# An Analysis of the Capital Structure of an Insurance Company

by

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Abstract

This paper attempts to analyze the capital structure of an insurance company in a way that: (1) views the insurance company as an ongoing enterprise; and (2) allows for the stochastic nature of insurance business. A model is developed. This model is used to analyze the effect of uncertainty in the loss reserves, the underwriting cycle and the cost of insurance regulation to the consumer. The paper considers both the investor's and the regulator's point of view.

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## 1. Introduction

An insurance company is in the business of transferring risk. It does this by accepting premium from policyholders and paying claims. It can happen that the premium collected is less than the total amount paid for claims. If this is the case, the insurer is expected to pay for the claims from the capital[1] of the insurance company.

This paper addresses the following question.

*How much capital will be invested in a given insurance company?*

The owners of (or investors in) the insurance company are concerned with the yield and the safety of their investment. The money they invest in the insurance company must be competitive with respect to the yield and safety of alternative investments.

The insurance regulator has a vital interest in this question. The concern is that the insurance company have enough money to fulfill its obligations to the policyholders when loss experience is worse than expected.

A deterministic analysis of the capital structure of an insurance company might proceed as follows.

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1. We shall use the terms "capital" and "surplus" interchangably to represent the owner's equity in the insurance company.

Let: $P$ = risk premium (or expected loss);

$L$ = security (or profit) loading;

$U$ = initial surplus; and

$i_u$ = interest rate earned on the surplus.

The expected rate of return on the owner's equity, $i$, satisfies the following equation:

$$U \cdot i = P \cdot L + U \cdot i_u. \tag{1.1}$$

Fix $P$, $L$ and $i_u$. It is easily seen that lowering $U$ will increase the rate of return, $i$. There are two forces which limit how low $U$ will go. First, the rate of return may get sufficiently high to attract more capital. For example let:

$P$ = \$20,000,000;

$L$ = .025; and

$i_u$ = .06.

Suppose the competitive rate of return is found to be $i = 12\%$ , we can solve Equation 1.1 for $U$ = \$8,333,333. If the surplus were to go less than \$8,333,333 then new capital would be attracted to the insurance company. Conversely, if the surplus were to go above \$8,333,333 the owners could invest the excess surplus elsewhere and obtain a greater return on their investment.

A second limiting force is that of regulation. Regulators are interested in assuring that the insurance company can fulfill its obligation to the policyholders. Putting a lower bound on U will help accomplish this purpose. However, it should be pointed out that this action is not without cost to the policyholders. Suppose, in the above example, the regulator decides to require a surplus of $9,333,333. If the competitive rate of return remains at 12%, the insurance company will be forced to raise its profit loading, L. Solving Equation 1.1 gives L = .028. Raising U by $1,000,000 will cost the policyholders $60,000.

While this analysis captures some essential points of insurance company operations, there are many other factors that should be considered. These factors include the following.

1. An insurance company is an ongoing operation.

2. The amount paid for claims varies from year to year.

3. The insurance industry is very competitive. The profit loading varies from year to year in a fashion described as the "underwriting cycle."

4. The ultimate claim cost is not determined at the end of the policy year. The result is uncertainty in the liabilities, and hence the surplus of the insurance company.

This paper analyzes the effect these factors will have on the capital structure of an insurance company. The analysis will consider the same questions as the deterministic analysis given above, namely: what surplus will give a competitive rate of return to the insurance company owners; and what is the cost to policyholders of minimum surplus regulation?

We begin with a model which describes how claim amounts vary.

## 2. The Collective Risk Model

We shall use the collective risk model to describe the incurred losses, $X_t$ in year t. This model assumes separate claim severity distributions and claim count distributions for each line of insurance written by the insurer. We shall use the version of the model described by Heckman and Meyers [1983] and Meyers and Schenker [1983].

This version of the model can be described by the following algorithm.

1. Select $\beta$ at random from an inverse gamma distribution with $E[1/\beta] = 1$ and $Var[1/\beta] = b$
2. For each line of insurance, k, do the following.
   - 2.1 Select $\chi$ at random from a gamma distribution with $E[\chi] = 1$ and $Var[\chi] = c_k$.
   - 2.2 Select a random number of claims, N, from a Poisson distribution with mean $\chi \cdot \lambda_k$.
   - 2.3 Select N claims at random from the claim severity distribution for line of insurance k.
3. Set $X_t$ equal to the sum of all claims selected in step 2, multiplied by $\beta$.

The parameter $c_k$, called the contagion parameter, is a measure of uncertainty in our estimate of the expected claim count, $\lambda_k$, for line k. The parameter b, called the mixing parameter, is a measure of uncertainty of the scale of the claim severity distributions. Note that the *random* scaling factor, $\beta$, acts on all claim severity distributions simultaneously.

For demonstration purposes we have selected a comparatively small insurance company writing a single line of insurance. The claim severity distribution is a Pareto distribution with c.d.f.

$$S(z) = 1 - (a/(a+z))^{\alpha}$$

where a = 10000 and $\alpha$ = 2. Each claim is subject to a $500,000 limit.

The expected number of claims, $\lambda$, is set equal to 2039.544. The parameters b and c are set equal to 0 and .04 respectively. The resulting risk premium for this insured is $20,000,000.

Exhibit I shows the resulting aggregate loss distribution as calculated by the Heckman/Meyers algorithm [1983].

We will refer to this example as the ABC Insurance Company in what follows.

## 3. The Distribution of Surplus

We will view the insurance company as an ongoing operation. It collects premiums, pays claims, pays dividends to the owners (or stockholders). Occasionally, the owners will be required to contribute additional capital in order to maintain the surplus at a level specified by the regulator.

The financial status of an insurance company is usually measured at year end. Accordingly, a discrete treatment of financial results is assumed, i.e. the state of a company's finances will be calculated at time $t = 0,1,2,...$ where $t$ is in years.

Let: $P$ = risk premium (assumed constant for all years);

$L_t$ = security loading for year $t$;

$X_t$ = incurred loss during year $t$;

$D_t$ = stockholder dividends paid at the end of year $t$;

$R_t$ = additional capital contributed at the end of year $t$;

$U_t$ = surplus at the end of year $t$; and

$i_u$ = interest rate (assumed constant) earned on the surplus.

