Title: AN INVESTIGATION OF METHODS, ASSUMPTIONS, AND RISK MODELING FOR THE VALUATION OF PROPERTY/CASUALTY INSURANCE COMPANIES

Author: Robert S. Miccolis

Biography: The author is a Fellow of the Casualty Actuarial Society (CAS), a member of the American Academy of Actuaries (AAA) and the International Association of Actuaries. He is a Consulting Actuary with Tillinghast/TPF&C and serves as Chairman of the CAS/AAA Casualty Loss Reserve Seminar Program Committee and as a member of the CAS Syllabus Committee and the CAS Committee on Valuation Principles and Techniques.

He holds a Bachelor of Science Degree in mathematics from Drexel University, Philadelphia, PA. He received the Woodward-Fondiller Award (1977) from the CAS.

Abstract: This paper discusses the various approaches used in the financial/actuarial evaluation of an insurance operation principally engaged in writing property and casualty insurance coverages. This paper explores some of the concepts presented by previous papers on this topic and investigates alternative methods of determining assumptions and values.

Since the insurance business is heavily influenced by the stochastic nature of insured losses, a large portion of the paper is devoted to the risk elements in terms of the identification and treatment of risk in the financial modeling of insurers and the determination of value.

INTRODUCTION

The intent of this paper is to discuss the major areas of valuation approaches and to explore the concepts involved in assessing risk and uncertainty in a valuation context. First, the current concepts of value are discussed and the major components of value are described. Next, the sources of risk affecting these components and the influence of external variables are reviewed. Then, alternate valuation models are presented and the concept of an insurance company risk model is developed. The potential applications of risk models are discussed such as for solvency testing, required surplus analysis, acquisition or new venture strategy analysis, and confidence level evaluations of economic value. Finally, suggestions are presented for the implementation of risk models to the various potential applications.

VALUATION CONCEPTS

The paper "Actuarial Valuation of Property/Casualty Insurance Companies" by Robert Sturgis[1] presents the essential and fundamental principles involved in the valuation process. The concept of economic value of a property/casualty insurer is defined by Sturgis drawing upon concepts presented in other papers on the valuation of life insurers. Economic value is computed as:

Current net worth (surplus),
plus adjustments for reserve adequacy,
plus non-admitted assets,
plus special liabilities (e.g. unauthorized reinsurance),
less excess of book over market value of bonds,
plus discounted value of future earnings,
less cost of capital.

This economic value could generally be considered an actuarial value for many uses. As Stephen Lowe[2] describes in his review of the Sturgis paper, the value computation can be divided into components such as "blocks" of business and it can be used to value alternative marketing, underwriting or financial strategies. This suggests that an actuarial value could be used for financial planning measurement. It could also be used as a means for :

- determining financial strength in measuring solvency or in the rating of insurers,
- measuring overall insurer performance based on the change in value, and
- providing the basis for a valuation actuary to assess the financial condition of an insurer.

Each of the items in the economic or actuarial value computation is discussed in more detail below.

A. Adjusted Net Worth (Surplus)

The statutory surplus of an insurer generally represents the book net worth of the company. However, insurers domiciled in the U.S. generally have financial factors which would increase or decrease this net worth but are not reflected due to regulation or accounting practices. Some of the major items are:

1. Book vs. Market Value of Bonds: This adjustment can be made directly by adjusting surplus by the full amount of any difference. Alternatively, the determination of future earnings could be based on projected future investment yields which reflect the company's current bond portfolio yields and maturities (in which case no adjustment is made to surplus). The concept of an "investment neutral" valuation, as described below, uses the following logic. A purchaser of an insurance company is buying all of the company's existing investment portfolio and therefore should not pay any more or less than the market value of those assets. Since U.S. insurers' bond portfolios are usually carried at their amortized value, this adjustment should be computed for all of a company's bonds for each bond issue. Obviously in times when financial markets are changing, this adjustment can change daily. Sturgis[1] suggests that a direct adjustment of surplus should be offset by a lower cost of capital (if market value is less than book value).

2. Non-Admitted Assets: Most state insurance regulations exclude certain assets (furniture & fixtures, agents' balances over 90 days due, etc.) from an insurer's balance sheet. To the extent that such assets are collectible or have financial value, they should be included in adjusted surplus.

3. Liabilities for Unauthorized Reinsurance: Reinsurance ceded to unauthorized reinsurers is not generally recognized by state regulations unless the recoveries of paid or unpaid losses is fully secured (usually by a letter of credit). Unsecured recoverables from unauthorized reinsurers can be 9added to surplus to the extent that such recoverables are judged to be collectible based on the financial strength of such reinsurers.

4. Uncollectible Reinsurance: Another adjustment related to the above liability is the potential uncollectibility of reinsurance recoveries from authorized reinsurers or from secured unauthorized reinsurance. The potential insolvency of one or more reinsurers should also be considered as a reduction in surplus. While such insolvencies are difficult to predict by individual reinsurer, a ranking of reinsurers can be made based on financial ratios and ratings of each reinsurer. If actuarial valuations of insurers and reinsurers are available, they would provide a means of assessing potentially uncollectible reinsurance.

Without such valuations, the deduction for estimated uncollectible reinsurance recoveries requires several assumptions and judgments where weak reinsurers are involved. The estimated amount of uncollectible reinsurance could be made as an indirect adjustment by reducing future earnings for the uncollectible reinsurance recoveries as they might emerge.

