ALLOCATION OF SURPLUS FOR A MULTI-LINE INSURER

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Abstract

Various methods have been advanced for allocating policyholders' surplus to lines of insurance. While these methods could be powerful analytical tools, there are practical and theoretical problems that limit their usefulness. These problems are due to both the functions and nature of surplus and the nature of the decision-making processes that might use an allocation method. The author reviews some proposed allocation methods and develops practical considerations for an allocation method. None of the proposed allocation methods considered meet these criteria.

These criteria address only the practical concerns about allocation methods, not the theoretical ones. So even proposed allocation methods that meet these standards have theoretical hurdles to overcome. Alternative analytical approaches are proposed as replacements for allocation methods.

"...I come to bury Caesar, not to praise him."

- Julius Caesar, Act III, Scene ii.

An understanding of how an insurer's surplus supports its various operations would be a valuable tool for making business decisions, such as evaluating performance, analyzing the capital structure of the firm, and recognizing opportunity costs. This understanding is generally absent.

Yet, casualty actuaries are familiar with the concept of allocation. For example, indicated rate level changes are often allocated to classes or territories. So, it is surprising that a familiar tool, the allocation of the amounts in financial accounts, has not been applied to solve an important problem addressing property-casualty insurers. Perhaps the explanation for this failure to associate surplus with the different operations of the insurer lies in the difficulty of the task. It may be an impossible task. However, if an allocation is to be useful we should first consider in detail the question of

WHY ALLOCATE SURPLUS?

The surplus of an insurer is a finite good. The limitations to surplus prevent the insurer from writing greater volumes of business, or larger risks, or business that has an expectation of higher profits. Thus surplus has a value beyond the insurer's liquidation value. That value is the opportunity to earn additional profits by writing more insurance.

Open-market pricing of coverage reflects this value and calls it "underwriting profit margin", "surplus charge", or "risk load". If the amount of surplus directly associated with insurance contracts were known, the calculation of prices that recognize the value of that surplus could be explicit, and perhaps more accurate, consistent and stable.

A second beneficiary of an allocation of surplus would be insurer management. Since the use of surplus has an opportunity cost, and since various underwriters can commit the insurer's surplus by writing insurance contracts, management needs to measure their relative success. If surplus could be allocated to underwriting decision-making units, the use of surplus could be evaluated. The management of each unit could then be held accountable for the value of the use of the surplus that it had committed.

If an allocation of surplus could be made to lines and states, then regulators, insurer management, and guaranty fund authorities could examine how the amount of surplus allocated to different lines varied between insurers, in addition to measuring the absolute levels. This could provide important information about solvency. It could also indicate any lines where competitive markets were not effectively regulating prices.

An allocation of surplus is desirable for consumers, management, guarantors and regulators for the same reason. The specific underlying purpose of an allocation of surplus is to make informed economic decisions, such as pricing, that consider how each portion of the insurer's book restrict its ability to write other risks. This essay will address only this purpose of an allocation, and will not address the total amount of surplus that an insurer should have.

The actuary trying to do such an analysis may be encouraged by some trivial cases where a clear allocation of an insurer's surplus is possible. These cases include the one-line insurer, the over-capitalized insurer and the insolvent insurer.[1]

The important case, the case where an allocation of the insurer's surplus to portions of its writings can influence decisions, is the complement of these cases: the solvent insurer, whose writings have several distinct sections, and whose underwriting decisions are constrained by the amount of its surplus.

1/ The one-line insurer. An insurer whose writings are uniform in a dimension (such as all in one line of business, or all in one state, or all in one time period) can of course allocate all of its surplus to the one value that its writings take on in that dimension. Unfortunately, this allocation provides no information about how the surplus may support the separate parts of the insurer's writings along the other dimensions. A truly trivial insurer that writes one line of insurance, in one state, during one time period, etc. can fully allocate its surplus to that one cell. However the company and its regulators have no further economic decisions to make, except the basic question of solvency. Thus the allocation of surplus possible in the simple case of the mono-line insurer either leaves important questions unanswered, or addresses a company so simple that the trivial allocation which is possible is useless.

The over-capitalized insurer. If an insurer has enough surplus, relative to its writings and other exposures to loss, that its decisions are not affected by considerations of surplus, then an allocation of surplus is both possible and irrelevant. In fact, several very different allocations of the insurer's surplus are possible. Here also the allocation does not provide information useful in economic decision-making. And the question of solidity, by definition, is foregone.

The insolvent insurer. The insurer without surplus has only a trivial allocation to make.

This type of insurer, which we will call the multi-line insurer, is the subject addressed in this essay. Since the obligation to pay contractual obligations does not pass to the owners of a corporation, this study will address insurers on a company, not group, basis. We will refer to the collection of the insurer's expired and in-force policies as its book of business, and will refer to a set of identifiable, distinct sections of that book. The sections are mutually exclusive and exhaustive. Sections may be identified by type of risk or by peril, by producer or by other geographic or temporal defining characteristics (to be called dimensions), or by a combination of dimensions. Surplus, of course, refers to the amount of policyholders' surplus evaluated on a particular date. Finally, an allocation of a financial account refers to assigning dollar amounts to one or more sections of the insurer's book, with the total of the amounts allocated less than or equal to the total amount of the financial account. Now let's see why.

ALLOCATING SURPLUS MAY BE DIFFICULT TO DO.

Every multi-line insurer regularly and publicly makes allocations of various underwriting accounts to lines of insurance and states. These accounts include: written, earned and unearned premiums, premiums-in-force, and paid, reported and unreported losses and loss adjustment expenses, reinsurance recoveries, dividends, investment and other income, general and acquisition expenses, commissions, and licenses, taxes

and fees.[2] Insurers generally do not report an allocation of their surplus, even though that allocation would provide useful information. The reason may be that surplus is different, in very fundamental ways, from the items allocated to lines or states in the Annual Statement, IEE or SEC filings.

