Life Contingencies Study Note for CAS Exam S

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(Revised 9/19/2015)

Introduction

"Life contingencies" is a term used to describe survival models for human lives and resulting cash flows that start or stop contingent upon survival. As such it is a central topic for life insurance actuaries. While there are many applications of these techniques to property and casualty insurance, life contingency calculations in P&C work are typically intermediate results used to estimate some other quantity of interest.

This note is intended to illustrate terminology and techniques in straightforward contexts, and to point out to the reader some places where further refinement is possible. A reader who needs more details or is interested in exploring these refinements should consult any of the many fine life contingencies texts that have been written over the years.

Chapter 1: Life tables and an extended example

A life table is simply a way of presenting a family of conditional survival random variables in a very concise way. Before giving a definition, let's look at an example.

Example 1: The SUX Vacuum Cleaner Company manufactures vacuums that last up to four years. One third of the company's vacuums fail in the first year, another third in the second year, one sixth in the third year, and the remaining sixth in the fourth year. On 01/01/XX the company sells 300 vacuums. How many of these are expected to fail in each of the next four years?

These quantities are easily seen to be 100, 100, 50, and 50, respectively. This type of data is very suitable for presentation in a table:

Year XX	01/01/XX Number Functioning	Number Failing During Year XX
00	300	100
01	200	100
02	100	50
03	50	50

After the first year, assuming that we saw the 100 failures that we expected, we could display the information about the remaining vacuums in another table:

Year XX	01/01/XX Number Functioning	Number Failing During Year XX
01	200	100
02	100	50
03	50	50

Obviously, the second table is exactly the last three rows of the first table, but there is one subtle difference. In the first table, the information is about all three hundred original vacuums. Whereas, in the second table, the information is only about those that survived the first year.

Results for businesses are often expressed on an annual basis, so the number expected to fail in the coming year is a quantity of interest. Initially, there are 300 functioning vacuums, and we expect one third of them (100) to fail during the first year and another third (100) to fail during the second year. Looking at the second of the two tables above, we see that of the 200 vacuums functioning at the start of Year 1, one-half of them are expected to fail during the year. This ratio, the number expected to fail during the coming year divided by the number still functioning at the start of that year, is the conditional one-period probability of failure sometimes called the mortality rate or even more succinctly the mortality.

The number functioning at the start of the year, the number failing during the year and the mortality are denoted by the letters, l, d, and q, respectively. Since these generally vary by year, a subscript is used to indicate which year, as in the following table:

Age x	l_x	d_x	q_x
0	300	100	0.333
1	200	100	0.500
2	100	50	0.500
3	50	50	1.000

Suppose that the company decides to offer a warranty that will pay $100 at the end of the year in which the machine fails. Since all of the machines fail by the end of year 4 and no machine can fail more than once, this will cost the company exactly $100/machine. Still, the timing of the payment is uncertain and the discounted value of the payment can be thought of as a random variable. In order to make sense of that, we need to know the rate(s) of interest for discounting.

In practice, yield curves are seldom level, but the formulas for classical life contingencies are much simpler in the case that there is a single deterministic interest rate. Hence, that assumption is generally made when introducing life contingencies, and we will make it too.

At the time of the vacuum's sale, the expected value of the warranty payment is:

$$APV = 100 * \sum_{j=0}^{3} v^{j+1} * \Pr(K = j)$$

where $v = \frac{1}{1+i}$, *i* is the interest rate, and K is the number of full years that the machine functions.

APV stands for actuarial present value, which is simply the present value of a stream of cash flows that depend upon the value of a random variable.

If the level interest rate is 5%, the reader should verify that the APV = $90.09.

The company decides that this is too much to give away. Instead it offers a subscription service so that purchasers of a new vacuum can make annual payments to buy the warranty. The company would like the payments to be level and, of course, customers will stop subscribing once they have collected on the warranty. How much should the company charge for this if it wants to break even on it?

The analysis is very similar to the above calculation:

$$APV = S * \sum_{j=0}^{3} v^{j} * \Pr(K \geq j)$$

Here *S* is the annual subscription payment. This payment is made at the beginning of each year that the machine functions, which explains the one less year of discount from the earlier calculation.

The reader should confirm that each dollar of subscription charged has an APV of $2.08, so to break even the company should set the two APV amounts equal and charge $43.30/year for the warranty (amounts are rounded).

The above example illustrates most of the major ideas in life contingencies. The remainder of this note fleshes them out a bit more.