5. Surplus Notes or Debentures: If some part of an insurer's capital is supported by surplus notes or surplus debentures, then all principal and interest owed on such financing should be deducted from surplus to the extent that there are no balance sheet liabilities for such notes or debentures.

6. Investment in Affiliates: If a company has unconsolidated insurance subsidiaries or affiliates, then the assets which represent the company's investment in those operations should be separately valued and then combined, or the valuation should specifically exclude these investments.

7. Reserve Adequacy: The accuracy of an insurance company's liabilities is certainly of major importance in the determination of an insurer's value. What is suggested here is that any deficiency or redundancy in loss and loss adjustment expense reserves should be adjusted directly against surplus rather than through emergence in future earnings. If reserve adjustments are made against future earnings, the

effect on value will be reduced as long as the discount rate is greater than the investment yield.

Premium reserves are generally unaffected except possibly for the inclusion of estimated retrospective additional premiums which are normally excluded from booked assets. Projected underwriting profits or losses from unearned premiums should be included in the future earnings calculations.

8. Tax Considerations: Several of the items listed above will have an impact on the company's current or future U.S. federal income tax liabilities. A complete discussion of U.S. taxation of property-casualty insurers is beyond the scope of this paper; however the tax effect of any surplus adjustments should be reflected in an after-tax valuation. In particular, the adjustments for unauthorized reinsurance, uncollectible reinsurance, and reserve adequacy will generally affect the company's taxes. How such tax adjustments are made will depend upon the overall tax situation of the current (or future) owner(s) and the projected timing of the tax effects associated with each surplus adjustment. In addition, any tax loss carry-forwards (also referred to as net operating losses) which are available to offset future income tax liabilities should be recognized in the projection of future after-tax earnings.

B. Value of Future Earnings

This part of the valuation process has several elements and requires many actuarial assumptions and judgments. The major components are:

- Runoff of Loss Reserves to reflect the investment earnings associated with the assets needed to pay loss and loss adjustment expense reserves as claims are settled,
- Runoff of Unearned Premium Reserves to reflect the projected earning of the premiums and the expenses and losses incurred as the inforce policies are earned, including the associated investment earnings from the runoff of losses and any policyholders' dividends,
- Renewals of Inforce Business to include the projected net income or loss from the company's renewal of business inforce based on an assessment of the company's expected retention of its policyholders, and
- New Business to recognize the projected net income or loss from both expected growth in business and replacement business from the non-renewal of inforce policies.

Each of these components of future earnings is discussed further below.

1. Runoff of Loss Reserves: Assuming that all loss and loss adjustment reserves have been adjusted to be fully adequate, the payout of these reserves must be projected. Each line of business or other segment of loss reserves that is believed to have different payment characteristics should be separately estimated. Allocated loss adjustment expenses

and unallocated loss adjusted expenses could be combined with losses in the estimation of payment patterns or they could be separately projected. The separation of loss adjustment expenses from losses may be necessary where these expenses are not expected to follow losses in a consistent manner or where the payment patterns are not likely to be consistent. In those cases, separate estimates of expense reserves (and possibly separate expense projections for future years) and separate payment patterns should be used. (Allocated loss adjustment expenses may also need to be treated separately from unallocated.)

Investment earnings should be computed based on the total amount of the reserves less projected annual payments. These investment earnings should be adjusted for an estimated percentage of the invested assets which correspond to the reserves. Payment projections can be developed from the company's paid development history (adjusted for any expected changes) and the estimated ultimate losses.

For a company which uses discounted loss reserves, the investment earnings would be computed on the value of the discounted reserves, adjusted for the percentage of corresponding invested assets. As losses are paid out and those discounted reserves runoff each year into the future, the discount will be amortized. The amortization of the loss reserve discount will offset the investment earnings generated from these reserves.

For an after-tax valuation the loss reserve runoff will generate additional income taxes from future investment earnings. In addition, under the Tax Reform Act of 1986, future tax losses will be created as the tax-dictated discounting of loss reserves is unwound (according to the prescribed tax rules). The future earnings projections should incorporate these tax effects associated with the company's loss reserve runoff.

2. Unearned Premium Reserves: Most of a company's unearned premiums are normally earned in the next 12 months unless longer term policies are issued. Projected loss ratios are applied to the premiums as they are earned to estimate the losses and loss adjustment expenses. These loss ratios can be selected from the company's past history of developed and trended losses and premiums adjusted to current rate levels.

While a company's acquisition expenses and a certain portion of general expenses would have been expensed, those additional underwriting expenses associated with servicing the inforce business should be added as an additional expense. For example, if underwriting expenses total 30% of premiums of which 25% is directly related to initial policy costs, then the remaining 5% should be expensed as the premiums are earned.

Investment income should be earned on the invested portion

of collected premiums needed to support the loss reserves until losses and loss adjustment expenses are paid similar to the runoff of loss reserves. This investment income plus any underwriting profit or loss from the unearned premiums is the projected net income from the runoff of unearned premium reserves. An after-tax valuation must reflect the recognition of premium income according to the prescribed tax treatment of unearned premium and the deductibility of incurred losses (and the corresponding loss reserve runoff) per the tax-required discounting formulas.

