

ROBUST CREDIBILITY VIA ROBUST KALMAN FILTERING

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ABSTRACT

Credibility theory is closely related to Kalman filtering. As a consequence, methods proposed for robustifying the Kalman filter can often be specialised to obtain robust credibility rating procedures. The application of one such method to several classical credibility models is shown in this paper.

1. INTRODUCTION

Credibility theory is a very old branch of risk theory and non-life insurance mathematics. Early results are by MOWBRAY (1914) and WHITNEY (1918). A theoretically elegant approach was given by BÜHLMANN (1967) and BÜHLMANN & STRAUB (1970).

The classical models presented by those authors can be generalised to regression models, hierarchical models and evolutionary models. Generalisations have been studied intensively in the actuarial literature over the past twenty years. Some key references are HACHEMEISTER (1975), TAYLOR (1979), SUNDT (1980, 1983), NORBERG (1980, 1986), KREMER (1988a, 1988b).

In later years several authors have investigated ways of robustifying credibility rating methods. The aim of robustification is to limit the influence of extremely large claim amounts on the estimated premium. The reader is referred to, e.g. GISLER (1980), BÜHLMANN et al. (1982), KREMER (1991), KÜNSCH (1992), GISLER & REINHARD (1993).

MEHRA (1973) pointed out that credibility estimation can be achieved by the Kalman filtering technique. DE JONG & ZEHNWIRTH (1983) explored the correspondence for the classical credibility models, and ZEHNWIRTH (1985) explored its implications for evolutionary models.

Robust versions of the Kalman filter have been studied for some time, for example by MASRELIEZ & MARTIN (1977), MEINHOLD & SINGPURWALLA (1989) and CIPRA & ROMERA (1991). Due to the close relation between Kalman filtering and credibility theory, it is obvious that corresponding robust versions of credibility rating techniques can be derived. In the present paper we specialise the method proposed by CIPRA & ROMERA (1991) to the three most important credibility models. The resulting robust credibility techniques turn out to be quite tractable.

2. PRELIMINARIES

Suppose that all probabilistic statements are based on a probability space (Ω, \mathcal{A}, P) and consider a risk during periods with indices $i = 1, 2, 3, \dots, n + 1$. Assume that the claims behavior of a risk over all periods can be described by a parameter θ . Suppose that the value of this parameter is unknown and interpret it as a realisation of a random variable:

$$\theta : (\Omega, \mathcal{A}, P) \rightarrow (\Theta, \tau),$$

with the parameter space Θ and the σ -algebra τ on Θ . Let the observed claims amounts (or loss ratios) of the risk be represented by the nonnegative random variables:

$$X_i, \quad \text{with} \quad i = 1, 2, 3, \dots, n + 1,$$

defined on (Ω, \mathcal{A}, P) . It is assumed that all X_i lie in the Hilbert space L_2 of measurable, square integrable functions f (identified with the equivalence class of all g which are P -a.e. equal to f) defined on \mathcal{A} with scalar product:

$$\langle f_1, f_2 \rangle = E(f_1 \cdot f_2)$$

and norm:

$$\|f\| = E(f^2)^{1/2}.$$

In the given insurance context the conditional expectation

$$m_i = E(X_i | \theta)$$

is called the **net premium** (or **net loss ratio**) in period no. i . Then the credibility estimator is nothing else but the linear-affine prediction of m_{n+1} from X_1, X_2, \dots, X_n . Defining the subspace A_n of all linear-affine combinations

$$f_n = a_0 + \sum_{i=1}^n a_i \cdot X_i,$$

the **credibility estimator** is defined as the projection of m_{n+1} on A_n , i.e. as the random variable $m_{n+1} \in A_n$ with

$$\|m_{n+1} - \hat{m}_{n+1}\| \leq \|m_{n+1} - f_n\|$$

for all $f_n \in A_n$.

