THE PROPENSITY TO CAUSE ACCIDENTS*

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1. INTRODUCTION

Information relating to the expected number of losses is of importance in automobile insurance systems. The distribution of risks by number of losses per year may be based on the following model

$$P(x) = \frac{\lambda^x}{x!} e^{-\lambda} \qquad x = 0, 1, \ldots \tag{1}$$

with λ representing the average number of losses per year. This distribution is the Poisson distribution. Tests of this model versus actual observations often indicate significant deviation. This discrepency can result from the constancy of λ which makes the model appropriate for an individual but would require an isohazardous population when applied to a group of individuals. In reality, however, λ will vary from individual to individual. A model accounting for this spread in λ is given in

$$Q(x) = \int_\lambda P(x) \cdot z(\lambda)\, d\lambda \qquad x = 0, 1, \ldots \tag{2}$$

where $z(\lambda)$ is a distribution describing the spread of λ. The results of model (2) certainly will depend on the form of $z(\lambda)$. It has been hypothesized that $z(\lambda)$ can be represented by (3)

$$z(\lambda) = \frac{a^b}{\Gamma(b)} \lambda^{b-1} e^{-a\lambda} \tag{3}$$

which is a Pearson Type III [1, 5, 7]. With this assumption model (2) becomes the negative binominal distribution

$$Q(x) = \left(\frac{a}{1+a}\right)^b \binom{-b}{x} \left(\frac{-1}{1+a}\right)^x \tag{2a}$$

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with a mean of

$$E(x) = \frac{b}{a}$$

and a variance of

$$\sigma^2 = \frac{b}{a} \cdot \frac{a + 1}{a}$$

If the observed mean is $\hat{\lambda}$ and the observed variance $\hat{\sigma}^2$ it is possible to determine a and b by solving the above equations for mean and variance. Thus

$$a = \frac{\hat{\lambda}}{\hat{\sigma}^2 - \hat{\lambda}}$$

and

$$b = \frac{\hat{\lambda}^2}{\hat{\sigma}^2 - \hat{\lambda}}$$

The results indicate an improved fit to actual observations [1, 5, 6].

From this it is concluded that

(a) there exists a spread of λ in a group of individuals

(b) the Pearson Type III represents $z(\lambda)$ satisfactorily.

If the observed data relate to accidents, model (2a) produces both an estimate of the expected number of accidents for a given population and the distribution of the propensity to cause accidents of that population. If, on the other hand, the observed data represent claims, (2a) will estimate the expected number of claims and the distribution of the population by propensity to file claims.

In reality the observed data will generally relate to claims [9] since most existing automobile insurance systems are characterized by merit rating and merit rating has the built-in incentive not to file a claim for every accident (bonus hunger [8]). The derivation of the distribution of the population by accident causing propensity requires to consider explicitly the claim behavior.

The purpose of this paper is to propose a model to be used in deriving the distribution of insurees by their accident causing propensity without making any assumptions about the type of the distribution.

2. The Automobile Insurance System

In automobile insurance systems the insured population is classified according to various criteria. Let index $j (j = 1, \ldots, J - 1)$ represent a risk category as defined by one or more demographic characteristics such as age, sex, marital status, driving intensity, type of automobile, territory of operation, etc. In systems with merit rating structures the insurees in a given risk category are also grouped according to some performance criterion, generally the claim experience. $n (n = 1, \ldots, N)$ identifies a rating class for a given claim experience. Each rating class n is characterized by a discount (bonus) from or a surcharge (malus) to a base premium. For each rating class n a set of transition functions $T_n(k)$ can be defined which specify the rating class in the next policy year after filing k $(k = 0, 1, \ldots, K(n))$ claims with the insurance company. Transition function $T_n(K(n))$ is the same for $k \geq K(n)$. It should be noted that $K(n)$ may vary from rating class to rating class, e.g. in rating class $n = 3$, $K(3)$ may be equal to two while it is four for $n = 7$. Furthermore, an insuree in risk category j and rating class n is characterized by his accident causing propensity λ following the hypothesis that a distribution $z(\lambda)$ exists. The accident causing propensity is subdivided into small intervals which are identified by an index $s (s = 1, \ldots, S)$.

