GLM with Correlated Frequency & Severity – An Ontario Personal Auto Application

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cutting through complexity

Poll Questions

Poll 1: Have you been involved in automobile rate filing in Ontario since 2010?

- Yes
- No

Poll 2: Have you ever carried on GLM analyses?

- Yes
- No

Agenda

Overview of Ontario Auto

- Market Size & Performance
- Impact of 2010 Reforms
- Changes since 2010

GLM Application

- Quick Overview of Basics
- Three Approaches to Reflect Dependencies

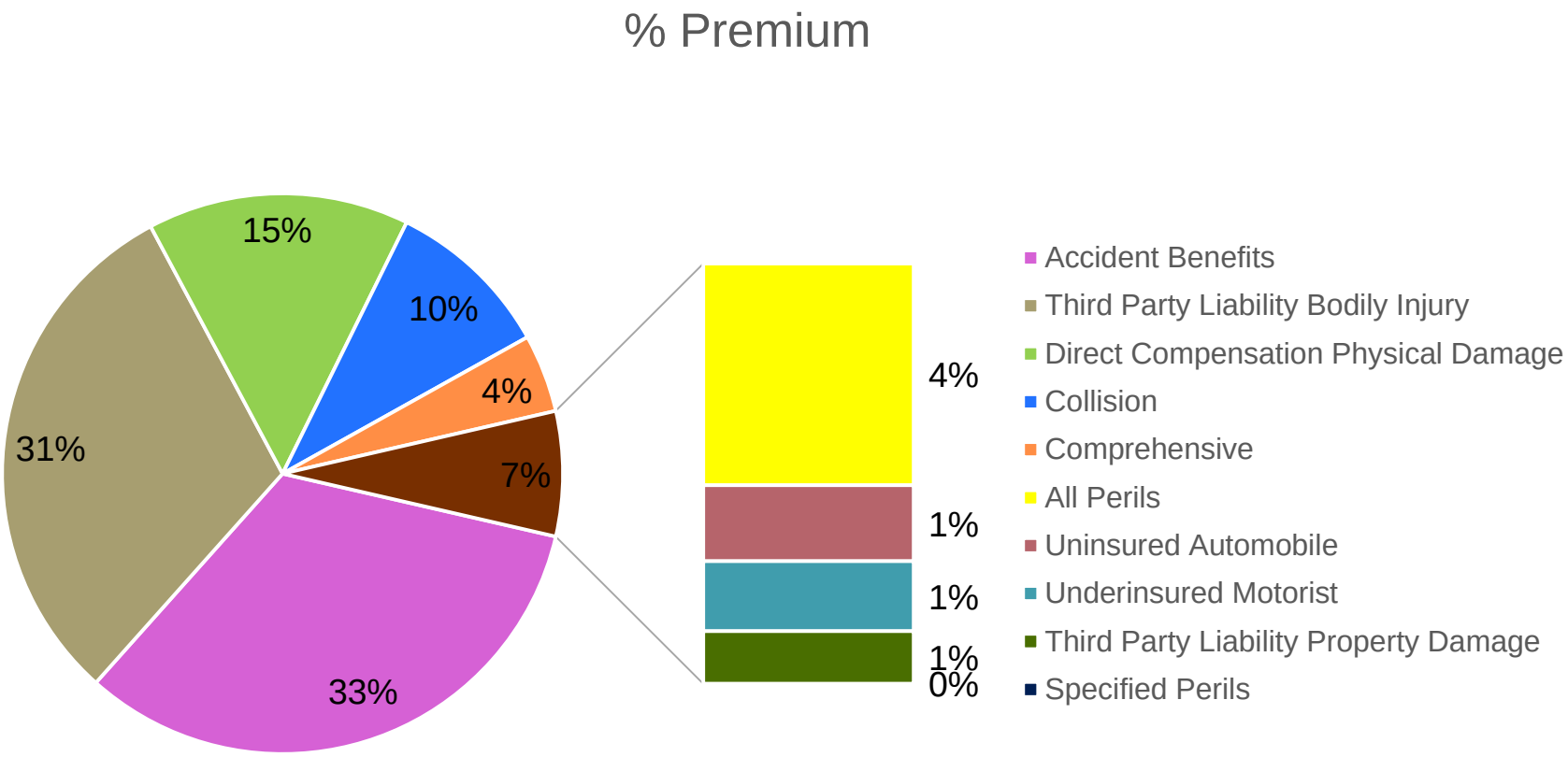
Ontario Automobile – Market Size and Performance

Private Passenger Automobile Excluding Farmers

- Over 6.9 million vehicles
- $10.5 billion of earned premium in 2014
- 69.7% Loss Ratio for AY 2014