In general the credibility estimator can be determined by solving certain normal equations. Under more special model assumptions one can derive explicit formulas for the credibility estimator. Three special models are given in the Section 4. Especially in the regression model one can calculate explicit credibility estimators, see e.g. NORBERG (1980).

It is well-known that the usual credibility estimators are not robust against extremely large claim amounts. This has led to attempts to construct robust versions of the classical credibility estimators. One method to robustify the credibility estimator was given by Gisler already in 1980.

Alternative robustifications, which also cover the general regression model, are given by KREMER (1991).

3. ROBUST KALMAN FILTERING

The KALMAN filter is a well-known instrument for recursive prediction in dynamic linear systems. A dynamic linear system is defined by the two stochastic linear recursions :

$$\begin{aligned} X_i &= H_i \cdot b_i + v_i \\ b_i &= F_i \cdot b_{i-1} + w_i, \end{aligned}$$

where X_i is a p -dimensional stochastic vector of observations, H_i a known (pxq) -dimensional design matrix, b_i a q -dimensional stochastic parameter vector and F_i a known (qxq) -dimensional transition matrix. The v_i, w_i are random disturbances with :

$$\begin{aligned} E(v_i) &= 0, & E(w_i) &= 0, \\ E(v_i \cdot v_j^T) &= 0, & E(w_i \cdot w_j^T) &= 0, & i \neq j \\ E(v_i \cdot v_i^T) &= R_i, & E(w_i \cdot w_i^T) &= Q_i \\ E(v_i \cdot w_j^T) &= 0, & i \neq j, \end{aligned}$$

where R_i, Q_i are known covariance matrices. The Kalman filter algorithm gives handy recursions for the optimal affine-linear predictor of b_i from X_1, X_2, \dots, X_{i-1} . For more details see Section 3 in DE JONG et al. (1983).

Like other standard methods the Kalman filter is not robust to outliers. As a consequence, several authors have proposed robust versions of the usual Kalman filter algorithm. Recently, a handy robustification was proposed by CIPRA & ROMERA (1991). We give a brief summary of their result.

Denote by \hat{b}_i , the one-step ahead prediction of b_i , based on the observations X_1, \dots, X_{i-1} , and define its error covariance matrix as

$$C_i = E(b_i - \hat{b}_i)(b_i - \hat{b}_i)^T.$$

For a given covariance matrix M , we define as $M^{-1/2}$ any matrix which satisfies $M^{-1/2} \cdot M \cdot (M^{-1/2})^T = I$.

With this convention we now introduce the matrices

$$\begin{aligned} A_i^{q \times q} &= C_i^{-1/2}, \\ D_i^{p \times q} &= R_i^{-1/2} \cdot H_i, \end{aligned}$$

and the random vectors

$$\begin{aligned} s_i^{p \times 1} &= R_i^{-1/2} \cdot X_i, \\ p_i^{q \times 1} &= A_i \cdot \hat{b}_i. \end{aligned}$$

Note that, conditional on b_i , the stochastic vector s_i has mean $D_i \cdot b_i$ and covariance matrix I . Further note that

$$E(p_i - A_i \cdot b_i)(p_i - A_i \cdot b_i)^T = I.$$

In the spirit of **M-Estimation**, CIPRA & ROMERA (1991) propose to determine the updated estimate \hat{b}_i^i of b_i , given all observations X_1, \dots, X_i , as the solution of a minimisation problem, namely

$$(3.1) \quad \text{minimise} \left\{ \sum_{k=1}^q \rho_{1k}(p_{ki} - a_{ki}^T \cdot \hat{b}_i^i) + \sum_{j=1}^p \rho_{2j}(s_{ji} - d_{ji}^T \cdot \hat{b}_i^i) \right\}$$

Here $\rho_{11}, \dots, \rho_{1q} \geq 0$ and $\rho_{21}, \dots, \rho_{2q} \geq 0$ are arbitrary robustifying functions, the k -th row of A_i is denoted by a_{ki}^T , and the j -th row of D_i is denoted by d_{ji}^T . Denote the derivatives of the functions ρ by Ψ .

