BRANCH OFFICE PROFIT MEASUREMENT FOR PROPERTY-LIABILITY INSURERS

Robert P. Butsic

Biographical Sketch

Current employer is Fireman's Fund Insurance Co.; prior affiliation was with CNA Insurance from 1969-1979. Earned BA in Mathematics in 1967 and MBA (Finance) in 1978, both from the University of Chicago. Associate in Society of Actuaries, 1975; Member of American Academy of Actuaries. Wrote papers for previous CAS Call Paper programs in 1979 ("Risk and Return for Property-Casualty Insurers") and 1981 ("The Effect of Inflation on Losses and Premiums for Property-Liability Insurers").

Abstract

Effective measurement of financial performance for individual branch offices is hindered by two major problems. The first is an appropriate definition of profit; this is addressed through an economic-value accounting method which minimizes distortions due to the timing of income recognition. Return on equity is the basic profitability gauge used to compare results between profit centers. The second problem is that, in comparing results between branches, different levels of risk will produce an uneven chance of error in measuring true performance vs. reported results. This problem is addressed through techniques which equalize systematic risk (by implying an equity value) and non-systematic risk (through internal reinsurance), for each branch. To develop the internal reinsurance concept, a Poisson claim frequency and a Pareto claim severity model is constructed. Finally, in order to recognize the credibility of each profit center's actual experience, a compromise is made to the equal-variance principle. The analysis concludes that branch office profit measurement is best served when the branch network has minimal variation in size and product mix.

BRANCH OFFICE PROFIT MEASUREMENT FOR PROPERTY-LIABILITY INSURERS

INTRODUCTION

The insurance industry is presently becoming less regulated, creating an increasingly competitive long-term environment. To effectively meet the challenge posed by this new climate, insurers must strengthen their marketing and get closer to the consumer. Consequently, a strong branch office network is needed in order to cope with the variety and complexity of local conditions.

A major factor in the development of a viable branch office organization is the principle that each branch is completely responsible for its own contribution to corporate profits. Hence, the profit center concept. Given the objective to drive corporate profits from the sub-units which are held accountable, an appropriate tool for measuring performance must be used.

The purpose of this paper, then, is to outline a general approach for measuring the performance of individual profit centers comprising a property-casualty company. The methods presented here could apply to line of business or regional definitions of "profit center", but it will be most useful to think of a profit center as a branch office.

The paper focuses on the risk aspects of profit measurement and presents methods which equalize both systematic and purely random variation in profit center results. Other important aspects of profit measurement, including the accounting treatment, are developed in lesser detail. Many of the thoughts presented here have evolved over time at the writer's own company and are now being brought to a practical application in its management reports.

PROFIT MEASUREMENT IN PROPERTY-CASUALTY INSURANCE

Corporate Profitability

Before addressing the profitability measurement of individual profit centers one must first define an appropriate yardstick for the corporation as a whole. For this important concept we will choose return on equity, or ROE for short. This measure is commonly used for other industries, and represents the return to an investor in the corporation (for further background on this topic, see references [4], [8] and [10]).

Although ROE is a simple concept, it must be carefully defined. The accounting conventions used must be suitable for performance measurement over a reasonable time frame such as a month, quarter or year. Stated differently, management reports should encourage behavior which will tend to maximize the value of the firm.

The return on equity measure can be separated into two components: net income (the numerator) and equity (the denominator).

Net income must reflect, to the extent possible, the current impact of all future transactions related to the premium earned in the current period. This means that:

1. Accident-period accounting is used, with all losses, premiums, expenses and dividends continually being restated as better estimates of their ultimate values become known.

2. All future investment income earned on cash flows arising from the current period must be recognized in the period. The usual device for collapsing an income stream is net present value.

3. The future cash flows are taken to present value using a market interest rate, and not the portfolio rate. It should be noted that the appropriate rate for corporate performance measurement may differ from that for profit centers, due to the separation of responsibility for investment and underwriting risk.

Equity, in a similar fashion, must be adjusted to reflect the above timing of income:

1. All assets must be evaluated at market prices and all liabilities must be discounted to present value (i.e., the market price in a portfolio reinsurance transaction).

2. Otherwise, normal GAAP accounting for equity should be used.

The preceding concepts attempt to recognize what is known in the accounting literature (see Lev [5]) as economic income -- the anticipated consequences of current decisions are directly reflected in current earnings. The notion of economic income can be further explained by a numerical example. Assume that a miniature "company" is formed under the following circumstances:

1. An annual policy of $100 is written on January 1, 1985.

2. A single claim of $68.20 occurs on January 1, 1985 and is paid over three years, as indicated in the following cash flow schedule.

3. Cash transactions occur on January 1 of each year; the last loss payment occurs in 1987.

4. Cash is invested at a yield of 10% per year.

5. All cash flows are certain.

6. There are no income taxes applicable.

7. Capital of $50 cash (initial equity) is available on December 31, 1984.

8. Expenses of $35 are paid on January 1, 1985.

The cash flows are shown below:

Cash Flow Schedule

	1/1/85	1/1/86	1/1/87	Total	1/1/85 Present Value
Premium	100			100	100
Losses	0	-44	-24.2	-68.2	-60
Expenses	-35			-35	-35
Total	65	-44	-24.2	-3.2	5

Here we see that the value of the policy to us at the time the premium was written is $5.00. Under economic-value accounting, the balance sheet would look like:

Balance Sheet: Economic Value

	1984	1985		1986		1987
	12/31	1/1	12/31	1/1	12/31	1/1
Assets	50.00	115.00	126.50	82.50	90.75	66.55
Loss Reserve	0	60.00	66.00	22.00	24.20	0
Equity	50.00	55.00	60.50	60.50	66.55	66.55
- Pres. Value (1/1/85)	50.00	55.00	55.00	55.00	55.00	55.00