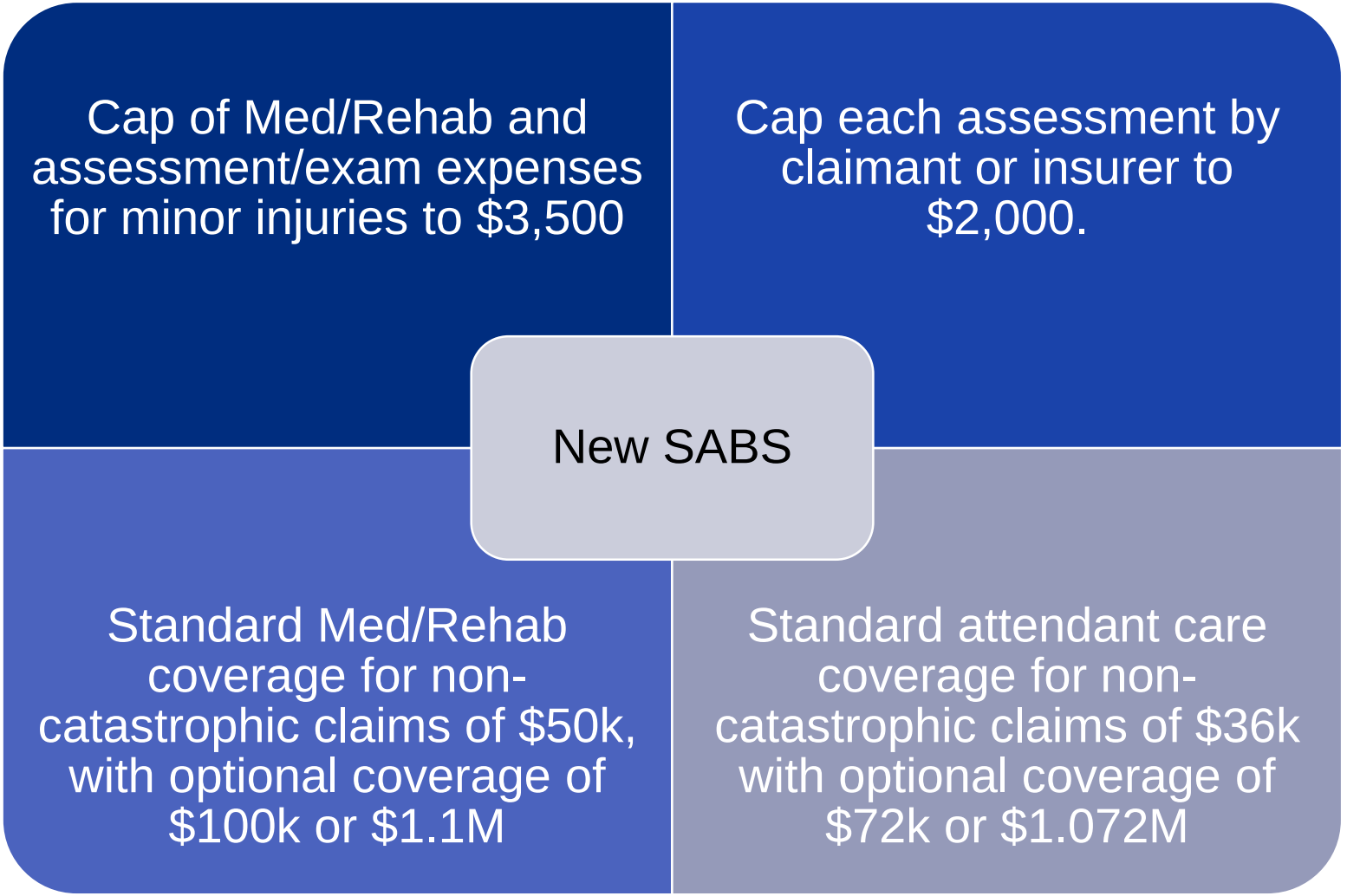
Source: GISA Data AUTO1010-ON_2014.pdf

Ontario Automobile – Premium Allocation



Source: GISA Data AUTO1010-ON_2014.pdf

Ontario Automobile – 2010 Reforms



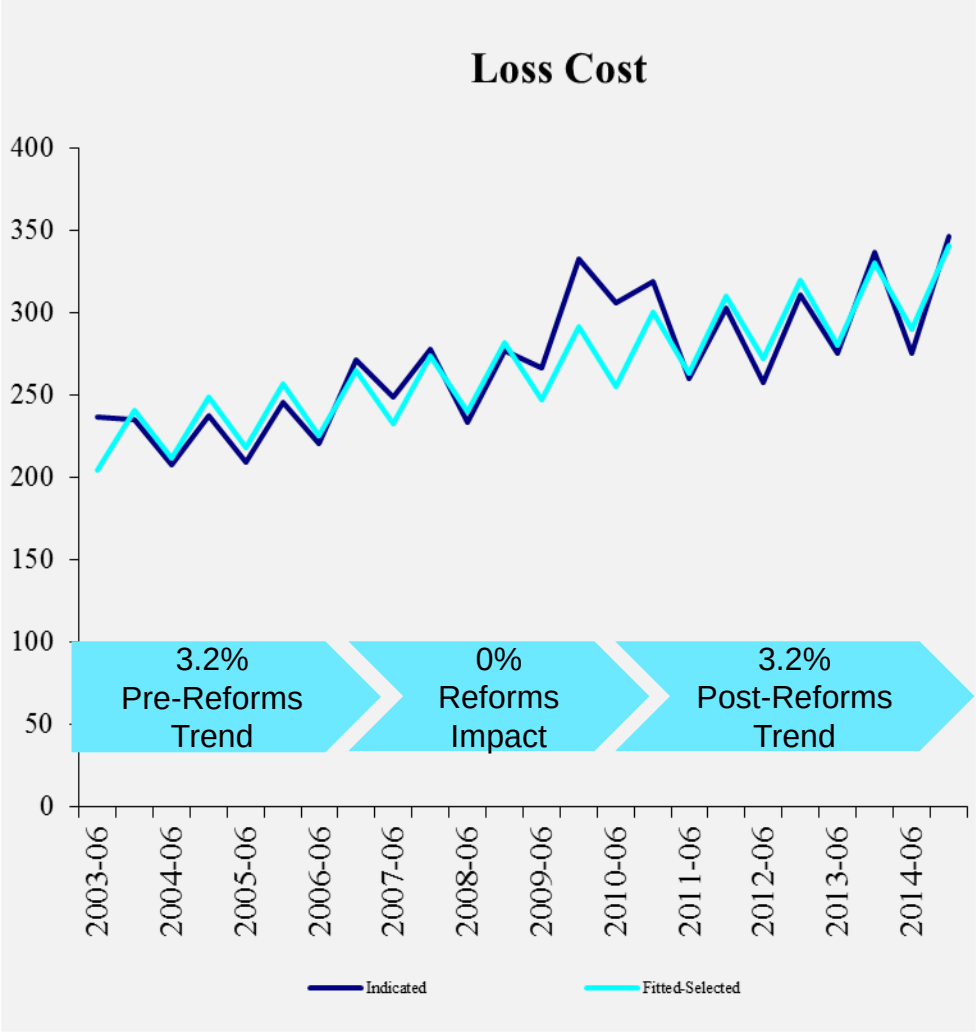
Source: https://www.fsco.gov.on.ca/en/auto/autobulletins/2010/Pages/a-01_10.aspx