Our model of insurance company operations can be described as follows. Given the surplus, $U_{t-1}$, define the random variable $V_t$ by

$$V_t = U_{t-1} \cdot (1+i_u) + P \cdot (1+L_t) - X_t.$$

Let Umax be the maximum surplus and Umin be the minimum surplus determined by the insurance company management and/or regulators. Then we define:

$$D_t = \max(V_t - Umax, 0);$$

$$R_t = \max(Umin - V_t, 0); \text{ and thus}$$

$$U_t = V_t - D_t + R_t.$$

This model is similar to that described by Beard, Pentikäinen and Pesonen [1984, p. 215].

Let $F_t(v)$ be the c.d.f. for $V_t$. Let $M = Umax\cdot(1+i_u) + P\cdot(1+L_t)$. M represents the maximum value of $V_t$. $F_t(v) = 1$ for $v \geq M$.

Let $u_t$, $d_t$, and $r_t$ represent the expected values of the surplus, $U_t$, the dividend, $D_t$ and the additional contributed capital, $R_t$ at time t respectively. We have:

$$u_t = \int_{Umin}^{Umax} v\cdot dF_t(v) + Umax\cdot(1-F_t(Umax)) + Umin\cdot F_t(Umin);$$

$$d_t = \int_{Umax}^{M} (v-Umax)\cdot dF_t(v); \text{ and}$$

$$r_t = \int_{-\infty}^{Umin} (Umin-v)\cdot dF_t(v).$$

Note that $u_t + d_t - r_t = E[V_t]$.

The requirement that additional capital be contributed applies even when the surplus is negative. It is possible for reinsurance companies or guaranty funds to contribute money to raise the surplus to 0. Cummins [1986] and Meyers [1987b] discuss some ways to price this reinsurance.

Also of interest are $p_t(D)$, the probability of receiving a dividend, and $p_t(R)$, the probability that additional capital will be required. These probabilities are calculated for each year t.

Some notes on the history of this operating strategy is in order. This dividend paying strategy originated in the risk theory literature in a paper by de Finetti [1957]. It has been discussed by Bühlmann [1970, p. 164] and Borch [1974, p. 225]. A more general version of this strategy has been discussed by Tapiero, Zuckerman and Kahane [1983]. They insert an additional level, Ulong, between Umin and Umax. When $V_t$ goes above Ulong then the amount, $V_t$ - Ulong, is put into long term investments.

Meyers [1986] addresses the same questions addressed by this paper with an operating strategy that does not require the contribution of additional capital.

## 4. Yield Rates

The yield rate of an investment is defined to be the interest rate at which the present value of the investments is equal to the present value of the returns.

Let T be the investor's time horizon. The investments consist of the initial surplus at time zero and the additional contributions to surplus at each time t. The returns consist of dividends payable at each time t, and the average surplus at time T. Any yield rate calculation must, of course, reflect the probability that the payments are actually made.

Let i be the yield rate. The yield rate must satisfy the following equation:

$$u_0 + \sum_{t=1}^{T} r_t \cdot (1+i)^{-t} = \sum_{t=1}^{T} d_t \cdot (1+i)^{-t} + u_T \cdot (1+i)^{-T}.$$

This equation can be solved for i by the Newton-Raphson method.

The methodology described above has been incorporated into a computer program called the "Insurer Surplus Model." Meyers [1987a] describes the mathematical techniques used in this program. This program makes repeated use of the Heckman/Meyers algorithm.

Let us now consider the case of ABC Insurance Company. We make the following (debatable) assumptions.

1. The investors in ABC Insurance Company are risk neutral.

2. The investors in ABC Insurance Company can easily shift their capital investments to seek the highest rate of return.

Suppose that the regulators require a minimum surplus of \$6,000,000, and that the market/regulators allow a security loading of .025. Suppose further that $i_u$ = .06 and the investors select a time horizon of T=25 years. The company management calculates the following yields for varying levels of initial surplus (= maximum surplus).

Table 1

| Surplus | Yield |
|---|---|
| \$12,000,000 | 10.80% |
| 10,000,000 | 11.66 |
| 8,000,000 | 12.79 |

To continue our example, let us suppose that the yield on alternative investments is 12% for T=25. It is a consequence of the above assumptions that the investors in ABC Insurance Company will adjust the surplus until a 12% yield is obtained. Thoughtful trial and error quickly gives an initial (= maximum) surplus of \$9,330,000. Note the the yield does vary with the time horizon, T, selected. The output the Insurer Surplus Model for this initial surplus is given in the following table.