The loss reserve here equals the present value of unpaid losses. Notice that beginning 1/1/85, the present value of equity is always $55.00, because no additional income is generated by the future cash flows arising from the insurance operation (as opposed to re-investment of equity):

Income Statement: Economic Value

	1985	1986	Total
Underwriting Gain	-1.00	-2.20	-3.20
Investment Income - Loss Reserves	6.00	2.20	8.20
Net Income - Insurance Operation	5.00	0	5.00
Investment Income - Equity	5.50	6.05	11.55
Net Income - Total	10.50	6.05	16.55

This tabulation clearly shows that <u>all</u> income arising from the policy is recorded in 1985, when the premium is earned. For comparison, the traditional accounting method would give the following balance sheet and income statement:

Balance Sheet: Traditional Accounting

	1984	1985		1986		1987
	12/31	1/1	12/31	1/1	12/31	1/1
Assets	50.00	115.00	126.50	82.50	90.75	66.55
Loss Reserve	0	68.20	68.20	24.20	24.20	0
Equity	50.00	46.80	58.30	58.30	66.55	66.55
- Pres. Value (1/1/85)	50.00	46.80	53.00	53.00	55.00	55.00

Income Statement: Traditional Accounting

	1985	1986	Total
Underwriting Gain	-3.20	0	-3.20
Investment Income - Loss Reserves	6.82	2.42	9.24
Net Income - Insurance Operation	3.62	2.42	6.04
Investment Income - Equity	4.68	5.83	10.51
Net Income - Total	8.30	8.25	16.55

Notice that by the end of 1986 (a moment before the last cash transaction on 1/1/87) the accumulated equity is equivalent under either accounting method. However, the timing of income recognition differs dramatically.

The preceding example illustrates some fundamental ideas which can be developed more fully. The first is the concept of the total profit margin, or TPM. In economic valuation, the issuance of the $100 policy created an instant "profit" of $5; we are indifferent to selling the policy or accepting $5 in cash. This total profit brought to present value is 5% of premium, hence a 5% total profit margin.

The second is the economic return on equity. We started with $50 of equity and one year later the economic value of the mini-enterprise is $60.50, yielding a 21% ROE.

More compactly, the ROE can be represented by

(1) $1 + R = (1 + i)(1 + km)$,

where R denotes the (economic) return on equity, i the market (riskless) interest rate, k the premium divided by initial equity, and m the total profit margin. To verify the preceding example, we get $1 + R = 1.1[1 + 2(.05)] = 1.21$.

Although traditional accounting may provide a reasonable means of aggregate performance measurement for an insurer under conditions of stable growth, interest rates and product mix, it can fail miserably at the individual profit center level due to more severe timing distortions of income recognition (for example, individual case reserve changes on prior period losses can be dramatic for a single branch).

PRODUCT LINE PROFIT MEASUREMENT

Having established the economic return on equity evaluation approach as a viable aggregate profitability measurement, its application must be extended to individual product lines.

Relative Risk

Although ROE is a good profit measure, there are problems associated with its use in comparing insurance (and other types of) companies or product lines. These difficulties arise because the amount of risk associated with an ROE measure varies significantly by line of business. For example, given the option of buying stock in a medical malpractice insurer with an expected ROE of 20% or in a personal lines company of the same size having a 15% ROE, it is unclear what choice to make. The medical malpractice insurer would be considered to have a riskier return. The returns must be adjusted to equalize risk between various types of coverage.

Here risk means systematic or process risk, which cannot be reduced (relative to premium) by increasing the number of individual exposures. Examples of systematic risk for property-liability insurance include:

1. uncertainty of ultimate losses due to length of time from occurrence to final settlement,

2. uncertainty of ultimate loss due to future costs (inflation) being higher than anticipated in pricing, and

3. uncertainty of loss costs arising from low-frequency events, such as in municipal bond or nuclear reactor coverages.

Further discussion of systematic risk can be found in references [2], [4] and [9]. Also, Appendix I provides a more rigorous treatment of this topic, showing the difference between systematic and non-systematic (random) risk.

To adjust the ROE measure for risk, consider Equation (1) with the ROE and TPM as <u>random variables</u>:

$$(2) \quad 1 + \tilde{R} = (1 + i)(1 + k\tilde{m}).$$

Here "$\sim$" denotes a random variable. The interest rate, being riskless, is not a random variable. Also, the premium/equity ratio k is a constant since it represents a known quantity. The variance and standard deviation of the ROE are given by

$$(3a) \quad \mathrm{Var}(\tilde{R}) = (1 + i)^2\, k^2\, \mathrm{Var}(\tilde{m}) \text{ and}$$

$$(3b) \quad \sigma(\tilde{R}) = (1 + i)\, k\,\sigma(\tilde{m}).$$

In order to proceed further, the <u>ROE risk</u> will be defined as being equal to its <u>standard deviation</u>. This is commonly used in financial theory (see Sharpe [9], for example) and has the intuitively appealing and important property that it is independent of the scale of operation: two companies identical in all other respects but size will have the same risk, measured in terms of variation from expected ROE. This definition is equivalent to that of systematic risk, which, being independent of the number of exposures, is constant for a particular product line, regardless of size.

For two different product lines (denoted by subscripts) one can equate the ROE risk using (3b);

(4a) $\sigma(\tilde{R}_1) = \sigma(\tilde{R}_2) = k_1 \sigma(\tilde{m}_1) = k_2 \sigma(\tilde{m}_2)$, or

(4b) $\sigma(\tilde{m}_1) / \sigma(\tilde{m}_2) = k_2/k_1$.