Chapter 2: Comments on Life tables

In this section we will discuss life tables a bit more, this time using human lives as our example instead of machine failure.

In the above example, all of our machines were manufactured on 01/01/XX and the table described their states on 01/01 of the following years. In other words, our machines were of exact age *x*. People have different birthdates, but for practical reasons we will use a table that assumes that their exact age in known. Even the notion of age requires a more careful definition: is it age nearest birthday (so that age 60.9 is considered age 61) or is it age last birthday (so that age 60.9 would be age 60)? We will not concern ourselves with these nuances, but you should be aware that they do exist.

Mostly we will be interested in individuals that are having their birthday today, so that they just turned age *x* and we will denote that person by (x). For example, (40) is an individual whose exact age is 40.

In the example we denoted the number of machines functioning at the start of year *x* by l_x. When this number is the number of individuals who are exact age (x), we use the same notation.

In the example, we denoted the number of machines that failed during the year by d_x. For individuals, this quantity is the number of people that died between exact age x and x+1. In other words:

$$d_x = l_x - l_{x+1}$$

We also defined:

$$q_x = \frac{d_x}{l_x}$$

These we called (annualized) mortalities. They can each be interpreted as the probability that a person age *x* dies before reaching age *x*+1.

In our example we started with 300 machines, but we could just as well have started with some other number, say 600. The number of machines at the start of each year doubles as does the number failing during the year, but the mortality stays the same as it is a ratio of these two. Typically we will be interested only in the mortality so the choice of l_0 does not matter any positive number will do. That quantity (called the radix) is usually picked to be some convenient round number, such as 100,000.

It is convenient to have a standard notation for surviving from age *x* to *x+1* and we denote this by p_x.

$$p_x = \frac{l_{x+1}}{l_x}$$

This follows from the above two relationships, as the reader should confirm.

We will also want a standard notation for surviving from age *x* to *x+n* and we denote this by ${}_n p_x$. If *n*=0, by convention we interpret this symbol as equal to one. Notice:

$$ {}_n p_x = p_x p_{x+1} p_{x+2} \cdots p_{x+n-1} = \frac{l_{x+n}}{l_x} $$

This last fact illustrates how convenient a life table can be for performing this type of calculation.

As one might guess, the symbol ${}_n q_x$ denotes $1 - {}_n p_x$.

Exercises:

Using the Illustrative Life Table for this exam, compute:

1) The probability that (50) lives to be at least 60 years old.
2) The probability that (50) dies at age 60 (meaning sometime during the year).
3) The probability that (40) lives to age (65) and an independent life (30) dies before age (50).
4) The expected number of deaths during the next 10 years of 1,000 people age (80).
5) The variance in 4)
6) Using the normal approximation, give a 95% confidence interval for the number of deaths in 4)

Chapter 3: Contingent Cash flows

Consider a contract that pays $1 at the end of the year of death of (x). We are interested in computing the actuarial present value of this payment. We will need to know the mortality assumptions (from the life table), the discount rate (we assume a level term structure of rates), and the timing of the payment (we are told that it occurs at the end of the year of death).

Such contracts are called life insurance contracts. Some life insurance contracts only remain in effect for a fixed number of years the result being that if the insured dies during the contract period, the $1 is paid, otherwise nothing is paid. Such contracts are called <u>term life insurance</u> and the length of the period is called the <u>term</u>. Policies with an unlimited term are called <u>whole life</u> policies.

Since all of the vacuums in our earlier example failed within four years, the warranty the company offered was effectively whole life insurance.

It is convenient to have a notation for the actuarial present value of a whole life insurance policy of $1 on (x), and we will introduce this shortly. This policy pays $1 at the end of the year of death of (x). Measuring years from the start of the policy, call that year *k* with the first year being *k*=1. Consider the end of the first year, one of two things has happened: either (x) has died --- this has probability q_x --- or (x) lived, which has probability p_x. So, the actuarial present value of the payment (if any) at the end of year 1 is vq_x. If there is no payment, the contract continues for another year, <u>but (x) is now one year older</u>. If we write A_x for the actuarial present value of a life insurance of $1 on (x), then we have:

$$A_x = vq_x + v\,p_x A_{x+1}$$

In other words, a whole life policy of $1 on (x) is equivalent to a dollar paid in a year if (x) dies and if he doesn't die the present value of a replacement policy delivered in one year on (x+1). The reader is urged to match the words in the interpretation to the symbols in the formula --- as you will see the order is slightly different.