One difference between surplus and these other accounts is that surplus is subject to demands from every section of the book. A second difference is that accounts allocated in financial statements record the flow of funds either from accepting or discharging the obligation to provide insurance coverage, or from holding funds until the discharge of that obligation. This can be contrasted with the role of surplus, which is to act as a catalyst, not as an ingredient or a product of the insurance contract. Since the total amount of insurance coverage, as represented by the book of business, has been divided into sections, underwriting accounts can be allocated directly to the section of coverage from which they arose. Surplus cannot. Other items, like directors' compensation and fees for licenses, arise from providing coverage to the entire book. Allocation of these overhead accounts is needed so that premium charges can be adequate to cover these expenses in the future. This is not true of surplus.

2/ For example, in the Annual Statement Investment and Underwriting Exhibit, parts 2 and 3, Page 14, and Schedules O and P, and in the IEE, part 2, these accounts are allocated to lines of insurance. Virtually no other accounts are allocated to line in the Annual Statement or IEE.

The premiums and losses associated with a particular policy are well-defined concepts. So, the total premiums and losses of an insurer can be broken down into groups as small and as finely defined as the policies can be broken down. It can make sense to talk about a single policy with one hundred dollars of premiums for a one million dollar limit of coverage. Because of the law of large numbers, it may be possible to speak of twenty thousand of those policies with two million dollars in surplus. But it makes no sense to speak of a single policy with one hundred dollars of surplus and a one million dollar limit. There is a limit to how fine a breakdown of surplus is possible. But, even on broad groups, breaking surplus apart poses problems.

Consider a two-line insurer that expects to incur ten million dollars of losses each year in each of two independent, normally-distributed lines. The incurred losses in both lines have a standard deviation of five million dollars and the insurer has fourteen million dollars of surplus. The probability that losses exceed their expected amount by more than the surplus is found by observing that the standard deviation of the total losses will be the square root of two times the standard deviation of each line's losses, or very near seven million dollars. So the probability of exhausting the insurer's surplus is approximately the probability that a normal variate exceeds its mean by twice the standard deviation, or 2.3%.

However, one of two independent one-line insurers expecting ten million dollars of normally-distributed losses with standard deviations of five million dollars, and holding seven million dollars of surplus each, faces a different future. Either of these two insurers will exhaust its surplus

with the same probability as that of a standard normal variate exceeding its mean by 7/5. That probability is 8.1%.

The two-line insurer can, if it wishes allocate seven million dollars of its surplus to each line. However, it faces a probability of failure much smaller than either of the two one-line companies'. So the surplus allocated to a section of the book cannot be considered by itself; the insurer's entire operations must be considered. What then does the allocation mean? The author can't answer the question, but an actuary attempting to make a sensible allocation of surplus must answer this question before beginning. Apparently, the NAIC agrees that, at least for Financial Guarantee insurance, a simple paper allocation of surplus is not sufficient. To insulate other policyholders from the volatility of that coverage, NAIC model legislation requires a separate mono-line carrier to write these guarantees.

Having considered some theoretical difficulties in achieving a meaningful allocation, let us move on to examine some attempts by ratemakers and regulators at allocating surplus.

ALLOCATION OF SURPLUS.

One early attempt to make a pricing decision that explicitly reflects the value of surplus was the New Jersey Automobile Remand Decision in 1972. The Supreme Court asked then Commissioner of Insurance Robert Clifford to determine a fair profit provision for automobile insurance. Clifford determined both the fair return to the insurers' equity and the means of

calculating that equity. After extensive hearings, Clifford found that only underwriting profits and the investment income earned from investing "policyholder supplied funds" were to be considered in ratemaking. A total return from these sources of 6% was appropriate on the "needed surplus", but only 1% was appropriate for "surplus surplus."

Clifford held that for automobile insurance, surplus was needed only up to one-half of written premiums, the excess was "surplus surplus." Using this reasoning, and a one-to-one industry average premium-to-surplus ratio he allocated both "needed" and "surplus" surplus in proportion to written premiums. Averaging the 6% and 1% returns, he found that a 3.5% return from insurance operations was appropriate then.[3] In the years since the decision, Clifford's arithmetic has not been revised to reflect changes in either the industry's capital structure or profit levels outside the industry. The 3.5% operating profit margin has been used as a fixed target.[4]

This appears to have been the understanding of Clifford's decision from the first. Dineen, quoting ISO, "summed up Commissioner Clifford's decision" as:

[3/] R. E. Dineen, "An Early Look at the Decision in the New Jersey Remand Case" NAIC Proceedings, 1974, Volume II.

[4/] See, for example, recent N.J. private passenger rate filings.

"The effect of Commissioner Clifford's Determination is to establish a 3.5% after Federal Income Tax provision for underwriting profit from which investment income on policyholder-supplied funds, variable by line, must be deducted and that result then increased to a pre-Federal Income Tax basis for inclusion in the ratemaking formula."

Even though Clifford's allocation method has not attracted much attention, it is still worth examining. Clifford considered the surplus of the industry, but the technique is applicable to a single insurer.

$$S_i = (S/WP) \times WP_i \qquad (1)$$

The surplus allocated to a section of the insurer's book (S_i) is found by multiplying that section's net written premium (WP_i) by the ratio of the insurer's total surplus to total net written premiums.

This is shown for some groups of lines of insurance for 1985 industry totals in Table I.

Table I -- Surplus Allocated on Annual Written Premium

Line	1985 Written Premiums	Allocated December 31, 1985 Surplus
Auto Liability	$ 37,576,765	$18,388,661
Auto Physical Damage	25,519,959	12,488,512
Homeowners	14,473,884	7,082,977
Other Property	12,196,058	5,968,294
Workers' Compensation	19,263,729	9,426,947
Medical Professional	3,218,076	1,574,806
Other Liability	16,048,871	7,853,716
Miscellaneous	26,008,331	12,727,503
Total	$154,305,673	$75,511,417

Note: Figures are in thousands of dollars. Unless otherwise noted, all industry totals are taken from the 1986 edition of Best's Aggregates and Averages.

One practical problem with this allocation is that changing the relative rate levels of the different sections changes the allocation. The amount of surplus allocated to a section decreases when the rates are decreased. This is counter-intuitive.

An alternative that avoids this problem is to allocate surplus on accident year incurred losses and loss adjustment expenses. However, this method has the problem of accurately estimating incurred losses during the accident year. If that estimate could reliably be made, surplus would hardly be needed.