In other words, two product lines have identical ROE risk when their premium/equity ratios are inversely proportional to their respective measures of systematic risk.

Risk Equalization

How do we measure systematic risk for property-liability lines of business? Unfortunately, there is no objective way to measure some of the risk components, such as the uncertainty of low-frequency events. Nevertheless, even a judgemental approach is better than to assume that all lines have equal risk.

A suggested method for subjectively balancing risk for various product lines is:

1. Select a product line, say Commercial Multiple Peril, with an average perceived risk. Assign to it an arbitrary premium/equity ratio in the neighborhood of the long-term industry average premium/equity ratio for all lines; e.g., 2.5-to-1.

2. Select another line, compare it to the standard line (CMP) and set a premium/equity ratio at which you would be indifferent to writing this line compared to the standard line. For example, Fire (having a fast loss payout and a relatively complete pricing data base) at a 4-to-1 premium/equity ratio might be considered equally risky as CMP at 2.5-to-1.

3. Repeat the process for all applicable product lines. Of course, the method can be extended to sublines or even new types of insurance.

This procedure, or one which actually attempts to measure the relative systematic risk (see Fairley [1]), will produce imputed equity values for each line based upon the respective premiums written. The aggregate all-lines imputed equity need not equal the "actual" equity reported externally, since our intent is to measure relative profitability between lines without having to be concerned about their different absolute levels of risk.

Returning to the earlier quandary (medical malpractice at 20% ROE vs. personal lines at 15% ROE), suppose that the medical malpractice ROE arises from a 5% TPM having a standard deviation of 3%, while the personal lines TPM is 2% with a standard deviation of 1%. The interest rate is 10%.

Applying Equation (2), the medical malpractice premium/equity ratio is 1.818 and the personal lines ratio is 2.273. Equation (3b) gives a 6% medical malpractice ROE standard deviation and a 2.5% value for personal lines. To equalize the ROE risk, we increase the personal lines premium/equity ratio to 2.273(6/2.5) = 5.455 (conversely, we could decrease the malpractice premium/equity ratio by a factor of 2.5/6), yielding an ROE standard deviation of 6%. However, the expected personal lines ROE now increases to 1.1[1 + 5.455(.02)] - 1 = 22%. Therefore the personal lines return would be superior to that of the medical malpractice insurer.

The preceding calculations are summarized in the following table:

			Before Equalization			After Equalization		
	$E(\tilde{m})$	$\sigma(\tilde{m})$	k	$E(\tilde{R})$	$\sigma(\tilde{R})$	k	$E(\tilde{R})$	$\sigma(\tilde{R})$
Medical Malpractice	5.0%	3.0%	1.82	20.0%	6.0%	1.82	20.0%	6.0%
Personal Lines	2.0	1.0	2.27	15.0	2.5	5.45	22.0	6.0

PROFIT CENTER RISK EQUIVALENCE

Having developed the basis for equalizing systematic risk between product lines, it is now possible to apply the method to a composite of various lines, namely the profit center.

Systematic Risk Equivalence Between Profit Centers

For the composite of two (or more) lines, there is an additional element which tends to increase systematic risk — correlation between total profit margins. Suppose that the implied equity for each line has been determined to equalize the respective ROE risk. Let f_1 and f_2 represent the respective proportions of the total ($f_1 + f_2 = 1$) implied equity for each line, ρ the correlation coefficient between $\tilde{m}_1$ and $\tilde{m}_2$, and the subscript t the results for the branch total. The variance of the branch ROE is

$$\text{(5)} \quad Var(\tilde{R}_t) = Var(\tilde{R}_1)[1 - 2(1-\rho)f_1 f_2].$$

Appendix I derives this result. Notice that $Var(\tilde{R}_1) = Var(\tilde{R}_2)$ since the ROE risk has been equalized.

The following numerical example illustrates the preceding result:

Line(i)	Premium	k_i	Equity	$E(\tilde{m}_i)$	$\sigma(\tilde{m}_i)$	$E(\tilde{R}_i)$	$\sigma(\tilde{R}_i)$	ρ
1	100	4.0	25	1.5%	1.0%	16.60%	4.4%	-
2	150	2.0	75	4.0%	2.0%	18.80%	4.4%	-
Total	250	2.5	100	3.0%	1.6%	18.25%	4.4%	1
					1.3%		3.5%	0

Notice that the standard deviation of the ROE for this branch will be reduced (down to 3.5%) if the lines have total profit margins which are not fully correlated.

Several observations can be made from the analysis so far:

1. To the extent that lines within a profit center are not perfectly correlated in their TPM's, the overall ROE risk is reduced.

2. For a given correlation structure, the profit center ROE risk reduction is a function of the line mix.

3. Maximum risk reduction is attained when the line mix is such that the implied equity amount is equal for each line (i.e., $f_1 = f_2$ in Equation (5)).

For comparing results between two different profit centers, the theoretically correct procedure would adjust the implied equity for each branch due to risk reduction from line mix. However, this would be a formidable computational task due to the large size of the relevant correlation matrix and the difficulty of estimating the line correlation coefficients from empirical data. A more practical approach is to assume that the line mix among branches is such that there will be a negligible variation in ROE risk due to intercorrelation.