This recursion relationship can be applied repeatedly to produce the following formula:

$$A_x = \sum_{n=0,1,2,\ldots} v^{n+1}\,{}_n p_x q_{x+n}$$

Which says: the actuarial present value of an insurance of $1 on (x) is the discounted value of a payment made at the end of year *n* if (x) lives to age *x+n* and then dies during the next year.

So far we have looked at whole-life insurance. Another commonly sold product is term (life) insurance. This product is just like whole-life insurance except that at a certain age the policy terminates (expires worthless); since some insureds are still alive at policy termination, there are fewer death claims and this results in a lower pure premium than that required for whole-life. A similar product is called endowment insurance, which is just like term insurance except that at termination the policy pays $1 instead of maturing worthless; like whole-life, every such policy eventually pays $1, and for insureds that are still alive at policy maturity it pays earlier than whole-life would, so such policies have a higher pure premium than whole-life. Finally, there is a pure endowment which is the difference between term insurance and endowment insurance, namely a payment at maturity if the insured is still alive. For example, a ten-year term insurance on (40) pays $1 if (40) dies in the next ten years. A ten-year endowment insurance on (40) pays $1 if (40) dies in the next ten years and if (40) is still alive at age 50, it pays $1 anyway. A pure ten-year endowment on (40) pays $1 if (40) lives to age 50. In these examples all death benefits are assumed to be paid at the end of the year of death and maturity payments are made at maturity.

In our example in the introduction, the company had the customers pay an annual fee in order to purchase the warranty. This is how life insurance is typically paid for. At the start of each year (x) pays a premium of *P* to the insurance company in exchange for a payment of $1 at the end of the year of death of (x). We need a notation for the actuarial present value of a stream of payments made at the beginning of each year while (x) is alive. Such a stream of payments is called a life-annuity-due and its actuarial present value is denoted by $\ddot{a}_x$. The double dot over the annuity symbol means that the payments are made at the start of the year. The reader may recall from interest theory that such an annuity is called an annuity-due. If the payment is made at the end of the year (provided that (x) is still alive then), it is called a life-annuity-immediate and its actuarial present value is denoted by a_x.

We would like to evaluate actuarial present value of a life-annuity on (x). This can be done by directly writing down the recursive formula, just as we did for the whole life policy on (x). In words: a life-annuity due of $1 on (x) is equivalent to an immediate payment of $1 and, if (x) is still alive in one year a replacement annuity on (x+1) deliverable in one year. In symbols:

$$\ddot{a}_x = 1 + v\, p_x \ddot{a}_{x+1}$$

Expressing this as a sum, as we have done for insurance above, is simple, and readers should do that for themselves as an exercise.

There is an extremely useful relationship between the value of a life insurance policy on (x) and the value of a life-annuity on (x). This relationship can be seen best through an arbitrage argument: an investor is ambivalent between having a dollar today and receiving a combination of one year's interest on that dollar and getting the dollar itself back at the end of one year. A moment's thought reveals that there is nothing special about one year, it could just as well have been three years or ten years or even a random number of years.

Recall from interest theory that interest paid at the beginning of the year is called discount and is denoted by *d*. (Not to be confused with d_x the number of deaths at age x in a life table from before.)

Here is the relationship in symbols:

$$1 = d\ddot{a}_x + A_x$$

In words: an investor is ambivalent between (LHS[1]) having $1 today and (RHS[2]) receiving a stream of *d* dollars at the start of each year while (x) is alive and then at the end of the year of the death of (x) getting back the dollar.

You can think of the annual payment of *d* as the rent on the dollar for the coming year.

We immediately obtain:

$$\ddot{a}_x = \frac{1 - A_x}{d}$$

This lets us determine the level annual premium for a whole life policy on (x) by setting:

$$A_x = P\ddot{a}_x$$

In other words, we want to find an annual premium payment, *P*, payable at the start of each year that (x) is alive which is actuarially equivalent to a dollar paid it the end of the year of death of (x). This is easily solved for *P*.

[1] Left Hand Side of the equation
[2] Right Hand Side of the equation

Just as we could limit the contract period for whole life and obtain term life, we can also limit the term of an annuity. We could pay until (x) died or *n* years, whichever comes first. Another popular structure is a life annuity with a guarantee of paying at least *n* years. Such a contract would pay until (x) died or until *n*-years had passed, whichever happens last.