3. Renewal of Business Inforce: Where an insurer has a history of retaining a significant portion of its policyholders at each renewal, there is a value which can be assigned to future renewals. To project the value of these renewals the first step is to estimate the lapse rate (percentage which do not renew). Note that lapse rate can vary by line, by source of business, and by number of years insured with the company. Assuming that the historical average policy premium is roughly the same for renewals and non-renewals, then policy counts can be used to estimate lapse rates. Alternatively, lapse rates can be developed in terms of the historical percentage of premiums renewed (after adjustment for past changes in average policy premium, e.g. rate increases, insurance to value, exposure changes, etc.). From these lapse rates the future renewals of inforce business are projected for subsequent renewals in future years until a certain minimum percentage remains or

until a specified number of years have elapsed. Note that the renewal premium projections should reflect future changes in average policy premium. Also, any operational changes (e.g. service capabilities, data processing facilities, change in management, pricing strategies, etc.) which could affect future lapse rates or renewal premiums should be incorporated into the projections.

The next step in valuing renewals is to project future loss ratios and expense ratios for this business. These loss and expense ratio assumptions can be derived by analyzing the company's historical results by line or business segment, considering pricing changes, inflationary trends, regulatory environment, historical industry results and trends, underwriting cycles, expected rates of return on equity by line of business, etc. For example, if a certain line has had very high loss ratios in recent years, then an assumption of large future rate increases would be appropriate. However, note that large rate increases may result in a significant loss of business and therefore affect the lapse rate assumption for the line.

In addition to the underwriting profit or loss from the projected renewals, estimated investment income on the net cash flows should be included. The timing of premium collections, percentage of positive net cash flow invested, loss payment patterns and investment yields are the other major assumptions needed for these projections. As with

the unearned premium runoff, similar tax computations would apply for an after-tax valuation of renewal business.

4. New Business: The value of new business is projected in a similar fashion to the renewal business. First, a review of premium or policy growth of the company can be made which analyzes overall growth divided by new vs. renewal business. The historical statistics may demonstrate the company's ability to attract both replacement business for non-renewals and for expansion. If a company has an erratic or inconsistent pattern of new business for certain lines or business segments, then such new business should be given little or no value. Assumptions similar to the renewal business projection are needed for loss and expense ratios, payment patterns, percentage of positive net cash flow invested, income taxes, etc. The number of years of projected new business should be limited in the value calculations to reflect the limitation on the accuracy of the various assumptions and the increasing uncertainty of the projections many years into the future.

The value of new business is a part of goodwill since it reflects management's ability to continue the profitable operation and growth of the company. The accounting and tax treatment of goodwill, and the definitions of identifiable vs. unidentifiable intangible assets, and amortizable vs. unamortizable intangible assets are discussed by Hall, Linden, Gerard and Heitz[3].

C. Discounted Value of Future Earnings

The various projections of future earnings from runoff, renewals, and new business result in annual estimates of cash flows and net income for future years. The income model valuation procedure calls for computing the present value of these future earnings. The appropriate discount rate should have the following properties:

- consistent with investment yield and inflation assumptions, tax considerations, and
- reflective of a rate of return on equity indicative of the uncertainty associated with realizing the future earnings projections.

The uncertainty of future earnings originates from several sources including:

1. Financial Market Risk: The determination of value is subject to the uncertainty inherent in the financial markets. For example, the valuation component for adjusted surplus reflects the expectations of the market in the valuation of the invested assets of the company. However, the prime rate, federal discount rate, yields on government securities and other financial values are subject to significant fluctuations and changes, particularly over long periods of time. The major influences of investor confidence arising from inflation, money supply, foreign exchange rates, etc. would certainly affect both asset values and investment yields.

2. Insurance Industry Risk: The property/casualty insurance industry has been through a variety of significant changes in recent history which illustrate substantial industry related risk. The two major sources of this industry related risk are underwriting cycles and claims cost inflation. The phenomenon of underwriting cycles in the property/casualty insurance industry can directly impact the value of an individual company. When the industry has excess capacity and recent profits are good, the industry can be fiercely competitive in prices, extension of coverage terms and types of risks accepted. Such conditions eventually produce poor underwriting results and reduction in surplus. Under low capacity conditions the industry typically increases prices, restricts coverage and reduces the types of risks which will be underwritten. This leads to improved underwriting results and increasing returns on equity. Of course the length and magnitude of such cycles are not completely predictable. In addition they are influenced by loss reserve adequacy, interest rates, inflation, asset values (in particular, book vs. market), regulation and other external factors.

Changes in economic and social inflation which affect insurance claims costs introduce uncertainty into the projection of future underwriting profits. Judicial decisions and increased utilization of insurance to compensate for injuries and damages can also produce unexpected

changes in claims. These uncertainties affect the projection of future loss ratios and the ultimate reserve adequacy especially where future settlements are not reflective of past history.

3. Insurance Company Risk: The particular company being valued can have many internal factors which pose substantial uncertainties to the valuation process. These factors can be divided into two categories - balance sheet factors and income statement factors. The major balance sheet factors which can have significant risk characteristics are:

   - Loss Reserve Adequacy: An analysis of confidence levels (intervals) for loss reserves could be used directly in assessing this risk. This is probably one of the most significant areas of balance sheet risk.
   - Assets Values: The difference between market and book value of bonds, the mix of maturities, the probability of default, and the fluctuation of market value of equities present additional risk elements which are unique to the company's portfolio.
   - Asset/Liability Mismatch: To the extent that the payout of liabilities exceeds the cash inflows generated by investment earnings and maturing bonds, a company would have to liquidate some of its invested assets. This could impair the company's overall yield on investments and may reduce the total asset values. However, the company's ability to attain higher

investment returns by the timing of bond purchases and a strategic mix of maturities could increase the company's effective investment yield.