Non-Systematic Risk: Poisson-Pareto Model

We have now reached the point where each profit center can be evaluated on the basis of its own ROE given the equalization of systematic risk. Ideally, we would like to remove all sources of chance variation, systematic or otherwise, in order to ascertain whether the measured result is the "true", or inherent result. However, as discussed earlier, systematic risk by its very nature is difficult to reduce since it is independent of the size of operation (loss ratio reinsurance would work somewhat, but at the expense of a lower return). On the other hand, non-systematic (or NS) risk can be trimmed more readily through internal reinsurance. Increasing the number of exposures will also reduce NS risk, but in practice the size of a profit center is severely constrained.

A major source of random risk for a profit center is large losses due to single occurrences. Here we define a large loss as arising from a single insured, to distinguish natural (i.e., ISO serial-number) catastrophes, which will be treated later.

For a network of profit centers, the NS risk arising from large claims can be formulated readily using some simplifying assumptions:

1. A large loss is denoted by the random variable $\tilde{X} \geq r$, where r is a reference point above which we are willing to establish an internal excess loss reinsurance pool. Losses below r are assumed to be fixed in number and amount, and do not contribute to the variance of total losses for the profit center. For simplicity of presentation we will henceforth assume that these losses are zero.

2. All individual losses arise from a single product line and have the same size distribution; however, the expected number N_i of large losses varies by branch i.

3. The number of large losses $\tilde{N}_i$ has the <u>Poisson</u> distribution with parameter N_i.

4. The loss amount $\tilde{X}$ has the <u>Pareto</u> distribution, with parameter $\underline{a}$ (Patrik[7] discusses the applicability of this assumption). Other functions, such as the log-normal, are less computationally tractable, besides being unsuitable for fitting the <u>tail</u> of loss size distributions. Basically, the $\underline{a}$ parameter indicates the "tail thickness" of the loss size distribution: the higher the value of $\underline{a}$, the less likely a loss will occur at a higher level relative to a lower level. Empirical evidence indicates that $\underline{a}$ varies between about 1.5 and 3, with low values for liability lines and high values for property coverages. Appendix II gives more detail regarding the Pareto distribution.

5. All losses have the same payment pattern. Thus, the present value of the loss $\tilde{X}$ can be represented by $d\tilde{X}$, where d is a constant.

6. The total premium for the profit center is collected when written and is proportional to the expected losses. Since the average loss size is the same for all branches, the premium, denoted by PN_i, is proportional to the expected <u>number</u> of large losses.

7. There are no expenses, only losses and premiums.

The total value of all losses in a particular branch i is (subscript i removed for clarity)

$$(6) \quad \tilde{S} = \tilde{X}_1 + \tilde{X}_2 + \ldots + \tilde{X}_{\tilde{N}} ,$$

where the number of losses $\tilde{N}$ is also a random variable with expectation N. Because $\tilde{S}$ is a compound Poisson distribution it has mean and variance (see Appendix III for derivation):

(7a) $E(\tilde{S}) = NE(\tilde{X}) = Nra/(a-1)$ and

(7b) $Var(\tilde{S}) = NE(\tilde{X}^2) + N^2C = Nr^2a/(a-2) + N^2C$,

where $C = Cov(\tilde{X}_k, \tilde{X}_j)$ is the covariance between two separate losses $\tilde{X}_k$ and $\tilde{X}_j$.

The total profit margin for the profit center is

(8) $\tilde{m} = (PN - d\tilde{S})/PN$.

The variance of the TPM is, from (7b) and (8),

(9) $$Var(\tilde{m}) = \frac{d^2}{P^2}\left[\frac{E(\tilde{X}^2)}{N} + C\right].$$

Notice that as N becomes infinite, the variance of the branch TPM is proportional to the covariance of individual losses; i.e., only systematic risk is present. This result is consistent with the basis for selection of the implied premium/equity ratios for different product lines. However, in the large loss model here, we have assumed a single line and therefore the covariance C is the same for all branches as is the premium/equity ratio. Consequently, equating the ROE risk for two branches implies that $E(\tilde{X}^2)/N$ must be the same for the branches.

Because the loss size distribution is the same for all profit centers, but the expected number of losses N may vary (in fact, N defines the size of the branch), the size of loss distribution must be transformed so that the second moment $E(\tilde{X}^2)$ can vary by branch. The common mechanism for achieving this goal is <u>excess reinsurance</u>.

To do this, we select a retention $br > r$, where b is a scaling factor. Now let $\tilde{Y} = \tilde{X}$ for $r \leq \tilde{X} \leq br$ and $\tilde{Y} = br$ for $\tilde{X} > br$. In other words, the loss is "stopped" at a value of br. For this protection, we charge the branch an amount such that its expected total losses remain equal to $N_iE(\tilde{X})$. As shown in Appendix II, the expected portion of a single loss retained in the interval r to br is

$$(10) \quad (1-b^{1-a})E(\tilde{X}) = (1-b^{1-a})\ ar/(a-1),$$

and the expected ceded amount above br is

$$(11) \quad b^{1-a}E(\tilde{X}) = b^{1-a}\ ar/(a-1).$$

Also shown in Appendix II, under the Pareto distribution, the second moment of the retained loss size becomes

$$(12a) \quad E(\tilde{Y}^2) = r^2(a - 2b^{2-a})/(a-2) \quad \text{for } a \neq 2,$$

$$(12b) \quad E(\tilde{Y}^2) = r^2[1 + 2\ln(b)] \quad \text{for } a = 2.$$

To determine the relative retentions which will equalize the NS risk for two branches, set $E(\tilde{Y}_1{}^2)/N_1 = E(\tilde{Y}_2{}^2)/N_2$, where the subscripts denote the respective branches. Letting $K = N_2/N_1$ be the ratio of the expected number of losses for the two branches, Equation (12) can be solved to produce the relationship between retentions b_1 and b_2:

$$(13a) \quad b_2 = [Kb_1{}^{2-a} - a(K-1)/2]^{\frac{1}{2-a}} \quad \text{for } a \neq 2,$$

$$(13b) \quad b_2 = b_1{}^K e^{(K-1)/2} \quad \text{for } a = 2.$$

The Frequency Problem

The above results have some interesting implications. From Equation (7b) we see that the non-systematic component of the variance of total losses is $NE(\tilde{X}^2)$ with no reinsurance protection. With excess reinsurance protection and a >2, the non-systematic variance ranges from $NE[\tilde{Y}^2] = Nr^2$ when $b = 1$, to $NE[\tilde{Y}^2] = NE[\tilde{X}^2] = Nr^2\, a/(a-2)$ when b is infinite. Thus, the non-systematic variance can only be reduced by a factor of $(a-2)/a$.