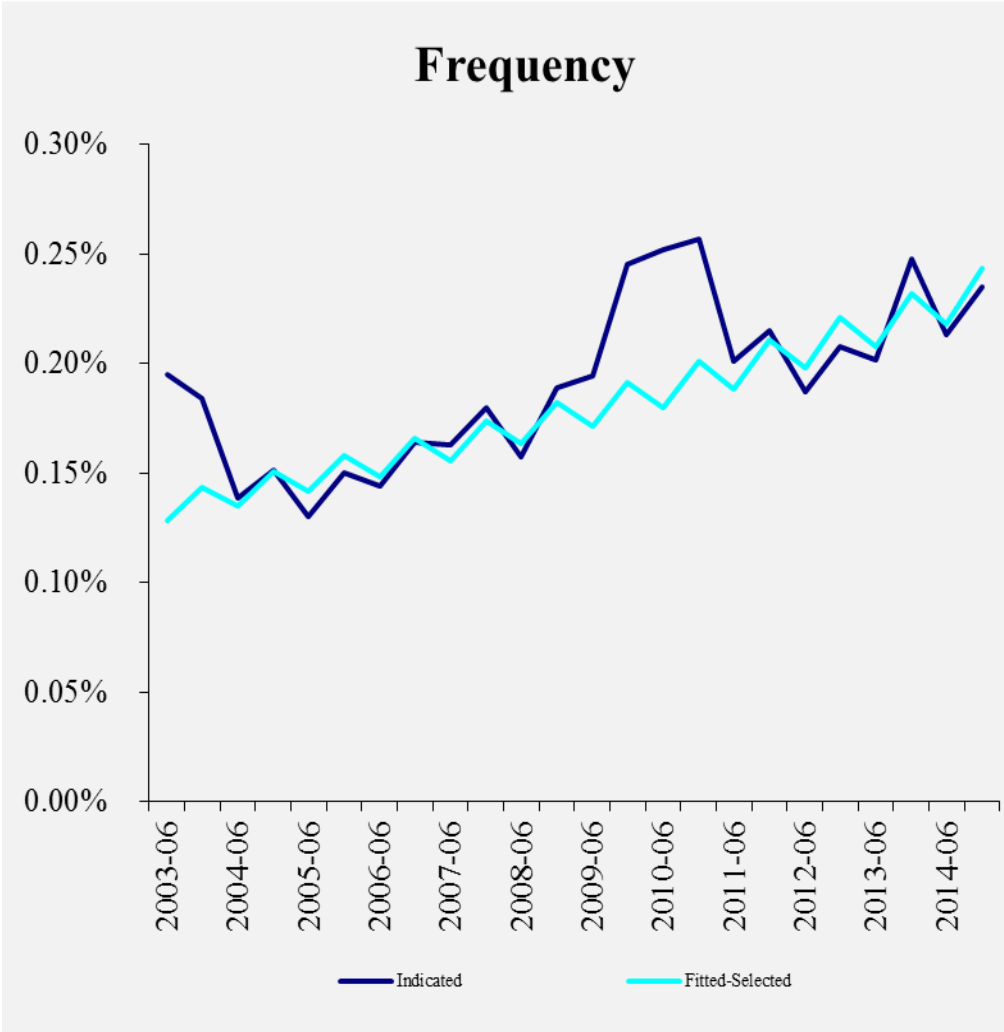
Quantitative Analysis – 2010 Reforms

Coverage	Trends Pre-2010 Reforms	Trends Post-2010 Reforms	Reforms Impacts on Claim Cost Levels
Third Party Liability	2.7%	2.5%	0%
Accident Benefits	15.2%	1.3%	-42.3%
Total Compulsory Coverages	9.9%	1.9%	-26.7%
Total Physical Damages	-1.9%	-1.9%	0%
Total All Coverages	8.1%	1.5%	-22.7%

Source: http://www.fin.gov.on.ca/en/autoinsurance/kpmg-expert-report-ar2015.html, sections 1.8.1 and 6.2