Table 2
Insurer Surplus Model
Standard Assumptions

| t | $p_t(R)$ | $r_t$ | $u_t$ | $d_t$ | $p_t(D)$ | $L_t$ | Yield |
|---|---|---|---|---|---|---|---|
| 1 | 0.14518 | 371690 | 8501385 | 2260106 | 0.62482 | 0.02500 | 11.36 |
| 2 | 0.20393 | 580225 | 8256856 | 1834837 | 0.54171 | 0.02500 | 11.59 |
| 3 | 0.22181 | 644846 | 8183506 | 1713608 | 0.51713 | 0.02500 | 11.72 |
| 4 | 0.22719 | 664276 | 8161486 | 1677306 | 0.50976 | 0.02500 | 11.80 |
| 5 | 0.22880 | 670109 | 8154875 | 1666409 | 0.50754 | 0.02500 | 11.85 |
| 6 | 0.22928 | 671860 | 8152891 | 1663137 | 0.50688 | 0.02500 | 11.88 |
| 7 | 0.22943 | 672386 | 8152295 | 1662155 | 0.50668 | 0.02500 | 11.91 |
| 8 | 0.22947 | 672544 | 8152116 | 1661860 | 0.50662 | 0.02500 | 11.92 |
| 9 | 0.22949 | 672591 | 8152062 | 1661772 | 0.50660 | 0.02500 | 11.94 |
| 10 | 0.22949 | 672605 | 8152046 | 1661745 | 0.50660 | 0.02500 | 11.95 |
| 11 | 0.22949 | 672610 | 8152041 | 1661737 | 0.50659 | 0.02500 | 11.96 |
| 12 | 0.22949 | 672611 | 8152040 | 1661735 | 0.50659 | 0.02500 | 11.96 |
| 13 | 0.22949 | 672611 | 8152040 | 1661734 | 0.50659 | 0.02500 | 11.97 |
| 14 | 0.22949 | 672611 | 8152039 | 1661734 | 0.50659 | 0.02500 | 11.98 |
| 15 | 0.22949 | 672611 | 8152039 | 1661734 | 0.50659 | 0.02500 | 11.98 |
| 16 | 0.22949 | 672611 | 8152039 | 1661734 | 0.50659 | 0.02500 | 11.98 |
| 17 | 0.22949 | 672611 | 8152039 | 1661734 | 0.50659 | 0.02500 | 11.99 |
| 18 | 0.22949 | 672611 | 8152039 | 1661734 | 0.50659 | 0.02500 | 11.99 |
| 19 | 0.22949 | 672611 | 8152039 | 1661734 | 0.50659 | 0.02500 | 11.99 |
| 20 | 0.22949 | 672611 | 8152039 | 1661734 | 0.50659 | 0.02500 | 11.99 |
| 21 | 0.22949 | 672611 | 8152039 | 1661734 | 0.50659 | 0.02500 | 11.99 |
| 22 | 0.22949 | 672611 | 8152039 | 1661734 | 0.50659 | 0.02500 | 12.00 |
| 23 | 0.22949 | 672611 | 8152039 | 1661734 | 0.50659 | 0.02500 | 12.00 |
| 24 | 0.22949 | 672611 | 8152039 | 1661734 | 0.50659 | 0.02500 | 12.00 |
| 25 | 0.22949 | 672611 | 8152039 | 1661734 | 0.50659 | 0.02500 | 12.00 |

One does not need the Insurer Surplus Model to find the yield for T=1.

$$u_0 + r_1/(1+i) = (u_1+d_1)/(1+i) \quad \Rightarrow$$

$$(1+i)\cdot u_0 = u_1+d_1-r_1 = E[V_1] \quad \Rightarrow$$

$$i = E[V_1]/u_0 - 1.$$

Now: $E[V_1] = u_0(1+i_u) + P\cdot L_1.$

Thus: $$i = i_u + P\cdot L_1/u_0. \tag{4.1}$$

Note that Equation 4.1 can also be derived from Equation 1.1.

## 5. Uncertainty in Loss Reserves

The time t=0 does not have to be the date the insurance company begins operation. The old advertising jingle "Today is the first day of the rest of your life" applies also to insurance companies. Applying the above approach to an ongoing insurance company presents a special problem which is discussed here.

Probably the largest and most uncertain liability for a property and casualty insurance firm is the loss reserve. This creates uncertainty in the initial surplus, $U_0$. We attempt to model this by making the additional assumption:

$U_0$ has a normal distribution with known mean and variance.

The debate concerning the variability of loss reserves has taken on new life within the last few years. Publications by the Casualty Actuarial Society Committee on the Theory of Risk [1986], De Jong and Zehnwirth [1983] and Taylor [1986] deal with this problem extensively. Even so, the author considers the problem far from being solved.

In our example, the ABC Insurance Company, we will use $1,790,035 as the standard deviation of the loss reserve i.e. the initial surplus. This figure was derived from the following assumptions.

1. The claim severity distribution is known.

2. Claims are paid out over a period of eight years. The paid to ultimate ratios are .05, .20, .40, .60, .75, .90, .96 and 1.00 respectively.

3. The smallest claims are settled first.

The details of this derivation are in the appendix.

Using the Insurer Surplus we calculate that a value of $9,340,000 for $u_0$ and Umax will result in a yield of 12% if all other inputs remain the same. Table 3 contains the output.

Table 3
Insurer Surplus Model
Uncertain Initial Surplus

| t | $p_t(R)$ | $r_t$ | $u_t$ | $d_t$ | $p_t(D)$ | $L_t$ | Yield |
|---|---|---|---|---|---|---|---|
| 1 | 0.16524 | 458453 | 8444587 | 2414266 | 0.61053 | 0.02500 | 11.35 |
| 2 | 0.20838 | 596836 | 8244475 | 1803622 | 0.53504 | 0.02500 | 11.61 |
| 3 | 0.22284 | 648744 | 8184631 | 1703257 | 0.51494 | 0.02500 | 11.74 |
| 4 | 0.22722 | 664559 | 8166623 | 1673646 | 0.50893 | 0.02500 | 11.81 |
| 5 | 0.22854 | 669323 | 8161202 | 1664741 | 0.50712 | 0.02500 | 11.86 |
| 6 | 0.22893 | 670757 | 8159570 | 1662061 | 0.50658 | 0.02500 | 11.89 |
| 7 | 0.22905 | 671188 | 8159079 | 1661254 | 0.50641 | 0.02500 | 11.91 |
| 8 | 0.22909 | 671318 | 8158931 | 1661011 | 0.50636 | 0.02500 | 11.93 |
| 9 | 0.22910 | 671357 | 8158886 | 1660938 | 0.50635 | 0.02500 | 11.94 |
| 10 | 0.22910 | 671369 | 8158873 | 1660916 | 0.50634 | 0.02500 | 11.95 |
| 11 | 0.22910 | 671373 | 8158869 | 1660909 | 0.50634 | 0.02500 | 11.96 |
| 12 | 0.22910 | 671374 | 8158868 | 1660907 | 0.50634 | 0.02500 | 11.97 |
| 13 | 0.22910 | 671374 | 8158867 | 1660907 | 0.50634 | 0.02500 | 11.97 |
| 14 | 0.22911 | 671374 | 8158867 | 1660906 | 0.50634 | 0.02500 | 11.98 |
| 15 | 0.22911 | 671374 | 8158867 | 1660906 | 0.50634 | 0.02500 | 11.98 |
| 16 | 0.22911 | 671374 | 8158867 | 1660906 | 0.50634 | 0.02500 | 11.98 |
| 17 | 0.22911 | 671374 | 8158867 | 1660906 | 0.50634 | 0.02500 | 11.99 |
| 18 | 0.22911 | 671374 | 8158867 | 1660906 | 0.50634 | 0.02500 | 11.99 |
| 19 | 0.22911 | 671374 | 8158867 | 1660906 | 0.50634 | 0.02500 | 11.99 |
| 20 | 0.22911 | 671374 | 8158867 | 1660906 | 0.50634 | 0.02500 | 11.99 |
| 21 | 0.22911 | 671374 | 8158867 | 1660906 | 0.50634 | 0.02500 | 12.00 |
| 22 | 0.22911 | 671374 | 8158867 | 1660906 | 0.50634 | 0.02500 | 12.00 |
| 23 | 0.22911 | 671374 | 8158867 | 1660906 | 0.50634 | 0.02500 | 12.00 |
| 24 | 0.22911 | 671374 | 8158867 | 1660906 | 0.50634 | 0.02500 | 12.00 |
| 25 | 0.22911 | 671374 | 8158867 | 1660906 | 0.50634 | 0.02500 | 12.00 |