However, the NS variance needs to be reduced by a ratio of N_1/N_2 if $N_2 > N_1$. Consequently, when a is large (indicating a thin-tail loss size distribution), the excess reinsurance program will be insufficient. One way to further reduce variance is to stop the number of large claims in addition to (or instead of) limiting individual loss amounts.

To further illustrate this point, we can separate the total NS loss variance into frequency and severity components. The frequency component is obtained by setting the loss equal to its expected value; the severity component arises from fixing the number of losses. This decomposition is derived in Appendix III with the following result for the Poisson-Pareto model:

Frequency	Severity	Total
$\frac{a\,(a-2)}{(a-1)^2}$	$\frac{1}{(a-1)^2}$	1

For example, if $a = 4$, an excess reinsurance program can only reduce the NS variance by 50%. Thus the maximum spread of branch sizes is 2-to-1 for risk equalization. However, since the frequency component of the variance is 89% of the total, it is possible to allow up to a 9-to-1 range of branch sizes by holding the number of claims at the expected value while allowing unlimited loss sizes.

Notice that if a <2, then the NS variance is infinite, but can be made finite with excess reinsurance. In this case the retention scaling factors b_i can be determined for any range of branch sizes.

Numerical Example

To illustrate the NS variance-equalizing choice of retentions, suppose that the lower limit r of the loss size distribution is $50,000 and the branch sizes range from 10 to 80 expected large losses. The following table shows two (of many possible) sets of equivalent retentions for three different values of a:

Equal-Variance Retentions (1,000's)

Branch Size	a					
(Exp. No. of Losses)	1.5		2.0		3.0	
10	50	100	50	100	50	100
20	78	216	82	165	100	-
40	153	580	224	448	-	-
80	378	1,838	1,656	3,312	-	-

For a = 3, it is not possible to find equal-variance retentions beyond a range of 3-to-1 in branch sizes. Notice that the range of retention amounts can be greater than the range of branch sizes if that range is large enough. Another observation is that, if a >2, setting the lowest retention above r further reduces the range of branch sizes which will equalize NS variance. In the example, when a = 3 and the lowest retention is 100,000 (instead of 50,000), the maximum range is 1.5-to-1.

The Credibility Problem

We have determined that the excess loss approach will equalize NS variance across branches provided that a is low enough or the spread of branch sizes is narrow enough. If not, limiting the number of losses will be required. But for now, assume that the excess method works. The preceding analysis has indicated that various sets of retentions will equalize variance. How do we choose the right set?

On one hand, we would like to minimize the NS variance for a particular branch. On the other hand, we would like to measure the true performance of each profit center to the extent that it differs from the average of all branches. This is the classical credibility problem.

Using the model developed in the preceding section, we can specify the problem more precisely. For a branch i, let N_i represent the true (but unknown) number of large losses (greater than r). Let $\hat{N}_i$ be the estimate of the true number. Recall that a is assumed to be the same for all branches, so that the expected loss size is identical by profit center.

Above the retention br, the reinsurance cost to the branch is

$$(14) \quad \hat{N}_i \, b^{1-a} \, E(\tilde{X})$$

when $\tilde{X}$ is a Pareto variable (from Equation (11)).

With no reinsurance, the expected true branch total loss cost is $N_iE(\tilde{X})$. Hence, the expected cost below the retention br is, from Equation (10)

$$(15)\quad N_i(1-b^{1-a})E(\tilde{X}),$$

and total expected cost is

$$(16)\quad [N_i + (\hat{N}_i-N_i)\ b^{1-a}\ ]E(\tilde{X}).$$

The <u>error</u> introduced by the reinsurance program is the expected total cost with reinsurance (Equation (16)) minus the true expected total losses of $N_iE(\tilde{X})$. Therefore the error for branch i is

$$(17)\quad (\hat{N}_i-N_i)b^{1-a}\ E(\tilde{X}).$$

Since N_i is an unknown parameter, it has a prior probability distribution. Let $h(N_i)$ denote the prior distribution. We want to minimize the variance of the error in this process as a function of b. Thus we integrate the square of equation (17) over all possible values of the parameter N_i:

$$(18)\quad EV(b) = \int_0^{\infty}(\hat{N}_i-N_i)^2h(N_i)b^{2-2a}\ E^2(\tilde{X})dN_i.$$

Bayesian credibility methods can be used to determine the $\hat{N}_i$ which will minimize the error variance for a fixed b. Since N_i is assumed to be a Poisson variable, a Gamma distribution could be used as the conjugate prior distribution (see Mayerson [6] for further information). Developing the optimal $\hat{N}_i$ estimate is an important practical problem, and needed for pricing internal reinsurance, but will be left for subsequent analysis.