It is seen that the resulting estimator \hat{b}_i^i will be the result of a compromise between the desire to minimise deviation from the one-step ahead prediction (note $p_i - A_i \cdot \hat{b}_i = 0$) and the desire to have \hat{b}_i^i reflect the information in the new data as represented by s_i .

Having solved (3.1) to obtain \hat{b}_i^i , one obtains the one-step ahead prediction to use in the next recursion by

$$\hat{b}_{i+1} = F_{i+1} \cdot \hat{b}_i^i.$$

For \hat{b}_i^i one has the normal equations (see (2.8) in CIPRA & ROMERA (1991)):

$$(3.2) \quad \sum_{k=1}^q a_{ki}^T \cdot \Psi_{1k}(p_{ki} - a_{ki}^T \cdot \hat{b}_i^i) + \sum_{j=1}^p d_{ji}^T \cdot \Psi_{2j}(s_{ji} - d_{ji}^T \cdot \hat{b}_i^i) = 0.$$

By approximating \hat{b}_i^i by \hat{b}_i (3.2) gives a certain approximate normal equation

$$(3.3) \quad \sum_{k=1}^q w_{1ki} \cdot a_{ki}^T \cdot (p_{ki} - a_{ki}^T \cdot \hat{b}_i) + \sum_{j=1}^p w_{2ji} \cdot d_{ji}^T \cdot (s_{ji} - d_{ji}^T \cdot \hat{b}_i) = 0$$

where the weights w_{1ki} , w_{2ji} are defined as:

$$w_{1ki} = \lim_{x \rightarrow 0} \left(\frac{\Psi_{1k}(x)}{x} \right)$$

$$w_{2ji} = \frac{\Psi_{2j}(s_{ji} - d_{ji}^T \cdot \hat{b}_i)}{s_{ji} - d_{ji}^T \cdot \hat{b}_i}.$$

So far for the general robustification of the Kalman-filter. Turn now to more practicable, special cases.

It is reasonable to assume that the disturbances w_i do not produce outliers. This results in the choice:

$$\Psi_{1k}(x) = x.$$

Furthermore one is willing to take :

$$\Psi_{2j}(\cdot) = \Psi(\cdot)$$

independent of j , e.g. :

$$\Psi_{2j}(\cdot) = \Psi_H(\cdot)$$

In the practical case $p = 1$ one can choose the so called (one-sided) **Huber-function** :

$$\begin{aligned} \Psi_H(z) &= z, & \text{for } z \leq c \\ &= c, & \text{for } z > c, \end{aligned}$$

where c is a given positive constant. CIPRA & ROMERA (1991) propose to take (3.2) in case of the Huber-function Ψ_H for Ψ and the approximation (3.3) in case of general Ψ . For general Ψ the formula (3.3) gives the following recursion in case that $p = 1$:

$$(3.4) \quad \hat{b}_i^j = \hat{b}_i + \left(\frac{C_i \cdot H_i^T}{H_i \cdot C_i \cdot H_i^T + R_i/k_i} \right) \cdot (X_i - H_i \cdot \hat{b}_i)$$

where :

$$(3.5) \quad C_i = F_i \cdot C_{i-1}^{-1} \cdot F_i^T + Q_i$$

$$(3.6) \quad C_i^j = C_i - \frac{C_i \cdot H_i^T \cdot H_i \cdot C_i}{H_i \cdot C_i \cdot H_i^T + R_i/k_i}$$

$$(3.7) \quad k_i = \frac{\Psi(R_i^{-1/2} \cdot (X_i - H_i \cdot \hat{b}_i))}{R_i^{-1/2} \cdot (X_i - H_i \cdot \hat{b}_i)}$$