This example suggests that the uncertainty in loss reserves has little effect on surplus levels from the investor's point of view. More will said about this later.

## 6. The Underwriting Cycle

We now consider the case when the security loading varies from year to year in a cyclic maner. This is a well established phenomenon in casualty insurance which is felt, at least by the author, to be caused by intense competition from within the insurance industry. Berger [1986] proposes a model whereby the underwriting cycle results from (1) the desire to maximize profits and (2) aversion to bankruptcy.

To model the underwriting cycle we assume that

$$L_t = L_0 + A \cdot \sin(\omega \cdot (t-1) + \phi).$$

This is a special case of the AR(2) model considered by Beard, Pentikainen and Pesonen [1984, p. 202 and p. 388] for cyclic variation.

To demonstrate the effects of the underwriting cycle on the ABC Insurance Company we set $L_0 = .025$, $A = .02394$ and $\omega = \pi/4$. These parameters will produce an eight year cycle with a reasonable amount of variation..

We first consider what happens when we catch the cycle on the way up. If we set $\phi = 0$ along with the assumptions stated immediately above and in Section 4, we calculate that a value of $10,600,000 for $u_0$ and Umax will result in a yield of 12%. The results of the Insurer Surplus Model for this case are in Table 4.

Table 4
Insurer Surplus Model
Underwriting Cycle on the Way Up

| t | $p_t(R)$ | $r_t$ | $u_t$ | $d_t$ | $p_t(D)$ | $L_t$ | Yield |
|---|---|---|---|---|---|---|---|
| 1 | 0.09049 | 215329 | 9643356 | 2307973 | 0.63153 | 0.02500 | 10.72 |
| 2 | 0.13201 | 355329 | 9381137 | 2034662 | 0.56686 | 0.04193 | 12.50 |
| 3 | 0.14135 | 387016 | 9323605 | 1986147 | 0.55316 | 0.04894 | 13.58 |
| 4 | 0.15099 | 418743 | 9258197 | 1882081 | 0.53551 | 0.04193 | 13.85 |
| 5 | 0.17115 | 486594 | 9124016 | 1676267 | 0.49982 | 0.02500 | 13.50 |
| 6 | 0.19766 | 579358 | 8956122 | 1456180 | 0.45728 | 0.00807 | 12.85 |
| 7 | 0.21712 | 649582 | 8838445 | 1325894 | 0.42894 | 0.00106 | 12.23 |
| 8 | 0.21700 | 648298 | 8839775 | 1338762 | 0.42969 | 0.00807 | 11.89 |
| 9 | 0.19766 | 577898 | 8958231 | 1489828 | 0.45896 | 0.02500 | 11.87 |
| 10 | 0.17253 | 490525 | 9118200 | 1706563 | 0.49970 | 0.04193 | 12.07 |
| 11 | 0.15641 | 436580 | 9224283 | 1856320 | 0.52740 | 0.04894 | 12.29 |
| 12 | 0.15683 | 438201 | 9220174 | 1834282 | 0.52578 | 0.04193 | 12.40 |
| 13 | 0.17355 | 494854 | 9108970 | 1659268 | 0.49611 | 0.02500 | 12.37 |
| 14 | 0.19868 | 582988 | 8949991 | 1449991 | 0.45582 | 0.00807 | 12.23 |
| 15 | 0.21755 | 651133 | 8835919 | 1323475 | 0.42835 | 0.00106 | 12.08 |
| 16 | 0.21718 | 648916 | 8838742 | 1337734 | 0.42944 | 0.00807 | 11.98 |
| 17 | 0.19773 | 578126 | 8957820 | 1489373 | 0.45886 | 0.02500 | 11.97 |
| 18 | 0.17256 | 490607 | 9118042 | 1706367 | 0.49966 | 0.04193 | 12.03 |
| 19 | 0.15642 | 436610 | 9224224 | 1856242 | 0.52739 | 0.04894 | 12.11 |
| 20 | 0.15684 | 438213 | 9220151 | 1834253 | 0.52577 | 0.04193 | 12.15 |
| 21 | 0.17355 | 494859 | 9108961 | 1659258 | 0.49610 | 0.02500 | 12.14 |
| 22 | 0.19868 | 582990 | 8949987 | 1449987 | 0.45582 | 0.00807 | 12.09 |
| 23 | 0.21755 | 651134 | 8835917 | 1323473 | 0.42835 | 0.00106 | 12.04 |
| 24 | 0.21718 | 648916 | 8838741 | 1337733 | 0.42944 | 0.00807 | 12.00 |
| 25 | 0.19773 | 578126 | 8957820 | 1489372 | 0.45886 | 0.02500 | 12.00 |

Let us next consider what happens when we catch the cycle on the way down. If we set $\phi = \pi$ along with the assumptions stated immediately above and in Section 4, we calculate that a value of \$7,975,000 for $u_0$ and Umax will result in a yield of 12%. The results of the Insurer Surplus Model for this case are in Table 5.

