year 01. Thus no dividends have been declared on these policies. Thus Statutory Accounting does not give any recognition to expected policyholder dividends from premiums earned during year 01 since they have not been declared. GAAP accounting principles do require recognition of these accrued but undeclared dividends. At time 1.0, all the premiums written during the fourth quarter of year 0.0 have been earned and have accrued dividends declared but unpaid. At the same time, half the premiums written during year 1.0 are earned and have accrued dividends undeclared and unpaid.

Another point illustrated in Fig. 2 is that the dividends paid in any given year are based on prior years written premiums. Thus we estimate the current dividend payout percentage by dividing dividends incurred by the prior years written premiums. Dividends incurred are equal to dividends paid plus the change in dividends declared but unpaid.

$$\text{DIVIDEND PAYOUT PCT}_{curr\ yr} = \frac{\text{DIVIDENDS INCURRED}_{curr\ yr}}{\text{WRITTEN PREM}_{prior\ yr}} \qquad (31)$$

We estimate the policyholder dividend reserve by multiplying the dividend payout percent by the estimated premiums written and earned since the third quarter of the prior year, as shown in Figure 2. The most convenient way to do this with data from the annual statement is to add half the premiums written during the latest year to one fourth of the prior years written premiums. Thus the policyholder dividend reserve is the policyholder dividend percentage times one half the latest year's written premiums plus one fourth the prior year's written premiums. The net policyholder dividend reserve is the same.

POLICYHOLDER DIV RES (32)

$$= \frac{\text{INCURRED POLICYHOLDER DIVIDENDS}_{\text{current year}}}{\text{WRITTEN PREMIUM}_{\text{prior year}}}$$

$$\times \ (1/2 \text{ WRITTEN PREM}_{\text{current year}} + 1/4 \text{ WRITTEN PREM}_{\text{prior year}})$$

TOTAL EXPENSE RESERVES

We calculate total expense reserves by various combinations of the formulae presented earlier. The unearned and net unearned expense reserves are as follows (shown with formula numbers):

UNEARNED EXPENSE = COMMISSION RESERVE (25) (33)
+ PREPAID ACQ (27)
+ PREPAID GEN EXP (28)
+ PREPAID PREM TAX (29)
+ POLICYHOLDER DIV RES (32)

NET UNEARNED EXPENSE = NET COMMISSION RESERVE (26) (34)
+ PREPAID ACQ (27)
+ PREPAID GEN EXP (28)
+ NET PREPAID PREM TAX (30)
+ POLICYHOLDER DIV RES (32)

We can plug these numbers in from the 1985 industry annual statement from A.M. Best's Aggregates and Averages.

$\text{UNEARNED EXPENSE}_{1985}$ = -5,646
+ -3,055
+ -1,540
+ -1,732
+ 1,911

-10,063

$$
\begin{aligned}
\text{NET UNEARNED EXPENSE}_{1985} = \; & -2{,}964 \\
& -3{,}055 \\
& -1{,}540 \\
& -68 \\
& 1{,}911 \\
\\
& -\ 5{,}716
\end{aligned}
$$

To estimate the lags, we need to know the total expenses incurred on 1985 earned premiums. These were $38,956 million. Thus the industry booked its expenses about (.258 = 10,063/38,956) years before they earned them. They paid them about (.147 = 5,716/38,956) years before they were earned. The investment income lost from the prepayment of expenses in 1985 was approximately ($284 = 38,956 x $1.051^{.147}$ - 1) million dollars. This was about 0.2% of the 1985 earned premiums.

It is the relative insignificance of this figure that leads to the conclusion that the worst estimation problems exist where they do the least harm. It is likely that the true percentage of investment income lost due to prepayment of expenses is somewhere between .0% and .4% of premiums.

It is now possible to apply the earned premium formulas, (13) - (20) to expenses. We substitute the ratios of prepaid and net prepaid expense reserves for the ratios of unearned and net unearned premiums. Thus we have:

$$
\begin{aligned}
\text{TOTAL EXPENSE PAID} \quad = \quad & \text{PAID COMMISSION} \\
+ \quad & \text{PAID OAE} \\
+ \quad & \text{PAID GENERAL EXPENSE} \\
+ \quad & \text{PAID TLF} \\
+ \quad & \text{PAID POLICYHOLDER DIVIDENDS}
\end{aligned} \tag{35}
$$

$$
\text{NET EXPENSE INCURRED} = \text{TOTAL EXPENSE PAID} + \Delta\text{NET UNEARNED EXPENSE} \tag{36}
$$

$$
\begin{aligned}
&\text{EVA INTEREST LOST} \\
&\quad = \text{UNEARNED PREMIUM} \times \text{TOTAL EXPENSE AND DIVIDEND RATIO} \\
&\quad \times (1+i)^{(.5-(\text{NET PREPAID/TOTAL EXPENSE})/3} - 1
\end{aligned} \tag{37}
$$

$$
\text{EVA UNEARNED EXPENSES} = \text{NET EXPENSE INCURRED} + \Delta\text{EVA INTEREST LOST} \tag{38}
$$

The interest rate to use in calculating interest earned is similar to that used for earned premiums:

$$\text{INT RATE}_{\text{curr yr}} \tag{39}$$

$$= \text{INT RATE}_{\text{prior yr end}} \quad \text{x} \quad \frac{\text{EVA UNEARNED EXPENSES}_{\text{beginning}}}{\text{EARNED PREMIUM}}$$

$$+ \text{INT RATE}_{\text{curr yr avg}} \quad \text{x} \quad 1 - \frac{\text{EVA UNEARNED EXPENSES}_{\text{beginning}}}{\text{EARNED PREMIUM}}$$

The interest lost during the year is:

$$\text{INT LOST}_{\text{curr yr}} = \text{INT RATE}_{\text{curr yr}} \quad \text{x} \quad \frac{\text{EVA UNEARNED EXP}_{\text{beginning}} + \text{EVA UNEARNED EXP}_{\text{end}}}{2} \tag{40}$$

The EVA Paid Expense is equal to the sum of the premiums collected during the year plus the interest earned on those premiums. Thus:

$$\text{EVA PAID EXPENSE} = \text{PAID EXPENSES} + \text{INT LOST}_{\text{curr yr}} \tag{41}$$

Finally, the EVA Incurred Expense is equal to the EVA Paid Expense plus the change in the EVA Unearned Expense. Thus:

$$\text{EVA INCURRED EXPENSE} = \text{EVA PAID EXPENSE} + \Delta\text{EVA UNEARNED EXPENSE} \tag{42}$$

LOSSES

About 10% of total industry losses and loss adjustment expenses (LAE)[6] are paid five years or more after they occur. Loss Reserves are paid about 2.3 years after the statement date. Loss and LAE average payment dates by line vary from .55 years for homeowners to 6 years for medical malpractice.

These facts indicate that loss flows generate a lot more investment income than premium or expense flows. The average time industry premium funds are invested is about .2 years. Expenses are paid, on average, about one sixth of a year before they are earned. Losses are invested for 1.7 years.

No direct information about actual premium and expense cash flows is available from current accounting data. Cash flow data on losses, however, is available from Schedules O and P of the annual statement.

These schedules are organized on a year of occurrence basis. Claims occurring in any given year are followed for ten years. Losses paid on these claims are recorded on a cumulative basis. Company estimates of future loss payments (reserves) are also recorded. Almost all amounts shown are nominal, but some workers' compensation reserves are discounted. Data recorded this way is called accident year data because it provides information on the group of claims (accidents) occurring in a given year.

Data in Schedules O and P record what has happened in the past. We wish to estimate what will happen in the future. These Schedules are useful only to the extent that they provide guidance about the future. We use these schedules to provide information about the expected amounts and timing of future loss flows.

Loss flows are stable. Both amounts and timing are predictable for large books of primary company business. A large book is one with more than $100 million of premium.

Exhibit VI records Schedule P data for the industry. The data shown is cumulative average payment dates for Schedule O, Schedule P and all lines. This type of data provides information about what is happening to each accident year on a cumulative basis which is easy to compare to other years. The formula for cumulative loss average payment dates is as follows:

---

6. All future references to losses in this paper will include LAE unless otherwise noted.

EXHIBIT VI
Page 1

INDUSTRY SCHEDULE O AVERAGE PAYMENT DATES

	AGE								
Year	2	3	4	5	6	7	8	9	10
1977	.XX	.XX	.XX	.XX	.XX	.XX	.XX	.XX	
1978	.XX	.XX	.XX	.XX	.XX	.XX	.XX		
1979	.XX	.XX	.XX	.XX	.XX	.XX			
1980	.XX	.XX	.XX	.XX	.XX				
1981	.30	.39	.42	.47					
1982	.33	.40	.42						
1983	.35	.42							
1984	.32								

INDUSTRY AUTO LIABILITY AVERAGE PAYMENT DATES

	AGE								
Year	2	3	4	5	6	7	8	9	10
1977	.43	.71	.94	1.12	1.24	1.31	1.35	1.37	
1978	.43	.71	.94	1.11	1.23	1.29	1.32		
1979	.43	.70	.94	1.10	1.21	1.28			
1980	.45	.71	.94	1.09	1.20				
1981	.45	.73	.95	1.11					
1982	.44	.73	.95						
1983	.46	.76							
1984	.47								

INDUSTRY GENERAL LIABILITY AVERAGE PAYMENT DATES

	AGE								
Year	2	3	4	5	6	7	8	9	10
1977	.53	1.08	1.68	2.21	2.65	3.03	3.26	3.49	
1978	.55	1.10	1.69	2.19	2.62	2.97	3.25		
1979	.55	1.12	1.65	2.20	2.60	2.96			
1980	.58	1.13	1.69	2.21	2.64				
1981	.58	1.18	1.72	2.21					
1982	.56	1.14	1.69						
1983	.58	1.16							
1984	.59								

INDUSTRY MEDICAL MALPRACTICE AVERAGE PAYMENT DATES

	AGE								
Year	2	3	4	5	6	7	8	9	10
1977	.65	1.39	2.13	2.78	3.40	3.86	4.19	4.53	
1978	.68	1.42	2.16	2.81	3.37	3.75	4.14		
1979	.69	1.45	2.15	2.78	3.30	3.70			
1980	.73	1.43	2.15	2.69	3.19				
1981	.74	1.45	2.12	2.70					
1982	.59	1.29	1.98						
1983	.60	1.33							
1984	.67								

EXHIBIT VI
Page 2

INDUSTRY WORKER'S COMPENSATION AVERAGE PAYMENT DATES

	AGE								
Year	2	3	4	5	6	7	8	9	10
1977	.49	.81	1.07	1.28	1.46	1.60	1.71	1.82	
1978	.49	.81	1.07	1.28	1.46	1.59	1.70		
1979	.49	.81	1.06	1.28	1.44	1.58			
1980	.48	.80	1.05	1.25	1.42				
1981	.48	.80	1.05	1.25					
1982	.48	.79	1.04						
1983	.50	.83							
1984	.51								

INDUSTRY MULTI PERIL AVERAGE PAYMENT DATES

	AGE								
Year	2	3	4	5	6	7	8	9	10
1977	.32	.43	.54	.65	.75	.82	.86	.92	
1978	.31	.43	.55	.66	.77	.84	.90		
1979	.30	.41	.52	.63	.72	.80			
1980	.30	.41	.52	.63	.73				
1981	.29	.41	.53	.66					
1982	.27	.40	.53						
1983	.34	.48							
1984	.30								

INDUSTRY TOTAL SCHEDULE P LINES AVERAGE PAYMENT DATES

	AGE								
Year	2	3	4	5	6	7	8	9	10
1977	.41	.67	.91	1.13	1.31	1.45	1.54	1.62	
1978	.41	.67	.91	1.13	1.31	1.43	1.53		
1979	.40	.65	.90	1.10	1.27	1.40			
1980	.41	.65	.89	1.09	1.27				
1981	.40	.67	.91	1.12					
1982	.39	.66	.90						
1983	.43	.71							
1984	.43								

INDUSTRY ALL LINES AVERAGE PAYMENT DATES

	AGE								
Year	2	3	4	5	6	7	8	9	10
1977	.XX	.XX	.XX	.XX	.XX	.XX	.XX	.XX	
1978	.XX	.XX	.XX	.XX	.XX	.XX	.XX		
1979	.XX	.XX	.XX	.XX	.XX	.XX			
1980	.XX	.XX	.XX	.XX	.XX				
1981	.36	.56	.74	.89					
1982	.37	.57	.74						
1983	.39	.60							
1984	.39								

$$\text{CUMULATIVE LOSS AVERAGE PAYMENT DATES}_{j:n} = \frac{\sum_{i=0}^{n} i \times \text{PAYMENT}_i}{\sum_{i=0}^{n} \text{PAYMENT}_i} \qquad (43)$$

Where PAYMENT_i is the payment made during year $i+j$ for accident year j.