Note that the 'ordinary' Kalman filter is just (3.4)-(3.6) with $k_i = 1$. For the special $\Psi = \Psi_H$ one gets from (3.2) in case $p = 1$ the recursion :

$$(3.8) \quad \hat{b}_i^j = \hat{b}_i + C_i \cdot H_i^T \cdot R_i^{-1/2} \cdot \Psi_H \left(\frac{R_i^{1/2} \cdot (X_i - H_i \cdot \hat{b}_i)}{H_i \cdot C_i \cdot H_i^T + R_i} \right)$$

Cipra & Romera propose to update C_i like in the original (nonrobust) Kalman-filter, i.e. :

$$(3.9) \quad C_i = F_i \cdot C_{i-1}^{-1} \cdot F_i^T + Q_i$$

$$(3.10) \quad C_i^j = C_i - \frac{C_i \cdot H_i^T \cdot H_i \cdot C_i}{H_i \cdot C_i \cdot H_i^T + R_i}$$

Obviously the resulting recursions are quite tractable.

4. ROBUST CREDIBILITY

We consider three well-known credibility models. For each model the robustifications (3.4)-(3.10) are presented, giving recursions for a robust credibility estimator.

4.1. Bühlmann-Straub (1970) model

Suppose that X_1, \dots, X_{n+1} are conditionally independent given θ . There exist measurable functions

$$\begin{aligned}\mu &: (\Theta, \tau) \rightarrow (IR, IB) \\ \sigma^2 &: (\Theta, \tau) \rightarrow (IR_+, IB_+)\end{aligned}$$

such that:

$$\begin{aligned}E(X_i | \theta) &= \mu(\theta) \\ \text{Var}(X_i | \theta) &= \sigma^2(\theta)/V_i,\end{aligned}$$

where V_i , $i \geq 1$ are known volume measures. Explicit formulas for the credibility estimator \hat{m}_{n+1} can be found in the original article, equivalent recursions for it based on the classical Kalman filter in DE JONG et al. (1983). Obviously a dynamic linear model like in Section 3 is given with:

$$\begin{aligned}p &= 1, & H_i &= 1, & R_i &= \sigma_0^2/V_i \\ q &= 1, & F_i &= 1, & Q_i &= 0,\end{aligned}$$

where:

$$\sigma_0^2 = E(\sigma^2(\theta)).$$

One has:

$$\begin{aligned}b_i &= m_i = \mu(\theta) \\ \hat{b}_i &= \hat{b}_{i-1}^i = \hat{m}_i, & C_i &= C_{i-1}^{i-1}\end{aligned}$$

implying from (3.4), (3.6), (3.7) in case of general Ψ :

$$\hat{m}_{i+1} = \hat{m}_i + \left(\frac{C_i \cdot V_i \cdot k_i}{C_i \cdot V_i \cdot k_i + \sigma_0^2} \right) \cdot (X_i - \hat{m}_i)$$

$$C_{i+1} = C_i - \frac{C_i^2 \cdot V_i \cdot k_i}{C_i \cdot V_i \cdot k_i + \sigma_0^2}$$

$$k_i = \frac{\Psi(V_i^{1/2} \cdot (X_i - m_i)/\sigma_0)}{V_i^{1/2} \cdot (X_i - m_i)/\sigma_0}$$

and from (3.8), (3.10) in case of special $\Psi = \Psi_H$:

$$\hat{m}_{i+1} = \hat{m}_i + C_i \cdot \left(\frac{V_i^{1/2}}{\sigma_0} \right) \cdot \Psi_H \left(\frac{\sigma_0 \cdot V_i^{1/2} \cdot (X_i - \hat{m}_i)}{C_i \cdot V_i + \sigma_0^2} \right)$$

$$C_{i+1} = (V_i/\sigma_0^2 + C_i^{-1})^{-1}.$$

Obviously one has quite handy recursions for the (robust) credibility estimator \hat{m}_i .