Table 5
Insurer Surplus Model
Underwriting Cycle on the Way Down

| $t$ | $p_t(R)$ | $r_t$ | $u_t$ | $d_t$ | $p_t(D)$ | $L_t$ | Yield |
|---|---|---|---|---|---|---|---|
| 1 | 0.22753 | 636215 | 7380114 | 2209600 | 0.61761 | 0.02500 | 12.27 |
| 2 | 0.30233 | 922729 | 7214041 | 1693095 | 0.52639 | 0.00807 | 10.41 |
| 3 | 0.32807 | 1026142 | 7158032 | 1536263 | 0.49623 | 0.00106 | 9.20 |
| 4 | 0.32173 | 1000594 | 7171898 | 1577697 | 0.50374 | 0.00807 | 9.00 |
| 5 | 0.29409 | 891738 | 7232678 | 1761271 | 0.53683 | 0.02500 | 9.64 |
| 6 | 0.26412 | 778592 | 7299746 | 1983999 | 0.57398 | 0.04193 | 10.64 |
| 7 | 0.24901 | 723287 | 7334002 | 2105747 | 0.59320 | 0.04894 | 11.49 |
| 8 | 0.25610 | 748834 | 7317772 | 2043617 | 0.58400 | 0.04193 | 11.95 |
| 9 | 0.28192 | 844836 | 7259561 | 1842113 | 0.55153 | 0.02500 | 12.01 |
| 10 | 0.31286 | 964985 | 7191167 | 1630440 | 0.51409 | 0.00807 | 11.81 |
| 11 | 0.33011 | 1034457 | 7153645 | 1524718 | 0.49389 | 0.00106 | 11.55 |
| 12 | 0.32211 | 1002136 | 7171065 | 1575421 | 0.50329 | 0.00807 | 11.42 |
| 13 | 0.29416 | 892006 | 7232524 | 1760810 | 0.53674 | 0.02500 | 11.47 |
| 14 | 0.26413 | 778637 | 7299718 | 1983909 | 0.57396 | 0.04193 | 11.66 |
| 15 | 0.24901 | 723295 | 7333997 | 2105730 | 0.59319 | 0.04894 | 11.86 |
| 16 | 0.25610 | 748835 | 7317771 | 2043614 | 0.58400 | 0.04193 | 11.98 |
| 17 | 0.28192 | 844836 | 7259561 | 1842112 | 0.55153 | 0.02500 | 12.00 |
| 18 | 0.31286 | 964985 | 7191166 | 1630440 | 0.51409 | 0.00807 | 11.94 |
| 19 | 0.33011 | 1034457 | 7153645 | 1524718 | 0.49389 | 0.00106 | 11.85 |
| 20 | 0.32211 | 1002136 | 7171065 | 1575421 | 0.50329 | 0.00807 | 11.81 |
| 21 | 0.29416 | 892006 | 7232524 | 1760810 | 0.53674 | 0.02500 | 11.82 |
| 22 | 0.26413 | 778637 | 7299718 | 1983909 | 0.57396 | 0.04193 | 11.88 |
| 23 | 0.24901 | 723295 | 7333997 | 2105730 | 0.59319 | 0.04894 | 11.95 |
| 24 | 0.25610 | 748835 | 7317771 | 2043614 | 0.58400 | 0.04193 | 11.99 |
| 25 | 0.28192 | 844836 | 7259561 | 1842112 | 0.55153 | 0.02500 | 12.00 |

## 7. Ruin Theory

Thus far our assumption has been that the investors in an insurance company will adjust the surplus so that the expected yield will be constant. An alternative to this assumption is provided by ruin theory. Ruin theory[2] makes the assumption that the investors in an insurance company will adjust the surplus so that the *probability of insolvency* (i.e. *the probability of ruin*) will remain constant. In this section we shall demonstrate that the two assumptions imply quite different results.

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2. See, for example, Beard, Pentikäinen and Pesonen [1984 ch. 4].

It is sufficient to consider the probability of ruin for a one year time span. Let $\epsilon$ be the selected probability of ruin. We have:

$$\Pr\{U_1 < 0\} = \epsilon \text{ if and only if } u_0(1+i_u) + P(1+L_1) = x_{1-\epsilon},$$

where $x_{1-\epsilon}$ is the $1-\epsilon^{th}$ percentile of the random loss X. If $\epsilon$ is fixed, it can be seen that a reduction in $L_1$ should be accompanied by a corresponding increase in $u_0$, and conversely an increase in $L_1$ should be accompanied by a corresponding decrease in $u_0$.

Equation 4.1 indicates the opposite behavior. If i is fixed, it can be seen that $L_1$ and $U_0$ move in the same direction. This behavior also holds in the multiyear analysis of the underwriting cycle. If the cycle is on the way down, Umax also goes down and the insurance company's surplus is reduced. The opposite happens when the cycle is on the way up.

The two assumptions have different implications when we consider uncertainty in loss reserves. It was demonstrated in the example above that uncertainty in the loss reserves has little effect on the surplus. The surplus raises from $9,330,000 to $9,340,000. Suppose we are satisfied with the probability of ruin for the standard assumptions (Table 2). Using the Insurer Surplus Model with Umin = 0, we calculate that the probability of ruin after one year is .0152. If the standard deviation of the loss reserve is $1,790,035, as in Table 3, it requires a surplus of $10,045,000 to maintain the probability of ruin of .0152 for the first year.

## 8. The Cost of Regulation

It is the regulator's job to impose standards which keep the number of insolvencies to a minimum. One way of doing this is to impose a minimum surplus so that the probability of ruin is acceptably low. It was demonstrated in the last section that such a regulatory strategy may not be in accordance with the wishes of insurance company owners.

The owners don't have any choice in the matter. The regulators set the standards and the insurance companies comply with them. A higher minimum standard will result in a higher level of surplus in the industry as a whole, and a higher profit loading will be demanded. The purpose of this section is find this additional cost of solvency regulation to insurance consumers.

Let us consider the example in Table 2. We will vary the minimum surplus and calculate the security loading that will result in a yield rate of 12% after 25 years. The results are in Table 6.

