Classification Ratemaking—Further Discussion

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Abstract

Classification Ratemaking is one of the most important elements in the process of a Property/Casualty rate calculation. It is here that the pricing actuary moves from a rate change that is appropriate for an entire portfolio of policyholders, to prices that attempt to be fair and equitable for each policyholder in the portfolio.

Classification Ratemaking is so important that is has its own complete chapter in the textbook "Foundations of Casualty Actuarial Science" (Chapter 6, authored by R. Finger). Other sources of P&C study material also present lengthy analysis of this topic (e.g. Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance (2^{nd} Edition) by Brown and Gottlieb (2001).

This paper illustrates that these two important references do not arrive at exactly the same results for a Classification Ratemaking situation where some cells have less than full credibility. The paper then goes on to attempt to isolate the reason for the differences and in so doing, sheds further light on the process itself.

I Introduction

For more than a decade now, students of the CAS syllabus have learned Classification Ratemaking from Chapter 6, Risk Classification (authored by R. Finger) in the textbook: "Foundations of Casualty Actuarial Science".

However, this is not the only source of study material on this topic. The Society of Actuaries also introduces their students to some P&C topics through their Part 5 course and they use the textbook: "Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance" authored by Brown and Gottlieb.

Interestingly, it will be shown that these two text references do not arrive at exactly the same solution for a Risk Classification Ratemaking question where some classes in the analysis do not have full credibility.

By analyzing the reason for the differences in the two answers, this paper attempts to shed new light on the entire process of Classification Ratemaking.

II The Problem by Illustration

In the text: "Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance (2nd Edition) by Brown and Gottlieb, the authors present an algebraic proof that the two classical methods to calculate Class differentials; namely, the Loss Ratio Method and the Loss Cost Method are equivalent. This example, however, only covers the case where all risk classes being analyzed have credibility equal to one (see Brown and Gottlieb (2001), Appendix A, pages 173-175).

It is also the case that for a portfolio of risks where credibility is one for every class, that the class relativities produced by Finger are equal to the relativities produced by Brown and Gottlieb.

This will now be illustrated with a simple example.

Example, Part I

The pricing actuary has decided upon a statewide adopted rate level increase of +6%. Given the following data, show the new adopted rates for Classes 1, 2, and 3.

The existing base rate is $100 in Class 1.

All Classes have full credibility ($Z = 1$).

You also have the following data by class:

Class	Existing Relativity	Exposure Units	Earned Premium	$Loss	Loss Cost	Loss Ratio
1	1.00	500	$50,000	$30,000	$60.00	0.6000
2	1.25	150	18,750	12,750	85.00	0.6800
3	1.50	200	30,000	15,900	79.50	0.5300
		850	98,750	58,650	69.00	0.5939

Method I

We will use the Loss Cost Method using Class 1 as the Base Rate for the calculation. Remember that Z = 1 throughout. We will use seven decimal accuracy in all calculations even if fewer decimal place accuracy is displayed.

Class	Existing Relativity	Loss Cost	Indicated Relativity
1	1.00	60.00	1.000
2	1.25	85.00	1.416
3	1.50	79.50	1.325

Since Z = 1 in all cells, the "Existing Relativity" does not have any impact on the answer and could be ignored (as it is in some examples below).

We have set the Class Relativity for Class 1 equal to 1.000. This means that our overall rate change may not balance to +6%. So, we need to balance back, as follows:

Old Average Relativity = [500(1.00) + 150(1.25) + 200(1.50)]/850 = 1.1617647

New Average Relativity = [500(1.00) + 150(1.416) + 200(1.325)]/850 = 1.15

Balance-back Factor = 1.1617647/1.15 = 1.0102302, giving us:

Class	New Rate	Exposure Units	Premium Income
1	$107.08	500	$53,542
2	151.70	150	22,755
3	141.89	200	28,377
			104,675

Now, $104,675 = $98,750 * (1.06),

so, everything is as it should be.

Method II

We will use the Loss Cost Method but the Base Class will be the State (Loss Cost).

Class	Loss Cost	Indicated Relativity	Relativity with Class 1 = 1.000
1	60.00	0.8695652	1.000
2	85.00	1.2318841	1.416
3	79.50	1.1521739	1.325
State	69.00		

So, this gives us the same relativities as does Method I and there is no reason to go further (i.e. there is no reason to do the balance back calculation).