Both allocation methods have several other practical problems. Neither method actually considers how much surplus is needed to support a section of the insurer's book. For example, a line of insurance that is no longer written may take many years to run off. No surplus would be allocated to a line running off because it has no written premiums and no accident year losses. However the danger of adverse runoff still limits the insurer's capacity and should be reflected in any allocation of surplus. Several major insurers were acutely aware of this in the 1980's as they experienced significant adverse development on Medical Malpractice reserves but had stopped writing this line in the seventies[5]. Clearly, surplus funded the development, whether allocated to the line or not.

5/ Source: A. M. Best Standard Computer Tapes, Schedule P.

The opposite problem can occur in a rapidly growing section of the book. Too much surplus would be allocated there. Since these methods do not distinguish between different sections of the book to reflect special circumstances, such as the type of reinsurance, growth patterns, and reserve margins, they are extremely limited.

Either allocation formula relies on an annual flow (either premiums or losses) to allocate year-end surplus. This choice of a one-year history of the flow of funds is arbitrary. The choice also causes a peculiar factor in the allocation formula. A section's surplus is found in equation (1) by multiplying the section's annual written premium by the quantity (S/WP). S is valued in dollars, and WP is valued in dollars-per-year, so (S/WP) must be valued in years. This is a peculiar result to at least one actuary.[6]

Calculation based on these allocations may not even need to consider surplus. Consider a pricing methodology that uses a return-on-net-worth calculation as a target.

$$\mathbf{RONW}_i = P_i/S_i \qquad (2)$$

6/ Charles Niles' remarks at the 1984 Casualty Loss Reserve Seminar (CLRS proceedings, p. 901.) addressed the premium-to-surplus ratio as a measure of leverage, but they should apply here as well.

The return-on-net-worth is found by comparing profit to surplus for a section of the book. If we substitute equation (1) in this expression, we find

$$RONW_i = P_i/[(S/WP_i) \times WP_i] \quad (3)$$

$$\text{or,} \quad RONW_i = (P_i/WP_i) \times (WP/S) \quad (4)$$

The return on net worth is seen as a profit-to-premium ratio (or operating margin) times the insurer's premium-to-surplus ratio. Any decision that would use an amount of surplus found from one of these allocation methods to compare relative returns on surplus is really only considering the return on written premium or incurred losses, adjusted for a scaling factor.

Classical micro-economic theory can give rise to another allocation formula. When a finite, rational firm in perfect competition can produce several products, the firm maximizes its profits when it produces less of the products that yield a smaller marginal return on input, and more of the products that yield a higher marginal return. When the firm is in equilibrium, the marginal expected return on the constraining inputs will be equal for each product that is produced.[7]

To apply this theory to allocating the surplus of a multi-line insurer, four assumptions are needed:

1. The various sections of the book are priced and sold independently,
2. The insurer is in equilibrium,

7/ See Samuelson for a derivation.

3. Surplus is the only constraining factor of production, and

4. Each section's marginal premium-to-surplus requirements is equal to its average ratio. (This assumption is reasonable if the relative mix of writings among sections is independent of surplus levels.)

The substantial inaccuracies in the first three assumptions will limit the accuracy of any results.

Randall Brubaker showed[8] that if total profit is a function of product mix and there is one constraining input, then the ratio of marginal profit to the marginal amount of input required to produce the product is equal for all products. In the case of an insurer, the marginal profit for each section of the book is in a uniform ratio to the section's marginal premium-to-surplus ratio.

$$\frac{dP_1/dWP_1}{dS_1/dWP_1} = \frac{dP_2/dWP_2}{dS_2/dWP_2} = \cdots \qquad (5)$$

We have assumed that each section's marginal premium-to-surplus ratio is equal to the average ratio, so

$$\frac{dS_i}{dWP_i} = \frac{S_i}{WP_i} \qquad (6)$$

For this illustration, the marginal profit ratio for a section of the insurer's book (denoted as r) is arbitrarily assumed to be the operating profit reported on the IEE plus ten percent of written premium (representing fixed expenses) divided by written premium:

$$r_i = dP_i/dWP_i = \frac{P_i + (0.10 \times WP_i)}{WP_i} \qquad (7)$$

8/ "A Constrained Profit Maximization Model for a Multi-Line Property/Liability company", 1979 CAS Call Paper Program.

Algebraically, we find

$$\frac{r_1 \cdot WP_1}{S_1} = \frac{r_2 \cdot WP_2}{S_2} = \frac{\sum_i r_i \cdot WP_i}{S} \qquad (8)$$

and each S_i can be found from,

$$S_i = r_i \cdot WP_i \cdot S \,/\left(\sum_k r_k \cdot WP_k\right) \qquad (9)$$

That is, the section's surplus is proportional to the section's total marginal profit. This is shown in Table II.

Table II -- Surplus Allocated on Annual Marginal Profit

Line	1985 Annual Operating Profit	Adjusted Annual Marginal Profit	Allocated December 31, 1985 Surplus
Auto Liability	-$ 4,255,298	$ 2,890,235	$12,077,892
Auto Physical Damage	463,638	5,401,067	22,570,312
Homeowners	- 1,066,863	1,732,632	7,240,430
Other Property	270,416	2,381,766	9,953,071
Workers' Compensation	- 671,399	2,958,186	12,361,850
Medical Professional	- 759,422	48,314	201,898
Other Liability	- 2,929,845	96,922	405,024
Miscellaneous	- 2,463,049	2,560,731	10,700,941
Total	-$11,411,822	$18,069,853	$ 75,511,417

Note: Figures are in thousands of dollars. Since 1985 results were unprofitable, even on this estimated marginal basis, an adjustment to a stable basis is needed. The analysis arbitrarily reduced the loss ratios for Other Liability by fifteen percentage points, Medical Professional by twenty percentage points, and those of other lines by ten points.

While economists would apply this method to expected, not actual, profits, an explicit calculation of expected marginal profits is generally not available. Some of the practical problems noted with the first two allocation methods apply here as well. Not all exposures to loss are considered; the use of a one-year allocation base is arbitrary; and growth patterns are not considered. A line that is running off will have no significant expected marginal profits, but will have a large potential loss.

The method really does not need surplus in its considerations, either. In the same way that amount of written premiums could be used instead of surplus in the first method, amount of marginal profits can be used here.