Table 6
The Cost of Regulation

| Minimum Surplus | Security Loading (%) | Security Loading ($) | Additional Security Loading |
|---|---|---|---|
| $6,000,000 | 2.500% | $500,000 | --- |
| 7,000,000 | 2.583 | 516,600 | $16,600 |
| 8,000,000 | 2.673 | 534,600 | 18,000 |
| 9,000,000 | 2.767 | 553,400 | 18,800 |
| 10,000,000 | 3.000 | 600,000 | 46,600 |
| 11,000,000 | 3.300 | 660,000 | 60,000 |

Note that if the minimum surplus goes above $9,330,000 the minimum surplus becomes the maximum surplus, and the security loading can be obtained by solving Equation 1.1.

The changes in the market conditions brought on by increasing the minimum surplus are clearly more complex than is assumed by the above example. However, this may be an indication that the cost of regulation is small if the minimum surplus is not too high.

## 9. Concluding Remarks

This paper has attempted to analyze the capital structure of an insurance company in a way that:

(1) viewed the insurance company as an ongoing enterprise; and

(2) allowed for the stochastic nature of insurance business.

When one attempts a simple one year deterministic analysis as was done in the introduction, it is possible to comprehend the implications instantly. However, when given a complex computer program like the Insurer Surplus Model, the best one can do is try some examples and draw tentative conclusions. This paper represents one such attempt. The main conclusions are listed below.

1. The underwriting cycle has a major effect on the amount of capital that will be invested in an insurance company. For example, an insurance company should lower its surplus in the down part of the cycle. In our examples, the goal was to obtain an expected yield of 12% over a 25 year period. One should not view this strategy as being shortsighted.

2. The uncertainty in loss reserves has little effect from the

investor's point of view. However, it can have a substantial effect from the regulator's point of view.

3. Whether the investors like it or not, the regulators may require a minimum surplus. If this minimum is below what the investors would voluntarily allow, the cost to the policyholders is relatively small. As this regulatory minimum increases, the cost to the policyholders becomes substantial.

There are several items that should enter this analysis, but did not. A discussion of some of these items follows.

We assumed that that the investor would seek the same expected yield in all circumstances. One could reasonably argue that the investor should seek a higher yield when the surplus is low because of the increased variability of the return. This is debatable. It is unlikely that the investor would invest all his/her assets in a single enterprise, and so the investor's risk aversion should not be much of a factor. However, the author would like to keep the debate open.

The issue of asset risk has been omitted from this entire discussion. It could very well be as important as any of the items mentioned above. Any analysis of asset risk must include strategies for asset/liability matching. A good place for casualty actuaries to start would be the paper "Duration" by Ronald E. Ferguson [1983]. Further research needs to be done in order to integrate asset risk into the above approach for analyzing the capital structure of an insurance company.

Exhibit I — Collective Risk Model — ABC Insurance Company

| Line | Expected Loss | Claim Severity Distribution | Contagion Parameter | Claim Count Mean | Claim Count Std Dev |
|---|---|---|---|---|---|
| 1 | 20000000 | Pareto | 0.0400 | 2039.544 | 410.401 |

Mixing parameter 0.0000
Aggregate mean 20000000
Aggregate std dev 4147667

| Aggregate Loss Amount | Entry Ratio | Cumulative Probability | Excess Pure Premium | Excess Pure Premium Ratio |
|---|---|---|---|---|
| 10000000.00 | 0.5000 | 0.0018 | 10001341.98 | 0.5001 |
| 11000000.00 | 0.5500 | 0.0056 | 9004791.48 | 0.4502 |
| 12000000.00 | 0.6000 | 0.0143 | 8014234.53 | 0.4007 |
| 13000000.00 | 0.6500 | 0.0315 | 7036273.36 | 0.3518 |
| 14000000.00 | 0.7000 | 0.0608 | 6081225.87 | 0.3041 |
| 15000000.00 | 0.7500 | 0.1055 | 5162980.37 | 0.2581 |
| 16000000.00 | 0.8000 | 0.1669 | 4297760.83 | 0.2149 |
| 17000000.00 | 0.8500 | 0.2440 | 3501985.98 | 0.1751 |
| 18000000.00 | 0.9000 | 0.3333 | 2789781.74 | 0.1395 |
| 19000000.00 | 0.9500 | 0.4295 | 2170838.94 | 0.1085 |
| 20000000.00 | 1.0000 | 0.5268 | 1649158.71 | 0.0825 |
| 21000000.00 | 1.0500 | 0.6195 | 1222902.24 | 0.0611 |
| 22000000.00 | 1.1000 | 0.7033 | 885220.10 | 0.0443 |
| 23000000.00 | 1.1500 | 0.7756 | 625708.80 | 0.0313 |
| 24000000.00 | 1.2000 | 0.8350 | 432075.05 | 0.0216 |
| 25000000.00 | 1.2500 | 0.8821 | 291655.47 | 0.0146 |
| 26000000.00 | 1.3000 | 0.9180 | 192575.14 | 0.0096 |
| 27000000.00 | 1.3500 | 0.9444 | 124471.15 | 0.0062 |
| 28000000.00 | 1.4000 | 0.9632 | 78814.92 | 0.0039 |
| 29000000.00 | 1.4500 | 0.9762 | 48928.01 | 0.0024 |
| 30000000.00 | 1.5000 | 0.9849 | 29802.53 | 0.0015 |
| 31000000.00 | 1.5500 | 0.9907 | 17824.96 | 0.0009 |
| 32000000.00 | 1.6000 | 0.9943 | 10476.32 | 0.0005 |
| 33000000.00 | 1.6500 | 0.9966 | 6054.93 | 0.0003 |
| 34000000.00 | 1.7000 | 0.9980 | 3443.74 | 0.0002 |
| 35000000.00 | 1.7500 | 0.9989 | 1928.67 | 0.0001 |
| 36000000.00 | 1.8000 | 0.9994 | 1064.35 | 0.0001 |
| 37000000.00 | 1.8500 | 0.9996 | 579.14 | 0.0000 |
| 38000000.00 | 1.9000 | 0.9998 | 310.86 | 0.0000 |
| 39000000.00 | 1.9500 | 0.9999 | 164.68 | 0.0000 |
| 40000000.00 | 2.0000 | 0.9999 | 86.18 | 0.0000 |
| 41000000.00 | 2.0500 | 1.0000 | 44.57 | 0.0000 |

Appendix The Variability of Loss Reserves

In Section 5 we studied how the variability of loss reserves affected the surplus. We assumed that the loss reserves were normally distributed with a standard deviation of $1,790,035. In this appendix we show how the standard deviation was derived.