4.2. Hachemeister's (1975) regression model

Conditionally given θ the X_1, \dots, X_{n+1} are independent. Suppose that there exist functions:

$$b : (\Theta, \tau) \rightarrow (IR^q, IB^q)$$

$$\sigma^2 : (\Theta, \tau) \rightarrow (IR_+, IB_+)$$

such that:

$$E(X_i | \theta) = a_i^T \cdot b(\theta)$$

$$\text{Var}(X_i | \theta) = \sigma^2(\theta)/V_i,$$

where a_i is a known q -dimensional vector and $V_i, i \geq 1$ are given volume measures. The credibility estimator \hat{m}_{n+1} of m_{n+1} is given by:

$$\hat{m}_{n+1} = a_{n+1}^T \cdot \hat{b}_{n+1},$$

where \hat{b}_{n+1} is the (vector-valued) credibility estimator of $b(\theta)$ based on X_1, \dots, X_n .

Explicit formulas for \hat{b}_{n+1} can be found in the original paper of HACHEMEISTER (1975), equivalent recursions based on the classical Kalman-filter in DE JONG & ZEHNWIRTH (1983).

Obviously the model fits into the framework of Section 3.

Choose simply there:

$$p = 1, \quad H_i = a_i^T, \quad R_i = \sigma_0^2/V_i$$

$$F_i = 1, \quad Q_i = 1,$$

where:

$$\sigma_0^2 = E(\sigma^2(\theta)).$$

One has:

$$b_i = b(\theta)$$

$$b_i = b_{i-1}^i, \quad C_i = C_{i-1}^{i-1}.$$

The recursion (3.4), (3.6), (3.7) give in case of general Ψ :

$$\hat{b}_{i+1} = \hat{b}_i + \left(\frac{C_i \cdot a_i \cdot V_i \cdot k_i}{a_i^T \cdot C_i \cdot a_i \cdot V_i \cdot k_i + \sigma_0^2} \right) \cdot (X_i - a_i^T \cdot \hat{b}_i)$$

$$C_{i+1} = C_i - \left(\frac{C_i \cdot a_i \cdot a_i^T \cdot C_i \cdot V_i \cdot k_i}{a_i^T \cdot C_i \cdot a_i \cdot V_i \cdot k_i + \sigma_0^2} \right)$$

$$k_i = \left(\frac{\Psi(V_i^{1/2} \cdot (X_i - a_i^T \cdot \hat{b}_i) / \sigma_0)}{V_i^{1/2} \cdot (X_i - a_i^T \cdot \hat{b}_i) / \sigma_0} \right)$$

and the recursions (3.8)-(3.10) in case of special $\Psi = \Psi_H$:

$$\hat{b}_{i+1} = \hat{b}_i + C_i \cdot a_i \cdot \left(\frac{V_i^{1/2}}{\sigma_0} \right) \cdot \Psi_H \left(\frac{\sigma_0 \cdot V_i^{1/2} \cdot (X_i - a_i^T \cdot \hat{b}_i)}{a_i^T \cdot C_i \cdot a_i \cdot V_i + \sigma_0^2} \right)$$

$$C_{i+1} = C_i - \left(\frac{C_i \cdot a_i \cdot a_i^T \cdot C_i \cdot V_i}{a_i^T \cdot C_i \cdot a_i \cdot V_i + \sigma_0^2} \right)$$

Obviously also these recursions for a (robust) credibility estimator are quite practicable.

4.3. Gerber & Jones' (1975) evolutionary model

Suppose that X_1, \dots, X_{n+1} are conditionally independent given θ . Furthermore assume that:

$$m_i = m_{i-1} + w_i,$$

where the random disturbances satisfy:

$$E(w_i) = 0, \quad E(w_i^2) = w$$

$$E(w_i \cdot w_j) = 0, \quad i \neq j, \quad E(w_i \cdot m_0) = 0.$$

Finally let:

$$\sigma^2 : (\Theta, \tau) \rightarrow (IR_+, IB_+)$$

such that:

$$\text{Var}(X_i | \theta) = \sigma^2(\theta) / V_i,$$

where $V_i, i \geq 1$ are given volume measures. Recursions for the credibility estimator are given e.g. in the paper SUNDT (1981).