3. The Claim Behavior

Merit rating structures in automobile insurance systems require from an insuree the decision whether to file a claim for an accident when he is at fault. This decision will be based on a critical accident size: If the actual amount of the accident falls below the critical accident size the insuree will incur the cost of the accident himself in order to maintain his preferred position. If the actual loss is above the critical accident size the insuree will file a claim with the consequence of being reclassified according to the merit rating structure. The critical accident size is a non-negative quantity which depends on the insuree's risk category j, his position in the merit rating system prior to the decision as expressed by the rating class n and the number of claims, k, already filed with the insurance company during the policy year, and his accident causing propensity s. The claim behavior can be formalized by decision rule (4)

$$L_t - L_t^k(j, n, s) \begin{cases} \geq 0 \text{ Claim} \\ < 0 \text{ Do not claim} \end{cases} \tag{4}$$

with L_t representing the actual loss in year t ($t = 1, 2, \ldots$) and $L_t^k(j, n, s)$ the critical accident size. Naturally, it is possible to determine the optimal value of the critical accident size, $\hat{L}_t^k(j, n, s)$, using a given optimization criterion [3, 4].

4. The Model

Let $W_t(j, n, s)$ be the proportion of individuals from a given population who belong to risk category j, rating class n and propensity interval s during period t. Clearly

$$\sum_j \sum_n \sum_s W_t(j, n, s) = 1. \tag{5}$$

$W_t(j, n, s)$ can be considered as the result of flows between risk categories, rating classes and of changes in the accident causing propensity adjusted by a birth and death process allowing for individuals to enter (accretion) and others to leave (attrition) the automobile insurance system. The aspect of accretion and attrition can be handled by an artificial risk category J which new insurees come from (births) and "retiring" insurees go to (deaths). Let $p_t(ij, mn, rs)$ be the probability of transition from $i (i = 1, \ldots, J)$ to j, from m $(m = 1, \ldots, N)$ to n and from $r (r = 1, \ldots, S)$ to s. $W_t(j, n, s)$ can then be expressed by

$$W_t(j, n, s) = \sum_i \sum_m \sum_r W_{t-1}(i, m, r)\, p_t(ij, mn, rs). \tag{6}$$

The critical step is to derive the stochastic matrix. A transition from risk category i to risk category j takes place as the respective demographic characteristics change. $\pi_t(ij \mid m, r)$ for $i = 1, \ldots, J - 1$ represents the probability that an insuree in risk category i moves to risk category j given he belongs to rating class m and propensity interval r. $\pi(ij \mid \cdot, r)$ for $i = J$ is the probability of a new insuree entering risk category i. These insurees enter the rating class for beginners and those without any information regarding the claim experience. Changes in the accident causing propensity result from driving skills improving, remaining the same or deteriorating. Let $g_r(s)$ be the probability that a randomly selected insuree in propensity interval r will be in interval s next period. The specific form of $g_r(s)$ will depend on r but is taken to be independent of risk category and rating class [1]. A transition from rating class m to rating class n occurs as the consequence of the insuree's claim behavior which is determined by the critical accident size, $L_t^k(i, m, r)$, used in decision rule (4). Clearly, $L_t^k(i, m, r) = 0$ for $k \geq K(m)$ since filing $K(m)$ or more claims results in the same rating class $n = T_m(K(m))$ during the following policy year. The values for $K(m)$, of course, depends on the merit rating structures of the automobile insurance system under consideration. If we assume $\max\{K(m)\} = 4$ and define $f_t^j(L)$ as the density function of the amount of an accident in risk category j during t and $F_t^j(L)$ as the corresponding distribution function it is possible to express the transition probabilities with $K(m) = 4$ by (7) and (8).

