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SPEED OF FINALIZATION OF CLAIMS AND CLAIMS RUNOFF ANALYSIS

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SUMMARY

To some extent, this paper provides theoretical back-up for TAYLOR's (1980b) see-saw method of analysis of outstanding claims.

Section 3 deals with the payments per claim finalized (PPCF) method. This assumes PPCF to be a function of development year only.

Section 4 carries out a theoretical investigation of the validity of this assumption. It is found that the assumption is not justified in general; that PPCF can (and usually will) be sensitive to changes in speed of finalization of claims; and that this sensitivity will depend on the convexity properties of PPCF as a function of development year. The analysis is made on the basis of the so-called hypothesis of invariant order.

Section 5 develops the see-saw method from this theoretical basis, although Section 7 is at pains to point to situations in which that method remains applicable despite violation of the hypothesis of invariant order.

Section 6 provides a numerical example.

1. INTRODUCTION

Few methods of claims runoff analysis attempt to make due allowance for speed of finalization.

A survey of a number of simple methods and some more sophisticated ones is given by SKURNICK (1973). A survey of "reasonably sophisticated" methods can be obtained by taking SAWKINS (1979b) and TAYLOR (1980b) together. An examination of these papers will vindicate the previous paragraph.

Of the "reasonably sophisticated" methods which do make allowance for speed of finalization there are essentially only two These are:

(i) Sawkins' payments per claim finalized method, or some fairly simple variant of it;
(ii) Taylor's Reduced Reid method.

The two methods are dealt with in some detail in SAWKINS (1979ab) and TAYLOR (1980ab). The method of REID (1978) is left out of account here, being regarded as "highly sophisticated".

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The basic assumption on which the payments per claim finalized method rests is that, for a given development year, claim payments (adjusted for inflation) are proportional to number of claims finalized. The constant of proportionality is supposed to be independent of year of origin

The Reduced Reid method shows how this assumption may be upset by changes in speed of finalization. The method uses the concept of operational time. It replaces the assumption stated in the preceding paragraph by the assumption that, *for a given operational time period*, claim payments (adjusted for inflation) are proportional to number of claims finalized.

Section 4 of this paper makes a more extensive theoretical investigation of the way in which claim payments vary with numbers of claims finalized. As was perhaps to be expected, more subtle forces than those allowed for in either the payments per claim finalized method or the Reduced Reid method are found to be at work.

Section 5 then develops a (rather simplified) method for dealing with the realistic situation.

2. NOTATION

As a general rule, random variables will be represented by upper case Latin letters; realizations of those random variables by the corresponding lower case Latin letters, and the expected values of those random variables by the corresponding lower case Greek letters

Consider an array of random variables C_{ij} for some given but arbitrary set S of ordered pairs (i, j). The random variable C_{ij} is the amount of *claims paid* in development year j of year of origin i. Here the *year of origin* of a claim is the year in which that claim originates; it may be the year in which the claim is incurred; or the year in which the insurer is notified of it; other definitions may be used. The year of origin is, by definition, *development year* 0. Succeeding years are defined to be development years 1, 2, etc.

Now consider a further array of random variables N_{ij}, also defined for $(i, j) \in S$. The random variable N_{ij} is the *number of claims finalized* in development year j of year of origin i.

Define

$$P_{ij} = C_{ij}/N_{ij}, \tag{2.1}$$

and call this *payments per claim finalized* (PPCF) in development year j of year of origin i.

Define

$$N_i = \sum_{j=0}^{\infty} N_{ij}, \tag{2.2}$$

which is the *total number of claims originating* in year of origin i. For the purpose of this definition, we assume N_{ij} to be defined for all nonnegative real j.

Generally, in this paper, the numbers N_i of claims incurred are treated as known As pointed out by the referee of the paper, this is not realistic as regards the most recent years of origin where there may be significant numbers of IBNR claims A proper treatment of this feature would incorporate stochastic modelling of the N_i and in particular modelling of the delay from date of occurrence to date of notification of claim.

In the class of business considered in the numerical example of Section 6 detailed statistics on this delay suggest that its distribution has been undergoing slow but sure secular change in past years with occasional major dislocations. The reasons for these changes are not known. In such circumstances, the required modelling of number of claims incurred becomes a virtual impossibility.

The class of business dealt with in Section 6 is the one involving the longest notification delays of all classes of direct business in Australia. Even in this case, experience indicates that serious error in N_i is likely to occur only in respect of the very latest year of origin.

In these circumstances, we prefer to dodge the problem of modelling claim numbers, making a mental reservation about the accuracy of N_i for the latest year of origin.

Define

$$F_{ij} = N_{ij}/N_i, \tag{2.3}$$

and call this *speed of finalization* in development year j of year of origin i.

Define

$$M_{ij} = \sum_{k=0}^{j} N_{ik}, \tag{2.4}$$

which is the number of claims originating in year i and finalized in development year j or earlier. It can be regarded as a kind of aggregate measure of speed of finalization up to the end of development year j. More pertinently, it may be used as a measure of *operational time* in the sense of REID (1978). In particular, the development of year of origin i in *real time* (development years) can be dropped, and this development measured instead in operational time

$$T_{ij} = M_{ij}/N_i, \tag{2.5}$$

which varies from 0 at the beginning of development year 0 to 1 at development year ∞. Incidentally, by (2.3) to (2.5),

$$T_{ij} = \sum_{k=0}^{j} F_{ik}.$$

Let $P_i\ (s, t)$ denote PPCF during operational time period (s, t), of year of origin i.