Method III

This method follows the Loss Ratio approach with the base class being Class 1.

Class	Loss Ratio	Existing Relativity	Indicated Change $[LR_i/LR_1]$	Indicated Relativity
1	0.6000	1.00	1.000	1.000
2	0.6800	1.25	1.133	1.416
3	0.5300	1.50	0.883	1.325

Again, the same answer. Thus, it has been shown that under the set conditions, the Loss Ratio method and the Loss Cost method do provide the same answer (as proven algebraically by Brown and Gottlieb).

Method IV

We will follow the Loss Ratio approach again, but now the base 'class' will be the State (Loss Ratio).

Class	Loss Ratio	Existing Relativity	Indicated Change $[LR_i/LR_S]$	Indicated Relativity	Indicated Rel with Class 1 = 1.000
1	0.6000	1.00	1.0102302	1.0102302	1.000
2	0.6800	1.25	1.1449275	1.4311594	1.416
3	0.5300	1.50	0.8923700	1.3385550	1.325
State	0.5939				

Again, the same answer.

Method V

Finally, we follow the template presented in Chapter 6 of the Foundations textbook (Finger). Remember that the overall rate change is +6%.

Class	Existing Relativity	Adjusted Exposures	Adjusted Loss Costs	Indicated Adjustment	Extension	Adopted Relativity*
(1)	(2)	(3)	(4)	(5)	(6)	(7)
		[(2)*Given Exp]	[$Loss/(3)]	[(4)/(4) Total]	[(5)*Old Rate *1.06]	
1	1.00	500	60.00	1.0102302	107.30	1.000
2	1.25	187.5	68.00	1.1449275	151.70	1.416
3	1.50	300	53.00	0.8923700	141.89	1.325
		987.5	59.39			

* This is not produced by Finger, but is clearly consistent.

Obviously, we got identical answers for the Adopted Relativities and the New Rates from all the approaches attempted. This should be gratifying and should create a level of comfort among users.

Example, Part II

We now stir the pot somewhat by stipulating credibility factors for the different classes where only Class 1 has full credibility.

We will use the following data in this illustration:

Class	Existing Relativity	Exposure Units	Earned Premium	$Loss	Loss Cost	Loss Ratio	Credibility Z
1	1.00	500	$50,000	$30,000	60.00	0.6000	1.000
2	1.25	150	18,750	12,750	85.00	0.6800	0.500
3	1.50	200	30,000	15,900	79.50	0.5300	0.600
State		850	98,750	58,650	69.00	0.5939	1.000

Again, we will find the new Class 1, 2, and 3 (base) rates with an overall +6% rate increase.

We will now repeat the original five methods of calculation to see if they again produce identical answers.

Method I*

Remember that this is the Loss Cost method with the Base Class being Class 1.

Class (1)	Existing Relativity (2)	Loss Cost (3)	Indicated Relativity (4)	Z (5)	Adopted Relativity [Z(4) + (1-Z)(2)]
1	1.00	60.00	1.000	1.000	1.000
2	1.25	85.00	1.416	0.500	1.333
3	1.50	79.50	1.325	0.600	1.395

Again we have created off-balance, so we balance back:

Old Average Relativity = [500(1.00) + 150(1.25) + 200(1.50)]/850 = 1.1617647

New Average Relativity = [500(1.000) + 150(1.333) + 200(1.395)]/850 = 1.1517647

Balance Back Factor = 1.1617647/1.1517647 = 1.0086823

This produces the following new "base" rates:

Class	New Rate	Exposure Units	Premium Income
1	$106.92	500	$53,460.16
2	142.56	150	21,384.06
3	149.15	200	29,830.77
			104,674.99

or, $104,675, which is what we want.

Method II*

This is the Loss Cost method, but with the "base" being the State (Loss Cost).

Class (1)	Existing Relativity (2)	Loss Cost (3)	Indicated Relativity (4) [(3)/(3)Total]	Ind. Rel. Class 1 = 1.00 (5) $[(4)/(4)_1]$	Z (6)	Adopted Relativity [Z*(5) + (1-Z)*(2)]
1	1.00	60.00	0.8695652	1.000	1.000	1.000
2	1.25	85.00	1.2318841	1.416	0.500	1.333
3	1.50	79.50	1.1521739	1.325	0.600	1.395
		69.00				

This is the same answer as ***Method I****

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However, it is possible to get an incorrect answer by changing the order of the arithmetic operations.