Exercises:

1) What is the level premium for a whole life of $1 on (40) using the Illustrative Life table?
2) Would an increase in interest rates cause A_x to increase or decrease? How about $\ddot{a}_x$?
3) You are given that a $1 whole life policy on (x) has an APV of $0.30 and that d = .035. What is the level annual premium for this insurance?
4) On a given individual, rank in order of increasing APV: 10-year term, 10-year endowment insurance, and whole life. Explain.
5) Repeat exercise 4) for annuities with guarantee periods, annuities with maximum payment periods, and life-annuities in place of the three types of insurance.
6) (Variances of insurances) Recall that the variance of a random variable (the second central moment) can be computed by computing the second raw moment and subtracting from it the square of the first moment. Show that the second raw of a whole life insurance of $1 on (x) with a discount rate of υ is, in fact, the first moment of a whole life insurance of $1 on (x) with a discount rate of υ^2. In other words, we obtain the second raw moment of an insurance if $1 by doubling the force of interest. Now produce a formula for the variance of such an insurance.
7) (Variance of annuities) Use the relationship between insurances and annuities along with Exercise 6 (above) to produce a formula for the variance of a life-annuity-due of 1 on (x).

Chapter 4: An example from workers' compensation insurance

An important line of commercial insurance in the US is workers' compensation insurance, which provides benefits to workers who are injured on the job. One application of annuities is to estimate the future costs of paying an annuity to an injured worker who is unable to return to work. The details of workers' compensation insurance are covered elsewhere in the CAS Syllabus; for our purposes it is enough to know that an injured worker could be paid a monthly stipend and that this stipend stops when the worker returns to work. Depending upon the type of injury, the stipend might have a maximum number of months of payment or the stipend might have a guaranteed minimum number of payments.

We will look at two simplified examples. In both we will make the unrealistic (but computationally easier) assumption of annual payments. We will also assume that the actuarial present values are discounted for the time value of money by a level interest rate of 3%. (Under the accounting system used by insurance regulators in the US, most reserves for future loss payments are not discounted for time value of money, but certain reserves for workers' compensation payments are.)

To compute the actuarial present values of the two annuities we will use the following table:

Age x	l_x	$\ddot{a}_x$
20	100,000	20.8952
25	95,000	19.8536
40	75,000	17.0563
65	37,750	11.4546

The "mortality" (which in this case represents returning to work) was selected to make the calculations more transparent. Only the rows that we will use are shown; a real table would have rows for every age.

For our first example, we use the table to compute an annual life annuity-due for a 20-year-old with a certain period of 5 years (i.e. it pays $1 at the start of each year until (20) goes back to work or five years, whichever is longer.) The payments from this annuity are exactly the same as the payments from the following: a five-year annuity certain and a life annuity on (25), deliverable in five years, if (20) is still alive. Recalling the notation for a five-year annuity-certain-due, $\ddot{a}_{\overline{5}|}$, we have in symbols:

$$\ddot{a}_{\overline{5}|} + \frac{l_{25}}{l_{20}}(1.03)^{-5}\ddot{a}_{25}$$

The five-year annuity-certain at 3% = 4.7171 and the other quantities are taken from the table.

For our second example, we will compute the APV of an annuity on (40) which pays until he returns to work or reach age (65), whichever comes <u>first</u>. Again, we find a replicating portfolio. Suppose that we gave (40) payments until he went back to work, but made him start to give those payments back to us if he was still not back to work at age 65. The net payments to him would be exactly the payments we owe him. In other words, give a life annuity today on (40) and take back in twenty-five years a life annuity on (65), if he has not returned to work. In symbols: $\ddot{a}_{40} - \frac{l_{65}}{l_{40}}(1.03)^{-25}\ddot{a}_{65}$

Here all of the numbers can be found in the table excerpt.

Chapter 5: Why P&C companies don't sell level premium products

Look back at the end of the first example where the company sold the warranty to the customers for a level annual premium. The first year, the warranty premium was \$43.29. What does the customer get for this? They get coverage for the first year and they also get the option to buy coverage for future years at that same fixed price. If they had only wanted to buy coverage for the first year, the APV of this coverage is \$31.75. The reader should check this and should compute the one-year term premium for each of the remaining three years. (Coverage for each year could be called one-year term insurance.)

We said that the purchaser got an option to purchase coverage in future years. True, our contract specified that they had to buy coverage for each year, but in reality there is no way to enforce that they actually purchase it. A purchaser that does not renew a policy is said to have lapsed.