Initially, we shall adopt the *hypothesis of invariant order* (TAYLOR, 1980a). That is, it is assumed that the claim payments of year of origin i will be made in a specific order irrespective of speed of finalization; that changed speed of finalization will simply compress or extend the time-line on which those claims are paid without affecting their order of payment or finalization. The effect of this assumption is to make operational time a variable rather than real time.

In this spirit, we define:

(2.6) $\sigma_{iF}(t)$ = expected size of a claim payment made at operational time t in finalization of a claim with year of origin i,

(2.7) $\sigma_{iP}(t)$ = expected size of a claim payment made at operational time t in respect of a claim which is not finalized by that payment;

(2.8) $\varphi_i(t)$ = expected number of such partial payments between successive finalizations at operational times t and $t+0$ respectively.

It is supposed that all claim payments have been adjusted for inflation, and are expressed in common dollar values.

For reasons which will become apparent later, we write:

$$I_i(t) = \sigma_{iF}(t) + \varphi_i(t)\sigma_{iP}(t). \tag{2.9}$$

3. PAYMENTS PER CLAIM FINALIZED METHOD

The PPCF method is described in detail by SAWKINS (1979ab). The major assumption is as follows:

Assumption (PPCF). For each $j = 0, 1, 2$ etc. there is a parameter π_j, dependent only on j, such that

$$E[P_{ij} \mid \{N_{i'j'}\}] = \pi_j, \tag{3.1}$$

where E is the expected value operator, where all claim amounts have been converted to current values, and $\{N_{i'j'}\} = \{N_{i'j'}: (i', j') \in S\}$.

In other words, the magnitude of expected payments per claim finalized is a function of development year only. More particularly, it is independent of the speed of finalization of claims in the development year in question; and independent of the speed of finalization in other development years.

The PPCF method consists essentially of the following steps:

(i) estimation of the parameters π_j;
(ii) estimation of the future runoff of numbers of finalizations;
(iii) combination of the estimates in (i) and (ii) to produce a runoff of future claim payments in current values;
(iv) adjustment from current values to allow for whatever extent of claims escalation and discounting is required.

It is not the purpose here to go into the details of how these steps are achieved. Such details are given by SAWKINS (1979ab). It is simply intended to point out the fundamental nature of the assumption described by (3.1) This assumption is in fact implied by Step (*i*) above. The validity of the assumption is examined in Section 4.

4. VARIABLES AFFECTING PPCF

Consider year of origin i and operational time period $(t, t+d]$. The proportion of claims finalized in this period is d. Therefore,

$$(4.1)\qquad \begin{array}{l}\text{number of claims finalized in}\\ \text{operational time period } (t, t+d]\end{array} = dN_i.$$

By (2.6) to (2.8), expected claim payments in operational time period $(t, t+d]$ =

$$(4.2)\qquad N_i \int_t^{t+d} [\sigma_{iF}(s) + \varphi_i(s)\sigma_{iP}(s)]\, ds$$

By (4.1) and (4 2), expected PPCF in operational time period $(t, t+d]$

$$\begin{aligned} &= EP_i\,(t, t+d)\\ (4\ 3)\qquad &= \frac{1}{d}\int_t^{t+d} [\sigma_{iF}(s) + \varphi_i(s)\,\sigma_{iP}(s)]\, ds\\ &= \pi(t, t+d), \text{ say.}\end{aligned}$$

Equation (4.3) shows that, for a given operational time period, PPCF is indeed a constant. Thus, the basic assumption of the Reduced Reid method (Section 1) follows from the hypothesis of invariant order.

Note that the basic assumption of the PPCF method (Section 1) does not follow. For, if development year j of year of origin i corresponds to operational time period $(t_{ij}, t_{ij} + d_{ij}]$, then

$$(4.4)\qquad \text{expected PPCF for development year } j = \frac{1}{d_{ij}} \int_{t_{ij}}^{t_{ij}+d_{ij}} I_i(s)ds,$$

which depends on t_{ij} and d_{ij}, i.e. on speed of finalization. Here and in the remainder of this section, "expected PPCF for development year j" must be read as that expectation conditional on the set $\{N_{i'j'}\}$.

Of course, in practice, the form of the data is such that one usually has to work with development years (real time) rather than operational time. The starting point in such an investigation will consist of quantities of the type (4.4).

Consider then the effect on PPCF for year of origin i, development year j of a small change in speed of finalization. After the change, development year j will correspond to operational time

$(t_{ij} + \varepsilon_{ij}, t_{ij} + d_{ij} + \eta_{ij}]$, where ε_{ij}, η_{ij} are small. Then

expected PPCF for development year j (after change)

$$= \pi(t+\varepsilon, t+d+\eta) = \frac{1}{d+\eta-\varepsilon} \int_{t+\varepsilon}^{t+d+\eta} I(s)ds, \tag{4.5}$$

where the subscripts i, j have been temporarily suppressed.