Three assumptions were made.

1. The claim severity distribution is known.

2. Claims are paid out over a period of eight years. The paid to ultimate ratios are .05, .20, .40, .60, .75, .90, .96 and 1.00 respectively.

3. The smallest claims are settled first.

We used the Pareto distribution for the claim severity. The c.d.f. is given by:

$$S(z) = 1 - (a/(a+z))^{\alpha}$$

with $a = 10000$ and $\alpha = 2$.

Let: $c(i)$ = maximum claim size settled in the $i^{th}$ prior year; and

$n(i)$ = number of claims remaining to be settled.

We have:

$$\frac{\int_0^{c(i)} z \cdot dS(z)}{E[Z]} = \text{paid to ultimate ratio for prior year } i\text{; and}$$

$$n(i) = (1 - S(c(i))) \cdot 2039.544.$$

Recall that 2039.544 is the annual expected number of claims for the ABC Insurance Company.

We then calculate the following values.

| i | c(i) | n(i) |
|---|---|---|
| 1 | 2844 | 1236 |
| 2 | 7947 | 633 |
| 3 | 16754 | 285 |
| 4 | 32912 | 111 |
| 5 | 60172 | 41 |
| 6 | 154844 | 8 |
| 7 | 276340 | 2 |

For prior year i, n(i) claims are selected at random from the claim severity distribution, S(z), conditioned on each claim being above c(i), The loss reserve is the total amount generated by this process. The distribution of loss reserves can be calculated by CRIMCALC, a computer program for the Heckman/Meyers algorithm. Exhibit II gives the output for CRIMCALC and Exhibit III shows that the distribution of loss reserves can be approximated by a normal distribution.

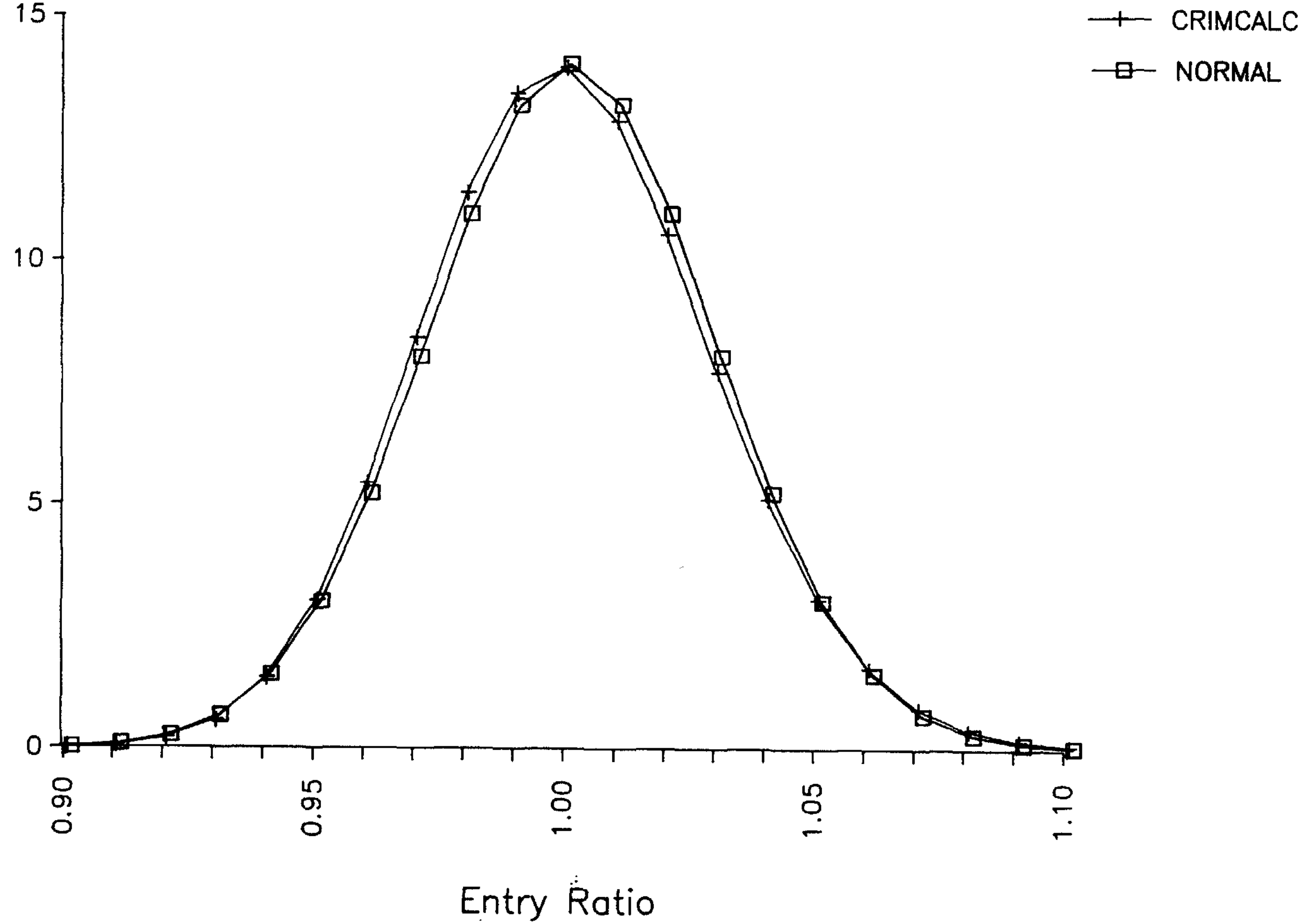
Exhibit II Collective Risk Model Reserve Risk

| Line | Expected Loss | Claim Severity Distribution | Contagion Parameter | Claim Count Mean | Claim Count Std Dev |
|---|---|---|---|---|---|
| 1 | 18991244 | Prior year 1 | -0.0008 | 1236.000 | 0.000 |
| 2 | 15991483 | Prior year 2 | -0.0016 | 633.000 | 0.000 |
| 3 | 12001336 | Prior year 3 | -0.0035 | 285.000 | 0.000 |
| 4 | 8018018 | Prior year 4 | -0.0090 | 111.000 | 0.000 |
| 5 | 4949249 | Prior year 5 | -0.0244 | 41.000 | 0.000 |
| 6 | 2131540 | Prior year 6 | -0.1250 | 8.000 | 0.000 |
| 7 | 803876 | Prior year 7 | -0.5000 | 2.000 | 0.000 |