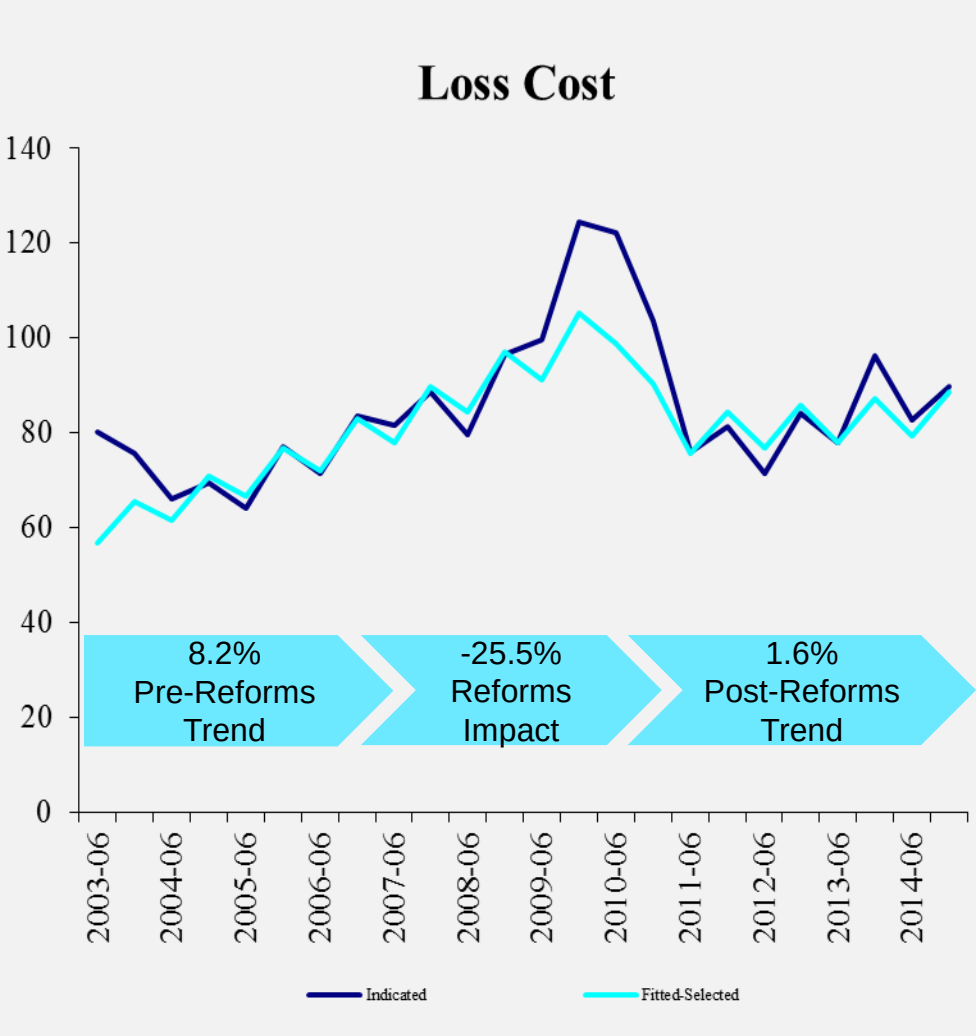
Ontario PPA – TPL – Bodily Injury

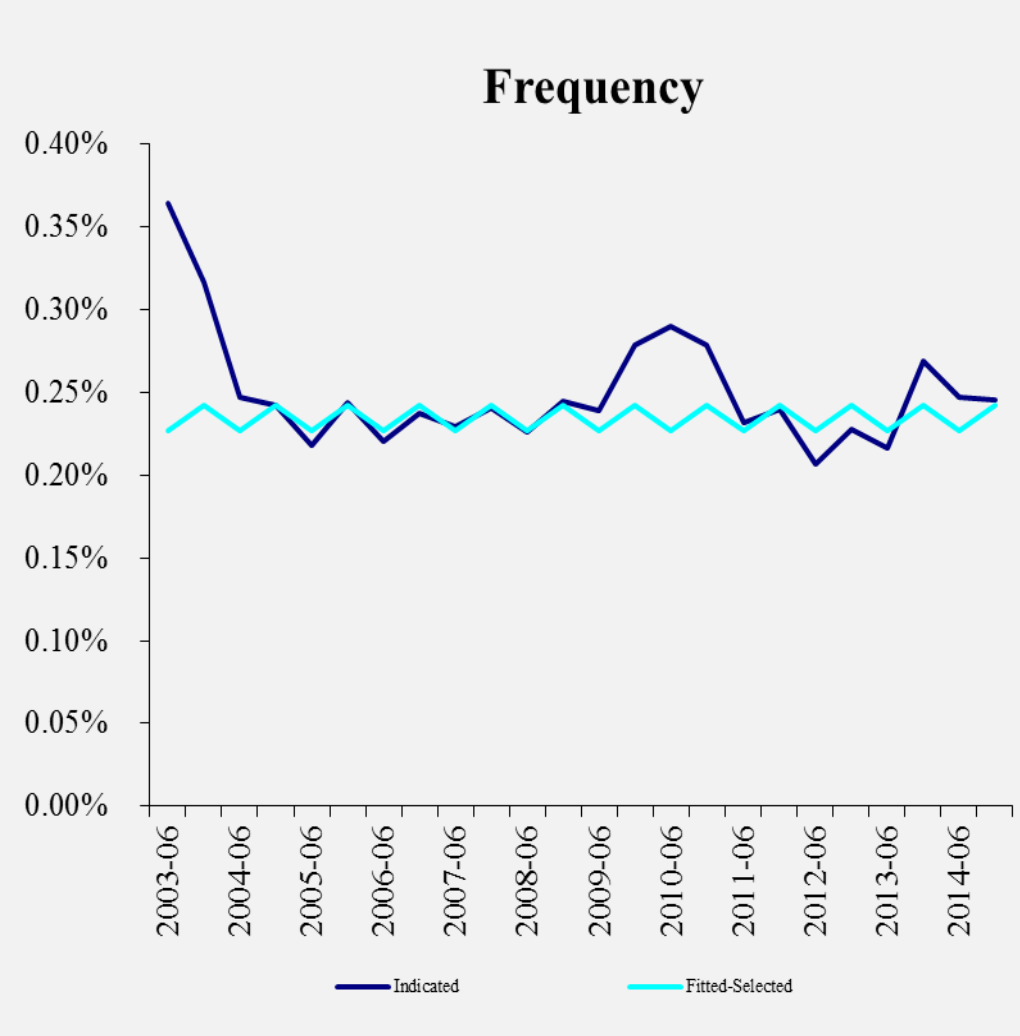




Source: http://www.fin.gov.on.ca/en/autoinsurance/kpmg-expert-report-ar2015.html, Appendix A, Exhibits, Segment I