Since ε, η are small, (4.5) can be written as:

$$\pi(t+\varepsilon, t+d+\eta) = (d+\eta-\varepsilon)^{-1} \{\int_t^{t+d} I(s)\, ds + \eta\, I(t+d) - \varepsilon I(t)\}$$

$$= d(d+\eta-\varepsilon)^{-1} \{\pi(t, t+d) + \frac{\eta}{d} I(t+d) - \frac{\varepsilon}{d} I(t)\} + o(\varepsilon) + o(\eta).$$

Expansion to first order of the terms preceding the braces gives:

$$\pi(t+\varepsilon, t+d+\eta) = \left(1 - \frac{\eta}{d} + \frac{\varepsilon}{d}\right) \{\ldots\}$$

$$= \pi(t, t+d) + \frac{\eta}{d} [I(t+d) - \pi(t, t+d)] + \frac{\varepsilon}{d} [\pi(t, t+d) - I(t)] + o(\varepsilon) + o(\eta). \tag{4.6}$$

Thus, an expression of the expected PPCF after the change in speed of finalization is obtained in terms of the expected PPCF before the change.

Now suppose that $\varepsilon = -\eta$. That is, the two operational time periods $(t, t+d)$ and $(t+\varepsilon, t+d+\eta) = (t-\eta, t+d+\eta)$ have the same midpoints, i.e. relate to the same average operational time. For this special case, (4.6) becomes:

$$\pi(t-\eta, t+d+\eta) = \pi(t, t+d) + \frac{\eta}{d} [I(t+d) + I(t) - 2\pi(t, t+d)]. \tag{4.7}$$

Suppose $\eta > 0$, i.e. the change in speed of finalization in development year j has been an increase.

Then

$$\pi(t-\eta, t+d+\eta) \gtreqless \pi(t, t+d) \tag{4.8}$$

according as

$$I(t+d) + I(t) \overset{>}{\underset{>}{=}} 2\pi(t, t+d) = \frac{2}{d} \int_t^{t+d} I(s)ds. \tag{4.9}$$

A sufficient condition for (4.9) to hold is that:

$$I(t) \text{ is } \begin{cases} \text{strictly convex to the t-axis} \\ \text{linear in } t \\ \text{strictly concave to the } t\text{-axis.} \end{cases} \tag{4.10}$$

If $I(t)$ is twice differentiable, this condition is equivalent to

$$I''(t) \begin{matrix} > \\ = \\ < \end{matrix} 0. \tag{4.11}$$

In the case of $I(t)$ being twice differentiable, and of (4.9) requiring to be satisfied for all t, d, condition (4.11) is in fact necessary.

Two comments can be made readily. Firstly, it would be highly fortuitous if $I_i(t)$ should turn out to be linear in t, which is the condition for expected PPCF to be invariant under changes in speed of finalization. Hence, Assumption (3.1) cannot be expected to hold normally. Consequently, the PPCF method described in Section 3 will require some modification to allow for this fact.

Secondly, since, according to (4.4) expected PPCF in development year j $= \frac{1}{d} \int_t^{t+d} I(s) ds$,

and, since by the mean value theorem,

$$\frac{1}{d} \int_t^{t+d} I(s)\, ds = I(t+a), \text{ for some } 0 < a < d,$$

it follows that:

expected PPCF in development year j of year of origin i

$$= I_i(t_{ij} + a_{ij}),\ 0 < a_{ij} < d_{ij}, \tag{4.12}$$

where subscripts i, j have now been reinstated.

Thus, condition (4.10) can be reformulated as follows: A sufficient condition to ensure that increased speed of finalization (in development year j)

$$\text{causes} \begin{cases} \text{increase in PPCF} \\ \text{no change in PPCF} \\ \text{decrease in PPCF} \end{cases}$$

is that PPCF, as a function of development year k, be a function which

$$\text{is} \begin{cases} \text{strictly convex to the } k\text{-axis} \\ \text{linear in } k \\ \text{strictly concave to the } k\text{-axis,} \end{cases} \tag{4.13}$$

whatever the correspondence between development years and operational time periods.

This condition will be considered again briefly in Section 6 in the context of a numerical example.

It is not difficult to think of circumstances in which the hypothesis of invariant order, which underlies all of the reasoning hitherto in this section, can be violated. For example, in longtail liability insurance, a certain proportion of notified claims are usually closed at no cost. Such claims could be settled earlier or later in their lifetimes, in which case nothing would be changed in the insurer's experience except the order of finalization of claims.

There seems little point in going into algebraic detail in such cases. The qualitative effects of such changes are clear. Suppose that, entirely through administrative decision, the finalizations of some zero claims occur earlier. The distribution of claim payments over development years will be unaffected. The numbers of finalizations, in respect of a given year of origin, will be higher in the early development years, and lower in the later development years. Hence PPCF will decrease in the early development years and increase in the later development years.

A danger is that, if the change has occurred during the last few years, the resulting decrease in PPCF may have been observed, but the compensating increase mentioned above might be still to emerge

It is impossible to generalize on situations arising when the hypothesis of invariant order is violated. This latter was adopted as a working hypothesis for the first part of this section, and results obtained on the basis of it. If one wishes to consider a situation in which it does not hold, then analytic results can be obtained only if it is replaced by an alternative hypothesis defining precisely the rules of the game.