The error variance, EV(b), is a <u>decreasing</u> function of the retention, and is zero with no reinsurance (infinite b). However, the NS variance due to randomness is an <u>increasing</u> function of b, being proportional to $a - 2b^{2-a}$. To illustrate the trade-off involved, we can scale both the NS and the error variance so that their maximum value is 1,000 units:

	a = 1.5		a = 2.0		a = 3.0	
b	EV	NS	EV	NS	EV	NS
1	1,000	120	1,000	194	1,000	333
2	500	320	250	463	63	667
4	250	602	63	731	4	833
8	125	1,000	16	1,000	0	917
	0	1,000	0	1,000	0	1,000

Actually, the NS variance is infinite for $a \leq 2$, so in the above table the maximum value of 1,000 has been set to occur at b = 8.

The non-systematic vs. error variance trade-off can be specified directly. First, we assume that the function of N_i in Equation (18) is proportional to N_i^2. This generally would be true when the number of large claims N_i equals an unknown parameter λ_i, having the same distribution for all branches, times an exposure measure for branch i. Thus the error variance becomes

(19) $EV(b) = N_i^2 c b_i^{2-2a} E^2(\tilde{X})$,

where c is a constant, equal for all branches. Next, assume that we are willing to trade v units of NS variance for one unit of error variance. This condition produces an objective function T_i which is a linear combination of the NS variance $N_i E(\tilde{Y}^2)$ and Equation (19):

$$(20) \quad T_i = N_i r^2(a - 2b_i^{1-a})/(a-2) \quad + \quad vcN_i^2 b_i^{2-2a} r^2 a^2/(a-1)^2.$$

By setting the derivative with respect to b_i equal to zero, the optimal b_i minimizing T_i is

$$(21) \quad b_i^* = [N_i vca^2/(a-1)]^{1/a}.$$

This result implies that $b_2/b_1 = (N_2/N_1)^{1/a} = K^{1/a}$, a more elegant form compared to Equation (13).

The objective function T_i may be thought of as a "credibility-weighted" NS variance. Therefore the ratio T_i/N_i^2 is equivalent to the NS variance ratio $E(\tilde{X}^2)/N_i$ in Equation (9). However, we no longer want to equalize the NS variance across branches since doing so would not give proper weight to each profit center's true result based upon N_i. Instead of equalizing ROE variance by branch we minimize the <u>sum</u> of the individual branch credibility-weighted ROE variances. Since the T_i for each branch are independent, separately minimizing each of them (by choosing the b_i^*) minimizes the sum of the branch variances.

Equation (21) can be used to establish a set of retentions without directly specifying the v and c parameters. We merely choose (judgementally), for the smallest branch, a retention which would provide a reasonable balance between NS variance reduction and credibility of actual losses incurred. Retentions for the remaining branches are then determined easily.

Catastrophe Losses

Natural catastrophe losses are inherently unpredictable at the profit center level due to their low frequency and high severity (empirical evidence indicates an $\underline{a}$ value near 1.0). Even at the corporate level, these losses have a high variance. Consequently, there is almost no information regarding the true catastrophe loss expectation in a profit center's own experience for a single year. Since there is virtually no credibility in branch experience and the variance of catastrophe losses is high, it is necessary to remove variance, rather than to equalize it.

A reasonable method of handling catastrophe losses is to charge each profit center with its annual aggregate catastrophe loss expectation, as a percentage of earned premium. All actual catastrophe losses incurred are absorbed by the internal reinsurance pool. Notice that this protection is an extreme form of reinsurance since the variance of losses charged to the profit center is zero.

SUMMARY

A workable approach to measuring profits in property-liability insurance is the return on equity concept, with income defined such that timing differences are minimal. For measuring profits at the branch office level, it is important to equalize the ROE variance between branches. Otherwise the effect of measurement error is not uniform by profit center and a haphazard incentive system may result.

The preceding analysis has shown, in general terms, how both the systematic and non-systematic risk components can be equalized for profit centers. However, when the credibility of branch results is considered, some equalization of non-systematic risk must be forgone. Based upon the complexity of the risk-equalizing problem, a key observation emerges: As the range of branch sizes expands, the difficulty of equitably measuring profit center results increases dramatically. Also, the difficulty is compounded if the branches have widely varying product mixes.

Practical applications of these risk-equalizing and profit-measuring techniques will require additional, more specific assumptions and much empirical work. Appendix IV provides an example of a profit center income statement which might arise from these efforts.

APPENDIX I: SYSTEMATIC RISK

Assume that a product line has N identically distributed exposures with profit margins $\tilde{m}_i$ for $i = 1$ to N. For simplicity, let each exposure have a premium of one unit. The profit margin for the line is

$$(1.1) \quad \tilde{M} = (\sum \tilde{m}_i)/N,$$

where the limits of summation are 1 to N. The variance of the line profit margin is

$$(1.2) \quad \mathrm{Var}(\tilde{M}) = \mathrm{Var}[(\sum \tilde{m}_i)/N] = \Big[\sum \mathrm{Var}(\tilde{m}_i) + \sum_{i \neq j} \mathrm{Cov}(\tilde{m}_i, \tilde{m}_j)\Big]/N^2$$

$$= [NV^2 + (N^2 - N)\gamma V^2]/N^2 = V^2[\gamma + (1-\gamma)/N]$$

where $V^2 = \mathrm{Var}(\tilde{m}_i)$ and γ is the correlation coefficient between two different $\tilde{m}_i$ and $\tilde{m}_j$. As the number of exposures becomes infinite, $\mathrm{Var}(\tilde{M}) = \gamma V^2$. This is called <u>systematic risk</u> because it cannot be reduced by the law of large numbers. The remainder of the profit margin variance, $(1 - \gamma)V^2/N$, becomes smaller as N increases. This portion is called the non-systematic, or diversifiable risk; i.e., adding more exposures to a portfolio will reduce overall variance. The classical risk theory model assumes that $\gamma = 0$.

