CREDIBILITY AND SOLVENCY

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Much work has been done in the past few years on the applications of Bayesian credibility to insurance pricing. This work has been born of necessity due to the failure of "classical" credibility theories. Recent work by Bühlman and Straub as well as Morris & Van Slyke incorporate the Bayesian concept of utilizing as much information available from historical data as possible in predicting behavior for a segment of a population.

The Bühlman-Straub work develops a method for individual risk rating using a credibility scheme for each risk based on historical data of the risk and the total population. The work by Mr. Heckman is an attempt at extending this approach to the problem of individual risk rating within the classification ratemaking exercise. In so doing he emphasizes the point that individual risk credibility can be approached in the same way as class credibility and he derives a very neat nested credibility structure. A second point which is driven home by this paper is the extent of work which remains to be done in investigation of suitable and workable estimation schemes.

The Model

Mr. Heckman begins his work by defining a model which assumes a certain structure for the first and second moments of the loss ratio distribution function as follows:

$$E\left(\tilde{x}_{Aa}(t)\right) = \xi \ ,$$

$$E\left[(\tilde{x}_{Aa}-\xi)(\tilde{x}_{Bb}-\xi)\right] = \delta_{AB}\sigma^2\left[\frac{1}{K} + \delta_{ab}\left(\frac{1}{K_A} + \frac{\delta_{tu}}{W_{Aa}(t)}\right)\right] .$$

This model structure assumes a global mean, though this is later shown to be an unnecessary assumption. The covariance can be given the following verbal interpretation. Classes are assumed to be independently distributed. Thus, there is no contribution to the covariance unless the two risks belong to the same class, therefore K is a measure of the homogeneity of the class. Similarly, K_A is a measure of the correlation of an individual risks' experience over time under the assumption that one risks' experience is independent of anothers (ie: no contribution to covariance between two risks). The $W_{Aa}(t)$ represents a random component of the model and σ^2 is a constant.

By defining $\tilde{x}_{Aa}(0) - \zeta = \sum_{b \in t} A_b^{Aa}(t)\left(\tilde{x}_{Ab}(t) - \zeta\right) + \tilde{\varepsilon}_{Aa}(0)$ where $b \in A$, a risk function can be defined as $\frac{1}{\sigma^2} E\left(\tilde{\varepsilon}_{Aa}(0)^2\right)$ It is this function which is to be minimized to arrive at the optimal set of coefficients $A_b^{Aa}(t)$. Mr. Heckman asserts that the solution which minimizes the individual risk also minimizes the global risk. Unfortunately, this result flows directly from this covariance model structure and is not true in general. The resulting nested credibility structure is very neat.

The credibility for a risk at time t is calculated to be $\frac{W_{Aa}(t)}{K_A + W_{Aa}(*)}$ and the class aggregate is a credibility weighted average of class experience with credibility

$$\frac{K_A Z_{A*}(*)}{K + K_A Z_{A*}(*)}$$

This workup is quite similar to the Bühlman-Straub methodology and the major obstacle now becomes the estimation of the parameters K and K_A.

Estimation of Parameters

A reasonable estimation procedure is presented by Mr. Heckman but nothing is said regarding the speed at which such an iteration converges, nor is it

clear that it must converge in all cases. Somehow building a theoretical model with all its niceties and then having to resort to a trial and error approximation routine seems less than perfect. It is in the area I would like to see some work done.

Mr. Heckman refers to the small sample corrections introduced in the Morris & Van Slyke work as being "gratuitously introduced". At this point I must take exception, the correction term is required due to the estimation procedures used. It is true that the current work does not require such correction factors but this is due to alternative, not superior, mathematics.

Implementation

At the end of the paper, Mr. Heckman asserts that the K and K_A values can be updated on a regular basis by bureaus who are "awash in" in the required data. While it is true that the operation of an experience rating plan is based on the correlation of premium and loss data for an individual risk, only Workers' Compensation experience rating modifications are computed from bureau data. The Statistical Plan for this line is

specifically designed so that the unit report data is on a policy basis. For other lines (ie: ISO lines) this level of detail is not possible nor is it necessarily desirable. Data for these lines is not compiled on a policy basis and cannot be retrieved on such a basis from bureau files.

Returning to Workers' Compensation, the only line where such a scheme may apply, the experience rating plan generates a single modification. The proposed scheme would generate separate credibilities by class for each risk thus adding a great deal more complexity to the plan. I suspect it would also add alot more variability to an individual risks' modification. Whether this is desirable is open to question.

All credibility work I have seen done by the Bayesian method in the last few years has concentrated on a one way class scheme. We are nearing the point of workable credibility methodology using this approach and it is time for thinking about the much more difficult problem of 2-way schemes. Hopefully more work will be done this year and next on both problems.