[1]) The model can easily be expanded to make $g_r(s)$ also a function of risk category i and rating class m.

$$
p_t(ij, mn, rs) =
\begin{cases}
\pi_t(ij \mid m, r)\, g_r(s) \sum\limits_{x=0}^{\infty} P(x \mid r)\, [F_t^j(L_t^0(i, m, r))]^x & n = T_m(0) \\
& j = 1, \ldots, J-1 \\[1ex]
\pi_t(ij \mid m, r)\, g_r(s)\, [1 - F_j^t(L_t^0(i, m, r))] \sum\limits_{x=1}^{\infty} P(x \mid r) \cdot & \\
\cdot \left\{ \sum\limits_{h=0}^{x-1} [F_t^j(L_t^0(i, m, r))]^h [F_t^j(L_t^1(i, m, r))]^{x-1-h} \right\} & n = T_m(1) \\
& j = 1, \ldots, J-1 \\[1ex]
\pi_t(ij \mid m, r)\, g_r(s)\, [1 - F_t^j(L_t^0(i, m, r))]\, [1 - F_t^j(L_t^1(i, m, r))] & \\
\qquad \sum\limits_{x=2}^{\infty} P(x \mid r) \cdot & \\
\cdot \left\{ \sum\limits_{h=0}^{x-2} \sum\limits_{l=0}^{x-2-h} [F_t^j(L_t^0(i, m, r))]^h\, [F_t^j(L_t^1(i, m, r))]^l \cdot \right. & n = T_m(2) \\
\left. \cdot\, [F_t^j(L_t^2(i, m, r))]^{x-2-h-l} \right\} & j = 1, \ldots, J-1 \\[1ex]
\pi_t(ij \mid m, r)\, g_r(s)\, [1 - F_t^j(L_t^0(i, m, r))]\, [1 - F_t^j(L_t^1(i, m, r))] \cdot & \\
[1 - F_t^j(L_t^2(i, m, r))] \sum\limits_{x=3}^{\infty} P(x \mid r) \cdot & n = T_m(3) \\
\cdot \left\{ \sum\limits_{h=0}^{x-3} \sum\limits_{l=0}^{x-3-h} \sum\limits_{v=0}^{x-3-h-l} [F_t^j(L_t^0(i, m, r))]^h \cdot \right. & j = 1, \ldots, J-1 \\
\left. \cdot\, [F_t^j(L_t^1(i, m, r))]^l\, [F_t^j(L_t^2(i, m, r))]^v\, [F_t^j(L_t^3(i, m, r))]^{x-3-h-l-v} \right\} & \\[1ex]
\pi_t(ij \mid m, r)\, g_r(s)\, [1 - F_t^j(L_t^0(i, m, r))]\, [1 - F_t^j(L_t^1(i, m, r))] \cdot & \\
\cdot\, [1 - F_t^j(L_t^2(i, m, r))]\, [1 - F_t^j(L_t^3(i, m, r))] \cdot & n = T_m(K(m)) \\
\cdot \sum\limits_{x=K(m)}^{\infty} P(x \mid r) \sum\limits_{k=K(m)}^{x} \left\{ \sum\limits_{h=0}^{x-k} \sum\limits_{l=0}^{x-k-h} \sum\limits_{v=0}^{x-k-h-l} \cdot \right. & j = 1, \ldots, J-1 \\
\cdot\, [F_t^j(L_t^0(j, m, r))]^h\, [F_t^j(L_t^1(i, m, r))]^l & \\
\left. \cdot\, [F_t^j(L_t^2(i, m, r))]^v\, [F_t^j(L_t^3(i, m, r))]^{x-k-h-l-v} \right\} & \\[1ex]
0 & n \neq T_m(k) \\
& j = 1, \ldots, J-1 \\[1ex]
\pi_t(ij \mid m, r)\, g_r(s) & j = J
\end{cases}
\tag{7}
$$

for $i = 1, \ldots, J-1$ for $s = 1, \ldots, S$

for $r = 1, \ldots, S$ for $m = 1, \ldots, N \mid K(m) = 4$

and

$$P_t(ij, .n^*, rs) = \begin{cases} \pi_t(ij \mid \cdot, r)\, g_r(s) & j = 1, \ldots, J - 1 \\ 1 - \sum\limits_{j=1}^{J-1} \pi_t(ij \mid \cdot, r)\, g_r(s) & j = J \end{cases} \quad (8)$$

for $i = J$ and n^* representing the rating class for beginners and those without any claim experience.

The system (7) can easily be used to derive the transition probabilities for rating classes with $K(m) < 4$. For example let $K(m) = 2$. Clearly, $L_t^k(i, m, r) = 0$ for $k \geq 2$ The transition probabilities follow system (7) with the following modifications

(a) the branches for $n = T_m(2)$ and $n = T_m(3)$ have to be eliminated and

(b) the terms $[F_t^j(L_t^2(i, m, r))]^v$ and $[F_t^j(L_t^2(i, m, r))]^{x-k-h-l-v}$ must be dropped from the bracket in the branch for $T_m(K(m))$.