However, the biggest problem with applying this method is stability. Profits of property casualty insurers are extremely volatile, even when positive!

A very different approach was taken by Robert Butsic. Butsic has developed a technique for considering the difference in riskiness between lines of insurance that uses "imputed equity values" to adjust for risk. In Branch Office Profit Measurement for Property-Liability Insurers (1985 CAS Call Paper Program), he wrote:

> "A suggested method for subjectively balancing risk for various product lines is:
>
> "1. Select a product line, say Commercial Multiple Peril, with an average perceived risk. Assign to it an arbitrary premium/equity ratio in the neighborhood of the long-term industry average premium/equity ratio for all lines; e.g., 2.5-to-1.
>
> "2. Select another line, compare it to the standard line (CMP) and set a premium/equity ratio at which you would be indifferent to writing this line compared to the standard line. For example, Fire (having a fast loss payout and a relatively complete pricing data base) at a 4-to-1 premium/equity ratio might be considered equally risky as CMP at 2.5-to-1.
>
> "3. Repeat the process for all applicable product lines. Of course, the method can be extended to sublines or even new types of insurance.
>
> "This procedure or one which actually attempts to measure the relative systematic risk ... will produce imputed equity values for each line based upon the respective premiums written. The aggregate all-lines imputed equity need not equal the "actual" equity reported externally, since our intent is to measure relative profitability between lines without having to be concerned about their different absolute levels of risk." [Emphasis in original.]

Butsic is using his technique to measure the relative success of profit centers. His calculations would assign as "imputed equity" amounts equal to 1/4 of each center's Fire premiums and 1/2.5 of its CMP premiums.

The calculation is really comparing profits to risk-adjusted premiums on the basis of these subjective weights. While risk-adjusted premiums are an excellent tool for gauging managers' performance, the concept of surplus is actually not needed here. Furthermore, like the other methods that used premiums in allocations, lines that are growing or shrinking may be misrepresented, changes in rate level can distort the allocations, and the method uses an arbitrary one-year period as its base.

Butsic suggests subjective risk adjustments. An objective quantified technique would prove to be difficult because the risk, as measured by the likely error in projecting losses, is not stable. This instability can be seen in recent actual results.

The ISO and the NAII jointly collect quarterly incurred losses and earned premiums from insurers that have elected to participate in the Fast Track Monitoring System. A time series of incurred losses and earned premiums from each quarter between the first quarter of 1975 and the second quarter of 1986 is available for a consistent set of insurers, for several lines of insurance[9]. The Fast Track System reports both losses from accidents in

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9/ The author wishes to thank Mr. John Pergola of Insurance Services Office, Inc. for providing the Fast Track data used in this analysis. Workers' Compensation results are taken from the A.M. Best quarterly underwriting results.

the current quarter, as well as changes in reserves for accidents in prior quarters. If surplus is to be allocated in proportion to the relative riskiness of the various lines, the unpredictability of these Fast Track losses can be used. To measure the error in projecting losses at various dates, the analysis divided the data into five-year periods ending in the fourth quarters of each year between 1979 and 1985, and in the second quarter of 1986. In each period, and for each line of insurance, a regression model developed using the SAS computer language calculated the total squared error in these models. The model for line #i in the period beginning in Quarter t_o:

Losses (Line #i, Quarter $t_o + t$) =

$$A_i + B_i \times t + C_i(t_o + t) + D_i \times \text{Premium(Line \#i, Quarter } t_o + t) \quad (10)$$

where A_i, B_i and D_i are fitted regression coefficients that vary by line. The $C_i(t_o + t)$ are fitted seasonality constants that take on one value if $t_o + t$ is the first quarter of a year and a different value if it is a second quarter, etc.

After SAS developed the total squared error in equation (10) for the eight time periods (results are shown in Table IIIA), Chi-Square statistics are developed and shown in Table IIIB. Since Workers' Compensation experience is at industry total levels, and Fast Track is based on a smaller sample, these results are only meaningful for comparisons between time periods and not between lines. Each line must be brought to a comparable level for that comparison.

An alternative presentation is shown in Table IIIC. There, December 31, 1985 surplus for the industry is allocated in proportion to the square root of the total squared error in equation (10) for each line of insurance, brought to 1985 industry loss volume levels as shown:

For the period beginning in quarter t_o, for line #i

$$\text{if} \quad U_i(t) = \sqrt{\sum_{t=t_o}^{t+19} (L_{i,t} - L^*_{i,t})^2} \tag{11}$$

$$\text{then} \quad S_i(t_o) = S \times \frac{\left[(L_i \times U_i(t_o)) \Big/ \sum_{t=t_o}^{t+19} L_{i,t} \right]}{\sum_{k=1}^{\#\text{ of lines}} \left[(L_k \times U_k(t_o)) \Big/ \sum_{t=t_o}^{t+19} L_{k,t} \right]} \tag{12}$$

Where $L_{i,t}$ refers to the actual incurred losses in the t-th quarter and $L^*_{i,t}$ refers to the estimate. L_i is the industry total losses incurred for 1985 from Best's Aggregates and Averages. S is December 31, 1985 industry surplus.

Table III-A -- Total Squared Error (in 10^{12} 's) From Regressing Quarterly Losses

	Time Period -- 20 Quarters Beginning							
Line	1/1/75	1/1/76	1/1/77	1/1/78	1/1/79	1/1/80	1/1/81	7/1/81
Auto Liability	2,183	3,871	12,606	5,927	9,130	10,458	5,959	5,936
Auto Physical Damage	2,474	6,710	2,287	2,126	6,406	6,546	5,082	5,557
Homeowners	1,501	3,714	3,162	4,489	5,368	9,418	7,810	11,945
Other Property	3,449	3,326	2,686	2,785	2,246	4,355	4,391	5,179
Workers' Compensation	51,639	43,249	35,550	37,083	157,624	336,807	287,964	219,313
Medical Professional	1,443	1,523	550	695	1,499	2,092	1,417	2,108
Other Liability	2,476	2,318	3,335	7,853	11,644	10,939	27,746	34,017
Miscellaneous	12,704	15,697	13,467	8,001	10,996	15,699	15,236	22,753