For example, one might do the following erroneous calculation:

Class	Existing Relativity	Loss Cost	Indicated Relativity	Z	Adopted Relativity	Rate Manual Relativity
(1)	(2)	(3)	(4)	(6)	(7)	(8)
			[(3)/(3)Total]		[Z*(4) + (1-Z)*(2)]	(Class 1 = 1.00)
1	1.00	60.00	0.8695652	1.000	0.8695652	1.0000000
2	1.25	85.00	1.2318841	0.500	1.2409421	1.4270834
3	1.50	79.50	1.1521739	0.600	1.2913043	1.4850000
		69.00				

This answer is different than those found in the previous two calculations, and it is wrong. It is wrong, because in the formula for the Adopted Relativity [Z*(4) + (1-Z)*(2)], you do not have the relativities on the same basis. Column (2) has the relativities "normalized" such that the relativity for Class 1 equals 1.000, but in Column (4) the data have not been "normalized". Thus, in the formula for the Adopted Relativity, we are taking the weighted average of "apples" from Column (2) and "oranges" from Column (4). One could extend the analogy to consider one vector as degrees Fahrenheit and the other, degrees Celsius. These should not be commingled in a weighted average. Obviously, this would lead to an incorrect result.

Method III*

This is the classical Loss Ratio method with Class 1 being the base.

Class	Loss Ratio	Existing Relativity	Indicated Change [LR_i/LR_1]	Indicated Relativity	Z	Adopted Relativity
(1)	(2)	(3)	(4)	(5)	(6)	(7)
						[Z*(5) + (1-Z)*(3)]
1	0.6000	1.00	1.000	1.000	1.000	1.000
2	0.6800	1.25	1.133	1.416	0.500	1.333
3	0.5300	1.50	0.883	1.325	0.600	1.395

This agrees nicely with all of our previous work.

Method IV*

Again, this is the Loss Ratio Method with the base "class" being the State (Loss Ratio).

Class (1)	Loss Ratio (2)	Existing Relativity (3)	Indicated Change $[LR_i/LR_S]$ (4)	Indicated Relativity (5)[(4) * (3)]	Indicated Rel. Class 1 = 1.00 (6)	Z (7)	Adopted++ Relativity
1	0.6000	1.00	1.0102302	1.0102302	1.000	1.000	1.000
2	0.6800	1.25	1.1449275	1.4311594	1.416	0.500	1.333
3	0.5300	1.50	0.8923700	1.3385550	1.325	0.600	1.395
State	0.5939						

++ [Z*(6) + (1-Z)*(3)]

Obviously, this is an acceptable answer. But, again, the order of calculation, and the use of factors that are "normalized" to the same base, is of the essence. For example, we could erroneously do the following:

Class (1)	Loss Ratio (2)	Existing Relativity (3)	Indicated Change $[LR_i/LR_S]$ (4)	Indicated Relativity (5)[(4) * (3)]	Z (6)	Adopted Relativity [Z(5)+(1-Z)*(3)]	Rate Manual Relativity (Class 1 = 1.000)
1	0.6000	1.00	1.0102302	1.0102302	1.000	1.0102302	1.000
2	0.5280	1.25	1.1449275	1.4311594	0.500	1.3405797	1.327
3	0.5400	1.50	0.8923700	1.3385550	0.600	1.4031330	1.389
State	0.5676						

This is an incorrect answer because in our Adopted Relativity calculation, Column (3) has been "normalized" so that Class 1 has a relativity equal to 1.00, but Column (5) has not. Thus, we are attempting to do a weighted average of "apples" and "oranges".

Method V*

This uses the template found in Chapter 5 of the "Foundations" text as authored by Finger (2001).