(8) remains unchanged.

The system (5) and (6) with stationary (time independent) transition probabilities (7) and (8) represents a regular Markov process. The steady state solution of the Markov process is the distribution of risks by their accident causing propensity. For non-stationary transition probabilities the system can be viewed as a linear flow model.

5. Illustration

In order to use the model the transition probabilities (7) and (8) must be determined. Naturally, this involves estimation problems. Some general remarks regarding the estimation are, therefore, in order.

We will begin with the function $g_r(s)$ describing the changes in the accident causing propensity. $g_r(s)$ can be determined by longitudinal studies through observing the accident frequency of individuals. Since each individual is characterized by his age it is possible to observe the changes in the accident causing propensity as these individuals age from τ to $\tau + 1$ which forms the basis for probabilities of changing from propensity interval r to s given age τ. This can be done for any age. Since the age distribution of a given population is easily available, one can determine $g_r(s)$ by computing an average of the above probabilities weighted by the age distribution. If the age distribution varies between risk categories and/or rating classes it may be necessary to make $g_r(s)$ a function of risk category j and/or rating class n.

The accretion and attrition probabilities $\pi(ij \mid m, r)$ must be estimated from observations as insurees move between risk categories. The specification regarding the accident causing propensity r can be obtained by relating the age of the moving insuree to the mean of the distribution by accident causing propensity for the given age [1]).

Finally, the transition between rating classes must be determined as a result of the insurees claim behavior according to (4). The critical accident size, $L^k_t(i, m, r)$, can be found through field studies. On the other hand optimal claim decisions according to [4] can be evaluated. The density function $f^j_t(L)$ can be derived from published statistics.

The model developed has been applied to the German Automobile Insurance System as it existed during 1967 in order to determine the distribution of insurees by their accident causing propensity for a given risk category (cars with horsepower 91-115). The analysis has been made with the simplifying assumption that insurees can cause at most one accident per year [2]). This assumption allows us to truncate the propensity to cause accidents at $\lambda = 1$. The accident causing propensity was divided into 20 intervals of the same length. The rating classes and the merit rating structures are described in Table 1.

Table 1

The merit rating structure

Rating Class	Discount	$T(0)$	$T(1)$
1	0	1	2
2	10%	1	3
3	30%	2	4
4	50%	3	4

[1]) It should be noted that a distribution of individuals with age τ by their accident causing propensity can also be derived from such longitudinal studies.

[2]) This assumption is realistic for more than 98% of the population.

The critical accident size to be used in (4) is described in (9)

$$L_t(j, n, s) = C_{t+1}(T_n(1)) - C_{t+1}(T_n(0)) \tag{9}$$

and represents the premium difference in the following year for filing and not filing a claim respectively. Since the reported data [2] reflect only the aggregate net of accretion and attrition and no information is available regarding $g_r(s)$, extensive simulation experiments have been carried out to produce a reasonable function $g_r(s)$ and accretion and attrition probabilities which are consistent with the population and average claim frequencies reported in [2] [1]).