In our example, if a purchaser lapses, the company has the extra \$11.54. This amount is non-negative because the mortality in our example is non-decreasing over the life of the policy. Human mortality beyond very early ages tends to be non-decreasing, so these amounts (sometimes called policy values or premium reserves) are positive for life insurance. In fact, these amounts can be so significant that state laws often require that some portion of it be shared with a lapsing policyholder.

On the other hand, suppose that mortality were decreasing over the coverage period. Now if we try to price a level premium policy the same way we find a problem. When the mortalitywas increasing, as above, the insured paid “extra” premium in year 1 which went to subsidize the insufficient premium in later years, but if mortality is decreasing the insured would pay too little the first year and we would rely on the “extra” premium from subsequent years to make up the shortfall. Unfortunately, we can’t force the insureds to renew and the rational thing for them to do is lapse (after obtaining as much subsidy as possible) and then purchase term insurance for future years to obtain the same coverage at lower cost.

That scenario is not as far-fetched as it sounds, because property and casualty risks tend to have decreasing hazard rates (a synonym for mortality rates) over time. This happens because the worst risks tend to have their accidents earlier and over time only better risks remain in the cohort.

Chapter 6) What happens mid-year?

So far we have only concerned ourselves with annual timing of events and payments. The life table tells us the probability of (x) dying in the next year, but it does not tell us when during the year that that might occur. Suppose that we wanted to do everything on a monthly basis. We could simply refine our life table be a monthly table with monthly mortalities. We would need to obtain the entries from somewhere. Typically this is done by some form of interpolation between the annual mortalities. There are many ways to interpolate, each with its own advantages and disadvantages. Methods used in practice are linear interpolation, exponential interpolation, and hyperbolic interpolation. The last one appears in practice because it is well-suited for the data that one sometimes gets from clinical trials.

It is also possible to do things in continuous time. In that setting we think of mortality in terms of survival functions and hazard rates. These are covered elsewhere in the CAS Syllabus, so we won’t develop them here. The reader should know that the hazard rate when used to describe mortality is called the "force of mortality".

There is one situation where the timing of events during the year might matter to us. Suppose that we have sold worker's compensation insurance. One of the coverages requires us to pay the salary (or a portion of it) to a worker that was injured on the job. We will pay this amount every month until the worker returns to work, reaches age 65, or dies, whichever comes first. In evaluating the actuarial present value of these payments, we need to consider the likelihood of the three possible events that could make us stop paying, namely the probability that the worker returns to work , the probability that the worker reaches age 65, and the worker's mortality. Notice that these are not independent of one another.

The more likely it is that the worker returns to work, the less likely it is that he reaches age 65 or dies while we are still paying the claim. We think of the claim as being in a paying status and that status has multiple forces acting on it. These forces are "competing" in the sense that only one of them will cause the claim to stop being paid. An increase in the likelihood that a worker returns to work will decrease the probability that the worker is still on claim at age 65.

Sometimes an injured worker has injuries so severe that the usual life table is not appropriate. This is often described as a percentage increase in mortality, although saying that the force of mortality has increased or that the odds-ratio of survival has decreased is often more accurate. For instance, if the probability of (x) dying in the next year is 10% and we are told that his mortality triples, we could use 30% as the probability of (x) dying in the next year. Of course that would not work if his probability of dying were already 50% as probabilities are limited to 100%. Still that phraseology is often used and since mortalities (fortunately) tend to be small we will follow custom and just multiply the mortalities when we are told that they are increased[3].

Finally, it is possible to have an annuity that pays based on the survival of more than one person (joint lives). Typically, there will be two people (x) and (y) and in practice they will often be a married couple. The annuity may pay until one of the two dies (denoted $\ddot{a}_{xy}$) or it may pay until both persons have died (denoted $\ddot{a}_{\overline{xy}}$). There are relationships between annuities on single lives and annuities on joint lives. One such relationship is easily seen through an arbitrage argument: A couple is ambivalent between (LHS) each receiving \$1 while they individually are alive and (RHS) receiving \$1 while either of them is alive and a second dollar while they are both alive:

[3] Survival one year can be thought of surviving the cumulative hazard rate for an additional year. Increasing this hazard rate from $\lambda(t)$ to $k\lambda(t)$ changes the survival probability from p_x to p_x^k and the probability of death from $1-p_x$ to $1-p_x^k$. If p_x is near 1, then $1-p_x^k$ (which is $1-(1-q_x)^k$) will be near kq_x.

$$\ddot{a}_x + \ddot{a}_y = \ddot{a}_{\overline{xy}} + \ddot{a}_{xy}$$

Exercise:

1) A life annuity of $1 on (40) costs $15 and one on (50) costs $13. If an annuity that pays $1 while either of them are alive costs $20, how much does one that pays $1 while they are both alive cost?