All that can be said in general is that the hypothesis of invariant order, like most working hypothesis, is an idealization; that it may well approximate reality in many cases; but that the user of it must be watchful for any evidence to the contrary and prepared to react accordingly.

5. THE SEE-SAW METHOD

Essentially, the burthen of Section 4 is that PPCF for given development years will vary with changes in speed of finalization. Hence the assumption (3.1) on which the PPCF method rests will be invalid, and so that method will be of unknown reliability. As pointed out in Section 4, the noncumulative version of the Reduced Reid method (TAYLOR, 1980a) uses PPCF for development years as its starting point, interpolating between these quantities to obtain estimates of PPCF for particular operational time periods. Hence, this method will also be disrupted by a failure to recognise the dependency of PPCF for a development year on speed of finalization.

Therefore the question arises as to whether this last-mentioned dependency can be modelled. That is, it is now desired to recognise (inflation-adjusted) expected PPCF, conditional on $\{N_{i'j'}\}$ and $\{F_{i'j'}\}$, as a function of both operational time and speed of finalization:

$$E[P_{ij} \mid \{N_{i'j'}\}, \{F_{i'j'}\}] = g(\{T_{ik}: j < k \leq j+1\}, F_{ij}). \tag{5.1}$$

A couple of simplifying assumptions are made in (5.1). Firstly, it is assumed that the set $\{T_{ik}: j < k \leqslant j+1\}$ of operational times corresponding to year of origin i, development year j is adequately represented as an independent variable by its mid-value:

$$\bar{T}_{ij} = \tfrac{1}{2}(T_{ij} + T_{i,j+1}). \tag{5.2}$$

On this assumption, (5.1) simplifies to:

$$E[P_{ij} \mid \{N_{i'j'}\}, \{F_{i'j'}\}] = g(\bar{T}_{ij}, F_{ij}). \tag{5.3}$$

Now, *a priori* the form of the function g is not restricted in any way at all. The most useful approach, therefore, appears to be to consider separate relatively small ranges of $\bar{T}_{ij}$, making linearity assumptions in each.

Suppose the entire range of operational time $(0,1]$ is partitioned into r subintervals $(u_k, v_k]$, $k = 1, 2, \ldots, r$, with $u_1 = 0$, $v_r = 1$, $u_{r+1} = v_r$. Suppose $(u_k, v_k]$ to be sufficiently short that g can be taken to be linear. Thus, the second simplifying assumption is that:

$$g(\bar{T}_{ij}, F_{ij}) = \alpha_k + \beta_k \bar{T}_{ij} + \gamma_k F_{ij} \text{ for } u_k < \bar{T}_{ij} \leqslant v_k, \tag{5.4}$$

with α_k, β_k, γ_k constants.

The approximation (5.4) with $\bar{T}_{ij}$ given by (5.2) is reasonable only if the operational time-intervals $(T_{ij}, T_{i,j+1}]$ are roughly in correspondence with the $(u_k, v_k]$. The points u_k, v_k would therefore usually be chosen so as to force such a rough correspondence.

Up to this point no relation has been imposed on the α_k, β_k, γ_k associated with the different ranges $(u_k, v_k]$. However, it is desirable, presumably, to model g as a continuous function.

Continuity in the operational time variable can be obtained, while retaining the general form (5.4), by writing:

$$g(\bar{T}_{ij}, F_{ij}) = \alpha + \sum_k \beta_k \bar{T}_{ij}^{(k)} + \sum_k \gamma_k F_{ij}^{(k)}, \tag{5.5}$$

where

α is now a constant independent of interval of operational time under consideration;

β_k is different from β_k in (5.4);

γ_k is the same as in (5.4),

$$\bar{T}_{ij}^{(k)} \begin{aligned}[t] &= u_k, \text{ if } \bar{T}_{ij} < u_k; \\ &= \bar{T}_{ij}, \text{ if } u_k \leqslant \bar{T}_{ij} \leqslant v_k; \\ &= v_k, \text{ if } \bar{T}_{ij} > v_k; \end{aligned} \tag{5.6}$$

$$F_{ij}^{(k)} \begin{aligned}[t] &= F_{ij}, \text{ if } u_k < \bar{T}_{ij} \leqslant v_k; \\ &= 0, \text{ otherwise.} \end{aligned} \tag{5.7}$$

Note that continuity in F_{ij} is not obtained by the form (5 5). This results from the fact that the domains on which g is taken to be linear have been taken to depend on operational time (through the bounds u_k, v_k) but not speed of finalization.

From a purely mathematical point of view, it might have been more natural to define:

$$g(\bar{T}_{ij}, F_{ij}) = \alpha + \sum_k \beta_k \bar{T}_{ij}^{(k)} + \sum_l \delta_l F_{ij}^{(l)} \tag{5.8}$$

with $F_{ij}^{(l)}$ defined not as in (5.7) but in the same form as (5 6). This would indeed ensure continuity of g $(\bar{T}_{ij}, F_{ij})$ in both its variables. It would also produce linearity on the rectangular domains

$$\{(\bar{T}_{ij}, F_{ij}) : u_k^T < \bar{T}_{ij} \leq v_k^T, u_k^F < F_{ij} \leqslant v_k^F\},$$

for constants u_k^T, v_k^T, u_k^F, v_k^F.