Schedules O and P currently show accident year data for ten years. Thus the value of n runs from 1 to ten. We assume that the average payment date during each year is mid year[7]. Thus the average payment date for the initial year with respect to the average date accidents occur is always 0 and is not shown. Exhibit VI shows average payment dates for the past eight years running from n=1 to n=10[8].

Average payment date data has two uses: The first is to provide information about the cumulative development of each accident year which can be compared to other years. Each of these average payment dates incorporates information about all the payment flows to date for that accident year. It reflects the net impact of any acceleration or deceleration of payment flows within any particular accident year.

When average payment dates increase over time they reflect a slowdown in the rate of paying losses. Decreases in the average payment date reflect an acceleration of loss payments.

The second use is to provide an index of the impact of loss payment pattern changes on investment income from loss funds. Average payment dates can be used to calculate the interest earned from investing loss funds. Changes in average payment dates are indicators of changes in investment opportunities. Increases in average payment dates over time indicate increased opportunities to earn investment income. Decreases indicate reduced opportunities.

7. We know that this is not strictly true. Losses occurred and paid during a given year are paid later, on average, than mid year. The delay is minimal, however, and is ignored.

8. Only four years are shown for Schedule O and for the all-lines data. Schedule O data was only available to us beginning in 1981.

The largest change in average payment dates in Exhibit VI is found in the medical malpractice line on page 1. The change from 1978 to 1982 in average payment dates during the third year is .18 years. Investment income earned on the malpractice loss fund at the time would have changed from 16.1% of the nominal loss to 14.2% at 15% interest and from 13.4% to 12.5% at 7% interest. This change is probably a reflection of the volatility of the line. Changes in other lines and in aggregate Schedule P lines are much smaller and have less impact on potential earnings.

Other evidence of loss payment patterns is provided in Exhibit VII. This is a series of graphs showing the development for recent accident years for Schedule O and P lines. This type of analysis is affected, to some extent, by the assumption that stability exists. Thus it is not quite as persuasive as the data in Exhibit VI. Given the corroborating evidence provided in Exhibit VI, however, the graphs provide a striking picture of stability.

This steadiness in past flows gives us confidence that we can make reasonable estimates of the amount and timing of future flows. We have used paid loss data to make these estimates. Incurred loss data is not as useful because it uses company reserve estimates which have been unreliable[9]. Paid loss data does have some problems. It can be quite volatile. This is why it may not be entirely usable for small books of business or for reinsurers[10]. We use incurred loss data as a guide in some of our estimates for these reasons.

We estimate the timing and amount of expected losses by identifying the loss payment patterns underlying past accident years. We examine them for consistency and choose the average of the past three years as our estimate of future payment patterns. We have modified these estimates on a judgmental basis when it seemed appropriate. Table IX shows the payment pattern for general liability lines.

[9] The industry recognized almost $9 billion in reserve deficiencies in 1984 reserves in 1985.

[10] There is an increasing tendency by some companies to make entries in Schedules O and P that destroy its use as a scorekeeper of cash flows. The biggest problem comes from items from portfolio transfers which are entered as paid losses. These entries are huge and create discontinuities in the data that threaten to make it unusable.