Class (1)	Existing Relativities (2)	Exposure Units (3)	Earned Premiums (4)	Adjusted Exposures (5) [(3)*(2)]	Adjusted Loss Costs (6) [$Loss/(5)]	Indicated Adjustment (7) [(6)/(6) Total]	Z (7)
1	1.00	500	50,000	500	60.00	1.0102302	1.000
2	1.25	150	18,750	187.5	68.00	1.1449275	0.500
3	1.50	200	30,000	300	53.00	0.8923700	0.600
		850	98,750	987.5	59.39		

Continuing with the template:

Class	Credibility Weighted Adjustment (9) [Z*(7) + (1-Z)]	Extension (10) [(9) * (4)]	Balanced Adjustment (11) [(9)/(9) Total]	New Rates (12) [(11)*Old*1.06]	Extension (13) [(12)*(3)]
1	1.0102302	50,511.51	1.0109174	107.16	53,578.62
2	1.0724638	20,108.70	1.0731934	142.20	21,329.72
3	0.9354220	28,062.66	0.9360583	148.83	29,766.65
	0.9993202*	98,682.87			104,674.99

*0.9993202 = 98,682.87/98,750

This all seems to check out just fine. The final answer ($104,675) is a +6% rate increase as requested. However, the "New Rates" are different than what we got in the other four Methods.

One can also that the new Class Relativities created by Finger (but never actually displayed) are as follows and differ from those calculated by Methods I* to IV*.:

Class	Relativity with Class 1 = 1.00
1	1.000
2	1.327
3	1.389

Why is this?

With a little bit of work (and some insight) the differences are easily reconciled.

In Methods I* to IV*, we calculated all relativities using a base relativity of 1.000 for Class 1. What Finger does is to calculate all relativities using a base relativity of 1.000 for the State. We can show that this is true by re-calculating Methods I* to IV* using a relativity of 1.000 for the State.

In our existing examples, the following hold

Class	Relativity
1	1.000
2	1.250
3	1.500
State	1.1617647

Switch these values to equivalent values with the State relativity equal to 1.000 and you get:

Class	Relativity
1	0.8607595
2	1.0759494
3	1.2911392
State	1.0000000

Now, calculate your credibility-weighted new relativities using the above as starting points. You will get the following:

Class	Existing Relativity	Loss Cost	Indicated Relativity	Z	Adopted Relativity
1	0.8607595	$60.00	0.8695652	1.00	0.8695652
2	1.0759494	85.00	1.2318841	0.50	1.1539167
3	1.2911392	79.50	1.1521739	0.60	1.2077600
		69.00			

With this change in format, you will arrive at the premiums and relativities derived by Finger. Just re-create the above "Adopted Relativities" with Class 1 = 1.000, and you get:

Class	Adopted Relativity	(Class 1 = 1.000)
1	1.000	
2	1.327	
3	1.389	

Thus, one cannot conclude that one Methodology is correct and the other incorrect. They are just two versions of the same analysis that happen to result in slightly different answers. However, there are some implications to these findings, including:

--regulators cannot guarantee that two actuaries will arrive at the same answer given the same data without prescribing the methodology in extreme detail;
--examiners cannot guarantee that there is a uniquely correct answer to an examination question unless they prescribe the methodology in extreme detail (or force the student into a mode such as: "according to Finger…").
--the pricing actuary who is aware of these differences might then be able to use them to his or her advantage. For example, assume you have two large classes (A and B) which are fully credible and a few smaller classes with little credibility. If we assume that A increases by 10% and B declines by 10%, then the choice of A or B as the base class will drive the rates of the classes with little credibility. If we choose A, their rates will go up

and if we choose B their rates will go down. If we choose the state-wide average, their rates will not change by much (all else being equal).

III Conclusion

As stated in the Introduction, classification ratemaking is one of the most important steps in arriving at new rate manual rates.

This topic has been presented in a variety of forms, templates and methodologies over the years. Unfortunately, the different methods presented to students do not necessarily produce the same unique result.

It is the belief and hope of this author that a full understanding of the consequences as presented in this paper will bring the level of knowledge of future students to a new high in this important area.

Bibliography

Brown, R. L. and L. R. Gottlieb (2001) *Introduction to Ratemaking and Loss Reserving for Property and Casualty Insurance. (2nd Edition).* ACTEX Publications Inc., Winsted, CT.

Finger, R.J., "Risk Classification," *Foundations of Casualty Actuarial Science (Fourth Edition*), Casualty Actuarial Society, 2001, Chapter 6, pp. 287-342.