- Non-admitted Assets and Agents Balances: These values can usually be estimated accurately based on history and external information. However, there is some risk where there is a concentration of receivables from one or a few sources or where non-admitted assets are concentrated in one or a few items or categories.
- Collectibility of Reinsurance: The potential uncollectibility of reinsurance can be substantial where the company's reinsurers are small, unauthorized, or financially weak. In cases where a company is particularly vulnerable to one or a few reinsurers, the company could be subject to a future collection problem on current or future recoveries.

The major sources of uncertainty in projecting a company's future income are profitability and cash flow. The estimates of future loss ratios and expense ratios are predicated on recent history and current information. The value of future earnings is uncertain due to the assumptions made to project those earnings and the predictability of future events several years ahead. In addition, the cash flows from premium collections, reinsurance recoveries and loss and expense payments are subject to change and variability. In particular, loss payments can be erratic where large individual claim payments may be necessary

or where legal or administrative changes impact the claim settlement or payment process, and consequently affect the projected investment earnings.

4. Timing Risk: In making a valuation at a particular point in time with the consideration of future earnings, a single value is computed by taking the present value of the projected future earnings. This procedure assigns increasingly smaller values to earnings further into the future. When the discount rate is set at the "risk free" rate, the resultant value reflects the time value of money alone and does not reflect the greater uncertainty of the projected future earnings. Consequently, a value adjustment is needed to account for the uncertainty of these projections. The traditional method is to use a "risk-related" rate of return for the discount rate. However, given the various risk elements to those projections, it is not clear how to define or select a risk margin for such a calculation. More importantly, the risk margin could be different for the various elements, e.g. runoff of losses vs. new business. Given the greater uncertainties for projected income several years ahead, the risk margin should increase for each year in the future. Before attempting to model this risk margin question directly, first consider the last item in the valuation model, the cost of capital.

C. Cost of Capital

The cost of capital is a reduction in the value of a company based on the premise that invested capital and surplus would not earn the same rate of return if those capital and surplus funds were freely invested. Sturgis[1] suggests a theoretical "regulatory statutory surplus" e.g., one-third of net written premium, and an after-tax investment yield differential to compute this cost of capital. For example, suppose an investor requires a certain rate of return (at a particular level of risk) which is higher than the yield which can be prudently earned by a property/casualty insurer. The yield differential represents the theoretical income lost from the opportunity to invest the required capital and surplus funds at a higher yield (and higher risk). However, unless there is a yield differential at the same level of risk, this "opportunity" cost of capital would not be risk-neutral. Since a property/casualty insurer can freely invest in risk-free securities, such as U.S. Treasury securities, an investor should not expect a yield differential solely from the capital and surplus funds. Thus, there is a zero cost of capital if the risk associated with the investment income on capital and surplus is directly reflected in the valuation. Further, the appropriate discount rate to apply to the annual cost of capital would appear to be the risk-free rate since there is no real risk involved in the cost of capital itself.

From the above discussion it appears that the cost of capital is ill-defined for valuation purposes. However, the cost of capital really represents an equivalency factor which equates the income valuation model to the terminal value model as described in the next section. In effect, the discount rate used for the present value of future earnings represents an average internal rate of return, required by the investor, for all income generated by the company from all equity needed to support the company's premium writings. Thus, while the investment of capital and surplus funds in themselves might be risk-free, the other income (or loss) is not. The cost of capital is an essential element to the income valuation model that produces the desired total average rate of return.

If one considers the valuation of a particular company for a particular buyer without the cost of capital, it would include adjusted surplus plus the discounted value of projected future earnings where the discount rate reflects the buyer's assessment of risk and the corresponding risk/return requirements. However, the discount rate selected for this value computation should be higher than the total internal rate of return required by the buyer.

ALTERNATE VALUATION MODELS

The valuation concepts discussed above illustrate a model of value using net income available for distribution to stockholders. A cash flow model has also been suggested by Rothman and Deutsch[4] which Sturgis[1] shows to be a special case of the income model (when the discount rate equals the investment yield). This cash flow model is generally not appropriate for valuation purposes since it implicitly assumes that all net cash flow can generate investment income at a rate of return equal to the discount rate. Another model of value could be defined as the ending assets after all liabilities have been paid. This "terminal" value can be cqnsidered as a "residual basis" valuation and the terminal value could be discounted to present value. This approach could either encompass the total investment earnings including reinvestment of earnings, or stockholder dividends could be paid according to some formula.

The concept of a terminal value would allow the computation of an internal rate of return by using the beginning adjusted surplus and the terminal value (if dividends are paid for various years they should also be included in the internal rate computation). Note that unlike the income model, investment earnings on all capital and surplus can be included. A comparison of the income model with the terminal value model would show that the income model is a special case of the present value of the terminal value where all future annual earnings are paid in dividends, only the minimum required surplus generates investment earnings at the projected investment yield, and the internal rate of return equals the income model's

discount rate. While the terminal value model may not be particularly useful for appraising an insurer's value, it can be helpful in developing more explicit risk models.

RISK MODELS

Risk models of insurer solvency have been used by European actuaries for many years. Recent work in England includes asset value and investment risk in these models as well as the insurance (liability) risk. A model described by Butsic[5] has been used to describe risk/return criteria using the variance in the total return (underwriting and investment). By expanding the notion of return to include the future earnings in the income model, terminal value or the internal rate of return, the valuation process can be viewed as a stochastic process with explicit recognition of the various risk factors which affect value (or return). By constructing probabilistic models of loss reserves, loss ratios, cash flow, investment yields, asset values, and inflation (with appropriate correlations accounted for) a multivariate distribution can be created for the terminal value or internal rate of return (at alternate amounts of initial investment).