Exhibit VII
Page 1

CUMULATIVE PERCENT OF LOSSES PAID
SCHEDULE O

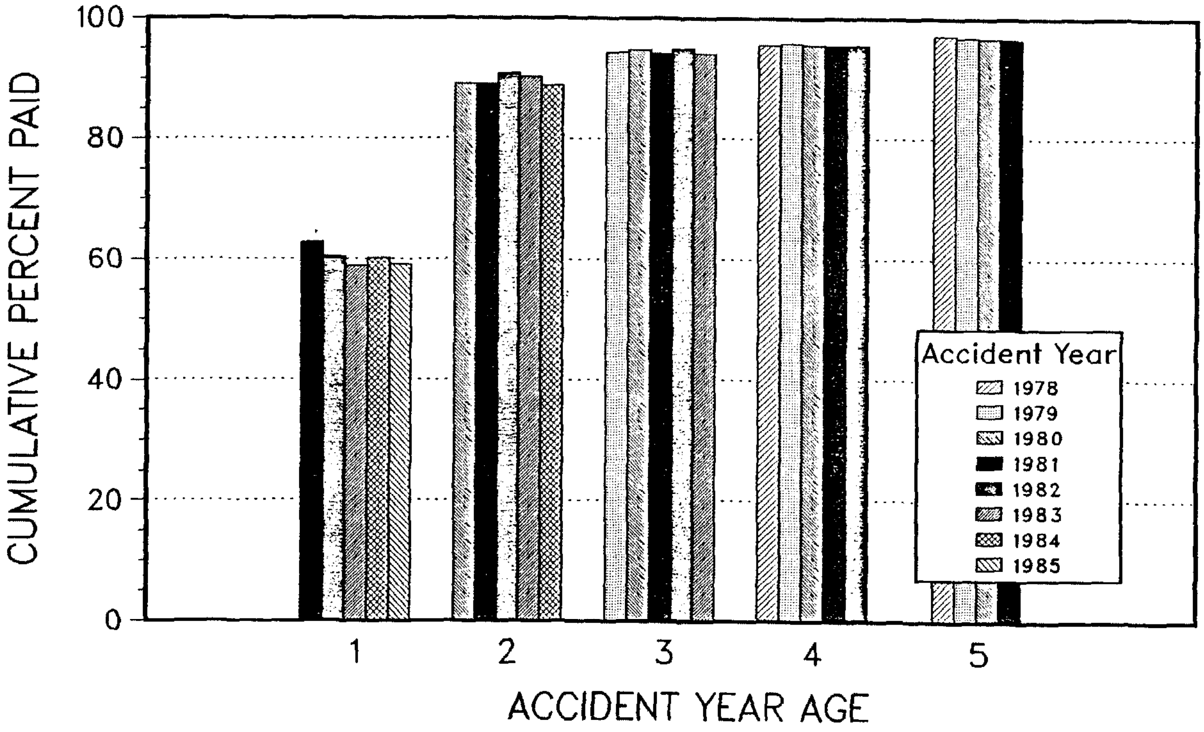


Exhibit VII
Page 2

CUMULATIVE PERCENT OF LOSSES PAID

AUTO LIABILITY

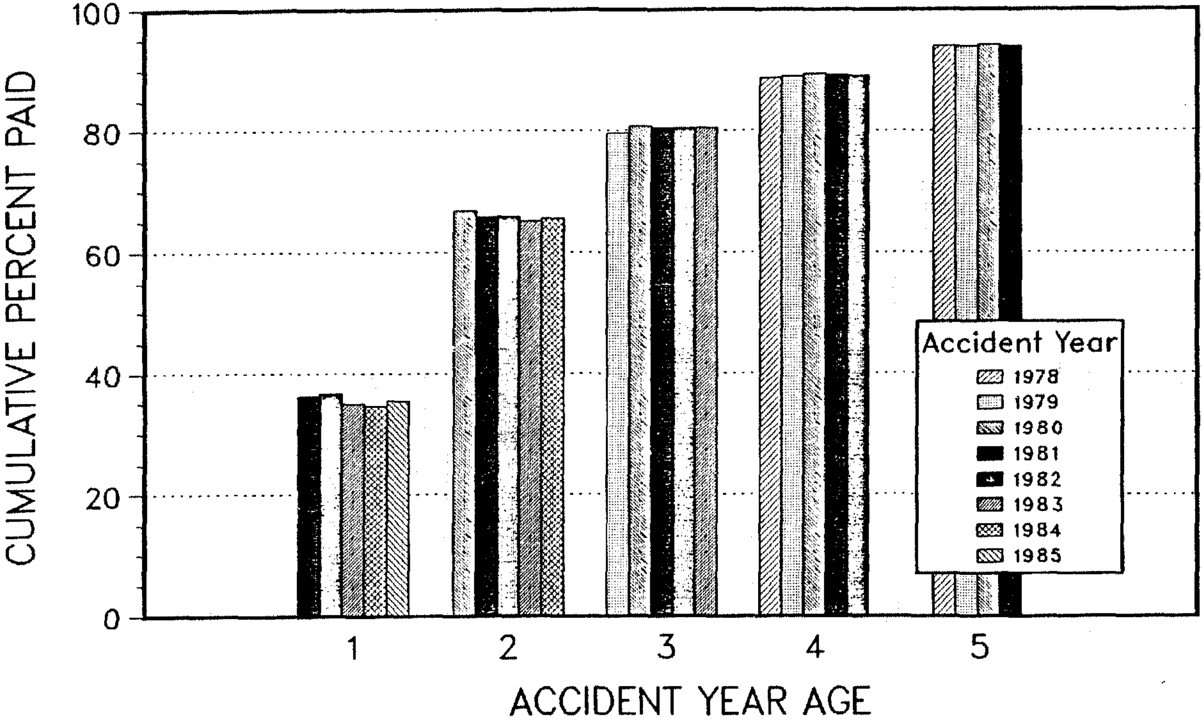


Exhibit VII
Page 3

CUMULATIVE PERCENT OF LOSSES PAID

GENERAL LIABILITY

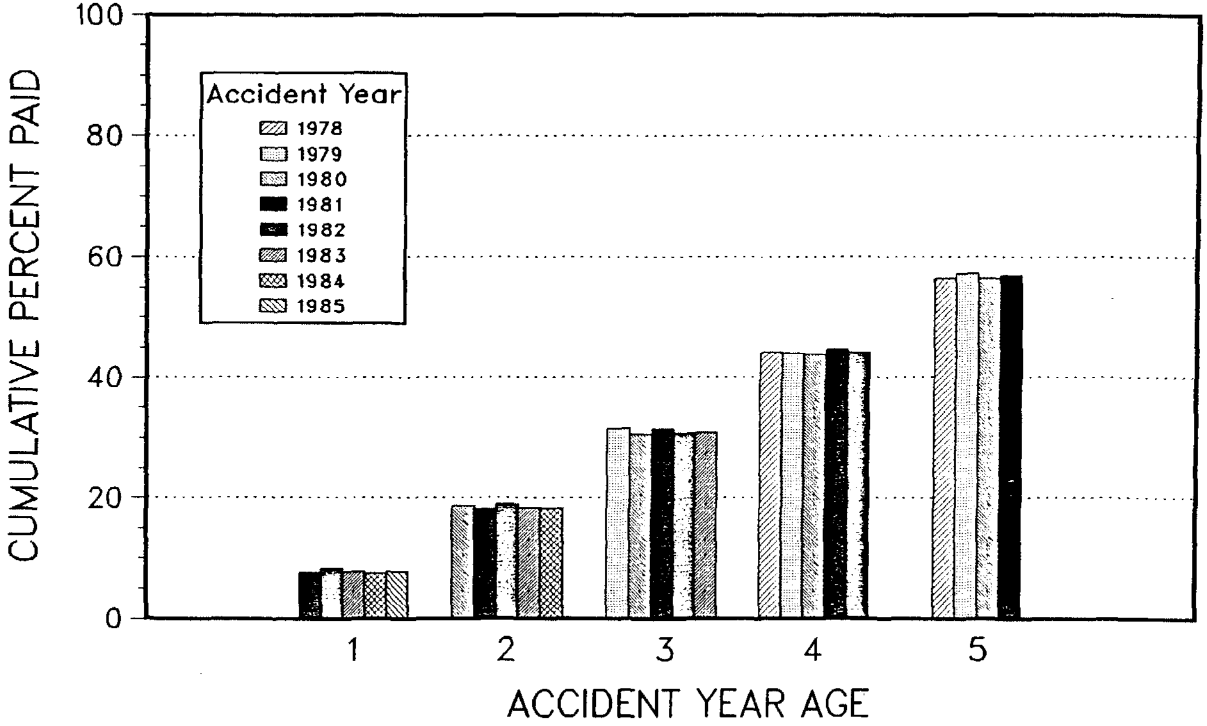


Exhibit VII
Page 4

CUMULATIVE PERCENT OF LOSSES PAID

MEDICAL MALPRACTICE

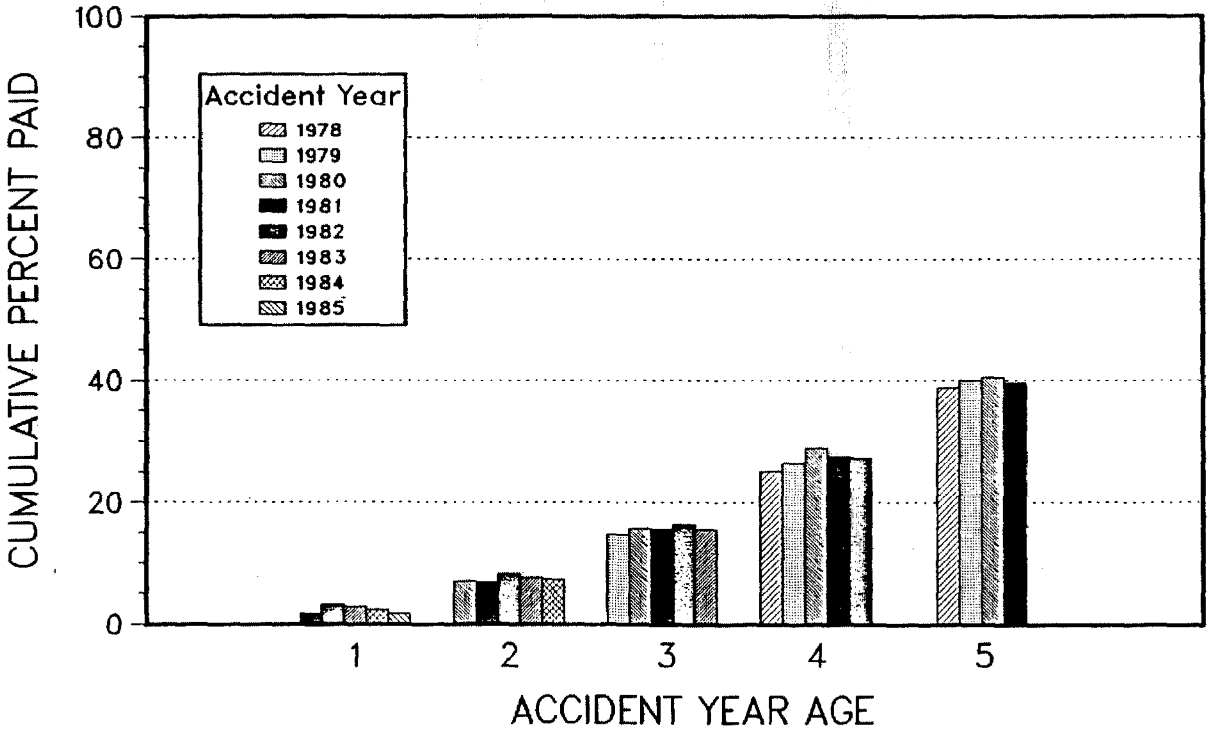


Exhibit VII
Page 5

CUMULATIVE PERCENT OF LOSSES PAID

WORKERS COMPENSATION

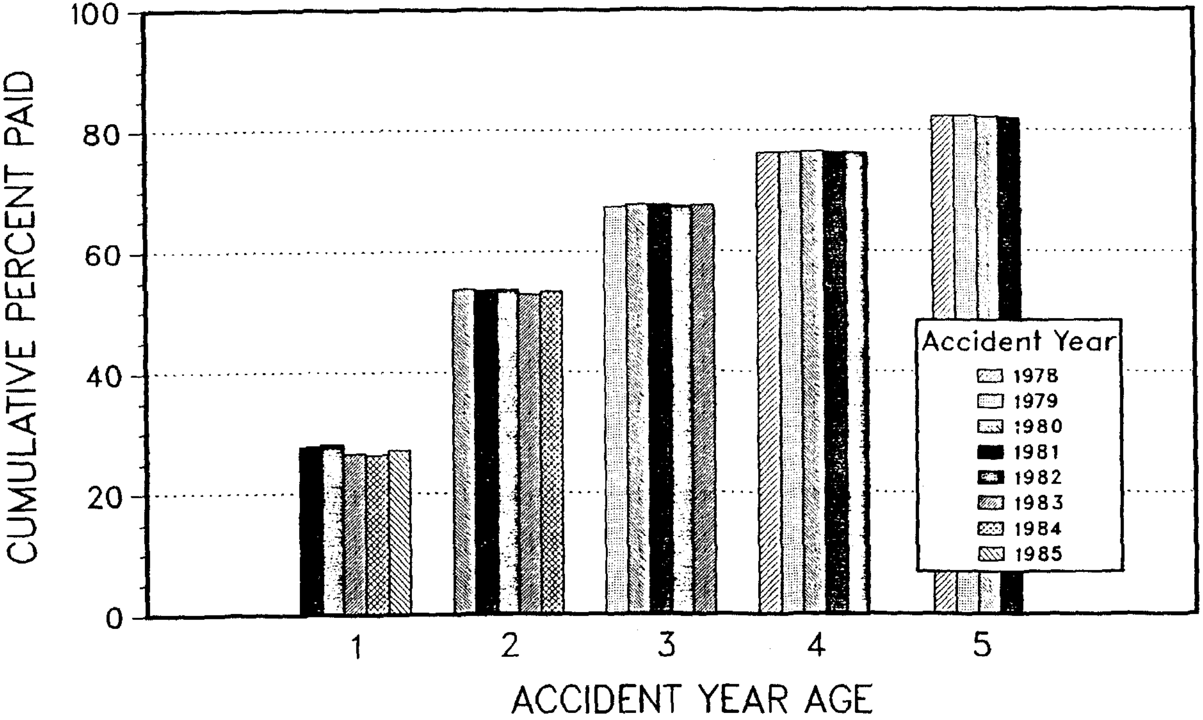


Exhibit VII
Page 6

CUMULATIVE PERCENT OF LOSSES PAID

MULTI-PERIL

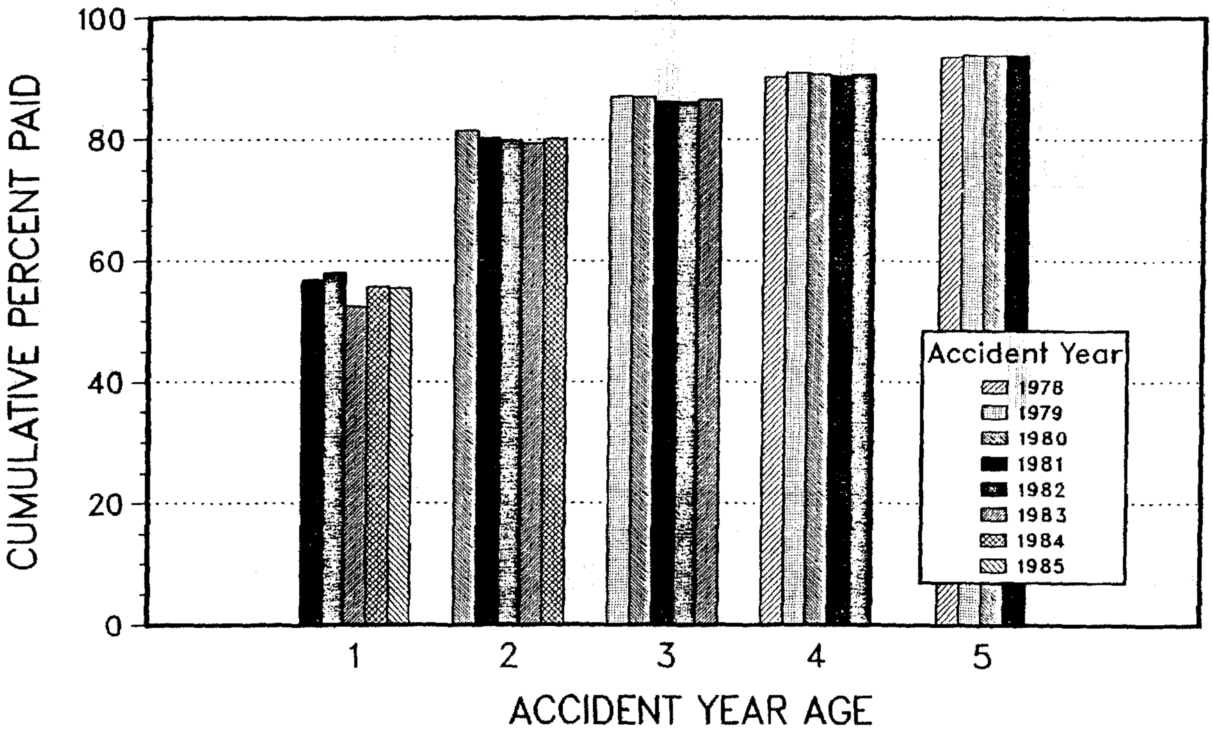


Exhibit VII
Page 7

CUMULATIVE PERCENT OF LOSSES PAID

SCHEDULE P

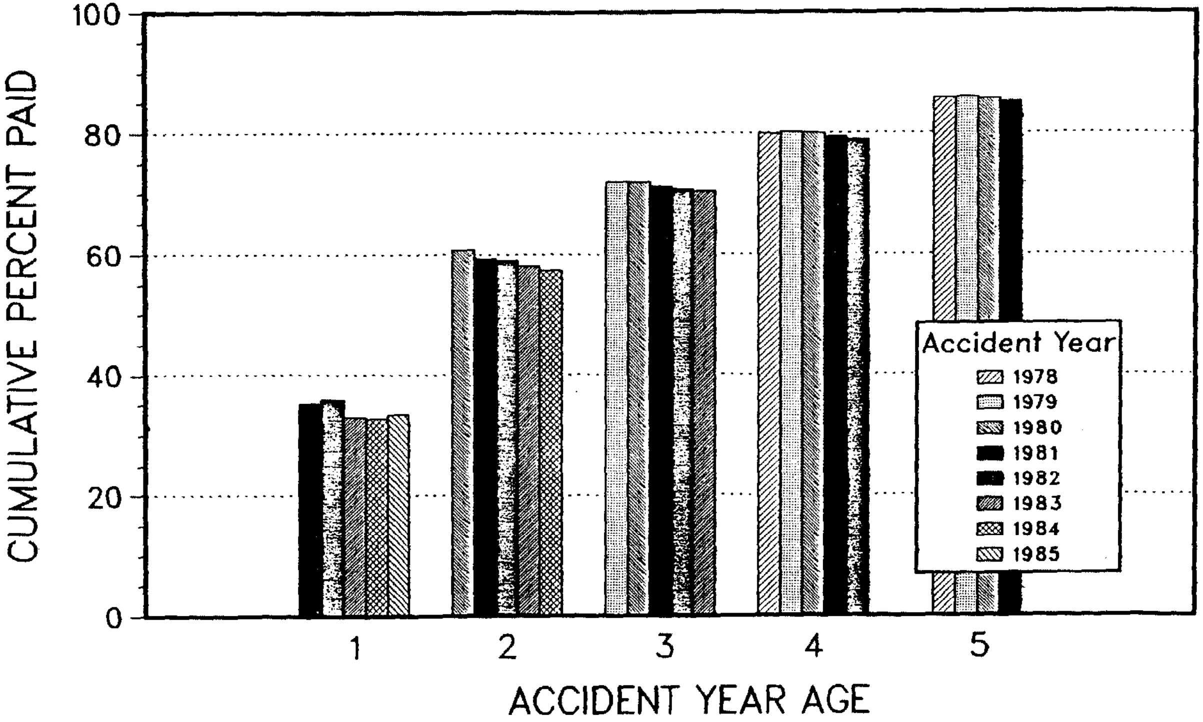


Exhibit VII
Page 8

CUMULATIVE PERCENT OF LOSSES PAID
ALL LINES

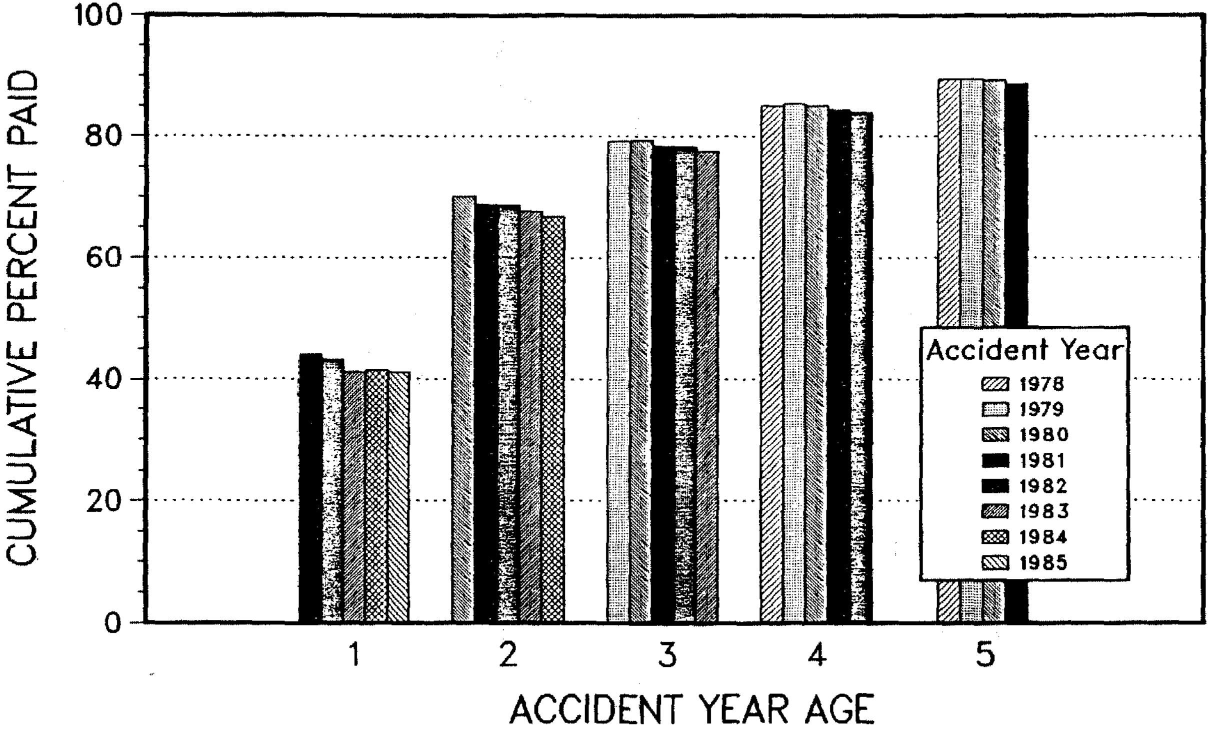


TABLE IX

GENERAL LIABILITY PAYMENT PATTERNS

	2/1	3/2	4/3	5/4	6/5	7/6	8/7	9/8	10/9	Ult
1976								.952	.961	
1977	.466	.630	.688	.772	.841	.887	.943	.949		
1978	.452	.618	.690	.785	.847	.896	.931			
1979	.448	.608	.718	.769	.856	.895				
1980	.425	.613	.698	.775	.845					
1981	.419	.578	.704	.787						
1982	.436	.617	.697							
1983	.423	.593								
1984	.410									
3 YR AVG	.423	.598	.703	.781	.851	.892	.933	.950	.961	
MCL	.115	.154	.192	.227	.233	.237	.199	.203	.211	.200
SELECTED	.423	.593	.700	.777	.850	.895	.937	.951	.961	.873

The data in Table IX represents the ratio of losses paid at age n divided by losses paid at age n-1[11].