The difference between (5.5) and (5.8) is of course that, for (5.5),

$$\partial g/\partial F_{ij} = \gamma_k \text{ for } u_k < \bar{T}_{ij} < v_k, \tag{5.9}$$

whereas for (5.8),

$$\partial g/\partial F_{ij} = \delta_k \text{ for } u_k^F < F_{ij} < v_k^F. \tag{5.10}$$

That is, in the first case the sensitivity of PPCF to changes in speed of finalization depends on operational time, whereas in the second this sensitivity depends on speed of finalization. The literature provides no guidance as to which of these factors is dominant. However, rather extensive experimentation on the part of the author suggests that, in most cases, the first of these alternatives seems nearer to reality. Hence, (5.5), though lacking in symmetry, is chosen as a better representation of PPCF than (5 8). In this choice, continuity of g in F_{ij}, though it would have been desirable, has been sacrificed.

As it happened, in the numerical example dealt in Section 6, γ_k did not vary greatly from one value of k to another, and so the sacrifice of continuity was not particularly important in numerical terms.

By (5 3) and (5.5), the adopted form for PPCF is:

$$E[P_{ij} \mid \{N_{i'j'}\}, \{F_{i'j'}\}] = \alpha + \sum_k \beta_k \bar{T}_{ij}^{(k)} + \sum_k \gamma_k F_{ij}^{(k)}, \tag{5.11}$$

with $\bar{T}_{ij}$, F_{ij} defined by (5.6) and (5.7) respectively.

The parameters α, β_k, γ_k may be estimated by regression of P_{ij} on $\bar{T}_{ij}$, F_{ij}. If any information is available (or can be guessed at) regarding the variances of the P_{ij}, the regression can be weighted to take account of it. The literature is virtually silent on the relation between Var P_{ij} and T_{ij}. But, as far as the relation between Var P_{ij} and F_{ij} is concerned, one would expect Var $[P_{ij} \mid \{N_{i'j'}\},$

$\{F_{i'j'}\}$ to be inversely proportional to N_{ij}. In the absence of further information, it might be reasonable to carry out a weighted regression of P_{ij} on $\bar{T}_{ij}$, F_{ij} with P_{ij} weighted by N_{ij}.

Empirically, it appears that the γ_k are negative. Thus, as speed of finalization increases, PPCF decreases. Because of this feature, the method developed above has been called the *see-saw method* (Taylor, 1980b).

6. NUMERICAL EXAMPLE

Appendix A contains data from a Compulsory Third Party (i.e. Motor – Bodily Injury) insurer in the Australian Capital Territory.

The PPCF are reproduced in Table 3 below. Operational times and speeds of finalization are given in Table 1 These figures are all taken from Taylor (1980b).

A weighted regression of P_{ij} on $\bar{T}_{ij}^{(k)}$ and $F_{ij}^{(k)}$ was carried out in the manner suggested in Section 5. The results were as follows:

TABLE 1 OPERATIONAL TIMES AND SPEEDS OF FINALIZATION

Accident year		(1) Average operational time (a), (2) Speed of finalization (b); during development year								
		0	1	2	3	4	5	6	7	8
1969	(1)	0.095	0.335	0.585	0.780	0.895	0.950	0.976	0.988	0.994
	(2)	0.19	0.29	0.21	0.16	0.07	0.040	0.013	0.010	0.004
1970	(1)	0.035	0.220	0.515	0.730	0.825	0.884	0.941	0.970	0.984
	(2)	0.07	0.30	0.29	0.14	0.05	0.067	0.042	0.014	0.014
1971	(1)	0.035	0.210	0.490	0.710	0.820	0.929	0.975	0.984	0.988
	(2)	0.07	0.28	0.28	0.12	0.14	0.077	0.015	0.003	0.005
1972	(1)	0.035	0.190	0.370	0.565	0.805	0.935	0.964	0.972	
	(2)	0.07	0.24	0.12	0.27	0.21	0.050	0.007	0.009	
1973	(1)	0.030	0.135	0.365	0.670	0.870	0.935	0.955		
	(2)	0.06	0.15	0.31	0.30	0.10	0.030	0.010		
1974	(1)	0.020	0.145	0.410	0.665	0.815	0.890			
	(2)	0.04	0.21	0.32	0.19	0.11	0.040			
1975	(1)	0.015	0.120	0.335	0.570	0.750				
	(2)	0.03	0.18	0.25	0.22	0.14				
1976	(1)	0.015	0.135	0.330	0.500					
	(2)	0.03	0.21	0.18	0.16					
1977	(1)	0.030	0.140	0.230						
	(2)	0.06	0.16	0.14						
1978	(1)	0.015	0.105							
	(2)	0.03	0.15							
1979	(1)	0.020								
	(2)	0.04								

(a) Average of operational times at beginning and end of year, (b) Increase in operational time, i e proportion of claims incurred finalized, during year

TABLE 2. REGRESSION COEFFICIENTS

Range of operational time k	from operational time	to operational time	β_k (a)	γ_k (a)	α (a)
			$	$	$
1	0	0 15	−2191	−33950	
2	0 15	0 35	+10410	−22090	
3	0 35	0 55	−1194	−30380	
4	0 55	0 75	−16720	−30870	−188100
5	0 75	0 85	−45 09	−35260	
6	0 85	0 95	+14660	−38010	
7	0 95	1 00	+208100	−145200	