Some of the potential uses of these risk models would include:

1. Analysis of risk and return tradeoffs;
2. Stochastic modeling of insurer operations;
3. Solvency testing, including liabilities and assets;
4. Determination of surplus requirements;
5. Acquisition or new venture strategy analysis, and
6. Confidence levels for insurer valuation.

Each of these uses is described below.

1. Risk/Return

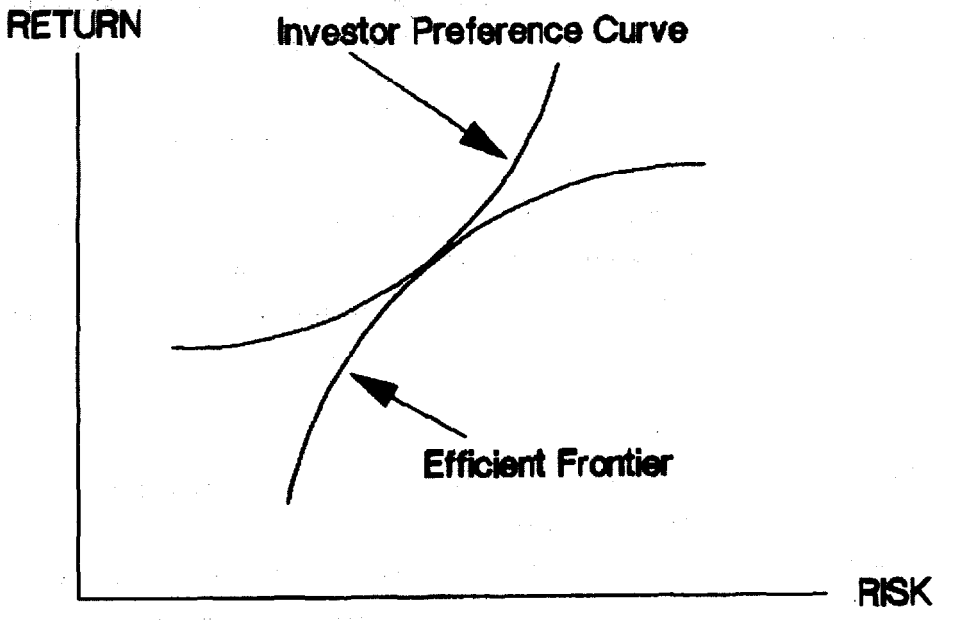
Several actuaries and economist's have studied risk vs. rate of return. Butsic[5] uses the variance to measure risk and describes the underwriting and investment components of return and the variances in return. The financial theory of such risk/return models relates the risk/return characteristics of an investment to the investor's utility as depicted in Figure 1 where the efficient frontier is the best set of expected risk/return results from alternative mixes of underwriting of risk types, coverages or geographic distribution with a portfolio of different types of invested assets. Butsic's representation of risk (variance) and return are based on the relationship:

$$\text{Return on Surplus} = \left[1 + \text{Premium to Surplus Ratio} \times \text{Reserves to Premium Ratio}\right] \times \text{Investment Rate of Return} + \text{Premium to Surplus Ratio} \times \text{Underwriting Return on Premiums}$$

From this he develops the expected value and variance relationships when the rates of return from investment and from underwriting are viewed as random variables.

Using these fundamental relationships the time horizon for the rate of return variable can be divided into the components previously described (in-force, renewals, and new). Also, the investment return variable can be modified to account for the current portfolio and the variability in loss payout. Reserve adequacy can also be included as a variable. In addition, by modeling the entire financial process for

FIGURE 1


RETURN
Investor Preference Curve
Efficient Frontier
RISK

an insurer, the distribution of return on surplus can be simulated (or possibly derived analytically or approximated numerically with techniques such as Heckman and Meyer[6] describe) to give a better representation of risk than through a single measure such as variance or standard deviation. For example, Figures 2 and 3 give two possible rate of return probability distributions. Both of these distributions have approximately the same variance and expected value but an investor would probably prefer Figure 3 given the likelihood of higher returns vs. the lower returns depicted in Figure 2. The variance measure of risk implicitly assumes the distribution of results is not skewed towards favorable results (higher returns). When the shape of the return distribution is likely to vary among alternative investments, a single risk measure such as variance may give invalid risk/return comparisons.

2. Stochastic Modeling

As described previously, many of an insurer's underwriting and investment parameters are subject to uncertainty which can impact financial results. Additionally, the value of both the assets and liabilities of a insurer's balance sheet are estimates and these values can vary, sometimes substantially, from their stated values. By constructing a stochastic model of an insurer's financial statements, a variety of measures of annual results can be modeled and studied.