Chapter 7: Life expectancy

For some purposes it is useful to be able to estimate the number of years an individual will live. Of course we can only answer this on average. We call this average the life expectancy. We might be interested in the number of full years or in the actual number of years. The expected number of full years is called the <u>curtate life expectancy</u>. It can be computed from the life table. If we are interested in the actual life expectancy (not just full years), it is called the complete expectation of life and it will be longer. To compute it we need to make an assumption about the timing of death during the year. If we assume that all deaths are uniformly distributed throughout the year, then we can get the complete expectation by adding 0.5 to the curtate expectation.

It should be mentioned that the curtate expectation itself need not be an integer; rather it is an average of integers.

Property & Casualty insurers pay claims for lines other than workers' compensation. For some of these lines, the payment may be made several years in the future. There is a cost to simply having a file open, in the past these were called unallocated loss adjustment expenses. Suppose that 55% of claims get closed in the first year, 25% in the second year, 10% in the third year and 10% in the fourth year. A new claim has just come in. How many full years do we expect the claim to be open?

We know that 100%-55%=45% of the claims remain open after one full year. Also, 20% remain open after an additional full year. Finally, 10% remain open after a third full year. So, the curtate expectation of a claim being open is 0.45+0.20+0.10 = 0.75 years. If we assume that when a claim closes, it on average closes in the middle of the year, we can compute the complete expectation of a claim being open as 0.75 + 0.50 = 1.25 years.

The calculation above illustrates how to compute the curtate expectation of life of (x) from a life table, namely compute ${}_n p_x$ (defined at the end of Chapter 1) and sum this over all *n* starting with n=1.

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Appendix: Solutions for selected problems from

Life Contingencies Study Note for CAS Exam S

Chapter 2 Exercises:

Using the Illustrative Life Table for this exam, compute:

1) The probability that (50) lives to be at least 60 years old.

$l_{50} = 8{,}950{,}901$ and $l_{60} = 8{,}188{,}074$ so the desired probability is: $\dfrac{8{,}188{,}074}{8{,}950{,}901} = 0.9148$

2) The probability that (50) dies at age 60 (meaning sometime during the year).

He needs to live to 60, and then die in the next year. $q_{60} = 0.01376$ so the desired probability is:

$0.9148 * 0.01376 = 0.01259$

3) The probability that (40) lives to age (65) and an independent life (30) dies before age (50).

The lives are independent, so we can compute the probabilities of the events separately and multiply them: $\dfrac{l_{65}}{l_{40}} * \left(1 - \dfrac{l_{50}}{l_{30}}\right) = 0.04687$

4) The expected number of deaths during the next 10 years of 1,000 people age (80).

This is $1000\,{}_{10}q_{80} = 1000(l_{80} - l_{90}) / l_{80} = 729.6$ directly from the tables.

5) The variance in 4)

Using a binomial model the variance is npq = 197.29

6) Using the normal approximation, give a 95% confidence interval for the number of deaths in 4)

Expected +/- 1.96*standard deviation = 729.6 +/- 1.96*sqrt(197.29) = (702.1,757.1)

Chapter 3 Exercises:

8) What is the level premium for a whole life of $1 on (40) using the Illustrative Life table?

   $A_{40} = 0.16132$ and $\ddot{a}_{40} = 14.8166$ from the table. Since $P\ddot{a}_{40} = A_{40}$, $P =$ 0.010888

9) Would an increase in interest rates cause A_x to increase or decrease? How about $\ddot{a}_x$?

   An interest rate increase decreases the present value of all future payments, contingent or not.

4) On a given individual, rank in order of increasing APV: 10-year term, 10-year endowment insurance, and whole life. Explain.

   Assuming that there is some chance that the individual lives more than ten years, 10-year term is the smallest, since it might not pay, and 10-year endowment is the largest since it might pay earlier than whole life, which is in the middle.

Chapter 6) Exercise:

2) A life annuity of $1 on (40) costs $15 and one on (50) costs $13. If an annuity that pays $1 while either of them are alive costs $20, how much does one that pays $1 while they are both alive cost?

   An easy arbitrage argument shows that this annuity must cost $15 + $13 - $20 = $8.

<End>