(a) Estimated by means of a weighted regression in accordance with (5 11) The weight associated with P_{ij} was N_{ij} The regression was carried out by means of the GLIM package The error terms were assumed to be normally distributed

TABLE 3 PAYMENTS PER CLAIM FINALIZED

Accident year		Payments (31/12/79 values) per claim finalized (1) actually recorded (a), (2) fitted (b), in development year								
		0	1	2	3	4	5	6	7	8
		$	$	$	$	$	$	$	$	$
1969	(1)	1950	10288	10285	9273	12843	6826	18373	14169	12539
	(2)	7282	9131	8386	6469	10110	7765	17100	20030	22150
1970	(1)	8184	8246	9195	11602	8509	8896	10003	12039	2570
	(2)	11490	7713	6686	8123	10340	10060	11840	15700	18620
1971	(1)	13338	8268	6621	9419	6872	4433	14780	28550	21610
	(2)	11490	8051	7020	9075	7172	10340	16600	20210	20750
1972	(1)	9528	5847	12255	5923	3751	13663	48599	49974	
	(2)	11490	8726	12020	6868	4705	11450	15470	16840	
1973	(1)	8813	8601	5538	3972	11297	10325	21906		
	(2)	11840	8552	6257	4187	8598	12210	13160		
1974	(1)	14089	6799	5986	5798	7818	8851			
	(2)	12540	6493	5900	7666	8230	11170			
1975	(1)	17651	7401	9197	7995	8872				
	(2)	12890	7566	10010	8328	12110				
1976	(1)	16725	9165	12287	5996					
	(2)	12890	6515	11510	10650					
1977	(1)	15143	9216	14118						
	(2)	11840	8201	11350						
1978	(1)	15660	9636							
	(2)	12890	8618							
1979	(1)	14852								
	(2)	12540								

(a) From Appendices A2 and A5, (b) Calculated from regression equation (5.11), using parameters of Table 2 and observed operational times and speeds of finalization tabulated in Table 1

Table 2 provides a model for the projection of future claim payments. This projection is carried through to its conclusion in Appendix B.

The fitted PPCF obtained from this regression appear in Table 3.

To a large extent the fitted PPCF appear in accord with the observed. There are deviations, however, and the quality of the fit *vis-à-vis* other methods is not clear without a good deal more work. Without going into the matter, it will simply be reported here that a thorough analysis of the fits obtained by seven different methods of analysis has been made by TAYLOR (1980b). The fit provided by the see-saw method appears clearly superior to the other six.

A significant feature of Table 1 is that $\gamma_k < 0$ for each k. That is, at every operational time, an increase in speed of finalization induces a decrease in PPCF A sufficient condition for this to occur was given by (4.13). It is interesting to investigate to what extent (fitted) PPCF, as a function of development year k, is strictly concave to the k-axis.

A brief investigation of fitted PPCF reveals that this condition does not hold whatever the correspondence between development years and operational time periods, as required by (4.13). However, it is perhaps more pertinent to examine the fitted PPCF for a typical runoff of claim numbers.

TABLE 4 PPCF FOR A TYPICAL RUNOFF OF CLAIM NUMBERS

Development year	Operational time at end of development year	Fitted PPCF	Second difference of fitted PPCF
		$	$
0	0 04	12,568	
1	0 30	8,106	+2,981
2	0 50	9,587	−2,593
3	0 70	8,475	−865
4	0 86	6,498	+5,921
5	0 92	10,442	−1,837
6	0 94	12,549	−1,814
7	0 96	12,842	+1,725
8	0 98	14,860	+2,556
9	0 99	19,434	−2,493
10	1 00	21,515	

It must be remembered of course that the fitted values of PPCF are subject to estimation error, and second differences even more so Nevertheless, it cannot be said that Table 4 provides compelling evidence that PPCF is a persistently concave function. The table does perhaps give the impression of a piecewise concave function with a couple of jumps from one concave piece to another at a higher level.

It is interesting that, although PPCF appear to fluctuate between concavity and convexity, still each $\gamma_k < 0$. This is rather unexpected in the light of Section 4. Two remarks can be made. Firstly, the see-saw method may well still be applicable (see Section 7). Secondly, the negativity of all the γ_k may be explicable as follows. TAYLOR (1980b, Section 12 and 13) showed that average claim sizes may have been increasing with increasing year of origin. But a glance at Table 1 shows that this would lead to a negative correlation between PPCF and speed of finalization.

The existence of superimposed inflation would manifest itself in a similar way.

It is worth recording that, for another CTP insurer (data not reproduced here), PPCF appeared quite clearly to be a monotone increasing and concave function of development year.

7. CONCLUDING REMARKS

Section 4 carries out an investigation of the way in which PPCF can be expected to respond to changes in speed of finalization. The investigation is based on the hypothesis that such changes merely have the effect of extending or compressing the time-line. They do not reorder nor change the nature of the events occurring on this time-line. This hypothesis is consistent with REID's (1978) use of operational time which, it appears from subsequent experience, is producing reasonably good results.