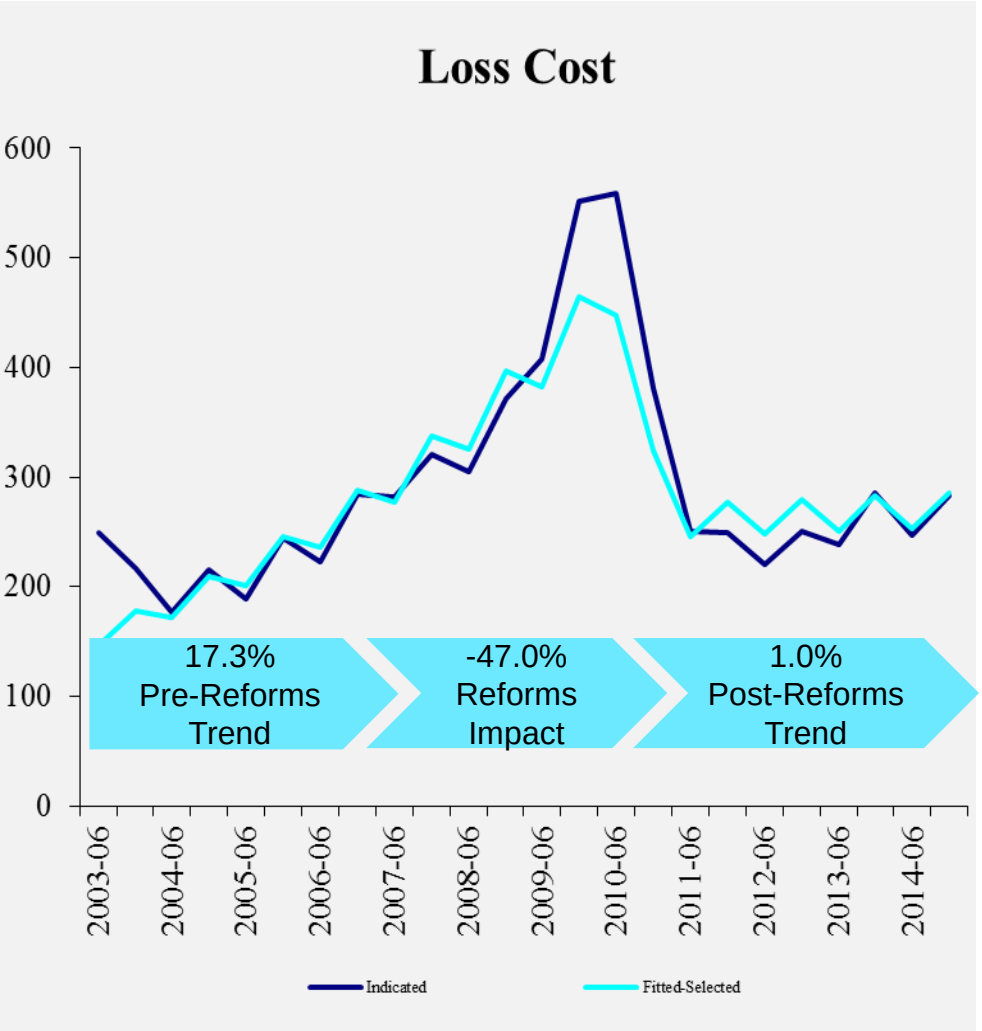
Ontario PPA – AB – Disability Income

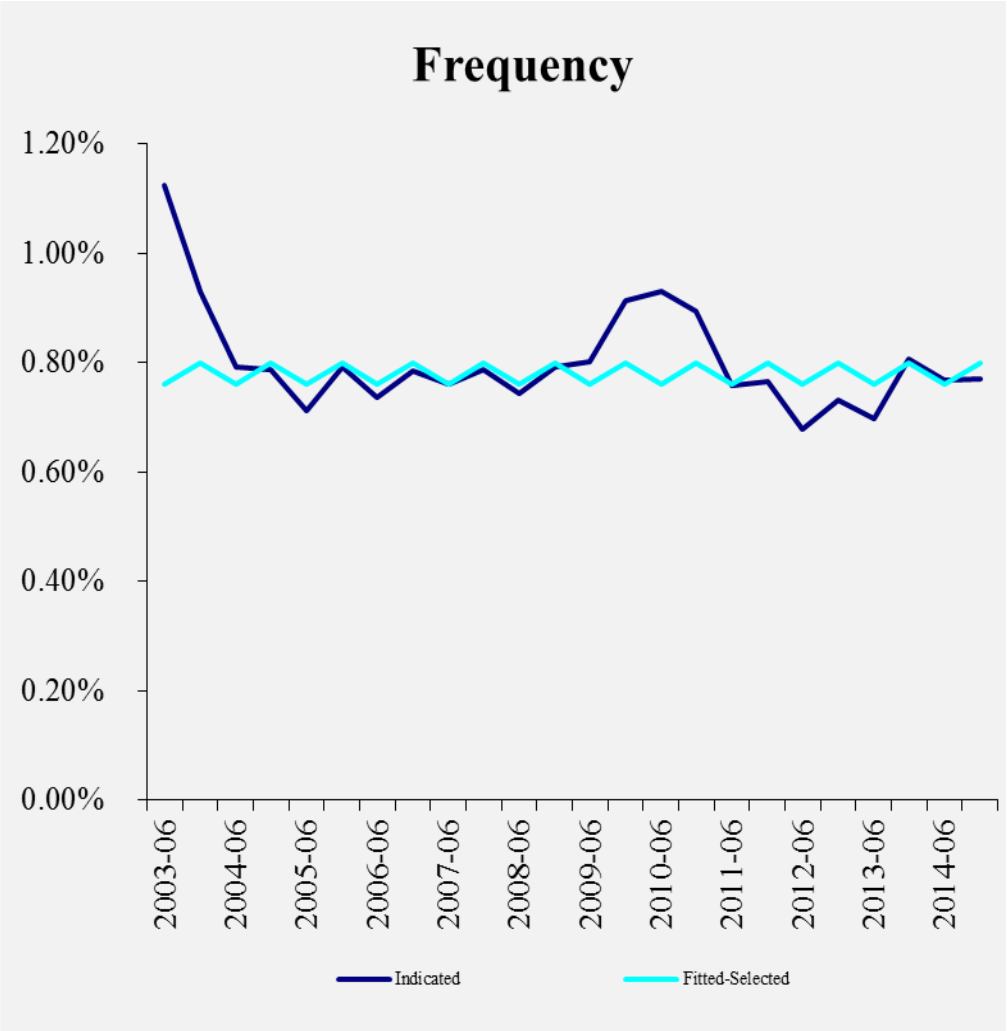




Source: http://www.fin.gov.on.ca/en/autoinsurance/kpmg-expert-report-ar2015.html, Appendix A, Exhibits, Segment IV

Ontario PPA – AB – Medical Expenses





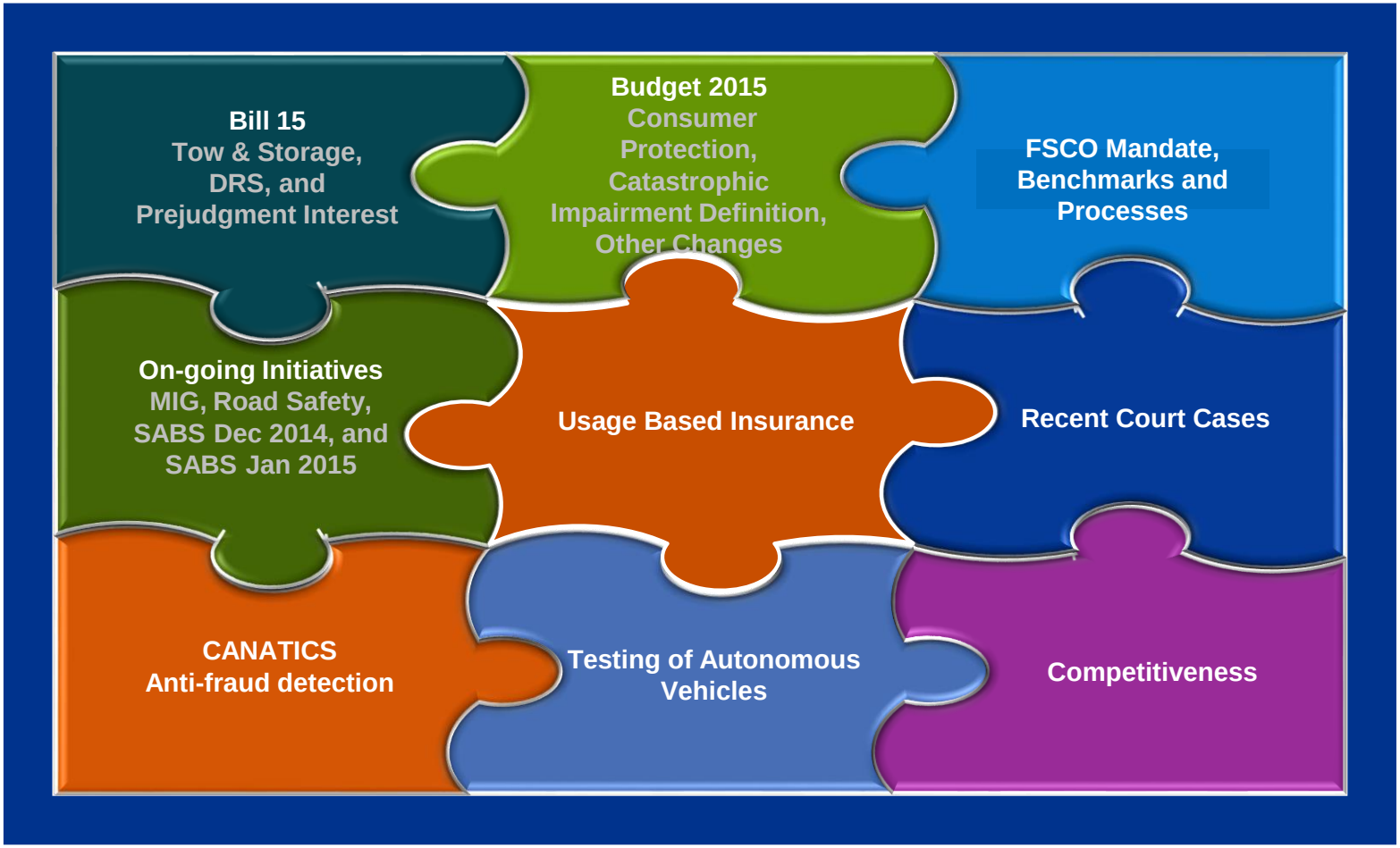
Source: http://www.fin.gov.on.ca/en/autoinsurance/kpmg-expert-report-ar2015.html, Appendix A, Exhibits, Segment VII

Quantitative Analysis – Sensitivities

Trends Post-2010 Reforms	Selected Model	Alternate Model
Third Party Liability – Bodily Injury	3.2%	3.7%
Third Party Liability – DCPD	1.1%	5.6%
Accident Benefits – Disability Income	1.6%	4.0%
Accident Benefits – Medical and Rehabilitation	1.0%	1.9%
Physical Damage – Collision	0.8%	4.4%
Physical Damage – Comprehensive	-2.2%	-1.5%

Source: http://www.fin.gov.on.ca/en/autoinsurance/kpmg-expert-report-ar2015.html, section 6.2.2

Qualitative Analysis – Development since 2014



Source: http://www.fin.gov.on.ca/en/autoinsurance/kpmg-expert-report-ar2015.html, section 5

Poll Questions

Poll 3: Have your company been able to use its own data to assess the impact of the 2015 Reforms?

- Yes
- No
- Not Applicable

Poll 4: How does your company's own assessment of the 2015 Reforms compare to FSCO's benchmarks?