Variance of ROE for Combination of Two Product Lines

For two separate product lines, denoted by subscripts 1 and 2, the return on equity for a composite of the two lines is

$$(1.3)\quad \tilde{R}_t = (1 + i)(1 + k_t\tilde{m}_t).$$

The composite premium/equity ratio k_t equals $f_1k_1 + f_2k_2$ and the composite TPM is $\tilde{m}_t = (f_1k_1\tilde{m}_1 + f_2k_2\tilde{m}_2)/k_t$. Thus

$$(1.4)\quad \tilde{R}_t = (1 + i)(1 + f_1k_1\tilde{m}_1 + f_2k_2\tilde{m}_2).$$

Now assume that each line has an infinite number of exposures so that only sytematic risk is present. The variance of $\tilde{R}_t$ is

$$\begin{aligned}(1.5)\quad \mathrm{Var}(\tilde{R}_t) &= (1 + i)^2[f_1{}^2k_1{}^2\mathrm{Var}(\tilde{m}_1) + f_2{}^2k_2{}^2\mathrm{Var}(\tilde{m}_2) + 2f_1f_2k_1k_2\mathrm{Cov}(\tilde{m}_1,\tilde{m}_2)]\\ &= (1 + i)^2k_1{}^2[f_1{}^2\mathrm{Var}(\tilde{m}_1) + f_2{}^2\mathrm{Var}(\tilde{m}_1) + 2f_1f_2\,\rho\mathrm{Var}(\tilde{m}_1)]\\ &= (1 + i)^2k_1{}^2\mathrm{Var}(\tilde{m}_1)[(f_1 + f_2)^2 - 2f_1f_2(1-\rho)]\\ &= \mathrm{Var}(\tilde{R}_1)[1 - 2f_1f_2(1-\rho)].\end{aligned}$$

Here ρ is the correlation coefficient between $\tilde{m}_1$ and $\tilde{m}_2$. Notice that $k_1{}^2\mathrm{Var}(\tilde{m}_1) = k_2{}^2\mathrm{Var}(\tilde{m}_2)$ by definition of the premium/equity ratios.

APPENDIX II: THE PARETO DISTRIBUTION

The Pareto distribution in its simplest form is useful for fitting the tails of loss size distributions. Its cumulative function is

$$(2.1)\quad F(x) = 1 - (r/x)^a \qquad \text{(where } a \geq 1 \text{ and } x \geq r\text{)}$$

with a density

$$(2.2)\quad f(x) = ar^a x^{-a-1}.$$

The mean and second moment are

$$(2.3)\quad \mu = \int_r^\infty xf(x)dx = ar/(a-1)$$

$$(2.4)\quad \mu_2 = \int_r^\infty x^2 f(x)dx = ar^2/(a-2).$$

Notice that the mean is infinite if $a \leq 1$ and the variance ($\mu_2 - \mu^2$) is infinite if $a \leq 2$. The expected portion of loss in the interval from r to br is

$$(2.5)\quad \int_r^{br} xf(x)dx = ar(1-b^{1-a})/(a-1), \text{ and}$$

the remaining segment of expected loss above br is

$$(2.6)\quad \int_{br}^\infty xf(x)dx = arb^{1-a}/(a-1).$$

The second moment of loss limited to a retention br is

$$(2.7) \qquad \int_{r}^{br} x^2 f(x)dx + \int_{br}^{\infty} b^2 r^2 f(x)dx = r^2(a - 2b^{2-a})/(a-2) \text{ for } a \neq 2,$$

$$= r^2[1 + 2 \cdot \ln(b)] \qquad \text{for } a = 2.$$

APPENDIX III: RANDOM SUMS

A common stochastic model for the claim-generating process is the <u>random sum</u>. This is discussed at length in probability theory (see Feller [3]). The total value of all losses occurring in a fixed time period is

$$(3.1) \quad \tilde{S}_N = \tilde{X}_1 + \tilde{X}_2 + \ldots + \tilde{X}_{\tilde{N}},$$

where the random variables $\tilde{X}_i$ are identically distributed and the number of claims $\tilde{N}$ is also a random variable independent of any $\tilde{X}_i$. Usually the $\tilde{X}_i$ are assumed to be independent of each other, but we will assume that correlation exists. The conditional expectation of the random sum given a <u>fixed</u> number of claims N is

$$(3.2) \quad E(\tilde{S}_N | N) = E(\sum_{i=1}^{N} \tilde{X}_i) = NE(\tilde{X})$$

The unconditional expectation is therefore

$$(3.3) \quad E(\tilde{S}_N) = \sum_{N=0}^{\infty} NE(\tilde{X})f(N) = E(\tilde{X})\, E(\tilde{N}),$$

where f(N) is the density function of N.