Table III-B -- Chi-Square Statistics From Regressing Quarterly Losses

Line	Time Period -- 20 Quarters Beginning							
	1/1/75	1/1/76	1/1/77	1/1/78	1/1/79	1/1/80	1/1/81	7/1/8
Auto Liability	0.00424	0.00515	0.00834	0.00516	0.00581	0.00568	0.00389	0.0036
Auto Physical Damage	0.00955	0.01458	0.00773	0.00661	0.01045	0.00963	0.00787	0.0080
Homeowners	0.01190	0.01663	0.01354	0.01398	0.01348	0.01616	0.01360	0.0165
Other Property	0.03140	0.02898	0.02413	0.02316	0.01981	0.02690	0.02695	0.0297
Workers' Compensation	0.00604	0.00481	0.00395	0.00385	0.00762	0.01044	0.00858	0.0073
Medical Professional	0.05507	0.04264	0.02292	0.02207	0.02919	0.03349	0.02835	0.0349
Other Liability	0.01861	0.01683	0.01833	0.02469	0.02591	0.02114	0.02689	0.0265
Miscellaneous	0.01958	0.01937	0.01613	0.01096	0.01139	0.01210	0.01053	0.0121

These results show substantial instability, as well as the problems noted with using one-year flows to allocate surplus.

Other allocation methods have avoided using one-year flows by examining ratios of surplus to unpaid losses, total reserves or total liabilities.[10] Table IV shows an allocation on loss reserves.

10/ See for example the 1984 New York Compensation Board Report to the New York Insurance Department, or Model D in Report of the NAIC Investment Income Task Force, Section III. The NYCB allocates surplus, first to fund any unearned underwriting losses, and the remainder in proportion to the sum of loss reserves and the expected loss component of the unearned premium reserves. The NAIC approach, which has been used in Texas, allocates surplus in proportion to estimated total liabilities.

Table III-C -- December 31, 1985 Surplus ($000's) Allocated on Unpredicability of Losses

Time Period -- 20 Quarters Beginning

Line	1/1/75	1/1/76	1/1/77	1/1/78	1/1/79	1/1/80	1/1/81	7/1/81
Auto Liability	$ 6,906,649	$ 7,982,392	$14,949,605	$10,605,507	$10,457,453	$ 9,670,360	$ 7,318,931	$ 6,552,944
Auto Physical Damage	8,702,880	12,638,699	7,755,077	7,599,107	10,513,919	9,169,970	8,294,501	8,031,665
Homeowners	6,539,381	8,697,385	8,185,686	9,686,180	8,186,675	9,276,336	8,638,232	9,895,271
Other Property	9,485,110	8,331,501	8,021,044	8,820,302	6,609,336	8,488,078	9,410,710	9,791,883
Workers' Compensation	4,796,212	3,637,168	3,454,903	3,855,124	6,680,872	8,659,635	7,872,335	6,360,699
Medical Professional	8,829,537	6,506,333	4,044,667	4,462,014	5,170,864	5,610,265	5,256,836	6,120,295
Other Liability	9,438,630	8,123,446	10,233,344	15,791,458	14,519,707	11,201,097	15,769,785	14,661,860
Miscellaneous	20,813,018	19,594,493	18,867,090	14,691,724	13,372,592	13,435,675	12,950,087	14,096,800

Table IV -- Surplus Allocated On Loss and Loss Expense Reserves

Line	December 31, 1985 Reserves	Allocated December 31, 1985 Surplus
Auto Liability	$ 40,583,226	$19,847,726
Auto Physical Damage	3,357,221	1,641,890
Homeowners	5,168,457	2,527,697
Other Property	3,510,750	1,716,975
Workers' Compensation	33,330,445	16,300,664
Medical Professional	10,074,143	4,926,884
Other Liability	26,123,563	12,776,050
Miscellaneous	32,252,599	15,773,530
Total	$154,400,404	$75,511,417

These methods have reversed the problems of growth and runoff that the other methods pose. A growing line presents an exposure to catastrophic loss that may be out of proportion to its reserves or other liabilities that arose earlier. A short-tailed line will be similarly under-represented even if the potential for very large losses is significant. In short, the volatility of a dollar of expected paid losses is not uniform among lines. Khury pointed this out for reserves in his paper, "Loss Reserves: Performance Standards" (PCAS, LXVII, 1980).

Clearly, the uncertainty in a dollar of windstorm unearned premiums, or casualty excess-of-loss loss adjustment expense reserves is higher than that of a dollar of automobile collision loss reserve. However, the principal difficulties in using these methods to make informed business decisions is the lack of support for these allocations. This is summarized in the NYCB Report.

"The hardest choices facing anyone wishing to evaluate insurance profits are those associated with returns on invested capital.

"(a) It is not possible to satisfactorily determine how much capital is associated with a particular group of policies.

"(b) It is even harder to determine how much capital should be associated with such a group of policies.

"(c) It is hardest of all to determine whether an observed rate of return on capital is satisfactory or not.

"...Any prospective method must deal with expected capital requirements and expected rates of return on that capital. Thus, the prospective approach adds its own set of significant estimation problems to the problems inherent in both methods." [Emphasis in original.]

Attempts to rationally allocate a risk margin to unpaid losses on the basis of expected variability cannot solve these problems. This is because the observed variability is neither constant in time, (as shown above) nor uniform among lines (as noted by Khury).

Without downplaying the specific problems of each at these allocation methods, there are general, practical problems that any method must address. Three problems deserve mention here. Sections of an insurer's book are often tied together in the marketplace, such as a piece of accommodation business written to obtain other, desirable business. In any adversarial decision there is an incentive to make the most favorable allocation, which may not be the most accurate. Finally, surplus is not in itself relevant. Equity is. While surplus is the key component of equity, pre-paid expenses, tax loss carry forwards, Schedule P excess reserves, non-authorized reinsurance, other non-admitted assets, and sunk costs such as training or software development costs have value and need to be reflected in any informed economic decisions, in addition to surplus. These other quantities are less subject to analysis.

Tables I, II, IIIc and IV each allocated the industry's surplus. The results, which are summarized in Table V, are substantially different. That is because, while each of these four methods reflected an allocation based on one variable that is associated with increased exposure, each method based its allocation on a different variable. Each method reflects some of the limitations on the insurers' capacity: none reflects all of them.