- Generally lower
- Generally in line
- Generally higher
- Not Applicable

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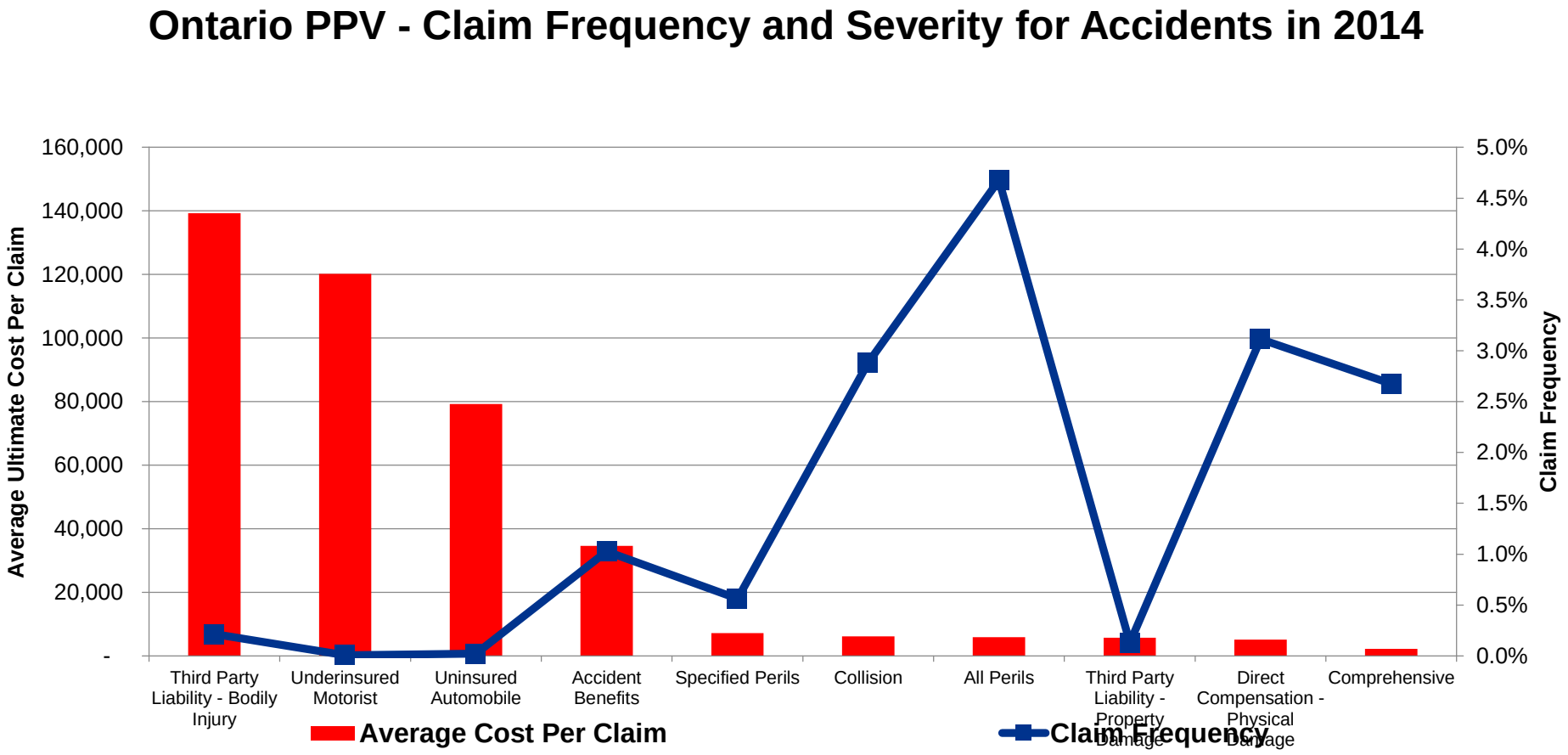
Ontario Automobile – Frequency & Severity



Source: GISA Data AUTO1010-ON_2014.pdf

Adding Segmentation Value by GLM

Poll Questions

Poll 5: Which GLM approach do you prefer?

- Frequency & Severity Approach
- Loss Cost Approach (Tweedie)
- Other
- Not Applicable

Poll 6: Have you ever considered correlation between Frequency and Severity in your GLM models?

- Yes
- No
- Not Applicable

Poll 7: Have you ever considered correlation between different coverages in your GLM models?

- Yes
- No
- Not Applicable

Traditional GLM Practice

1. Frequency & Severity Approach

- Build a frequency model with Poisson GLM
- Build a severity model with Gamma GLM
- Combined the two models by assuming the independency of frequency and severity

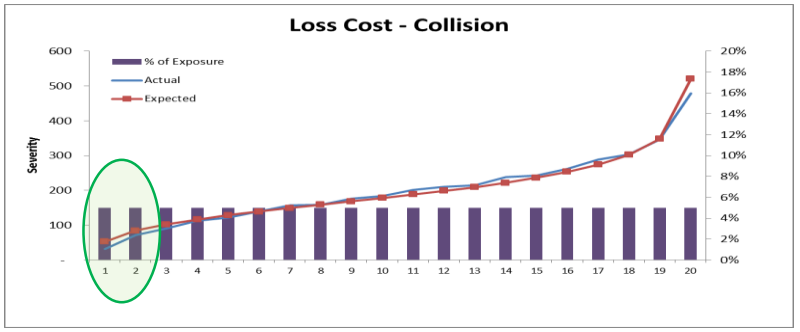
2. Pure Premium Approach

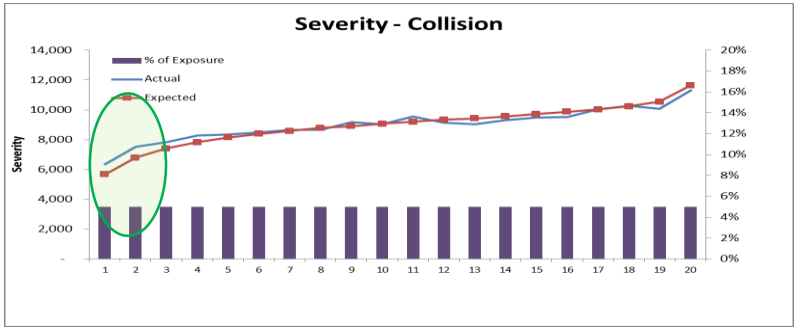
- Build a pure premium model with Tweedie GLM

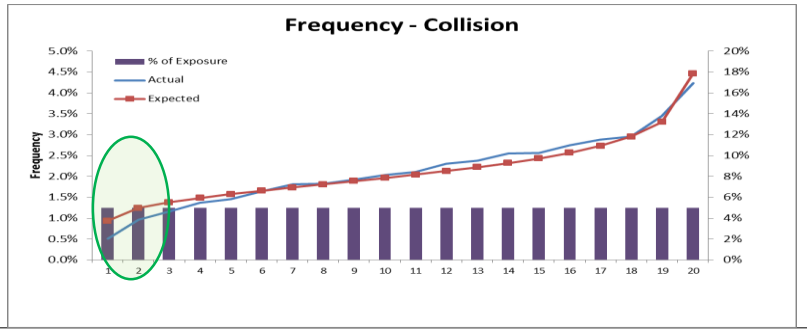
3. Compare Pros and Cons

- Frequency & Severity Approach:
  - ✓ Provide a better understanding of the way in which factors affect the cost of claims
  - ✓ Can more easily allow the identification and removal of certain random effects from one element of the experience
- Pure Premium Approach: reduce the amount of iterative modeling work