FIGURE 2

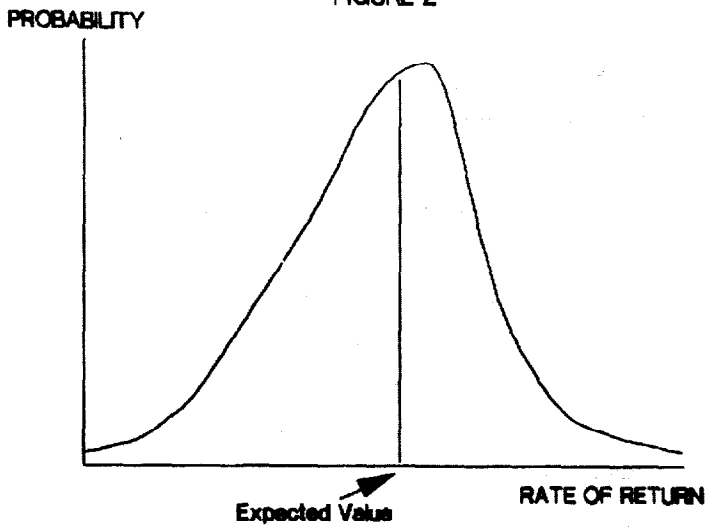
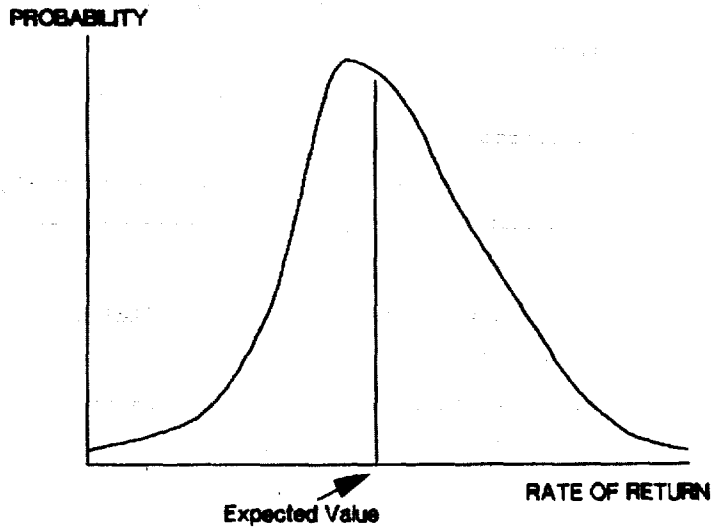


FIGURE 3



The major input variables to such a model would be:

Balance Sheet

- Premiums receivable
- Agents balances
- Market value of investment portfolio
- Value of non-admitted assets
- Recoveries from unauthorized reinsurers
- Collectibility of ceded reinsurance
- Adequacy of loss and loss adjustment expense reserves
- Other assets (e.g., investments in affiliates)
- Other liabilities

Underwriting Income

- Written premiums by line by year
- Earning distribution of written premium
- Underwriting expenses by line by year
- Losses and loss adjustment expenses by line by year
- Reinsurance ceded premiums
- Reinsurance ceded losses and loss adjustment expenses

Investment Income

- Payout of losses and loss adjustment expenses by line
- Initial and future asset distribution by type and duration
- Income from bonds
- Income from equities (dividends, capital gains/losses)
- Premium collections
- Income from short-term investments (cash)
- Asset selling rules

Income Taxes

Current taxes

Net operating losses (carry-forward/carry-back)

Other

Other income/loss

Income/loss from currency fluctuations

Most of these input variables have some degree of uncertainty in their actual current and/or future values. Also, many of the variables are correlated in varying degrees to each other and to external factors such as inflation and investment yields. By explicitly modeling each variable as a random variable and each combination of random variables which are believed to be highly correlated, a sophisticated insurance company financial computer simulation model can be developed to approximate the probability distribution of key financial results such as surplus, annual total return on surplus, internal rate of return, etc. This computer model can be used for various types of analyses such as:

- Analyzing the sensitivity of the return on surplus to underwriting results, interest rates, inflation, etc.
- Investigating the impact of uncertainty in loss reserves on current and future surplus and net income in conjunction with uncertainty in interest rates, cash flow demands, new business growth and profitability.
- Evaluating the effects of including or excluding certain types of business,or market segments with different financial characteristics and degrees of uncertainty.
- Assessing uncertainty on a total company basis by reflecting the independent and dependent random variables with dif-

ferent probability distributions, and by comparing results with different assumptions about the expected value of each variable along with their variability and probability characteristics (distribution type, scale and shape).

The use of stochastic financial models of insurer operations should add an important dimension to decision-making. While many assumptions are involved in implementing this approach, these models can provide better insights into how various sources of uncertainty can impact current and projected financial performance and how surplus and cash flow function in support of an insurance operation.

3. Solvency Testing

The stochastic model described above has recently been studied as a means of determining an insurer's solvency and financial strength by several authors notably Coutts and Devitt[7] and the General Insurance Study Group's Working Party on Solvency[8]. These authors developed and tested a computer simulation model principally oriented towards investigating the solvency implications of variability for both assets and liabilities. The modeling of cash flows is shown to be critical in solvency or financial strength analysis, particularly with respect to the asset structure, investment/reinvestment strategy and asset selling rules. A generalized stochastic model based on the principles described in this paper should provide similar analytical results for evaluating solvency and financial strength.

4. Surplus Requirements

Several approaches have been used to determine the required amount of capital and surplus. Leverage ratios (premium to surplus, reserves to

surplus, etc.) typically dominate the estimation of surplus needs or the rating of a company. In certain European countries solvency margins are required and they are generally computed from a ruin theory model (capital and surplus must be sufficient to maintain a low probability of insolvency). Finger[9] explains the application of a ruin theory model in the determination of minimum surplus requirements including recognition of the uncertainty in estimating the mean reserve. Arrata[10] describes a stochastic approach to determining loss reserving policy for a small, new insurer based on solvency considerations.