We have seen several practical concerns which would limit the usefulness of each allocation method for economic decision-making. But this is not to conclude that every allocation method must be impractical. However, before practical allocation methods can be discussed, there is a need to understand

THE FUNCTIONS OF SURPLUS.

Surplus serves only as a catalyst that allows coverage to be provided. There is no expectation that the surplus will be used up in providing the coverage. This is because any funds that are expected to be used to pay losses or expenses cease to exist as surplus and become loss and loss adjustment expense reserves as soon as the underpriced coverage is earned. How then does surplus act as a catalyst? By providing the guarantee that coverage can be provided if the total call on the insurer's assets are greater than the income. Three specific contingencies are often described in the literature. For example, in Insurance, Government and Social Policy (Edited by Dennenberg and Kimball),

Table V -- December 31, 1985 Surplus Allocated:

Line:	On 1985 Written Premiums	On Adjusted Marginal Profit	On Unpredicability of losses - 7/1/81 to 7/1/86	On 12/31/85 Reserves
Auto Liability	$18,388,661	$12,077,892	$ 6,552,944	$19,847,726
Auto Physical Damage	12,488,512	22,570,312	8,031,665	1,641,890
Homeowners	7,082,977	7,240,430	9,895,271	2,527,697
Other Property	5,968,294	9,953,071	9,791,883	1,716,975
Workers' Compensation	9,426,947	12,361,850	6,360,699	16,300,664
Medical Professional	1,574,806	201,898	6,120,295	4,926,884
Other Liability	7,853,716	405,024	14,661,860	12,776,050
Miscellaneous	12,727,503	10,700,941	14,096,800	15,773,530

"The 'required surplus,' which should be assured by regulation, is easy to state in the abstract, but difficult to implement in practice. It calls for analysis of the variables that the surplus to policyholders is expected to cover. Essentially they are three. First, the surplus must absorb any basic insurance costs (losses and expenses) which are in excess of the premiums charged. Second, the surplus must absorb any undervaluation of loss or claim reserves. Third, the surplus must absorb any declines in asset values."[11]

If surplus is seen as the difference between assets and liabilities, and the reserves as being the only uncertain liabilities, this view of surplus follows. The expected value of surplus is known:

Surplus = Assets - Loss and LAE Reserves - Unearned Premiums
- Other Liabilities.

or $$S = A - LR - UEPR - OL \quad (13)$$

But under uncertainty, using the operator "m" to represent the maximum adverse change to a variable,[12] we have:

$$S - mS = (A - mA) - (LR + mLR) - (UEPR + mUEPR) - OL \quad (14)$$

To be sure of solvency, we must require

$$S - mS > 0 \quad \text{or} \quad (15)$$

$$S > mS = mA + mLR + mUEPR \quad (16)$$

11/ Chapter 6, page 66, Excerpted from the Report of the New York Special Committee on Insurance Holding Companies. Note that the first purpose of surplus is described as "basic insurance costs... in excess of the premium...". This could be taken to include future losses that are expected. However, surplus is also addressed as a "variable that the surplus to policyholders is expected to cover", or a contingency. Considering the historical perspective of the New York Report (1968) the author believes that the Report didn't consider business knowingly written at an underwriting loss. In the thirty preceding years the stock industry had a combined loss and expense ratio at or below 100 in twenty-five years; the highest ratio was 102.9; the lowest was 87.6.

12/ The symbol "m" preceding an expression is used to represent the positive-valued maximum adverse change in the value of the expression, over the near term at a very high level of

In words, the surplus must be adequate to protect against the maximum probable fall in asset values, adverse loss reserve runoff, and unearned premium reserve inadequacy.

However, these three familiar sources of loss to the insurer are not the only ones. Other contingencies include default on premium balances, failure of reinsurers or reinsurance intermediaries, changes in income tax treatment, unfunded pension benefits, guarantee fund assessments, casualty losses such as thefts, foreign exchange rate changes, and unexpected increases in operating expenses. While the insurer has established a variety of liabilities and contra-assets to reflect the expected losses from those items, a buffer against these miscellaneous contingencies is also needed, if the promise of insurance is to be realized. The insurer's surplus serves that function as well as the three described in Dennenberg and Kimball. So equation (14) becomes

$$S-mS=(A-mA)-(LR+mLR)-(UEPR+mUEPR)-(OL+mOL) \quad (17)$$

and we require $$S > mA + mLR + mUEPR + mOL \quad (18)$$

12/ (Continued)

probability. That level of probability is unknown and unknowable. It varies between insurers, and within insurers it varies between accounts. For example, Loss Reserves and Asset value levels are not perfectly correlated so the probability of a drop in the value of the surplus of mLR + mA is not equal to the probability of the loss reserves increasing by mLR or the probability of the assets decreasing by mA. But we can write mS = mA + mLR with a higher degree of probability reflected in mS than in mA or in mLR. This concept of the maximum adverse change is analogous to the concept of the Probable Maximum Loss i derwriting.

Because of market share concerns, the desire to maintain customer and public acceptance, or regulatory requirements, insurers may decide to write some part of their book at inadequate rates. Since policyholder contributions will not fund the losses and expenses, surplus must. This is a use of surplus distinct from the types of contingency buffers described in Kimball and Dennenberg, because the adverse outcome is certain, not contingent. This constitutes a fifth purpose of surplus: funding underpriced business, without jeopardizing the ability of surplus to act as a contingency buffer. So inequality (15) becomes:

$$S - mS - PV(\text{expected losses}) > 0 \qquad (19)$$

and, $$S > mA + mLR + mUEPR + mOL + PV(EL) \qquad (20)$$

PV(expected losses) or PV(EL) represents the present value of the expected operating losses on the underpriced business that management has decided to accept.

The insurer also may desire to maintain dividend payments to its stockholders or policyholder when income is not available to fund them. For example, in 1985, of the approximately 1300 insurers reporting financial operating results to ISO and the NAII, 433 had negative net income after tax. Of the insurers with negative net income after tax, 92 declared dividends to shareholders and 169 declared dividends to policyholders.[13]

13/ Totals drawn from the financial operating results database maintained by ISO. Results are reported by insurers on either a group or individual company basis.