On the basis of this hypothesis it is concluded that whether PPCF increases or decreases with increasing speed of finalization depends on the convexity properties of PPCF (over infinitesimal time intervals) as a function of operational time.

Emphasis is given to the fact that this conclusion may require amendment should the hypothesis on which it is based be violated. As an example of the type of violation which can occur, reference is made to the possible situation in which an increase in speed of finalization is "not real" in the sense that it is induced merely by a change in administrative practice as to when certain claims are regarded as finalized.

Another possibility, not mentioned in Section 4, is that a change in speed of settlement does actually change claim sizes (apart from inflationary effects). For example, it may be argued that the insurer's earlier achievement of settlement with the insured produces a decrease, in real terms, in the amount of the claim.

The likelihood and effect of such distortions of the model deserve some comment. Firstly, as regards the "administrative change" distortion, Reid seems to be achieving reasonable results without bringing this possibility into account. Secondly, as regards the "real reduction in claim size" distortion, it is difficult to determine to what extent this is taken into account in Reid's method. If the suggested phenomenon were present, its effect would presumably be to influence Reid's inflation factors. If the phenomenon were significant, Reid would observe a bias in these factors in years in which a change in speed of finalization occurred. I am not aware that such a bias has been observed, but on the other hand there may not have been any particular reason for seeking it

All things considered, it does seem that the evidence for any important departure from the hypothesis of invariant order is rather slim.

In any event, it should be noted that, although Section 4 depends on the hypothesis of invariant order, Section 5 in which the see-saw method is worked out does not. Section 5 is based simply on the assumption that for small changes in speed of finalization and small ranges of operational time, PPCF is approximately a linear function of these two variables. Even if the hypothesis of invariant order were violated in both of the ways described above, this linearity assumption would still remain appropriate.

In other words, although Section 4 provides some interesting details of the claim payment mechanism, the see-saw method does not depend on the intricacies of this mechanism.

APPENDIX A: DATA

A1.

Numbers of claims incurred (n_i in the notation of Section 2)

Accident year	Number of claims incurred (a)
1969	523
1970	643
1971	676
1972	673
1973	809
1974	669
1975	513
1976	543
1977	622
1978	703
1979	743

(a) These numbers consist of numbers of claims reported to the end-1979 plus estimated numbers incurred but not reported (IBNR) Because the second of these components has been estimated, the above numbers are strictly subject to estimation error However, the scope for error is quite small For years other than the very latest it is very small indeed Throughout this paper, estimation error in the N_i has been disregarded

A2.

Numbers of claims finalized (n_{ij} *in the notation of Section 2*)

Accident year	Number of claims finalized in development year								
	0	1	2	3	4	5	6	7	8
1969	99	154	112	84	37	21	7	5	2
1970	46	193	187	89	34	43	27	9	9
1971	44	191	193	78	99	49	10	2	3
1972	45	166	78	185	136	36	5	6	
1973	52	115	256	240	78	27	9		
1974	25	140	216	129	70	31			
1975	16	93	126	113	71				
1976	16	114	99	88					
1977	37	102	84						
1978	23	105							
1979	30								

A3.

Claim payments (c_{ij} *in the notation of Section 2*)

Accident year	Claim payments in development year								
	0	1	2	3	4	5	6	7	8
	$	$	$	$	$	$	$	$	$
1969	57369	514096	418034	305728	208822	77754	82811	52596	20405
1970	122161	559595	674860	453750	156919	246291	200516	88166	20440
1971	212991	619776	561523	398485	438079	161268	120262	50451	61742
1972	168274	426536	518479	705550	378684	400212	214705	285566	
1973	201373	536473	912775	707685	716962	246320	187767		
1974	191049	612854	956866	611637	484568	260301			
1975	181845	511027	942889	798263	599887				
1976	198670	850127	1074810	502516					
1977	455912	830559	1129424						
1978	318237	963590							
1979	424327								

A4.

Inflation index (λ_{i+j})

Calendar year	Claims inflation index (a)
1969	0 297229
1970	0 324488
1971	0 374397
1972	0 392473
1973	0 439437
1974	0 537819
1975	0 643884
1976	0 742414
1977	0 813682
1978	0 883572
1979	0 952377

(a) Base value of 1 0 at 31/12/79 The index is proportional to Average Weekly Earnings for ACT In years 1973 and earlier, this statistic was not published It has been taken as 120% of NSW AWE Inflation of AWE from mid-1979 to end-1979 has been taken as 5%.

A5

Claim payments in 31/12/79 values (c_{ij}/λ_{i+j})

Accident	Claim payments (31/12/79 values) (a) in development year								
year	0	1	2	3	4	5	6	7	8
	$	$	$	$	$	$	$	$	$
1969	193013	1584331	1151882	778980	475203	143352	128612	70845	25077
1970	376473	1541950	1719509	1032570	289305	382508	270087	108354	23133
1971	568891	1579158	1277822	734670	680369	217221	147800	57099	64829
1972	428753	970640	955898	1095771	510072	491853	242995	299845	
1973	458252	989072	1417606	953222	881133	278778	197156		
1974	355229	948807	1292900	748003	547288	274367			
1975	282419	688332	1158793	903450	629983				
1976	267600	1044790	1216437	527644					
1977	560307	940002	1185899						
1978	360171	1011773							
1979	445545								

(a) Derived from Appendices A3 and A4.