As mentioned above, stochastic modeling of assets, liabilities and future business should provide better measures of required surplus than those presently used. In addition, through stochastic modeling new approaches can be investigated for determining surplus allocations to lines of business or blocks of business which reflect both profitability and cash flow risks for each business segment as well as the effect of these individual risks on the company's overall surplus requirements and return on surplus. It would also be possible to assess the effectiveness of surplus utilization by comparing the variability of returns on surplus by business segment vs. overall.

5. Acquisition and New Venture Strategy

The analysis of alternative strategies for insurance company acquisitions, for major business expansion, or for joint ventures is usually based on a valuation approach which evaluates the internal rate of return or the discounted present value of future earnings. However, because such acquisitions or new ventures have many unknown variables,

various scenarios are devised to demonstrate the sensitivity of the analytical results to various key assumptions. Each alternative strategy is then evaluated under each scenario to develop a matrix of results for selecting a particular strategy or devising a new one. This approach assumes that the individual scenarios are representative of the range of likely values of all the important variables affecting the results of each strategy. However, the development of such scenarios is highly judgmental, particularly with respect to whether the number of scenarios is sufficient and whether the range and combination of variable values is fully representative. Also, while the uncertainty of results is demonstrated by these scenarios, there is no assignment of probabilities to outcomes which could aid in the decision-making process.

The acquisition/new venture problem could be analyzed using a stochastic model of the insurance company which could provide a more comprehensive method of illustrating the uncertainty of the results of each strategy. This would greatly reduce or eliminate the need for building scenarios since each key variable could be described by a probability distribution. Using this approach the matrix of results will describe the estimated probability of attaining various financial results under each alternative strategy. Also, if the risk/return preferences of the investor(s) can be described by a single utility function, then the assessment of the various strategies is further simplified.

6. Valuation Confidence Levels

The concept of confidence levels for loss reserves can be extended to confidence levels for an actuarial valuation. This allows for the recognition of the risk elements described above, including the investment related factors. The stochastic modeling approach can produce the necessary probability statistics for representing confidence levels. The analysis of confidence levels for the valuation of an insurer would give an actuary a mechanism for assessing the insurer's total financial strength and for demonstrating the risks involved in the values from net worth vs. runoff vs. renewals vs. new business.

Note that there can be a fairly high degree of correlation between the value components. Consequently the risk associated with the total overall value might be greater than that indicated by the sum of the components. This should be expected where strong positive correlations exist.

The introduction of confidence levels into the actuarial valuation process would require recognition of explicit and implicit assumptions and variables which could affect the variability of projected results. The stochastic modeling used to develop such confidence levels should incorporate both the "process" and "parameter" risk. The parameter risk elements reflect the uncertainty in the various assumption "parameters" including the types of probability distributions used to model the various random variables which determine value.

IMPLEMENTATION OF RISK MODELS

The implementation of the stochastic models described require the construction of computer simulations or numerical approximation techniques for deriving the results. Before these computer models can be developed, the input variables and output requirements should be specified. Some of the important implementation considerations include determining the variables and their relationships, isolating the principal variables for analysis, and evaluating the results of the models. These considerations are discussed below.

Variables and Their Relationships

The fundamental accounting and actuarial variables need review to evaluate interaction variables and probabilities.

1. Surplus

Consider the relation:

SURPLUS = ASSETS - LIABILITIES

As mentioned previously, both assets and liabilities have several component variables which are subject to uncertainty. However, those explicit or implicit variables which affect both assets and liabilities need to be identified in the model. Certainly future inflation can impact the ultimate cost of current liabilities for unpaid claims and future inflation will affect interest rates and the value of insurer's investment portfolio. Also, increases in liabilities over the current (expected) value might increase reinsurance recoveries and thus decrease the probability of collecting those reinsurance recoveries. If assets are valued at book rather

than market value, the assets could decrease if the liabilities mature faster than the cash available from interest, dividends and bond maturities, and if investments are liquidated for less than their book value.

2. Loss Payout

The payout of liabilities for unpaid claims may require a separate formulation. The future payout of these loss reserves should be expressed as a random variable for each future calendar year. However, the two factors which must be included are:

- the time of payment, and
- the amount of payment (which could be expressed as a percentage of ultimate losses).

The time factor is really a parameter since it is not a random element. The payment amount variable is a random variable which is dependent upon the time parameter, and may also be dependent upon other time and/or structure parameters (e.g., time since occurrence, time since claim reported, year of occurrence, claims administration and settlement practice indicator, inflation rate, legal environment indicator).

Stochastic reserving models such as those developed by Zehnwirth[11], Taylor[12] and others could be adapted to represent the random future payout of liabilities for losses by accident year and year of payment. It would probably be helpful to include both the variable for the amount of the ultimate liabilities and the variable for the future payout by calendar year in such a model.

3. Renewals and New Business

The key actuarial assumptions for future business are the loss and expense ratios, the lapse rate for renewals, and the overall growth rate. In practice these assumptions are based on the company's historical experience, industry experience, recent rate activities and industry trends, e.g. underwriting cycle. In implementing a risk model of these elements one should consider those factors affecting the uncertainty of:

- Future losses, such as social and economic inflation, mix of exposures, accident frequency trends, volume of business, reinsurance retentions, etc;
- Future premiums, such as changes in prior experience used for ratemaking, (including loss reserve adequacy) the size and timing of pricing changes (reflecting the company's prior operating results and competitive environment factors), and the mix of exposures, limits, deductibles, etc;
- Future expenses including the mix of fixed and variable expenses, expenses on new vs. renewal business, inflation in operating costs, etc;
- Lapse rates such as changes in pricing and underwriting criteria (which are dependent upon prior experience), the degree of competition, and the mix of policies by number of previous renewals; and
- Growth in volume including prior lapses (non-renewals), pricing changes, inflation in exposures (insured values, payrolls, sales, revenues, etc.), and the degree of competition.