Estimating the percentage of losses that will be paid after ten years is the most difficult part of the procedure. One method commonly used is to assume that reserve estimates on accident years that have developed for ten years are both accurate and an unbiased estimator of that percentage. Another method is to assume that losses are paid at a constant exponential rate[12]. We take both methods into consideration along with judgment. Our estimate of the percentage of general liability losses unpaid after ten years is 12.7%. This means that 87.3% of the losses are paid within ten years.

These development factors and final cumulative payment percentage describe a complete payment pattern. The cumulative percent paid as of age n years can be calculated from the following recursive formula:

$$\text{CUMULATIVE LOSS PAID}_n = \text{CUMULATIVE LOSS PAID}_{n+1} \times \text{DEVELOPMENT FACTOR}_n \qquad (44)$$

---

[11] S. Philbrick suggested this way of representing loss development patterns in "Brainstorms", <u>The Actuarial Review</u>, Nov. 1984.

[12] See C.A. McClenahan, "A Mathematical Model for Loss Reserve Analysis" <u>Proceedings of the CAS</u>, Vol. LXII, 1975. 134-153. The MCL factor in Table IX is the payment rate actually generated by that data. It is introduced and discussed on the next page.

The cumulative payment pattern for general liability losses using the selected factors shown above is:

TABLE X

GENERAL LIABILITY: CUMULATIVE PAYMENT PATTERN

Age	Cumulative Loss Paid$_n$		Development Factor$_n$		Cumulative Loss Paid$_{n+1}$
1	.078	=	.423	x	0.183
2	.183	=	.593	x	0.309
3	.309	=	.700	x	0.442
4	.442	=	.777	x	0.569
5	.569	=	.850	x	0.669
6	.669	=	.895	x	0.748
7	.748	=	.941	x	0.798
8	.798	=	.951	x	0.839
9	.839	=	.961	x	0.873
10	.873	=	.873	x	1.000

Table X shows that 7.8% of general liability losses are paid during the year they occur. About two thirds of the losses are paid within five years.

We use the assumption that losses are paid at a constant rate to extrapolate payments after ten years. This is necessary for discounting and for final average payment dates. Most of the losses paid after ten years are paid well after ten years. It is simple to calculate the average payment date for losses paid after ten years using the constant rate assumption.

The formula for the percent of losses paid at any time when losses are paid out at a constant rate is as follows:

$$\text{AMOUNT PAID} = \text{FORCE OF PAYMENT} \times \int_0^\infty (1 - \text{MCL})^t$$

The MCL value is the amount of losses paid during the year as a percent of the amount payable at the beginning of the year[13]. Table IX shows the MCL values for the industry general liability experience. We have selected a factor of .2 for losses paid 10 years or more after they occur. The force of payment is as follows:

$$\text{FORCE OF PAYMENT} = -\ln (1 - \text{MCL})$$

---

[13] This idea was taken from C. McLenahan's paper cited earlier (12). The acronym was taken from the first three letters of his last name.

This result comes from the fact that the sum of the amounts paid is equal to the total losses that are eventually paid. This means that the sum of all payments from the formula is one. Thus we have:

$$\text{FORCE OF PAYMENT} \times \int_0^{\infty} (1 - \text{MCL})^t \, dt = 1$$

The value of this integral is $(-\ln (1 - \text{MCL}))^{-1}$. Thus the value of the force of payment is:

$$\text{FORCE OF PAYMENT} = -\ln (1 - \text{MCL}) \qquad \text{QED}$$

The average payment date for any stream of payments is the amount paid at any time multiplied by the time paid. We assume that the stream of payments after the tenth year is constant. Thus the average payment date for these losses is the amount of the final payment from Table IX, times the average date it is paid:

$$\begin{aligned} \text{AVERAGE PAYMENT DATE} &= \text{FORCE OF PAYMENT} \times \int_0^{\infty} t\,(1 - \text{MCL})^t \, dt \\ &= \text{FORCE OF PAYMENT}^{-1} \\ &= -(\ln (1 - \text{MCL}))^{-1} \qquad (45) \end{aligned}$$

The final average payment date with respect to the mid-point of the initial year is:

$$\text{FINAL APD} = 10 - (\ln (1 - \text{MCL}))^{-1} - .5 \qquad (46)$$

The MCL value for general liability is .2. Thus losses paid after 10 years are paid 4.48 years later according to this formula. They are paid 14.48 years after the beginning of the accident year. This is 13.98 years after the middle of the first year, the average date losses occured.

Thus we have a loss payout pattern consisting of the percentage of losses paid out at each age from 1 through 10 years. We have a final payment factor which is paid out from the end of the tenth year until all losses are paid. We discount the loss stream by discounting each percentage paid during the first ten years by (n-1) where n represents the age. We discount the final payment percentage by the final average payment date.

Thus the present value of losses incurred during a given year is estimated as follows:

$$\text{EVA INCURRED LOSS} = \left(\sum_{j=1}^{10} pd_j \times (1+i)^{1-j} \right) + pd_{ult} \times (1+i)^{.5-10-\text{Final APD}} \qquad (47)$$

Where pd_j = CUMULATIVE LOSS PAID$_{j+1}$ - CUMULATIVE LOSS PAID$_j$

These methods have been applied to each of the lines shown in Exhibit VI. The results are shown in Exhibit VIII. This exhibit shows the complete payment patterns estimated by year from 1977 through 1985. It also shows the ultimate average payment dates and the investment income expected on each flow as of the end of each year. Two year after-tax treasury bill rates have been used to discount the loss streams.

Exhibit VIII demonstrates the amount that companies can expect to invest at the time of the loss to have enough money when they can expect to pay it. Page 2 of the Exhibit shows that an investment of 93.1 cents for each dollar of losses incurred in 1985 for all lines combined will be worth a dollar at the time they are paid if invested at prevailing after-tax interest rates.

The data in Table X and Exhibit VIII are only applicable to losses incurred during any given year. They represent the time value of losses at the moment they occur. They do not apply to loss reserves, the stock of losses unpaid at any given time.

RESERVE FLOWS

We wish to estimate the amount that will be earned in the future on loss reserves outstanding at any given time. This means we need to estimate future flows on prior accident years. We represent this flow for reserves outstanding at the end of 1985 as follows:

TABLE XI

EXPECTED LOSS FLOW FROM 12/1985 RESERVES

${}_{<76}pd_{86}$	${}_{<76}pd_{87}$	${}_{<76}pd_{88}$	${}_{<76}pd_{89}$	${}_{<76}pd_{90}$	${}_{<76}pd_{91}$	${}_{<76}pd_{92}$	${}_{<76}pd_{93}$	${}_{<76}pd_{94}$	.	.
${}_{76}pd_{86}$	${}_{76}pd_{87}$	${}_{76}pd_{88}$	${}_{76}pd_{89}$	${}_{76}pd_{90}$	${}_{76}pd_{91}$	${}_{76}pd_{92}$	${}_{76}pd_{93}$	${}_{76}pd_{94}$	.	.
${}_{77}pd_{86}$	${}_{77}pd_{87}$	${}_{77}pd_{88}$	${}_{77}pd_{89}$	${}_{77}pd_{90}$	${}_{77}pd_{91}$	${}_{77}pd_{92}$	${}_{77}pd_{93}$	${}_{77}pd_{94}$	.	.
${}_{78}pd_{86}$	${}_{78}pd_{87}$	${}_{78}pd_{88}$	${}_{78}pd_{89}$	${}_{78}pd_{90}$	${}_{78}pd_{91}$	${}_{78}pd_{92}$	${}_{78}pd_{93}$	${}_{78}pd_{94}$	.	.
${}_{79}pd_{86}$	${}_{79}pd_{87}$	${}_{79}pd_{88}$	${}_{79}pd_{89}$	${}_{79}pd_{90}$	${}_{79}pd_{91}$	${}_{79}pd_{92}$	${}_{79}pd_{93}$	${}_{79}pd_{94}$	.	.
${}_{80}pd_{86}$	${}_{80}pd_{87}$	${}_{80}pd_{88}$	${}_{80}pd_{89}$	${}_{80}pd_{90}$	${}_{80}pd_{91}$	${}_{80}pd_{92}$	${}_{80}pd_{93}$	${}_{80}pd_{94}$	.	.
${}_{81}pd_{86}$	${}_{81}pd_{87}$	${}_{81}pd_{88}$	${}_{81}pd_{89}$	${}_{81}pd_{90}$	${}_{81}pd_{91}$	${}_{81}pd_{92}$	${}_{81}pd_{93}$	${}_{81}pd_{94}$	.	.
${}_{82}pd_{86}$	${}_{82}pd_{87}$	${}_{82}pd_{88}$	${}_{82}pd_{89}$	${}_{82}pd_{90}$	${}_{82}pd_{91}$	${}_{82}pd_{92}$	${}_{82}pd_{93}$	${}_{82}pd_{94}$	.	.
${}_{83}pd_{86}$	${}_{83}pd_{87}$	${}_{83}pd_{88}$	${}_{83}pd_{89}$	${}_{83}pd_{90}$	${}_{83}pd_{91}$	${}_{83}pd_{92}$	${}_{83}pd_{93}$	${}_{83}pd_{94}$	.	.
${}_{84}pd_{86}$	${}_{84}pd_{87}$	${}_{84}pd_{88}$	${}_{84}pd_{89}$	${}_{84}pd_{90}$	${}_{84}pd_{91}$	${}_{84}pd_{92}$	${}_{84}pd_{93}$	${}_{84}pd_{94}$	.	.
${}_{85}pd_{86}$	${}_{85}pd_{87}$	${}_{85}pd_{88}$	${}_{85}pd_{89}$	${}_{85}pd_{90}$	${}_{85}pd_{91}$	${}_{85}pd_{92}$	${}_{85}pd_{93}$	${}_{85}pd_{94}$	.	.