Mixing parameter 0.0000
Aggregate mean 62886746
Aggregate std dev 1790035

| Aggregate Loss Amount. | Entry Ratio | Cumulative Probability | Excess Pure Premium | Excess Pure Premium Ratio |
|---|---|---|---|---|
| 56000000.00 | 0.8905 | 0.0000 | 6886755.06 | 0.1095 |
| 56500000.00 | 0.8984 | 0.0001 | 6386768.85 | 0.1016 |
| 57000000.00 | 0.9064 | 0.0002 | 5886823.18 | 0.0936 |
| 57500000.00 | 0.9143 | 0.0006 | 5387011.46 | 0.0857 |
| 58000000.00 | 0.9223 | 0.0019 | 4887592.52 | 0.0777 |
| 58500000.00 | 0.9302 | 0.0050 | 4389195.07 | 0.0698 |
| 59000000.00 | 0.9382 | 0.0117 | 3893164.53 | 0.0619 |
| 59500000.00 | 0.9461 | 0.0252 | 3402048.32 | 0.0541 |
| 60000000.00 | 0.9541 | 0.0492 | 2920121.58 | 0.0464 |
| 60500000.00 | 0.9620 | 0.0881 | 2453739.46 | 0.0390 |
| 61000000.00 | 0.9700 | 0.1452 | 2011241.67 | 0.0320 |
| 61500000.00 | 0.9779 | 0.2219 | 1602207.69 | 0.0255 |
| 62000000.00 | 0.9859 | 0.3162 | 1236084.24 | 0.0197 |
| 62500000.00 | 0.9939 | 0.4229 | 920494.91 | 0.0146 |
| 63000000.00 | 1.0018 | 0.5341 | 659738.75 | 0.0105 |
| 63500000.00 | 1.0098 | 0.6415 | 453969.36 | 0.0072 |
| 64000000.00 | 1.0177 | 0.7375 | 299305.76 | 0.0048 |
| 64500000.00 | 1.0257 | 0.8175 | 188784.16 | 0.0030 |
| 65000000.00 | 1.0336 | 0.8796 | 113786.09 | 0.0018 |
| 65500000.00 | 1.0416 | 0.9246 | 65488.00 | 0.0010 |
| 66000000.00 | 1.0495 | 0.9552 | 35975.26 | 0.0006 |
| 66500000.00 | 1.0575 | 0.9748 | 18860.86 | 0.0003 |
| 67000000.00 | 1.0654 | 0.9865 | 9438.13 | 0.0002 |
| 67500000.00 | 1.0734 | 0.9931 | 4509.60 | 0.0001 |
| 68000000.00 | 1.0813 | 0.9967 | 2059.08 | 0.0000 |
| 68500000.00 | 1.0893 | 0.9985 | 900.03 | 0.0000 |
| 69000000.00 | 1.0972 | 0.9993 | 377.99 | 0.0000 |
| 69500000.00 | 1.1052 | 0.9997 | 153.68 | 0.0000 |
| 70000000.00 | 1.1131 | 0.9999 | 61.63 | 0.0000 |

Exhibit III



## References


1. Beard, R.E., Pentikäinen, T. and Pesonen, E., *Risk Theory*, Chapman and Hall (1984)

2. Berger, Lawrence, "A Model of the Underwriting Cycle in the Property and Casualty Insurance Industry," submitted for publication (1986)

3. Borch, Karl, *The Mathematical Theory of Insurance*, D.C. Heath and Company, (1974)

4. Bühlmann, Hans, *Mathematical Methods of Risk Theory*, Springer-Verlag, (1970)

5. Casualty Actuarial Society Committee on the Theory of Risk, "Risk Theoretic Issue in Loss Reserving," Casualty Actuarial Society, (1986)

6. Cummins, J. David, "Risk Based Premiums for Guaranty Funds," *Proceedings of the International Conference on Insurer Solvency*, June 1986, (To be published)

7. De Jong, Piet and Zehnwirth, B. "Claims Reserving, State-Space Models and the Kalman Filter," *Journal of the Institute of Actuaries*, V. 110, (1983)

8. Ferguson, Ronald E., "Duration," *Proceedings of the Casualty Actuarial Society*, Vol LXX (1983)

9. Finetti, B. de, "Su un' impostazione alternativa della teoria collectiva del rischio," *Transactions of the International Congress of Actuaries*, 2(1957), p. 433.

10. Heckman, Philip E. and Meyers, Glenn G., "The Calculation of Aggregate Loss Distributions from Claim Severity Distributions and Claim Count Distributions," *Proceedings of the Casualty Actuarial Society*, Vol LXX (1983)

11. Meyers, Glenn, "Equilibrium in the Capital Structure of an Insurance Company," *Proceedings of the International Conference on Insurer Solvency*, June 1986, (To be published)

12. Meyers, Glenn, "The Distribution of Surplus via Finite Ruin Theory," In preparation, (1987a)

13. Meyers, Glenn, "The Pricing of Insolvency Insurance via Finite Ruin Theory," In preparation, (1987b)

14. Meyers, Glenn and Schenker, Nathaniel, "Parameter Uncertainty and the Collective Risk Model," *Proceedings of the Casualty Actuarial Society*, Vol LXX (1983)

15. Tapiero, Charles S., Zuckerman, Dror and Kahane, Yehuda, "Optimal Investment-Dividend Policy of an Insurance Firm under Regulation," *Scandanavian Actuarial Journal* (1983), p. 65.

16. Taylor, G.C., *Claims Reserving in Non-Life Insurance*, North Holland, (1986)