Frequency & Severity Approach – Collision







1. Loss Cost:

i. Slightly overpriced for small loss cost risks

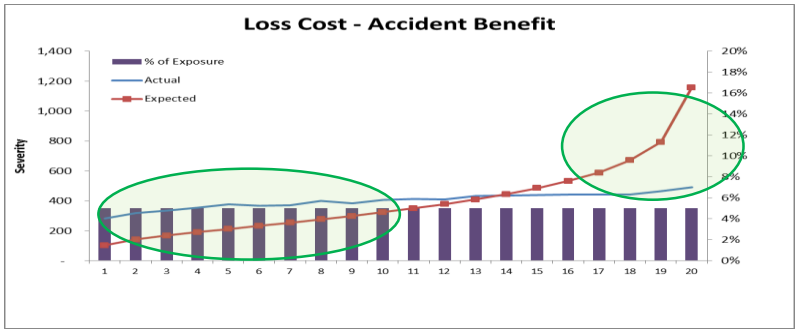
2. Severity:

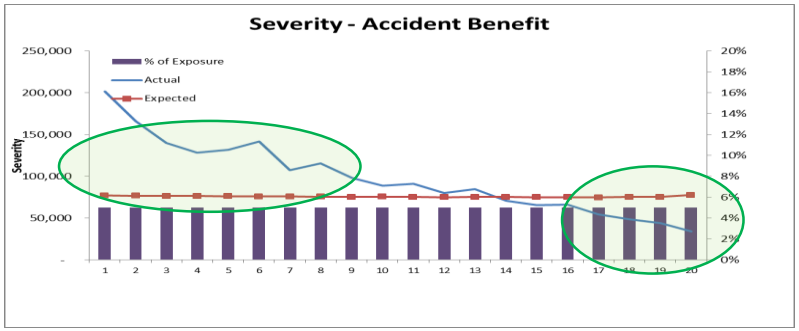
i. Under estimated for small loss cost risks

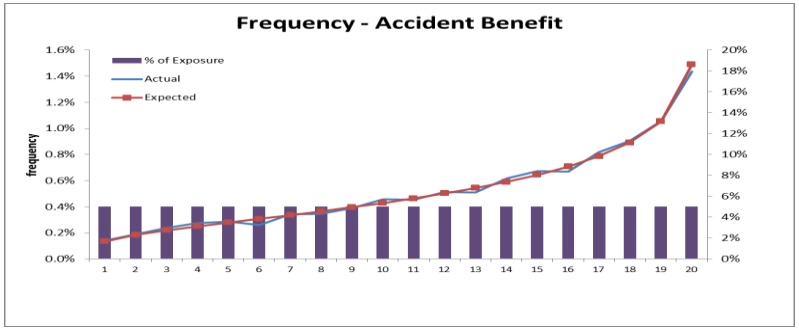
3. Frequency:

i. Over estimated for small loss cost risks

Frequency & Severity Approach – Accident Benefit







1. Loss Cost:

i. The model cannot really differentiate the high and low loss cost risks.

2. Severity:

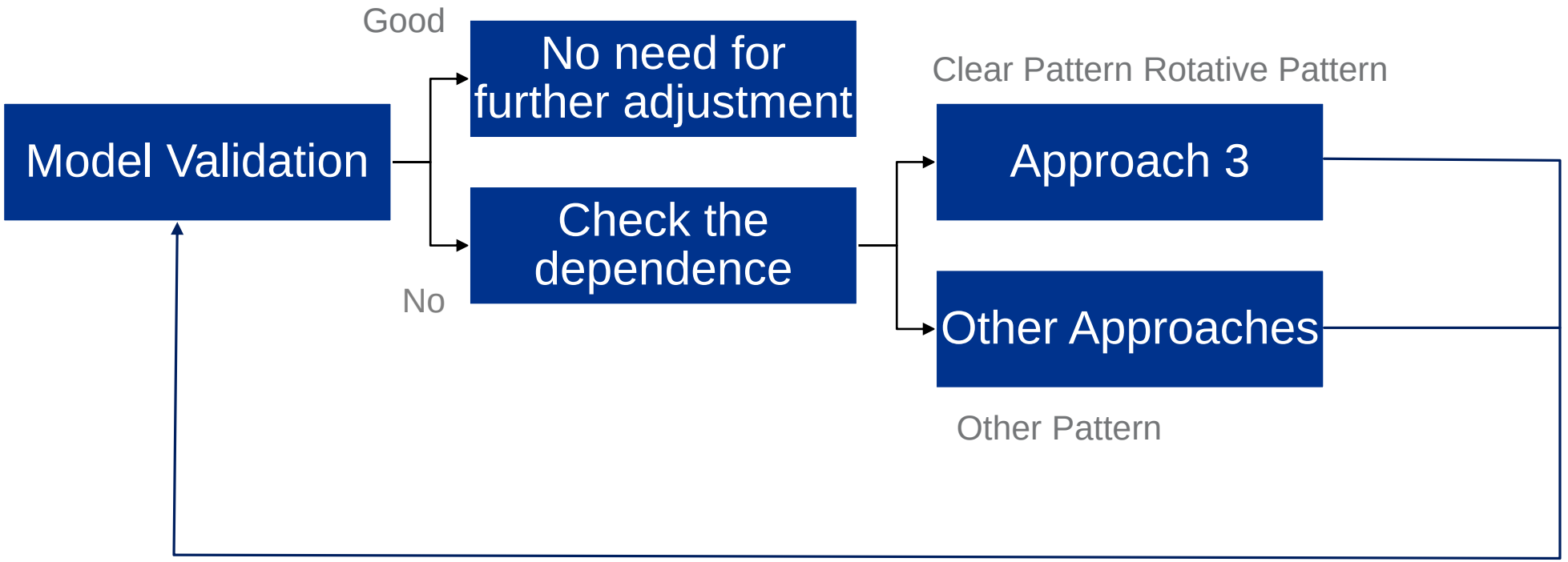
i. Model ***seems*** cannot really differentiate the high and low severity risks.