Exhibit VIII
Page 1

INDUSTRY LOSS AND LAE CASH FLOWS
SCHEDULE O

	AGE											
Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1977	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	X.XX	X.X%
1978	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	X.XX	X.X%
1979	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	X.XX	X.X%
1980	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	X.XX	X.X%
1981	.628	.891	.941	.955	.967	.978	.986	.992	.996	.998	0.67	4.3%
1982	.593	.897	.949	.955	.967	.978	.986	.992	.996	.998	0.68	3.3%
1983	.600	.902	.941	.955	.967	.978	.986	.992	.996	.998	0.69	3.6%
1984	.601	.889	.941	.955	.967	.978	.986	.992	.996	.998	0.70	3.7%
1985	.591	.889	.941	.955	.967	.978	.986	.992	.996	.998	0.71	3.0%

AUTO LIABILITY

	AGE											
Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1977	.373	.652	.793	.884	.938	.969	.984	.990	.993	.996	1.43	5.1%
1978	.373	.659	.799	.888	.940	.970	.983	.988	.993	.996	1.42	6.4%
1979	.377	.662	.796	.890	.939	.967	.981	.988	.993	.996	1.42	7.7%
1980	.369	.669	.807	.894	.941	.967	.981	.988	.993	.996	1.40	8.7%
1981	.363	.658	.805	.892	.939	.967	.981	.988	.993	.996	1.42	9.1%
1982	.367	.659	.805	.891	.939	.967	.981	.988	.993	.996	1.42	6.8%
1983	.350	.651	.805	.891	.939	.967	.981	.988	.993	.996	1.44	7.5%
1984	.346	.656	.805	.891	.939	.967	.981	.988	.993	.996	1.44	7.6%
1985	.355	.656	.805	.891	.939	.967	.981	.988	.993	.996	1.43	6.0%

GENERAL LIABILITY

	AGE											
Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1977	.087	.188	.298	.433	.560	.666	.751	.796	.839	.873	5.02	16.2%
1978	.085	.189	.305	.442	.564	.665	.743	.798	.839	.873	5.00	20.2%
1979	.086	.192	.316	.440	.573	.669	.748	.798	.839	.873	4.97	23.7%
1980	.080	.188	.306	.438	.565	.669	.748	.798	.839	.873	5.00	26.5%
1981	.076	.182	.315	.447	.569	.669	.748	.798	.839	.873	4.99	27.3%
1982	.083	.190	.308	.442	.569	.669	.748	.798	.839	.873	4.99	21.0%
1983	.078	.184	.309	.442	.569	.669	.748	.798	.839	.873	5.00	22.7%
1984	.075	.183	.309	.442	.569	.669	.748	.798	.839	.873	5.00	23.0%
1985	.078	.183	.309	.442	.569	.669	.748	.798	.839	.873	5.00	18.9%

MEDICAL MALPRACTICE

	AGE											
Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1977	.020	.059	.130	.242	.371	.514	.623	.697	.765	.827	6.44	20.6%
1978	.019	.058	.134	.252	.388	.521	.609	.692	.765	.827	6.43	25.5%
1979	.019	.060	.146	.264	.401	.525	.614	.692	.765	.827	6.38	29.8%
1980	.019	.070	.157	.288	.406	.519	.614	.692	.765	.827	6.34	32.9%
1981	.018	.068	.155	.275	.396	.519	.614	.692	.765	.827	6.36	34.0%
1982	.033	.082	.163	.272	.396	.519	.614	.692	.765	.827	6.33	26.2%
1983	.030	.075	.155	.272	.396	.519	.614	.692	.765	.827	6.35	28.4%
1984	.024	.073	.155	.272	.396	.519	.614	.692	.765	.827	6.35	28.7%
1985	.018	.073	.155	.272	.396	.519	.614	.692	.765	.827	6.36	23.7%

EXHIBIT VIII
Page 2

INDUSTRY LOSS AND LAE CASH FLOWS
WORKERS' COMPENSATION

AGE

Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1977	.271	.528	.673	.764	.821	.863	.892	.910	.926	.938	2.74	8.9%
1978	.272	.532	.675	.764	.823	.866	.892	.910	.926	.938	2.73	11.1%
1979	.271	.534	.674	.764	.823	.862	.887	.910	.926	.938	2.74	13.2%
1980	.279	.537	.678	.766	.822	.861	.887	.910	.926	.938	2.72	14.7%
1981	.278	.536	.678	.765	.821	.861	.887	.910	.926	.938	2.72	15.1%
1982	.282	.537	.676	.764	.821	.861	.887	.910	.926	.938	2.72	11.6%
1983	.265	.529	.677	.764	.821	.861	.887	.910	.926	.938	2.75	12.6%
1984	.264	.534	.677	.764	.821	.861	.887	.910	.926	.938	2.74	12.8%
1985	.271	.534	.677	.764	.821	.861	.887	.910	.926	.938	2.74	10.5%

MULTIPLE PERIL

AGE

Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1977	.557	.813	.871	.910	.941	.963	.976	.983	.990	.994	1.01	3.5%
1978	.555	.801	.864	.904	.936	.960	.973	.983	.990	.994	1.05	4.7%
1979	.570	.815	.871	.911	.939	.959	.973	.983	.990	.994	1.00	5.4%
1980	.567	.815	.870	.908	.939	.960	.973	.983	.990	.994	1.01	6.1%
1981	.569	.803	.862	.906	.939	.960	.973	.983	.990	.994	1.03	6.5%
1982	.581	.799	.862	.908	.939	.960	.973	.983	.990	.994	1.02	4.8%
1983	.525	.795	.866	.908	.939	.960	.973	.983	.990	.994	1.08	5.5%
1984	.559	.802	.866	.908	.939	.960	.973	.983	.990	.994	1.04	5.3%
1985	.557	.802	.866	.908	.939	.960	.973	.983	.990	.994	1.04	4.3%

TOTAL SCHEDULE P LINES

AGE

Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1977	.355	.598	.715	.799	.858	.900	.928	.943	.955	.965	2.10	7.1%
1978	.354	.598	.715	.799	.858	.900	.924	.941	.955	.965	2.10	9.0%
1979	.363	.606	.719	.802	.859	.898	.923	.941	.955	.965	2.08	10.6%
1980	.361	.608	.719	.800	.856	.895	.923	.941	.955	.965	2.09	11.9%
1981	.352	.593	.711	.794	.851	.895	.923	.941	.955	.965	2.13	12.6%
1982	.359	.590	.706	.789	.851	.895	.923	.941	.955	.965	2.14	9.5%
1983	.331	.580	.703	.789	.851	.895	.923	.941	.955	.965	2.18	10.5%
1984	.329	.574	.703	.789	.851	.895	.923	.941	.955	.965	2.19	10.7%
1985	.335	.574	.703	.789	.851	.895	.923	.941	.955	.965	2.18	8.7%

TOTAL ALL LINES

AGE

Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1977	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	X.XX	X.X%
1978	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	X.XX	X.X%
1979	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	X.XX	X.X%
1980	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	.XXX	X.XX	X.X%
1981	.441	.688	.785	.845	.888	.922	.944	.958	.968	.976	1.66	9.9%
1982	.433	.687	.783	.841	.888	.922	.944	.958	.968	.976	1.67	7.5%
1983	.412	.677	.775	.841	.888	.922	.944	.958	.968	.976	1.71	8.4%
1984	.411	.668	.775	.841	.888	.922	.944	.958	.968	.976	1.72	8.5%
1985	.412	.668	.775	.841	.888	.922	.944	.958	.968	.976	1.72	6.9%

The total loss and LAE reserve as of 12/1985 is the sum of this flow. This sum can be represented as follows:

$$\text{LOSS RESERVE}_{12/t} \quad = \quad \sum_{i=(<t-10)}^{t} \quad \sum_{j=t+1}^{\infty} \quad {}_{i}pd_{j} \qquad (48)$$

The reserve flow is the sum of losses paid each year in the future on the reserves. It represents each column in Table XI. This flow is represented as follows:

$$\text{LOSS RESERVE FLOW}_{12/t;i} \quad = \quad \sum_{j=(<t-10)}^{t} \quad {}_{j}pd_{t+i} \ / \ \text{LOSS RESERVE}_{12/t} \qquad (49)$$

Some of the losses expected in these formulae are paid more than ten years after they occur. In addition, the reserves include losses paid more than ten years ago. In all these cases we use the MCL factor approach to project losses using the computer.

The amount we expect to pay during year j on losses occurring during year i where (10<j-i) is represented as follows:

$$\begin{aligned} {}_{i}pd_{j} \quad &= \quad \text{ULTIMATE LOSSES PAID}_{i} \qquad (50) \\ &\times \quad (1 \ - \ \text{CUMULATIVE LOSS PAID}_{10}) \times \text{MCL} \times (1 - \text{MCL})^{j-i-1} \end{aligned}$$

We estimate the losses paid on accident year 1985 general liability losses during 1996, for example, as follows:

$$\text{ULTIMATE LOSSES PAID}_{1985} \quad = \quad \$10,623 \text{ million}$$

$$(1 \ - \ \text{CUMULATIVE LOSS PAID}_{10}) \quad = \quad 1 \ - \ .873 \ = \ .127$$

Thus:

$$\begin{aligned} {}_{1985}pd_{1996} \quad &= \quad \$10,623 \ \times \ .127 \ \times \ .2 \ \times \ (.80)^{1996-1985-1} \\ &= \quad \$270 \text{ million} \end{aligned}$$

This procedure can be repeated using the computer for all desired values of i and j.

The procedure for losses occurring more than ten years before the evaluation date is essentially the same for loss reserve data as it is for accident year data. The estimate of the ultimate losses paid, however, is the estimate of losses yet to be paid for all accident years more than ten years prior. A nice quality of the exponential assumption is that the payout pattern for the sum of several accident years is the same as it is for one accident year.

A convenient way of presenting reserve flows is provided by Part 2 of Schedule P. Instead of stopping at the current year as Schedule P does, we project out all accident years for n years beyond the current year. Flows *occurring more than 10 years after they occur are calculated using formula* (50). The $_{t}pd_{j}$ values calculated using the methods just described are cumulated to form the Schedule P, Part 2 matrix.

This technique has been used to construct Table XII which shows the payment schedule for all industry loss reserves outstanding at the end of 1983 - 1985. The dollars of loss payments expected for each of the past ten years plus all other years are shown on a cumulative basis.

TABLE XII

TOTAL ALL LINES

CUMULATIVE LOSS PAYMENTS

	PAYMENT YEAR												
ACC YR	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1993	1995	ULT
<1983	564	584	597	609	617	623	627	630	632	634	635	636	639
1983	39	64	73	79	84	87	89	90	91	92	93	93	94
<1984	602	647	670	688	701	710	716	720	723	725	727	729	733
1984		44	71	82	89	94	98	100	102	103	104	104	106
<1985		691	741	771	790	804	814	820	825	828	831	833	839
1985			47	77	89	97	102	106	109	110	112	112	115
<1986			789	847	879	901	916	926	934	939	942	945	954

The reserve for any accident year or group of accident years as of any payment year is the amount shown for that payment year subtracted from the amount shown in the ULT column. The reserve for the end of 1985, for instance, is ($165 = $954 - $789).

The differences between each column represent the payments made in each year on outstanding reserves. These differences are shown in Table XIII.

TABLE XIII
TOTAL ALL LINES
RESERVE LOSS PAYMENTS

	PAYMENT YEAR										
ACC YR	1983	1984	1985	1986	1987	1988	1989	1990	1991	1992	1994	ULT
<1983	20	14	12	8	6	4	3	2	2	1	1	3
1983	25	9	4	6	3	2	1	1	1	1	0	1
<1984	45	23	18	13	9	6	4	3	2	2	1	4
1984		27	11	7	5	4	2	1	1	1	1	2
<1985		51	29	20	14	10	7	5	4	3	2	6
1985			30	12	8	5	4	3	2	1	1	3
<1986			59	32	21	15	11	7	5	4	3	9

Table XIII informs us that the amount paid during 1983 on losses occurring before 1983 was $20 billion. It tells us that the amount that is expected to be paid during 1986 on losses occurring before 1985 is $20 billion. We can use this information to discount loss reserves.

We transform this information into a payment pattern by dividing the summation lines by the outstanding reserve. The percent of 1985 outstanding losses, for example, that will be paid in 1986 is (.355 = 59/165). This calculation has been made for all the Schedule P and O lines and is shown in Exhibit IX. Exhibit IX shows the payment pattern expected from loss reserves outstanding at the end of 1983 - 1985 for each of the lines we have been using.