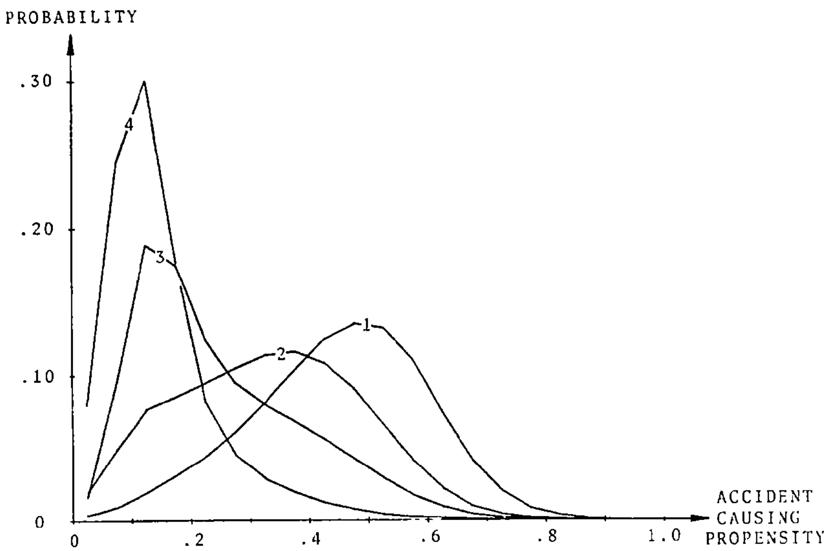

Fig. 1. Distributions (normalized) of individuals for rating classes 1 through 4.

The descriptive measures of the resulting distributions of individuals by their accident causing propensity are given in Table 2.

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1) No claim is being made that these values describe the underlying process perfectly though they proved accurate on a predictive basis.

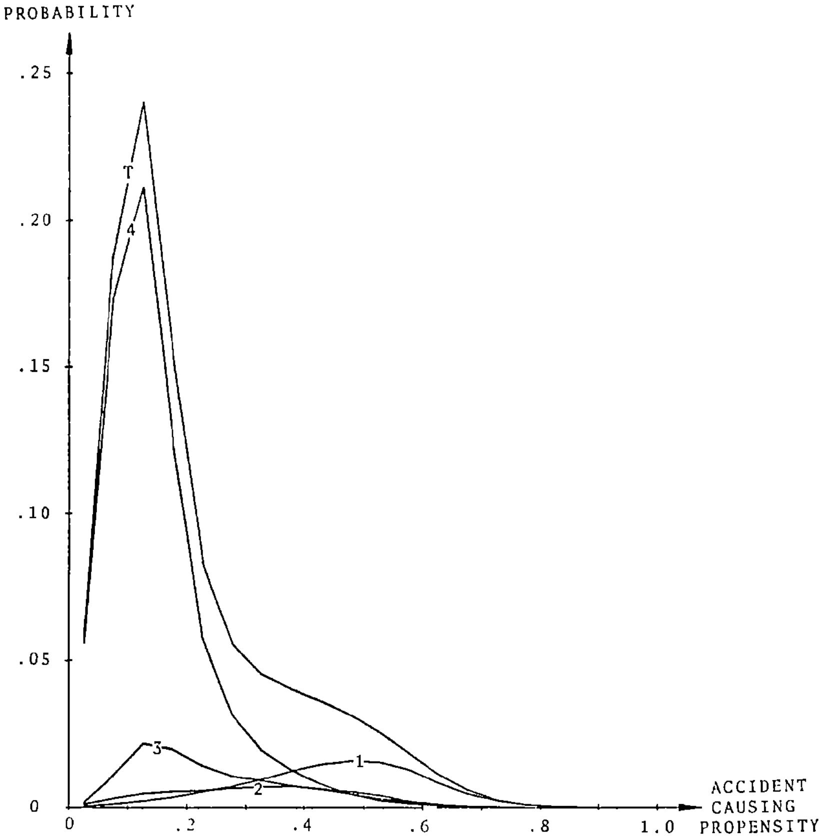

Fig. 2. Distributions of individuals for rating classes 1 through 4 and total.

Table 2

The descriptive measures of the resulting distributions

Descriptive Measures	Rating Class 1	2	3	4	Total
Mean	.4478	.3298	.2454	.1476	.2056
St. Deviation	.1470	.1526	.1393	.0927	.1507
Skewness	-.2845	.0890	.8280	1.5547	1.2589
Kurtosis	2.7825	2.3662	3.0958	6.4123	3.9698

Though the means of these distributions vary significantly thus reflecting a segregating effect through the merit rating structures, the standard deviations indicate a considerable overlap. The merit rating system, therefore, does not separate the total into fully homogeneous subgroups. This confirms previous findings [1]. The overlap and the degree of segregation is illustrated in Figures 1 and 2.

6. Conclusion

The purpose of this paper was to develop a model which can be used in deriving the distribution of risks by their accident causing propensity in automobile insurance systems with merit rating structures. The problem could be formulated and solved as a regular Markov process with the claim behavior being integrated in the analysis. The approach has been illustrated for the German automobile insurance system.

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