This model is a special case of the dynamic linear model of Section 3. Choose simply there:

$$p = 1, \quad H_i = 1, \quad R_i = \sigma_0^2 / V_i$$

$$F_i = 1, \quad Q_i = w,$$

with:

$$\sigma_0^2 = E(\sigma^2(\theta)).$$

The recursions (3.4), (3.6), (3.7) give in case of general Ψ :

$$\hat{m}_{i+1} = \hat{m}_i + \left(\frac{C_i \cdot V_i \cdot k_i}{C_i \cdot V_i \cdot k_i + \sigma_0^2} \right) \cdot (X_i - \hat{m}_i)$$

$$C_{i+1} = C_i - \frac{C_i^2 \cdot V_i \cdot k_i}{C_i \cdot V_i \cdot k_i + \sigma_0^2} + w$$

$$k_i = \frac{\Psi(V_i^{1/2} \cdot (X_i - \hat{m}_i)/\sigma_0)}{V_i^{1/2} \cdot (X_i - \hat{m}_i)/\sigma_0}$$

and the recursions (3.8), (3.10) in case of special $\Psi = \Psi_H$:

$$\hat{m}_{i+1} = \hat{m}_i + (C_i + w) \cdot \left(\frac{V_i^{1/2}}{\sigma_0} \right) \cdot \Psi_H \left(\frac{\sigma_0 \cdot V_i^{1/2} \cdot (X_i - \hat{m}_i)}{C_i \cdot V_i + w \cdot V_i + \sigma_0^2} \right)$$

$$C_{i+1} = C_i - \frac{C_i^2 \cdot V_i}{C_i \cdot V_i + \sigma_0^2} + w.$$

5. A SIMULATION STUDY

For the model of Section 4.1 data was simulated with the choice $V_i = 1$ for all i . The conditional distribution of X_i given $\theta = \vartheta$ was assumed to be given according

$$(5.1) \quad P(X_i = k \mid \theta = \vartheta) = (1 - \pi) \cdot p_{\vartheta}(k) + \pi \cdot p_{\vartheta_0}(k)$$

(for $k = 0, 1, 2, 3, \dots$)

with the Poisson-probabilities:

$$p_{\vartheta}(k) = \left(\frac{\vartheta^k}{k!} \right) \cdot \exp(-\vartheta)$$

and a probability π . For $\vartheta_0 > \vartheta$ π can be interpreted as the probability of an outlier occurring according to the probabilities $p_{\vartheta_0}(k)$. The risk parameter ϑ was simulated according to the Gamma-model with density on $(0, \infty)$:

$$(5.2) \quad f^{\vartheta}(\vartheta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} \cdot \vartheta^{\alpha-1} \cdot \exp(-\beta \cdot \vartheta)$$

where α and β are the nonnegative parameters. One gets for $\pi = 0$ as credibility estimator:

$$(5.3) \quad \hat{m}_{n+1} = \frac{\alpha + \sum_{i=1}^n X_i}{\beta + n}$$

what shall be compared with a robust variant calculated recursively according to the formulas :

$$(5.4) \quad \hat{m}_{n+1} = \hat{m}_n + (C_n / \sigma_0) \cdot \Psi_H \cdot \left(\frac{X_n - \hat{m}_n}{(C_n / \sigma_0) + \sigma_0} \right)$$

$$C_{n+1} = \frac{\sigma_0^2}{1 + (\sigma_0^2 / C_n)}$$

where $\sigma_0^2 = (\alpha/\beta)$ and Ψ_H is the Huber-function with $c = 1.645$. The recursions start with :

$$\hat{m}_1 = (\alpha/\beta), \quad C_1 = (\alpha/\beta^2).$$

The aim of the study was to compare the results of (5.3), (5.4) with the 'true' value ϑ and to see which one gives the smaller mean squared error :