It can be seen from Exhibits VIII and IX that the average payment dates and the expected interest receivable from losses occurring in 1985 and from reserves outstanding at the end of 1985 are as follows:

TABLE XIV
LOSSES INCURRED AND LOSS RESERVE FLOWS
1985 AVERAGE PAYMENT DATES AND INVESTMENT CREDIT

	LOSSES INCURRED		LOSS RESERVES	
Line	Average Payment Date	Investment Credit	Average Payment Date	Investment Credit
Schedule O	0.71	3.0%	1.11	4.6%
Automobile Liability	1.43	6.0%	1.26	5.3%
General Liability	5.00	18.9%	3.99	15.2%
Medical Malpractice	6.36	23.7%	4.57	17.3%
Workers' Comp.	2.74	10.5%	3.64	13.4%
Multi-Peril	1.04	4.3%	1.62	6.7%
Total Sched. P Lines	2.18	8.7%	2.58	10.6%
Total All Lines	1.72	6.9%	2.33	9.2%

EXHIBIT IX
Page 1

SCHEDULE O LINES

AGE

Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1983	.589	.701	.801	.873	.922	.955	.976	.988	.994	.997	1.21	6.1%
1984	.550	.724	.812	.878	.925	.956	.977	.988	.994	.997	1.20	6.2%
1985	.593	.747	.826	.886	.929	.959	.978	.989	.994	.997	1.11	4.6%

AUTOMOBILE LIABILITY

AGE

Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1983	.447	.686	.824	.901	.944	.967	.980	.988	.993	.996	1.28	6.6%
1984	.454	.692	.828	.903	.945	.967	.980	.988	.993	.996	1.26	6.6%
1985	.452	.691	.827	.903	.945	.967	.980	.988	.993	.996	1.26	5.3%

GENERAL LIABILITY

AGE

Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1983	.161	.314	.460	.579	.671	.741	.794	.836	.869	.895	4.10	18.7%
1984	.164	.327	.470	.587	.678	.746	.798	.839	.872	.897	4.03	18.7%
1985	.173	.334	.476	.592	.681	.749	.800	.841	.873	.898	3.99	15.2%

MEDICAL MALPRACTICE

AGE

Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1983	.106	.276	.403	.519	.619	.699	.765	.819	.859	.888	4.50	20.5%
1984	.151	.276	.403	.519	.619	.699	.765	.819	.860	.888	4.45	20.5%
1985	.123	.255	.387	.507	.609	.692	.760	.815	.856	.885	4.57	17.3%

EXHIBIT IX
Page 2

WORKERS' COMPENSATION

	AGE											
Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1983	.272	.439	.562	.650	.715	.766	.806	.838	.864	.886	3.79	16.7%
1984	.279	.457	.577	.663	.726	.775	.814	.845	.870	.890	3.67	16.4%
1985	.285	.462	.581	.666	.729	.777	.815	.846	.871	.891	3.64	13.4%

MULTIPLE PERIL

	AGE											
Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1983	.443	.638	.759	.842	.898	.936	.961	.976	.986	.992	1.58	8.0%
1984	.445	.628	.753	.838	.896	.934	.960	.976	.986	.991	1.61	8.2%
1985	.439	.624	.750	.836	.895	.933	.959	.976	.985	.991	1.62	6.7%

TOTAL SCHEDULE P LINES

	AGE											
Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1983	.302	.491	.632	.732	.802	.851	.886	.913	.933	.949	2.67	12.7%
1984	.307	.506	.643	.741	.809	.856	.889	.915	.935	.951	2.61	12,5%
1985	.314	.511	.647	.743	.811	.857	.891	.916	.936	.951	2.58	10.6%

TOTAL ALL LINES

	AGE											
Year	1	2	3	4	5	6	7	8	9	10	APD	INV
1983	.344	.521	.658	.754	.821	.867	.900	.925	.943	.958	2.43	11.7%
1984	.341	.537	.670	.765	.828	.872	.904	.927	.945	.959	2.38	11.6%
1985	.355	.547	.677	.768	.831	.875	.906	.929	.946	.960	2.33	9.2%

Table XIV demonstrates that there are significant differences between the reserve payment patterns and the incurred loss payment patterns. The average payment date on Schedule O loss reserves, for example, is about 1.10 years. Schedule O incurred losses, however, have an average payment date of only .7 years. Workers' Compensation Incurred Loss average payment dates are about 2.75 years whereas the reserves have an average payment date of about 3.6 years. This makes sense when we recognize that the Worker's Comp. line is a little like a barbell. A large proportion of losses are paid early in the form of medical bills. A large part of the remainder are paid late in the form of annuities. The early payments are not fully reflected in reserves and so the annuity payments get disproportionate weight.

Over all lines, the average payment date for reserves is roughly a half year longer than it is for incurred losses.

The data shown in Table XIII can be used to develop the investment receivables for loss reserves as shown in Exhibit IV. We wish to determine investment receivables separately each year for the current accident year and for all prior years. The first row in Table XIII shows the development from the end of 1983 onwards for all claims occurring prior to 1984. It represents the payment pattern for those losses. The second row of the exhibit shows the development of the reserves on accident year 1984 from the end of the year onwards. The third row shows the development of 1984 year end reserves. Dropping the first term in this row gives us the development of claims occurring before 1984 as of the end of 1984. Thus it gives us the development of prior year reserves for 1984. Table XIV shows the data rearranged and normalized for discounting purposes. Present Value factors are also shown. This demonstrates the calculation of present values used in Exhibit IV.

TABLE XV

CALCULATION OF INVESTMMENT RECEIVABLE FACTORS FOR INDUSTRY LOSS DATA

ALL LINES

	<1983		1983		<1984		1984		<1985		1985	
Age	AV	PV	AV	PV	AV	PV	AV	PV	AV	PV	AV	PV
1	0.27	0.26	0.45	0.44	0.27	0.26	0.44	0.42	0.30	0.29	0.44	0.43
2	0.19	0.17	0.17	0.15	0.21	0.19	0.18	0.17	0.20	0.19	0.18	0.17
3	0.15	0.13	0.11	0.10	0.15	0.13	0.11	0.10	0.14	0.13	0.11	0.10
4	0.11	0.09	0.08	0.07	0.10	0.08	0.08	0.07	0.10	0.08	0.08	0.07
5	0.07	0.06	0.06	0.04	0.07	0.05	0.06	0.04	0.07	0.06	0.06	0.05
6	0.05	0.05	0.04	0.03	0.05	0.04	0.04	0.03	0.05	0.04	0.04	0.03
7	0.04	0.03	0.02	0.02	0.04	0.03	0.02	0.02	0.04	0.03	0.02	0.02
8	0.03	0.02	0.02	0.01	0.03	0.02	0.02	0.01	0.03	0.02	0.02	0.01
9	0.02	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.02	0.01	0.01	0.01
10	0.02	0.01	0.01	0.01	0.02	0.01	0.01	0.01	0.02	0.01	0.01	0.01
AD	0.05	0.02	0.03	0.01	0.05	0.02	0.03	0.01	0.05	0.03	0.03	0.02
SUM	1.00	0.84	1.00	0.88	1.00	0.84	1.00	0.88	1.00	0.88	1.00	0.90
PRF		0.16		0.12		0.16		0.12		0.12		0.10

The present value (PV) column is the actual value (AV) column times the discount factor appropriate for the year. The all-other row (AO) has an average payment date of approximately 13.5 years. The present-value factor (PRF) is the sum of the actual value column less the sum of the present value column.

In formal terms, Table XII can be described as follows: The first row of the table is composed of the following entries:

$$\text{pdsum}_{<1983;i} = \sum_{j=(<1974)}^{1983} {}_{j}\text{pd}_{1982+i}$$

Where i is the age.

The next row of the table is the paid development of accident year 1983. The first entry is the losses paid on accident year 1983 during 1983. The second is losses paid on accident year 1983 during 1984, etc. Thus we have:

$$\text{Entry}_{i} = {}_{1983}\text{pd}_{i}$$

The next row is the first column plus the second. The next row is accident year 1984 and the process is repeated.

Thus, in summary, the procedure for discounting reserves in the EVA approach is to determine the payment patterns for each accident year using the techniques described in the preceding section. Once this is done, they are arranged in the form shown in Tables XII and XIII. The next step is to take this data and arrange it in the form shown in Table XV. The reserve discount factor is derived as in Table XV.

The other step necessary to produce the results shown in Exhibit IV is to calculate the interest earned during the year on EVA reserve funds. An intuitive method is to take the average reserves available during the year by averaging the EVA reserves outstanding at the beginning and the end of the year. This is similar to formulas used earlier in this paper. This was the procedure used in the Exhibit. Thus the formula for interest earned during the year on loss reserves is:

$$\text{INTEREST EARNED DURING YEAR} = i \times \frac{\text{EVA RESERVE}_{beg} + \text{EVA RESERVE}_{end}}{2} \quad (51)$$

Where i is the interest rate. It is equal to the prior year end interest rate for prior accident years and is equal to the current year average interest rate for the current accident year.

ALLOCATING EXPECTED INVESTMENT INCOME TO SUBLINES

Schedule P gives us information about the development of losses on a restricted set of lines. It does not provide direct information about such lines as homeowners and personal auto. We need a way to estimate investment income on these lines.

Formula (45) shows that the average payment date for an exponential cash flow is equal to -1/ln(1 - MCL). Shown below is the formula for the total reseves outstanding at a given time for a company with no growth and with an exponential loss payment pattern:

$$\text{TOTAL\_RESERVES\_OUTSTANDING} = \text{CURRENT\_YEAR\_LOSS} \times \int_{0}^{\infty} (1 - \text{MCL})^{t}\, dt$$

$$= \text{CURRENT\_YEAR\_LOSS} \times -1/\ln(1 - \text{MCL}) \qquad (52)$$

Thus the ratio of reserves outstanding to current year losses is:

$$\text{RESERVE\_OUTSTANDING\_RATIO} = -1/\ln(1 - \text{MCL})$$

This is the same as the average payment date.

This identity is a special case of the more general identity between the average payment date of any cash flow and the ratio of total reserves outstanding to accident year incurred loss. It can be proven that the two are always the same if there is no change in losses or payment patterns over time. That is:

$$\text{APD} = \text{RESERVE\_OUTSTANDING\_RATIO}$$

If steady state conditions apply.

What is more interesting is to examine the difference between the reserve outstanding ratio and the average payment date if payment patterns are held constant but there is premium growth. This is the normal state of affairs.

C. McClenahan proposed a simple exponential model for analyzing loss incurred and paid patterns in a paper appearing in the 1975 CAS Proceedings.[14] The model in that paper states that losses occurring m years prior to the current (or base) year are paid off at a constant rate once they are reported. He also assumes that there is a some average delay, d, before losses are reported. The rate at which losses are paid off is affected by inflation which is assumed to be constant.

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[14] C, McClenahan, Op. Cit.

McClenahan's model is stated in formula (1) of his paper and has been restated here in simpler terms.

$$ {}_{n}pd_{m} = c\ v_{a}^{m}\ p\ (1-p)^{n-d} \qquad (53) $$

Where

${}_{n}pd_{m}$ = Payment at time m+n on losses occurring at time m

c = Constant representing base month losses plus effect of inflation during time losses are unreported.

p = Rate at which losses are paid as a percent of amount remaing to be paid.

m = Number of months before base year that losses occurred.

n = Number of months elapsed since losses occurred

d = Average delay before losses are reported.

v_a = Growth rate of losses due to exposures, increases in severity, and changes in frequency expressed as a discount rate. That is, $v_o = (1+g)^{-1}$ where g is the growth rate.

The values of c,p,q are different than in the McClenahan formulation. Please refer to Appendix I for details on the derivation used in this paper.

The McClenahan formula is basically a geometric density function describing the probability that a loss occurring at time m will be paid at time m+n. It is much simpler to work with the continuous analog of the geometric density function, the exponential density function. This can be described simply as:

$$ g(t) = \lambda \exp^{-\lambda t} $$

We substitute (-ln(1-MCL)) as used earlier for λ, giving:

$$ g(t) = (-\ln(1-MCL))(1-MCL)^{t} $$

The continuous analog of the McClenahan model becomes:

$$ {}_{t}pd_{m} = \begin{cases} c\ v_{a}^{m}\ (-\ln(1-MCL))\ (1-MCL)^{t} & , \ t > d \\ 0 & , \ \text{otherwise} \end{cases} $$

The losses occurring at time m which are still unpaid at time m+t are represented as follows:

$$ {}_{t}\text{OS LOSSES}_{m} = \begin{cases} c\ v_{a}^{m} & , \ t > d \\ c\ v_{a}^{m}\ (1-MCL)^{t-d} & , \ \text{otherwise} \end{cases} $$

No losses are paid until they are reported. After that, they are paid according to the continuous payment function. Thus we can describe all outstanding losses at time 0, as follows:

$$\text{OS LOSSES} = c \int_0^{\infty} {}_0\text{OS\_LOSSES}_m \, dm$$

$$= c \left\{ \int_0^d v^m \, dt + \int_d^{\infty} v \;(1-\text{MCL})^{m-d} \, dt \right\}$$

The ratio of losses outstanding to current losses incurred is:

$$\text{OS RATIO} = \int_0^d v^m \, dt + v^d \int_d^{\infty} v^{m-d} (1-\text{MCL})^{m-d} \, dt$$

$$= \int_0^d v^m \, dt + v^d \int_0^{\infty} v^m \;(1-\text{MCL})^m \, dt$$

$$= (v^d-1) \ln^{-1}(v) + \{ \ln(v) + \ln(1-\text{MCL}) \}^{-1}$$

The first term represents the losses which have not yet been reported while the second represents losses which have been reported but are still outstanding.

Since $\ln(1-\text{MCL}) = -\text{APD}_d^{-1}$ where APD_d represents the average payment date for a loss payment pattern as of the date the losses are reported, we substitute APD_d in the second term as follows:

$$\text{OS\_RATIO} = (v^d-1) \ln^{-1}(v) + \{ \text{APD}_d^{-1} - \ln(v) \}^{-1}$$

Solving for APD_d^{-1}, we have:

$$\text{APD}_d^{-1} = \ln(v) + (\text{OS\_RATIO} + (1-v^d)/\ln(v))^{-1}$$

Since the average payment date with respect to the time of loss is $\text{APD}_d + d$, we solve for the average payment date as follows:

$$\text{APD} = d + \{ \ln(v) + (\text{OS\_RATIO} + (1-v^d)/\ln(v))^{-1} \}^{-1} \qquad (54)$$

When d=0, this simplifies to:

$$\text{APD} = (\ln(v) + \text{OS\_RATIO}^{-1})^{-1} \qquad (55)$$

Table XVI shows the results obtained using formula (55) on industry Schedule P data. The report delay, d, is assumed to be zero for all lines other than General Liability and Medical Malpractice. Values of .75 and 2.0 were used for these two lines respectively.