Formulas for the interdependency of many of these elements could be constructed by combining several assumptions about the insurer's operating behavior and industry practices with an economic model of industry underwriting cycles (such as described by James[13]). These formulas need not be deterministic but could be stochastic.

Isolation of Variables

A thorough comprehensive risk model of an insurer's current and future financial variables can be quite complex. Consequently, the implementation of such a model should only include those variables, factors, parameters, and relationships which are determined to have a significant effect on the results, particularly the uncertainty in insurer value, internal rate of return, and surplus. In order to make such determinations the model should have the capabilities of sensitivity testing. Such testing would isolate individual variables and demonstrate the effect of the value and uncertainty of each variable. In addition, the relationships and interdependency formulas should be similarly tested.

This testing is important for reviewing the various assumptions needed for a comprehensive risk model. Those assumptions which prove to be unnecessary can be discarded. Those which are the most significant can be studied in more detail. Additional assumptions or variables can be introduced and tested within the context of the other variables and relationships.

Evaluation of Results

Since there are several possible ways of viewing an insurer's current

value and future earnings, there should be several measures available from a valuation model. The principal measures which should be included are:

- Present value of future earnings at several discount rates,
- Terminal value of net worth after a selected period (e.g. 10 years) of new and renewal business, and
- Internal rate of return generated by future earnings and a final net worth (after 30 years for example) based on several initial investment assumptions.

For a risk model each of these measures would be computed as a probability distribution resulting from the various component variables both stochastic and deterministic). It could also be useful to compute the probability distributions of the annual projections for key financial results such as surplus, net income (investment vs. underwriting), by line results, etc.

Note that in these stochastic projections it might also be useful to introduce additional variables such as the difference between carried reserves (per the company's future estimates) and the fully adequate reserves (unknown by the company at the time of estimation). This difference could be dependent on other variables, such as changes in inflation, or simulated management behavior (e.g., increasing reserve levels when net income is high, decreasing reserve levels when there are operating losses or low profits). This type of analysis would provide evaluations of the effects of adding (or deleting) variables from projections.

CONCLUSION

This paper has attempted to describe a generalized approach to analyzing the economic value of a property-casualty insurer. The stochastic modeling of an insurer's operation can provide a method for assessing the risk and uncertainty associated with an insurer's economic value, including underwriting and investment risk, and for evaluating the effects of alternative operational strategies or other environmental changes.

These methods and approaches can provide some of the tools necessary for a valuation actuary to give an opinion of the financial strength of an insurer. The analysis of risk is fundamental to the concepts of solvency and required surplus. Risk analysis is also important to the stockholders of an insurance company and to potential investors. The actuarial profession should develop the models, methods and standards for risk analyses of economic value, rates of return, and financial strength for insurers. This paper has attempted to describe the principles and concepts which are fundamental to stochastic model development and the analyses of model results.

REFERENCES

1. Robert W. Sturgis, "Actuarial Valuation of Property/Casualty Insurance Companies", PCAS Vol. LXVIII (1981).

2. Stephen P. Lowe, Discussion of "Actuarial Valuation of Property/Casualty Insurance Companies" by Robert W. Sturgis, PCAS Vol. LXIX (1982).

3. James A. Hall III, Orin M. Linden, Stephen Gerard, Michael Heitz, "The Valuation of an Insurance Company for an Acquisition Involving a Section 338 Tax Election", PCAS Vol. LXXII (1985).

4. Robert Rothman and Robert V. Deutsch, Discussion of "Actuarial Valuation of Property/Casualty Insurance Companies" by Robert W. Sturgis, PCAS Vol. LXIX (1982).

5. Robert P. Butsic, "Risk and Return for Property-Casualty Insurers", 1979 Call Paper Program, Casualty Actuarial Society.

6. Philip E. Heckman and Glenn G. Meyer, "The Calculation of Aggregate Loss Distributions from Claim Severity and Claim Count Distributions", PCAS Vol. LXX (1983).

7. Stewart Coutts and Russell Devitt, "The Assessment of the Financial Strength of Insurance Companies - A Generalised Cash Flow Model", Presentation at the International Conference on Insurance Solvency (June, 1986) Philadelphia, PA.

8. C. D. Daykin, G. D. Bernstein, S. M. Coutts, E. R. F. Devitt, G. B. Hey, D. I. W. Reynolds and P. D. Smith, "Assessing the Solvency and Financial Strength of a General Insurance Company"' Report of Solvency Working Party, General Insurance Study Group (September 23, 1986).

9. Robert J. Finger, "A Model For Calculating Minimum Surplus Requirements", 1979 Call Paper Program, Casualty Actuarial Society.

10. David A. Arrata, "Loss Reserving for Solvency", PCAS Vol. LXX (1983).

11. Ben Zehnwirth, Presentation on Advanced Loss Reserving Techniques at the 1986 Casualty Loss Reserve Seminar, Washington, D.C.

12. Gregory C. Taylor, "Regression Models in Claims Analysis", presentation on Advanced Loss Reserving Techniques at the 1986 Casualty Loss Reserve Seminar, Washington, D.C.

13. Kaye D. James, "Underwriting Cycles in the Property-Casualty Insurance Industry", 1981 Discussion Paper Program, Casualty Actuarial Society.