APPENDIX B: PROJECTIONS OF FUTURE CLAIM PAYMENTS

B1.

Operational times and speeds of finalization

Accident year		(1) Average operational time (a) (c), (2) Speed of finalization (b) (c), during development year										
		0	1	2	3	4	5	6	7	8	9	10
1969	(1)	0.095	0.335	0.585	0.780	0.895	0.950	0.976	0.988	0.994		
	(2)	0.19	0.29	0.21	0.16	0.07	0.040	0.013	0.010	0.004		
1970	(1)	0.035	0.220	0.515	0.730	0.825	0.884	0.941	0.970	0.984		
	(2)	0.07	0.30	0.29	0.14	0.05	0.067	0.042	0.014	0.014		
1971	(1)	0.035	0.210	0.490	0.710	0.820	0.929	0.975	0.984	0.988	0.993	0.998
	(2)	0.07	0.28	0.28	0.12	0.14	0.077	0.015	0.003	0.005	0.005	0.005
1972	(1)	0.035	0.190	0.370	0.565	0.805	0.935	0.964	0.972	0.983	0.993	0.998
	(2)	0.07	0.24	0.12	0.27	0.21	0.050	0.007	0.009	0.014	0.005	0.005
1973	(1)	0.030	0.135	0.365	0.670	0.870	0.935	0.955	0.965	0.980	0.993	0.998
	(2)	0.06	0.15	0.31	0.30	0.10	0.030	0.010	0.010	0.020	0.005	0.005
1974	(1)	0.020	0.145	0.410	0.665	0.815	0.890	0.915	0.940	0.975	0.993	0.998
	(2)	0.04	0.21	0.32	0.19	0.11	0.040	0.010	0.040	0.030	0.005	0.005
1975	(1)	0.015	0.120	0.335	0.570	0.750	0.845	0.885	0.920	0.960	0.985	0.995
	(2)	0.03	0.18	0.25	0.22	0.14	0.050	0.030	0.040	0.040	0.010	0.010
1976	(1)	0.015	0.135	0.330	0.500	0.665	0.775	0.840	0.900	0.950	0.985	0.995
	(2)	0.03	0.21	0.18	0.16	0.17	0.050	0.080	0.040	0.060	0.010	0.010
1977	(1)	0.030	0.140	0.230	0.420	0.565	0.715	0.815	0.880	0.945	0.985	0.995
	(2)	0.06	0.16	0.14	0.12	0.17	0.130	0.070	0.060	0.070	0.010	0.010
1978	(1)	0.015	0.105	0.240	0.350	0.500	0.675	0.785	0.860	0.935	0.980	0.995
	(2)	0.03	0.15	0.12	0.10	0.20	0.150	0.070	0.080	0.070	0.020	0.010
1979	(1)	0.020	0.110	0.240	0.350	0.500	0.675	0.785	0.860	0.935	0.980	0.995
	(2)	0.04	0.14	0.12	0.10	0.20	0.150	0.070	0.080	0.070	0.020	0.010

(a) Average of operational times at beginning and end of year, (b) Increase in operational time, i.e. proportion of claims incurred finalized, during year, (c) Entries below the heavy diagonal line are derived from the illustrative predicted operational times given in Section 9.2 (first table) of TAYLOR (1980a)

B2.

Projected future numbers of finalizations and payments per claim finalized

Accident year		Projected. (1) Number of claims finalized (a), (2) Payments per claim finalized (b), in development year										
		0	1	2	3	4	5	6	7	8	9	10
1971	(1)										4	3
	(2)										21824	22864
1972	(1)									9	4	3
	(2)									18438	21824	22864
1973	(1)								8	16	4	4
	(2)								15273	16942	21824	22864
1974	(1)							6	26	20	3	3
	(2)							12709	11936	14449	21824	22864
1975	(1)						26	16	21	21	5	5
	(2)						10373	11509	11643	9876	19433	21514
1976	(1)					92	27	43	22	32	5	5
	(2)					8314	10375	9316	11349	11322	19433	21514
1977	(1)				75	106	81	44	37	44	6	6
	(2)				11994	9986	8712	9669	10295	10868	19433	21514
1978	(1)			84	70	141	105	49	56	49	14	7
	(2)			11927	13514	9468	8764	9671	9242	10721	16941	21514
1979	(1)		104	89	74	149	111	52	59	52	15	8
	(2)		7618	11927	13514	9468	8764	9671	9242	10721	16941	21514

(a) Derived in accordance with TAYLOR (1980a, first table in Section 9.2) The predicted numbers of future finalizations are needed for illustrative purposes only No importance attaches to them in the context of the principle involved in the see-saw method, (b). Derived from (5 11) using the parameters displayed in Table 2 and future operational times and speeds of finalization from Appendix B1

B3.
Outstanding claims

Accident year	Outstanding claims at 31/12/79 (in 31/12/79 values)
	$M
1971	0 156
1972	0 322
1973	0 572
1974	0 810
1975	1.110
1976	2.262
1977	4 194
1978	6 108
1979	7 269
Total (1971-1979)	22 803

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