The conditional second moment, given a fixed N is

$$(3.4) \quad E(\tilde{S}_N{}^2 | N) = E\Big[\sum_{i=1}^{N} \tilde{X}_i{}^2 + \sum_{i \neq j} \tilde{X}_i \tilde{X}_j\Big] = NE(\tilde{X}^2) + (N^2 - N)E(\tilde{X}_i \tilde{X}_j)$$

for $i \neq j$. The unconditional second moment becomes

$$(3.5)\quad E(\tilde{S}_N{}^2) = \sum_{N=0}^{\infty} [NE(\tilde{X}^2) + (N^2 - N)E(\tilde{X}_i\tilde{X}_j)]f(N)$$

$$= E(\tilde{N})E(\tilde{X}^2) + [E(\tilde{N}^2) - E(\tilde{N})]\ [\mathrm{Cov}(\tilde{X}_i,\tilde{X}_j) + E^2(\tilde{X})]$$

since $\mathrm{Cov}(\tilde{X}_i,\tilde{X}_j) = E(\tilde{X}_i,\tilde{X}_j) - E(\tilde{X}_i)E(\tilde{X}_j)$. The variance of the random sum is

$$(3.6)\quad \mathrm{Var}(\tilde{S}_N) = E(\tilde{S}_N{}^2) - E^2(\tilde{S}_N)$$
$$= E(\tilde{N})\mathrm{Var}(\tilde{X}) + E^2(\tilde{X})\mathrm{Var}(\tilde{N}) + [\mathrm{Var}(\tilde{N}) + E^2(\tilde{N}) - E(\tilde{N})]\mathrm{Cov}(\tilde{X}_i,\tilde{X}_j),$$

after some manipulation of terms. If $\tilde{N}$ is a Poisson variable, then

$\mathrm{Var}(\tilde{N}) = E(\tilde{N})$ and (3.6) simplifies to

$$(3.7)\quad \mathrm{Var}(\tilde{S}_N) = E(\tilde{N})E(\tilde{X}^2) + E^2(\tilde{N})\mathrm{Cov}(\tilde{X}_i,\tilde{X}_j).$$

Letting the premium charge to cover the aggregate losses be PE(N), and letting ρ represent the correlation coefficient between $\tilde{X}_i$ and $\tilde{X}_j$, the variance of the loss ratio becomes

$$(3.8)\quad \mathrm{Var}[\tilde{S}_N/PE(\tilde{N})] = \mathrm{Var}(\tilde{S}_N)/P^2E^2(\tilde{N}) = \frac{1}{P^2}\left[\frac{E(\tilde{X}^2)}{E(\tilde{N})} + \rho\,\mathrm{Var}(\tilde{X})\right].$$

Frequency vs. Severity Components of Variance

Equation (3.6) can be separated into distinct frequency and severity components by alternately fixing $\tilde{N}$ and $\tilde{X}$ at their respective means (variances of the fixed variables are zero):

(3.9) $V[\tilde{S}_N | \tilde{X} = E(\tilde{X})] = E^2(\tilde{X})Var(\tilde{N})$ (Frequency Variance)

(3.10) $V[\tilde{S}_N | \tilde{N} = E(\tilde{N})] = E(\tilde{N})Var(\tilde{X}) + [E^2(\tilde{N}) - E(\tilde{N})] \rho \, Var(x)$ (Severity Variance)

When $\rho = 0$, the sum of the frequency and severity variances equals the total variance $Var(\tilde{S}_N)$. For a Poisson $\tilde{N}$ and a zero covariance between claim amounts, we find that the ratios of the variance components to the total variance are a sole function of the loss size distribution:

(3.11) Frequency Ratio: $\dfrac{E^2(\tilde{X})}{E(\tilde{X}^2)}$ Severity Ratio: $\dfrac{Var(\tilde{X})}{E(\tilde{X}^2)}$.

APPENDIX IV: PROFIT CENTER INCOME STATEMENT

Illustrative Example

Branch A: Second Quarter 1985 ($1,000's)

	Amount	%*	Comments
Premium Written	$1,150	8.2	Growth %
Premium Earned	1,000	5.4	Growth %
Gross Losses	650	65.0	Before internal reinsurance
Excess Losses	-45	-4.5	Amount recovered
Reinsurance Charge	40	4.0	For excess reinsurance
Catastrophe Charge	25	2.5	Gross losses exclude catastrophes
Net Losses	670	67.0	
Allocated Loss Expense	50	5.0	
Commissions	150	15.0	
Taxes and Fees	30	3.0	
Branch Overhead Expense	95	9.5	Actual branch costs
Home Office Overhead Exp.	65	6.5	Allocated as a fixed % of earned premium
Underwriting Result	-60	-6.0	
Total Profit	25	2.5	After income tax
Return on Equity	--	15.5	Implied; based on required equity

*of premium earned, except for premium

REFERENCES

1. Fairley, William B., "Investment Income and Profit Margins in Property-Liability Insurance: Theory and Empirical Results," The Bell Journal of Economics, X (Spring, 1979), 192-210.

2. Fama, Eugene F., and Miller, Merton H. The Theory of Finance, Hinsdale, Il.: Dryden Press, 1972.

3. Feller, William, An Introduction to Probability Theory and Its Applications, Vol. I., New York: John Wiley & Sons, Inc., 1968.

4. Kahane, Yehuda, "The Theory of Insurance Risk Premiums--A Re-examination in the Light of Recent Developments in Capital Market Theory," The ASTIN Bulletin, Vol. 10, Part 2, 223-239.

5. Lev, Baruch, Financial Statement Analysis: A New Approach, Englewood Cliffs, N.J.: Prentice-Hall, 1974.

6. Mayerson, Allen L., A Bayesian View of Credibility," Proceedings of the Casualty Actuarial Society, LI, 85-104.

7. Patrik, Gary, "Estimating Casualty Insurance Loss Amount Distributions," Proceedings of the Actuarial Society, LXVII, 57-109.

8. Plotkin, Irving H., Rates of Return in the Property and Liability Insurance Industry: 1955-1967, Cambridge: Arthur D. Little, Inc., 1969.

9. Sharpe, William R., Portfolio Theory and Capital Markets, New York: McGraw-Hill, 1970.

10. Williams, C. Arthur, Jr., "Regulating Property and Liability Insurance Rates Through Excess Profits Statutes," Journal of Risk and Insurance, L, No. 3, 445-472.