Each of these payments were funded out of the insurer's surplus. Funding these payments must also be a purpose of surplus. So we now have:

$$S - mS - PV(EL) - PV(\text{unfunded dividends}) > 0 \quad (21)$$

and, $$S > mA + mLR + mUEPR + mOL + PV(EL) + PV(UD) \quad (22)$$

Insurance is only practical when the insurer faces exposure to loss from a large number of independent causes. To the policyholder, this means that coverage is only available if there are many other distinct risks. Maintaining confidence in the insurer's solidity among consumers, producers and regulators is essential to attracting a broad book of independent business. A key component in this endeavor is maintaining adequate surplus. While, and indeed since, surplus provides for certain or contingent unfunded payments, it also serves to maintain confidence in the insurer. This represents a seventh function of surplus.

Table VI -- Functions of Surplus

1. To provide protection against unexpected increases in losses and expenses,
2. To provide protection against adverse reserve runoff,
3. To provide protection against downward asset value fluctuations,
4. To provide protection against all other adverse financial contingencies,
5. To fund business written at a loss,
6. To fund dividend payments when income cannot, and
6. To maintain confidence in the insurer among producers, regulators and consumers.

Knowing what surplus does, and having seen the problems in some allocation techniques, we can now develop

PRACTICAL CONSIDERATIONS FOR MAKING MEANINGFUL ALLOCATIONS.

These functions of surplus are diverse, but a meaningful allocation method must consider them. This is because the underlying purpose of allocating

surplus is to make economic decisions that consider how each portion of the insurer's book restricts its ability to write other risks. Each of these seven functions (and perhaps others that the author did not consider) contribute to limitations on the insurer's capabilities and must therefore be considered in making these decisions or in making a meaningful allocation of the insurer's surplus. This conclusion provides the first consideration:

1. The allocation must consider each function that surplus is performing.

Other considerations follow from the intended purpose of the allocation and from what we mean by an allocation.

The definition of allocation used in this essay only requires that the sum of the amounts that are allocated does not exceed the total of the account being allocated. That is, four apples cannot be allocated into three apples and two apples. However the definition does not forbid allocating four apples into one apple and two apples (and leaving an unallocated apple) or into five apples and a negative apple (by lending between the allocated pieces). However, the surplus of a multi-line insurer must meet more demanding standards than apples do.

If the allocated amounts of surplus are less than the total amount of surplus, part of the surplus is not allocated. (This follows from the exhaustive, mutually-exclusive definition of the sections.) The unallocated part of surplus is still available to support the writings of the insurer.

Ignoring this piece of surplus would understate the ability of the insurer to write more business and distort economic decisions based on the perception of that ability. So the amount of surplus allocated must be no less than the total surplus of the insurer. This requirement and the definition of allocation, produce the second practical consideration:

2. The sum of the amounts of surplus allocated to the various sections of the insurer's book must be exactly equal to the insurer's total surplus.

By cancelling policies it is possible to write a negative amount of premium. In a similar situation when an the insurer cancels policies after the last surplus evaluation date, the equity in the unearned premium reserve that became income on cancellation may exceed the maximum adverse runoff on the reserves and other adverse contingencies. So it may be possible to look back and determine that the worst outcome on a group of cancelled policies is still a gain. Negative surplus equal to the minimum gain could be allocated to the cancelled policies, allowing an equal amount of additional, positive surplus to be allocated to other portions of the insurer's book.

The idea of negative surplus on an active portion of the insurer's book doesn't make as much sense. Surplus exists partly to convince potential policyholders of the solidity of the insurer's promise of coverage. A zero or negative amount cannot perform that function. The surplus allocated to a section of the insurer's book records the limitations that that section puts on the ability of the insurer to write additional business. The reason for making an allocation is to make reasonable decisions that recognize the limitation of the insurer's resources. Negative amounts allocated to a section of the book that present an exposure to loss would not express this exposure to loss. It must therefore distort any decisions that rely on it.

So a meaningful allocation could not allocate a non-positive amount of surplus to a section of the book that presents any probability of unfunded loss. This becomes our third practical consideration:

3. The amount of surplus allocated to any section of the insurer's book that presents an exposure to loss must be positive.

Since each section of the insurer's book that presents an exposure to loss limits the insurer's ability to write additional business, we have seen that sensible allocations of surplus will record that exposure to loss by assigning surplus. Moreover, if at a given level of probability, one section presents a higher maximum adverse outcome than another section, then the first section should be associated with a greater amount of surplus available to fund the adverse outcome. The amount of surplus available to fund adverse outcomes excludes the amount allocated to fund under-priced business or unfunded dividends. This constitutes the fourth practical consideration:

4. Among sections, the amount of allocated surplus, less amounts that fund under-priced business or unfunded dividends, must increase as any section's exposure to unfunded losses increases.

These four considerations only examine the practical requirements that an allocation method must meet which are dictated by the nature of the economic questions the allocation is intended to answer. Other requirements come from the nature of the decision-making process.

Decisions must be made for the present and future. Data are only available for the past, and are not always available for the recent past. If a decision that uses a surplus allocation is to be informed and reasonable, the allocations must be estimatable from old results: they must be

relatively stable. They must not change substantially due to small changes in the capital structure of the insurer or in its results over the near term.

If the allocations are to be valuable, the allocation must be available in situations where consensus is lacking. If a consensus were available then pricing, performance levels, profitability and solidity could be assessed without resulting to allocations of surplus. The formulas used to make the allocations must be explicit, objective and justifiable. These two observations complete the list of practical considerations[14].

The six practical considerations are shown in Table VII.

Table VII -- Practical Considerations In Allocating Surplus

1. The allocation must consider each function that surplus is performing,
2. The sum of the amounts of surplus allocated to the various sections of the insurer's book must be exactly equal to the insurer's total surplus,
3. The amount of surplus allocated to any section of the insurer's book that presents an exposure to loss must be positive.
4. Among sections, the amount of allocated surplus, less amounts that will fund underpriced business or unfunded dividends, must increase as the section's exposure to unfunded losses increases.
5. The allocated amounts must not change substantially due to small changes in the capital structure of the insurer, or of its results over the near term, and
6. The formulas used to make allocations must be explicit, objective and justifiable.