TABLE XVI

ESTIMATES OF AVERAGE PAYMENT DATES USING RESERVE RATIOS

SCHEDULE O AND P LINES

Line	Avg 8 Year Growth	Avg Reserve Ratio	Estimated Avg. APD	APD From Cash Flow
Schedule O	10.8%	0.69	0.71	0.71
Automobile Liability	11.3%	1.28	1.48	1.43
General Liability	15.8%	3.33	4.97	5.00
Medical Malpractice	25.5%	3.50	6.46	6.36
Workers' Compensation	10.7%	2.09	2.66	2.74
Multiple Peril	14.2%	0.92	1.04	1.04
Schedule P Lines	13.0%	1.73	2.18	2.18
Total All Lines	12.3%	1.44	1.68	1.72

These two estimates of the average payment dates are quite close, as shown in Table XVI. This closeness exists even though the actual payout patterns are not strictly exponential and even though actual growth varies over time. We conclude that insurance payment patterns and growth rates for the industry behave in a way that is reasonably similar to what is expected by the McClenahan model. This similarity allows formula (54) to give reasonably accurate estimates of average payment dates.

The advantage of using reserve ratios to estimate average payment dates is that far more information about reserves is available than information about loss payment patterns. Table XVII shows estimates of average payment dates and investment income expected for a more useful set of line definitions:

The data for Table XVII comes from pages 9, 10, and 14 of the Annual Statement and the Insurance Expense Exhibit. Reserves have been adjusted for adequacy using Schedules O and P.

The loss average payment dates have been used to estimate the expected investment income percentages shown in Table XVII. The approach is similar to that used in formula (55). We model the discount process as follows:

$$\frac{\text{INCURRED\_LOSS}_{PV}}{\text{INCURRED\_LOSS}} = \text{FORCE OF PAYMENT} \times \int_0^\infty v^t \, (1 - \text{MCL})^t \, dt$$

$$= \text{APD}^{-1} \, / \, (\text{APD}^{-1} - \ln(v))$$

Where $v = (1 + i)^{-1}$

The expected investment percent is one minus this amount:

$$\overset{*}{\text{INV\_INCOME\_PCT}} = -\ln(v) \;/\; (\text{APD}^{-1} - \ln(v)) \qquad (57)$$

$$= -\ln(v) \;/\; \text{RSR}^{-1} - \ln(g) - \ln(v) \qquad (58)$$

Where RSR = Ratio of Reserves to Incurred Losses.

TABLE XVII

ESTIMATES OF AVERAGE PAYMENT DATES USING RESERVE RATIOS

ALTERNATE LINE DEFINITIONS

Line	Avg Growth	Reserve Ratio	APD Estimated	Investment Income
Personal Lines				
Personal Auto	10.5%	.79	.89	3.9%
Homeowners	12.5%	.47	.55	2.4%
Total Personal	10.9%	.72	.82	3.6%
Commercial Property				
Fire and Allied	4.6%	.78	.81	3.6%
Commercial MP	16.5%	1.23	1.60	6.8%
Marine	12.2%	.09	1.27	5.5%
Total	11.7%	1.07	1.28	5.5%
Commercial Casualty				
Commercial Auto	11.1%	1.13	1.32	5.7%
GL (incl. MM)	18.7%	3.48	5.48	20.0%
Workers' Comp.	10.3%	2.09	2.64	10.7%
Other Casualty	10.2%	1.80	2.07	8.6%
Total	13.4%	2.45	3.56	14.0%
Total Commercial	12.7%	1.92	2.57	10.5%
Other	10.0%	1.46	1.79	7.5%
Total All Lines	11.7%	1.35	1.67	7.1%

An even simpler formula produces similar results using actual discount and growth rates:

$$\text{INV\_INCOME\_PCT} = (1 - v) \;/\; \text{RSR}^{-1} - g + 1 - v \qquad (59)$$

This equation uses the fact that g and (1 - v) are approximately equal to ln(1 + g) and -ln(v) respectively for small values of g and values of v close to one.

These formulas are particularly useful for estimating investment income by line and state where no payment pattern data exists or is likely to exist. At the same time, there are definite limits to the data which can be used in this way. If there are sharp discontinuities in loss growth the results will be biased. Loss growth must be known for several years, seven or eight, if possible.

There must be some confidence in the adequacy and degree of stability in the loss reserves. Reserves that are not accurate will produce inaccurate estimates of expected investment income. Loss payment patterns must also be stable. Data presented earlier illustrates the degree of payment pattern stability that has existed at the industry level. Similar degrees of stability in payment patterns exist for large individual companies, such as the top fifty, where there is still enough volume to produce reasonabe stability. As the size of the losses and loss reserves decrease, these methods may become subject to too much fluctuation. It is possible to use average reserve ratios for several years in some cases, however, to overcome volatility in payment patterns.

Finally, there must be confidence that there have not been fundamental changes in the underlying payment pattern. It would not be appropriate, for example, to use these formulas on business that has changed from occurrence based to claims made liability. Claims made loss payment patterns are distinctly different than occurrence loss payment patterns.

It is not appropriate to use these formulas to discount loss reserves. We have shown earlier that there are significant differences in the investment income which can be expected from incurred losses and from loss reserves.

Methods exist for estimating incurred loss payment patterns in those cases where the actual payment patterns are not available. The payment patterns thus estimated can then be built up in the Schedule P, Part 2 matrix as shown earlier and can then be used to discount reserves. A complete treatment of how to do this, however, is presently beyond the scope of this paper. We have, however, created estimated payment patterns for the set of lines shown in Table XVII using interpolation techniques. These patterns agree reasonably well with internal data available from a prior employer.

IV. INVESTMENT INCOME

Liabilities do not generate investment income. Assets do. Up to this point we have concentrated on insurance funds flows so that we could learn what opportunities exist for generating investment income. The amount of investment income that is generated depend on both that opportunity and the type of assets that are put to work.

We stated earlier that investment income that can be earned from investing risk free assets is relevant to the evaluation of insurance profitability. The market risk premium for investment risk has little relevance to the profit earned from insurance operations.

It is to be expected that a company that takes investment risk will earn a higher return from its investments than one which does not. One might argue that underwriting profits for companies that take investment risk should be lower than for companies that do not. This is what happens when actual investment returns are used to evaluate underwriting results. This implies, however, that the company should take risk but not be rewarded for it. Underwriting operations should not be evaluated on a basis of risky investment returns.

It is quite true that an investor should evaluate the total operation of an insurance company since his reward is the result of both investment and insurance risk. He would be well advised, however, to evaluate both the insurance and investment operations separately. Policyholders, by the very nature of insurance, are interested in eliminating risk and would generally not want to see the price they paid for insurance depend on the investment risk taken by the insurer. They do not want to participate in that risk. Regulators need to be concerned about the total operation of the insurance company when evaluating solvency but will learn more by looking at the underwriting and investment operations separately. When examining the adequacy of rates, however, it is appropriate to look only at the insurance operation rather than the banking operation.

These examples show that any appropriate analysis of insurance companies by any of its stakeholders should always focus on underwriting operations and should sometimes focus on investment operations as well. There should always, however, be separate recognition of both insurance and investment operations.

The way to meet this need is to demonstrate the insurance profit that would be earned if the company invested exclusively in risk free assets. This provides an appropriate evaluation of the performance of the insurance operation. Then the difference between the investment income actually earned and that which has been earned at risk free rates should be presented. This spread represents the realization of the risk taken in the banking function. It will be far more volatile than the investment return available from risk free investments. This volatility is the accompaniment of the reward for taking risk.

The risk free rate should, of course, be as free of risk as possible. This means it should be free of market risk, interest rate risk, and default risk. While it is probably true that no real world asset is completely riskless, an immunized portfolio of U.S. securities is reasonably risk free in all three aspects. They are as free from default as any asset can reasonably be. They have effectively zero market risk. Immunization makes them virtually free of interest rate risk.

IMMUNIZATION

There is a large and growing literature on duration and immunization. This reflects both the usefulness and complexity of the subject. In essence, however, immunization seeks to create portfolios of assets and liabilities which will have the same changes in value when interest rates change. Table VI and its attendant discussion demonstrated how the surplus of an insurance company with a simple immunized portfolio was unaffected by changes in interest rates. Exhibit III shows how surplus is affected by changes in interest rates when it is not immunized.

Many of the complexities that are addressed in the literature deal with the problem of staying immunized. It is usually possible to stay immunized over time if the asset and liability portfolios have matching cash flows. As long as they are matched, all changes in interest rates will affect the values of both equally. When durations are matched but there are substantial cash flow mismatches, a single change in interest rates will be immunized. This change in interest rates, however, will cause a different change in the duration of the assets than in the duration of liabilities. Thus future changes in interest rates will not be immunized unless the portfolio is rebalanced. It is necessary to rebalance the portfolio by appropriate sales or purchases to keep durations matched and to remain immunized.

Property/Casualty insurance companies have minimal rebalancing problems. The steadiness of loss payment patterns demonstrated earlier is accompanied by a reasonably stable duration. Table XVIII shows how interest rate changes over the past nine years would have effected the 1985 industry total lines reserve payment pattern. The interest rates used are after-tax year-end two year Treasury Bond rates.

TABLE XVIII

INSURANCE INDUSTRY: ALL LINES

LOSS RESERVE DURATION CHANGES DUE TO CHANGES IN INTEREST RATES

Year	1977	1978	1979	1980	1981	1982	1983	1984	1985
Int. Rate	3.9%	5.0%	6.2%	7.2%	7.5%	5.3%	5.9%	6.0%	4.7%
Duration	2.00	1.91	1.84	1.78	1.76	1.89	1.86	1.85	1.94

As a going concern, insurance companies need to continually rebalance by making purchases of assets which will keep the asset portfolio in balance with the liability portfolio. As the biggest liability, this means staying in balance with the loss reserve portfolio. Table XVIII shows that year end Treasury rates almost doubled between 1977 and 1981, but the duration of industry loss reserves decreased by about 12.5%. Thus the duration of the industry cash flows has not been heavily affected by some of the largest interest rate changes that have taken place in recent history. The all lines cash flows for medium to large companies is just about as steady as it is for the industry. Thus individual insurance groups have about the same relative immunity to duration changes as the industry.

The perspective of immunization is from the total company portfolio of assets and liabilities. A company is protected from interest rate risk when its total portfolio is protected. It does not matter to the company whether it is immune because a drop in interest rates creates a profit from long tail lines balanced by a loss from short tail lines or whether an increase in interest rates has the opposite effect.

In any event, the range of durations by line is surprisingly limited. Table XIX shows the durations by line of industry Schedule P cash flows for the last three years. The underlying cash flows are those used to construct Exhibit IX.

TABLE XIX

INSURANCE INDUSTRY: SCHEDULE P AND O LINES

LOSS RESERVE DURATION CHANGES DUE TO CHANGES IN INTEREST RATES

Year	Sched O	Auto Liability	General Liability	Medical Malpractice	Workers' Comp	Multi Peril	Sched P	Total
1983	1.02	1.12	3.23	3.61	2.72	1.35	2.13	1.95
1984	1.02	1.10	3.17	3.54	2.61	1.36	2.07	1.90
1985	0.96	1.14	3.27	3.83	2.76	1.43	2.14	1.94

In this paper we have used two year Treasury yields as a proxy for the yield that would be earned from an immunized industry portfolio of Treasury Securities. Financial theory suggests that the rate that is promised by an immunization strategy is the average yield on pure discount bonds with maturities equal to the duration of the liability portfolio[15]. Thus yields on two year zero coupon Treasuries might be slightly more appropriate.

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15. See, for instance, Platt, et. al, Controlling Interest Rate Risk, John Wiley and Sons, 1986.

THE EVA APPROACH (CONT.)

Changes in the market values of risk free assets are an integral part of the determination of insurance profits as defined in this paper. Exhibit IV shows gains and losses on changes in the value of investment income expected from assets underlying loss reserves. We see, for example, that the change in investment income expected from prior year reserves was only -$82 million in 1984 but was $-2,333 million in 1985. The change in 1985 came about because the discounted value of the liabilities decreased significantly when interest rates dropped during 1985. The change in EVA liabilities had little effect in 1984 but generated a significant loss in 1985. These changes, however, are offset by equivalent changes in the market value of the assets funding these liabilities.

Table XVII shows the Treasury Bill interest rates used in this paper. The year end rates were used on reserve funds and the average rates were used for the paid flows in Exhibit IV and the incurred loss flows in Exhibit VIII.