$$(5.5) \quad \frac{1}{n - n_0 + 1} \cdot \sum_{i=n_0}^n (\hat{m}_{i+1} - \vartheta)^2,$$

where n_0 is an adequate number smaller than n . The claims data had to be simulated with a sufficiently large ϑ_0 and an adequate small π . The author chose $\pi = 0.05$ and for ϑ_0 the values 20, 25, 30, whereas he took $\alpha = 100$, $\beta = 10$, giving for $E(\theta)$ the value 10. With these parameter choices he simulated 100 risk parameters ϑ_j , $j = 1, \dots, 100$ according to the model (5.2) and for each $\vartheta = \vartheta_j$ independently $n = 9$ values X_i according to (5.1) (with $\pi = 0.05$). In (5.5) he took $n_0 = 6$. He got for the overall mean squared error

$$MSE = \frac{1}{500} \cdot \sum_{j=1}^{100} \sum_{i=6}^{10} (\hat{m}_{i+1}^{(j)} - \vartheta_j)^2$$

the results of the following table :

	$\vartheta_0 = 20$	25	30
with			
(5.3)	0.956	1.231	1.593
(5.4)	0.806	0.807	0.807

showing the strong superiority of (5.4) for situations where bigger outliers can occur with small probability but one wants to rate the normal risk (i.e. case $\pi = 0$). For further illustration the simulation results shall be given for two typical cases. In the first row of the following tables the simulated X_i are given, in the second the \hat{m}_i of (5.3) and in the third the \hat{m}_i of (5.4).

$\vartheta_0 = 20$:

$\vartheta_j = 11.30$

9	13	11	22	13	15	14	14	16	—
10.00	9.91	10.17	10.23	11.07	11.20	11.44	11.59	11.72	11.95
10.00	9.91	10.17	10.23	10.63	10.79	11.05	11.23	11.38	11.62

$\vartheta_j = 8.42$

21	8	12	9	4	8	9	19	8	—
10.00	11.00	10.75	10.85	10.71	10.27	10.13	10.06	10.56	10.42
10.00	10.52	10.31	10.44	10.34	9.91	9.80	9.75	10.05	9.95

$\vartheta_0 = 25 :$

$\vartheta_j = 9.60$

7	19	11	11	11	33	12	11	11	—
10.00	9.73	10.50	10.54	10.57	10.60	12.00	12.00	11.94	11.89
10.00	9.73	10.20	10.26	10.31	10.36	10.70	10.78	10.80	10.81

$\vartheta_j = 10.71$

12	8	24	12	15	15	10	13	11	—
10.00	10.18	10.00	11.08	11.14	11.40	11.63	11.53	11.61	11.58
10.00	10.18	10.00	10.43	10.55	10.84	11.10	11.03	11.15	11.14

$\vartheta_0 = 30 :$

$\vartheta_j = 10.77$

11	7	13	6	7	28	7	40	11	—
10.00	10.09	9.83	9.79	9.60	9.60	10.75	10.53	12.17	12.11
10.00	10.09	9.83	10.08	9.79	9.60	9.95	9.77	10.08	10.13

$$\vartheta_j = 8.42$$

31	8	12	9	4	8	9	29	8	—
10.00	11.91	11.58	11.62	11.43	10.93	10.75	10.65	11.67	11.47
10.00	10.52	10.31	10.44	10.34	9.91	9.80	9.75	10.05	9.95

6. FINAL REMARKS

By applying robustifications of the Kalman-filter to credibility models one can derive fairly practicable recursions for a (robust) credibility estimator. For the Bühlmann-Straub and Hachemeister models one gets an alternative to an already existing approach to robust credibility (see KREMER (1991), KÜNSCH (1992)). For practical application of the above robustified recursions one needs (robust) estimators for the unknown model parameters. Desirable would be such estimators in a recursive form. Obviously here is something left for further research.

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