14/ No attempt is made here to show that these practical considerations constitute sufficient standards. Indeed, because of the philosophical and practical problems discussed earlier, the author feels that an allocation of the surplus of a multi-line insurer to lines of insurance need not be possible.

Marc Anthony's remarks cited at the beginning of this essay prefaced a eulogy that lauded Caesar in life and death. However, this essay contains no hidden praise. The author's intent is, as stated, to demonstrate practical and philosophical problems with making an allocation of surplus, and to propose alternatives to making allocations.

Having considered several possible methods of allocating surplus and establishing minimum standards for allocation methods, we can now draw

CONCLUSIONS.

None of the allocation methods presented in this essay satisfy the practical considerations. For example, most of the methods do not consider that surplus must serve both as a buffer against windstorm catastrophes that will exceed the unearned premiums and as a buffer against adverse liability reserve development. The method that does consider both, the NYCB method, doesn't reflect any difference between the two functions. None of the methods considers the potential of asset values falling. None consider the differences between certain losses and contingent ones. No method is developed in such a way that, in general, it will allocate more surplus to a section of the book that presents a greater exposure to unfunded losses. An equally serious concern is that none of the methods addresses the philosophical questions that underlie any attempt to allocate surplus.

Does this mean that the actuary is left without sound, practical analytical tools for comparing performance, for pricing and for analyzing profitability? No.

The concept of the insurance operating profit margin can be used to answer the same questions as an allocation of surplus is intended to answer. The insurance operating profit margin is the contribution to the insurer's

annual profits from a section of the book (earned premiums less incurred losses, expenses and dividends plus investment income) compared to the annual earned premiums for that portion of the book:

$$IOPM_i = (EP_i - IL_i - IE_i - Div_i + II_i)/EP_i \quad [15] \qquad (23)$$

The insurance operating profit margin can be calculated on a calendar, accident, or policy year basis. The use of the insurance operating profit margin in decision-making depends only on knowing the value that the open marketplace currently puts on insurance coverage.

To develop a price for an insurance contract that reflects the value of the use of the insurer's surplus, the price should be set to make a fair operating profit. That profit margin can be derived from industry or insurer results, adjusted for later changes in market conditions, or from corporate objectives. This calculation is certainly more simple, objective and empirically based than any analysis of risk, beta, returns on net worth and premium-to-surplus ratios. It is therefore more likely to produce stable and accurate prices.

If management believes that investment income opportunities are fairly stable, an additional complication can be eliminated. Pricing can be determined by using a marketplace underwriting profit or loss as a target. This analysis requires no allocation of investment income to sections of the book.

15/ The operating profit margin can be seen in careful, practical application as the Argonaut Return in David Skurnick's Measuring Division Operating Profitability, 1985 CAS Discussion Paper Program.

Management performance can also be evaluated using operating or underwriting profit margins. If the various underwriting management centers write books of business that are similarly distributed, and make similar demands on the insurer's capabilities, then the total insurance operating profit margin can be directly compared between centers. If there are differences between the profit center's various books, their relative performance can be compared on separate sections. Perhaps Homeowners to Homeowners, and Medical Malpractice to Medical Malpractice, etc. A different approach is to calculate a single result over all portions of a center's book by setting a company-wide target operating profit margin for each line of insurance, and comparing each center's results against what it would have produced if each section of its book had earned the company-wide target. In symbols, for profit center #j, for line of insurance #i:

$$\text{Result}(\text{Center \#j}) = \sum_i EP_{ij} \times (\text{Target}_i - IOPM_{ij}) \qquad (24)$$

The insurance operating profit margin can be used in economic decision-making whenever an allocation of surplus could be used. It also brings a parsimony and understanding that formal allocation techniques lack.

If for some reason a method of pricing or performance appraisal that uses an allocation of surplus in its calculations is essential, it is sensible to use an analytical tool that is fairly insulated from the vagaries of the allocations. One such tool is the Myers-Cohn pricing model that has been employed by the Massachusetts Auto Rating Bureau.

The MARB described the model in a recent paper[16] as:

"The basic premise underlying the Myers-Cohn model can be stated this way: a fair premium must be equal to the expected losses and expenses, discounted to present value at a risk-adjusted rate, plus the present value of the federal income taxes on underwriting and investment income, discounted at a risk-free rate. Premiums calculated this way preserve the equity invested in the company and give the investor a fair reward for the risk of underwriting.

Myers-Cohn Formula

$$\begin{aligned} PV\ (Premiums) = {} & PV\ (Losses\ and\ Expenses) \\ & + PV\ (Federal\ Tax\ on\ Investments) \\ & + PV\ (Federal\ Tax\ on\ Underwriting) \end{aligned} \quad [(25)]$$

"The discount factor applicable to losses and expenses first reflects the investment income on the cash flow at current risk-free rates. The Myers-Cohn model thus was consistent with prior models which included investment income at a risk-free rate of return. Although the model assumes that investment income can be earned at the risk-free rate, the company and its stockholders take the risk and receive the reward for any alternate risky strategy. Additionally, the discount factor applicable to losses and expenses reflects a risk adjustment that is chosen to yield a reasonable compensation for the uncertainty in both the estimates of losses and expenses and in their realization -- or in other words, for the risk of underwriting. The fair premium can then be calculated by including the present value of the federal income taxes on investment and underwriting income. The inclusion of the present value of income taxes on investment income requires the use of some method of allocating surplus to each line. The Myers-Cohn paper suggests allocating the surplus roughly in proportion to total outstanding reserves...

"In the Massachusetts applications to date the risk adjustment was chosen from among CAPM estimates but, unlike the Fairley models, nothing in the model requires the use of the CAPM or any other specific model of risk."

16/ Taken from "The Use of Investment Income in Massachusetts Private Passenger Automobile and Workers' Compensation Ratmaking" by Richard Derrig.

The allocated surplus is only important to develop the amount of taxes incurred as a result of investing the surplus. While the treatment of surplus has varied in applications of the Myers-Cohn Model, results are relatively stable. Calculations presented at ISO's Actuarial Research Committee in 1985 showed that for one set of parameters, a sixty point change in the premium-to-surplus ratio created a change of less than five points in the indicated Auto BI profit provision.