TABLE XX

AFTER TAX TREASURY BILL RATES

Year	Average Rates	Year End Rates
1977	3.5%	3.9%
1978	4.5%	5.0%
1979	5.5%	6.2%
1980	6.4%	7.2%
1981	7.9%	7.5%
1982	6.9%	5.3%
1983	5.4%	5.9%
1984	6.2%	6.0%
1985	5.1%	4.7%

Exhibit IV, Page 4 is a reconciliation of insurance cash flows for the industry for 1984 and 1985. It can be seen that change in the market value of the assets supporting insurance funds is the sum of the following items:

ΔINSURANCE FUNDS = NEW FUNDS REQUIRED (60)
+ INSURANCE CASH FLOW
+ INTEREST DIVIDENDS AND RENT EARNED DURING YEAR
+ CAPITAL GAINS (LOSSES) ON INSURANCE ASSETS

The amount of new funds required is equal to the sum of the EVA values of Premiums, Expenses, Current Year Losses, and changes in older years losses. We see from Exhibit IV, Page 3, that the industry incurred nominal losses of $3.4 and $8.9 billion respectively in 1984 and 1985 on prior years losses. These losses are expected to be paid over time just as other losses are and so an investment receivable is shown as an offset to the losses incurred. The sum of the incurred loss and the associated receivable represents the net new funding required. Thus the net amount required to fund losses from prior years was $3.2 and $8.3 billion for 1984 and 1985 respectively.

It can be seen from Exhibit IV, Page 4, for instance, in 1985 the net amount of new assets needed to fund the EVA losses and expenses on a pre-tax basis exceeded premiums by $17.7 billion. This is the insurance profit before taxes[16]. The insurance profit in 1985 before capital gains is a loss of $20.0 billion but this reflects the effect of new funding required because of the change in interest rates from 6.0% to 4.7% during the year. This change increased loss reserves by $2.3 billion and EVA incurred losses before taxes by the same amount. At the same time, however, the market value of the insurance assets also increased as a result of the interest rate change. The change in the value of the assets was equal to the value of the liabilities because they both had the same duration. The change in the value of the assets would have been different from the change in liabilities hao they had different durations. This is usually the case with real world portfolios which reflect the strategy adopted by the investment operation to gain a profit on the spread between the risk-free rate and the rate of return they actually receive.

The insurance cash flow is the sum of collected premiums and paid losses and expenses. Paid losses and expenses exceeded collected premiums by $7.9 billion in 1984. Collected premiums exceeded paid losses and expenses by $3.3 billion in 1985. Thus there was negative cash flow from insurance operations in 1984 and positive cash flow in 1985.

The actual interest earned during each year was $7.5 billion and $8.3 billion respectively. This plus capital gains or losses represents the change in funds attributable to the time value of insurance flows at risk free rates. The net investment income adds to this the effect of changes in investment receivables.

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16. Since we have used after-tax discount rates this is not strictly true, but we have found that the impact of federal taxes on interest earned before taxes is effectively the same as using after-tax discount rates. In addition, the approximation formulas used in the loss section are more accurate when after-tax discount rates are used than when pre-tax discount rates are used and then taxes are backed out. Thus the before-tax profit is before underwriting taxes. Investment taxes are passed through.

APPRAISING INVESTMENT PERFORMANCE

The next step is to develop the investment income actually earned after taxes by the company on insurance funds. Ideally, this should be that portion of the interest, dividends and rent earned by the company on insurance funds; plus all capital gains. This should include changes in market value on bonds.

There are two possible approaches to looking at actual portfolio results. If one adopts the idea that insurance funds are backed by fixed income assets, as suggested by the rule of thumb cited earlier, than the return from the bond portfolio is the best portfolio to use to evaluate investment performance. Otherwise the overall portfolio return can be used instead.

Techniques can be worked out for estimating the market value of bonds, but are beyond the present scope of this paper. Thus it is not really useful to compare the figures shown in Exhibit IV with actual industry investment results. It can be stated that the nominal after-tax return on bonds during 1985, for example, was about 6.5%[17]. This is clearly superior to the 5.1% earned from Treasuries during the year. If, however, bonds were stated at market values this return would almost certainly be lower. Thus it can be assumed that actual investment performance was probably higher than the risk-free rate but we do not know how much higher.

It is equally certain that capital gains and losses on the actual portfolio were much larger than the immunized risk-free portfolio, since the duration of the actual portfolio has been about seven to eight years[18]. The liability portfolio, as we have seen, has a duration of about two years. Thus the capital gain on the actual bond portfolio during 1985 was almost certainly much larger than that for the risk-free portfolio.

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[17] This is the after-tax interest on bonds divided by the average bonds held during the year less 0.3% for investment expenses. See A.M. Best's Aggregates and Averages, 1986.

[18] We did analyses during 1984 which showed that industry bond portfolio durations were in that range from 1977 through 1983. We have not had the time to update this analysis through 1985.

When the rate of return on the market value of the actual portfolio is available, it is appropriate to compare the interest earned and the capital gains on losses from the actual portfolio to the interest earned and capital gains on the risk-free portfolio during the year. We stated earlier that the interest receivable item on the EVA balance sheet represented *expected* invested income from future years. A non-immunized non-default-free portfolio has much less certainty about its expected returns than the Treasury Portfolio. The problem of discounting on the basis of actual portfolios is highlighted when a portfolio including stocks is used. Common stock returns are a mixture of dividends and capital gains. Discounting on the basis of dividend yields is clearly inappropriate, but discounting on the basis of dividends plus capital gains is absurd.

Thus we would recommend that the contribution of the investment operation be measured when it is earned rather than on a present value basis. It is true, however, that this shifts the recognition of the financial intermediary profit from the year investments are made to the time they are realized.

Thus we recommend that a full recognition of the profit earned by an insurance company due to its being in the insurance business should be made by measuring the insurance profit on a risk-free basis as shown in Exhibit IV, Page 3, and then adding the difference between the risk-free investment income and the actual interest plus gains realized during the year.

The remaining investment income is attributable to surplus.

V. FEDERAL INCOME TAXES

The Federal Income Tax Reform Act of 1986 has made significant differences in the timing of Property/Casualty insurance company federal income taxes. The biggest differences are the 20% tax on the change in unearned premiums and the fact that taxes will now be determined on the basis of discounted reserves.

Earlier we discussed the fact that the tax on unearned premiums was a way to tax prepaid expenses. The figures put together in this paper indicate that the 20% number might not be appropriate for every company. We are treating the federal taxation of expenses as if it were being done on an appropriate basis. Thus no recognition is given to any timing mismatch on the taxation of expenses.

Taxation of losses is handled on an EVA Incurred Loss basis. By doing so, we are again making the presumption that taxes will be assessed on an appropriate basis. We will examine the Federal approach by starting with an excellent description by Snader:[19]

> "Under the bill the deduction for unpaid losses is limited to the annual increase in discounted unpaid losses. Unpaid Losses include unpaid loss adjustment expense. Discounting is on a line-by-line, accident-year-by-accident-year basis. The discounted values are determined from assumptions regarding the pattern of payments, the duration over which claims will be paid, and the interest rate employed. These assumptions are used to calculate 'discount factors' applicable to each accident year for each line of business. The discount factors are a composite of the interest rate and payment pattern assumptions. <u>Once a series of discount factors is applied to a particular accident year it continues to be used without change as the accident year ages. In the terminology of the bill, each accident year is 'vintaged' and subsequent redeterminations of interest rates or payment patterns based on actual experience are not permitted."</u>

> "In the Senate Bill the initial interest rate was 5 percent, applicable to accident years beginning before or in 1987. For accident years beginning after 1987, the applicable interest rate was 75 percent of the so-called annual Federal mid-term rates (AFR). The AFR is a composite yield of federal instruments, currently running at about 7.5 percent. 'Mid-term' refers to the duration of the instruments, contrasted with 'short-term' or 'long-term.' A specified published interest rate was selected to avoid arguments over the determination of a company's yield on its own portfolio."

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19. R. H. Snader, "Federal Tax Reform Bill Resolves Reserve Discounting Issue, Dictates Methodology," <u>The Actuarial Review</u>, Nov. 1986, page 1.

> "The House-Senate Conferees amended this provision to 100 percent of the AFR."

> "The bill requires the Secretary of the Treasury to determine a loss payment pattern for each line of business in Schedules O and P. The first such determination will be made in 1987 using the latest available data published by A.M. Best & Co. The payment pattern is to be redetermined every five years after 1987." (Emphasis Supplied)

The assessment of taxes on an AFR rate is consistent with the taxes shown in Exhibit IV, Page 3. Reserves in that Exhibit have been discounted using duration-matched Federal rates.

The provision for not changing interest rates is substantially met when both capital gains from insurance assets and changes in present values of the reserves from a risk free portfolio are taken into account. While companies themselves will have to follow the actual procedures mandated in the tax bill, analysts should obtain reasonable results using the EVA approach.

The procedure used to determine the Federal Taxes in Exhibit IV, Page 3, is as follows:

$$\begin{array}{llll} \text{FED TAX CASH FLOW} = t \times (& \text{EARNED PREMIUMS} & + & (61) \\ & \text{EARNED EXPENSES} & + & \\ & \text{EVA PAID LOSS} &) & \end{array}$$

$$\begin{array}{llll} \Delta\text{FED TAX RESERVES} = t \times (& \Delta\text{UNEARNED PREMIUMS} & + & (62) \\ & \Delta\text{UNEARNED EXPENSES} & + & \\ & \Delta\text{EVA LOSS RESERVES} & + & \\ & \text{GAINS ON RISK FREE ASSETS} &) & \end{array}$$

The EVA Method reaches its results in a different fashion than the Federal Tax Reform Bill. We believe that it produces results which will be reasonably similar to those achieved by the Bill for the reasons discussed above. The offset of gains with changes in EVA receivables is certainly different than the freezing of old accident years and the use of different discount rates on each accident year. When prior year reserves are substantially misestimated, the two approaches will produce different results since we assume that the funding requirements will be met with assets bearing current interest rates. The Federal Tax Bill presumably assumes that assets bearing the original accident year discount rates will be resurrected from somewhere and used instead.

During 1984 and 1985, the industry increased reserves from prior years by $12.3 billion. Assuming that all of this increase came from losses occurring during 1981, the 1986 Tax Reform Act would require reserves to be discounted by the interest rates prevailing in 1981. The investment receivable posted in 1984 and 1985 as shown in Exhibit IV was $781 million. This should have reduced the tax credit from these incurred losses by 46% of that amount, or $359 million.. The tax reform act would have required posting an investment receivable of $937 million, assuming interest rates had been 20% higher in 1981. The tax liability from this receivable would have been $431 million. This is $72 million too high given the reduction in interest rates and the impossibility of funding these reserves in 1984 and 1985 with assets bearing 1981 rates of interest.

This example uses an unusually large change in interest rates and a very large increase in prior year loss reserves. The increase in taxes due to the vintaging approach in this case is about $72 million, about .1% of the industry 1985 earned premium. Thus the flaw in the methodology may not be too important.

VI. CONCLUSION

We have examined insurance accounting data to see what can be learned about improving the analysis of insurance company operations. We have shown that a reasonable determination can be made of overall insurance company performance in both of its roles: Insurance Operations and Financial Intermediation. We have found much less information available on a by-line basis, especially with regard to personal lines cash flows.

We have shown that estimates of incurred loss cash flows can be made by line using the exponential model and that these estimates can be very accurate when appropriate conditions exist. These conditions are primarily the presence of reasonably stable growth rates and reasonable reserve accuracy.

We have also shown that accident year payment patterns have been very stable at an industry level. This stability is also exhibited by all the large companies we have analysed.

The methods used to produce Exhibit IV are reasonably simple and can be extended to a line-by-line basis using approximation techniques. Some of these have been discussed in this paper. Others have not because we have not had the time to develop them to the extent required for presentation.

It is our fervent hope that this paper will inspire other analysts and actuaries to examine alternative accounting approaches to examining insurance company operations. It is also our hope that better data will eventually be made available by companies.

APPENDIX I

DERIVATION OF MODIFIED FORM OF McCLENAHAN MODEL

Formula (1) of McClenahan's Paper is a statement of his model. It goes as follows:

$$ {}_nP_m = c\ p\ q^{n-d}\ (1+x)^{m+n}\ (1+y)^m $$

$$ = \{\ c\ (1+z)^m\ (1+x)^d\ \}\ (1-q)\ (q(1+x))^{n-d} $$

Where:

$_nP_m$ = Payment on losses occurring during accident month m in month m+n

c = Constant representing base month losses

p = Payment rate with no inflation

q = Loss decay rate with no inflation. Note $p = 1-q$

x = Severity growth (assumed to be uniform over time)

y = Frequency growth (assumed to be uniform over time)

d = Delay in reporting claimes

z = $(1+x)(1+y) - 1 = x + y + xy$

We can restate this as follows

$$ {}_nP_m = c'\ (1+z)^m\ (1-q')\ q'^{\,n-d} \qquad (63) $$

Where:

$c' = c\ (1+x)^d\ (1-q)\ (1-q')^{-1}$

$q' = q\ (1+x)$

Formula (63) is in the exponential form used in the paper. The constant term is c', and the growth term is $(1+z)^m$.

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