3. Frequency:

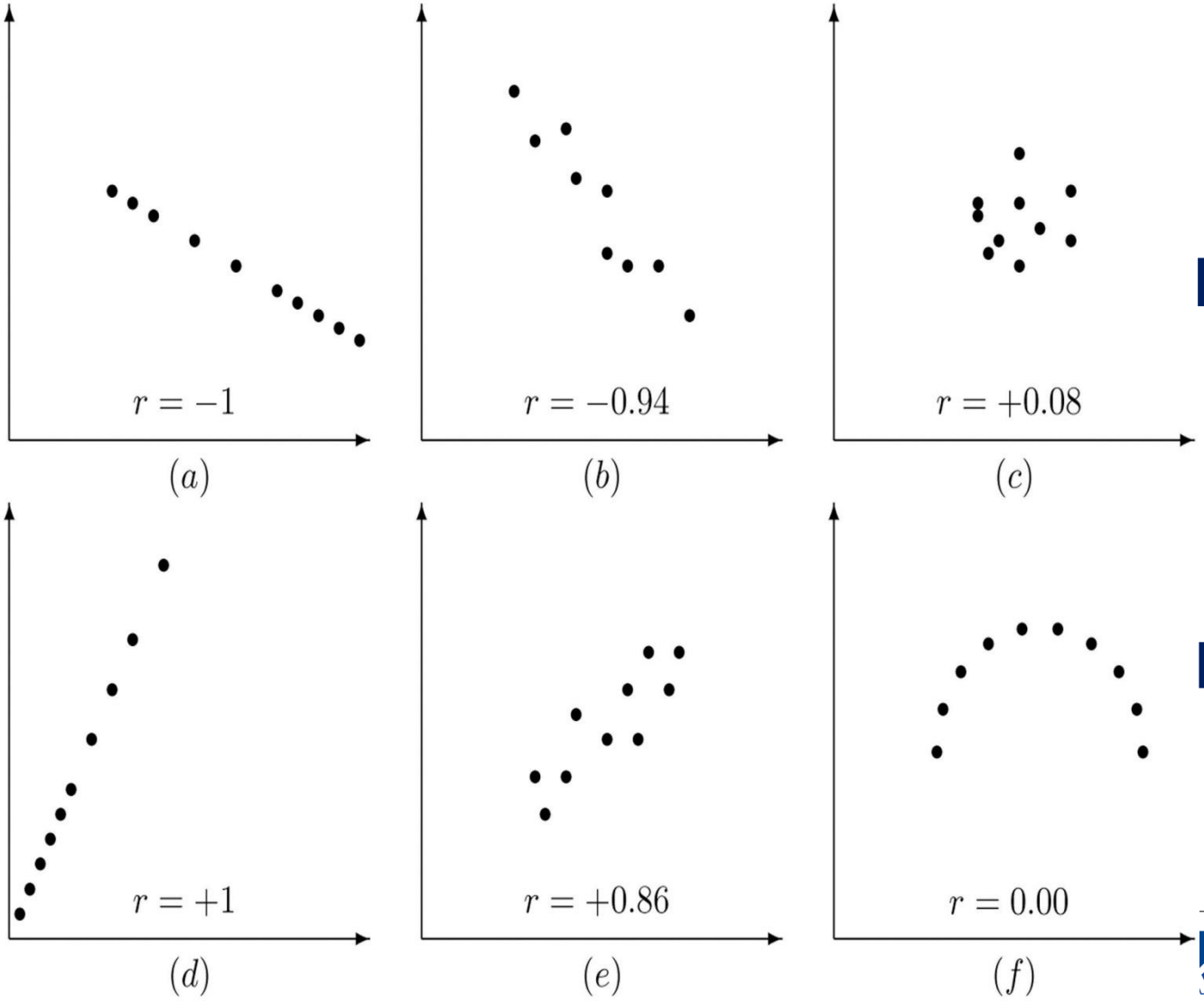
i. Model fit pretty well.

Significant opportunity for improvement.

Adjustment for Dependence



Adding Segmentation Value by Considering Dependence



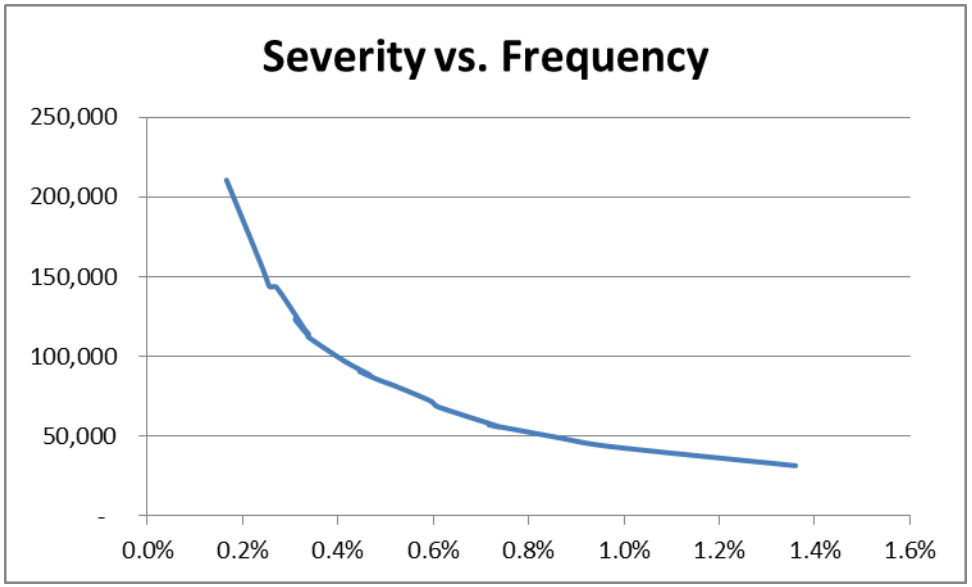
Quiz 1: Do you agree that a 0 correlation means no dependence

- ○ Yes
- **No**

Quiz 2: Claim frequency and severity are often

- ○ Negatively dependent
- ○ Positively dependent
- **Can be either way**

Empirical Dependence between Frequency and Severity – Accident Benefit



Observations:

a. There is a clear negative correlation between the frequency and Severity, i.e. the lower the frequency the higher the average severity and vice versa.

b. The correlation is NOT linear

Quiz:

1. What is the main driver of the loss cost:
   - ○ Frequency
   - ○ Severity
   - ○ Dependency

Three Approaches to Model Dependency

Approach 1: Link Marginal Frequency and Conditional Severity Model through a multiplicative factor

Approach 2: Link Marginal Frequency and Marginal Severity model through a copula

Approach 3: Link Marginal Frequency and Marginal Severity model through a Rotative Factor

Approach 1 – Marginal Frequency and Conditional Severity Model Concept

When Poisson counts are assumed and a log-link is used, the pure premium from this approach can be view as a production of the following three items:

Items	Formulae
Marginal Mean Frequency	$\boldsymbol{E[N]}$
Modified Marginal Mean Severity	$\boldsymbol{E[Y]}$
Dependence Multiplicative Factor	$\boldsymbol{e^{E[N]*(e^{\theta}-1)+\theta}}$

Key Point is item 3, which is indexed by a real-valued parameter that accounts for the association between the frequency and severity component of the model.

- **Aggregate Model:** $\boldsymbol{S = \sum_{j=1}^{N} Y_j}$
- **Independent model:** $\boldsymbol{E[S] = E[N] * E[Y]}$
- **Dependent Model:** $\boldsymbol{E[S] = E[N] * E[Y] * e^{E[N]*(e^{\theta}-1)+\theta}}$

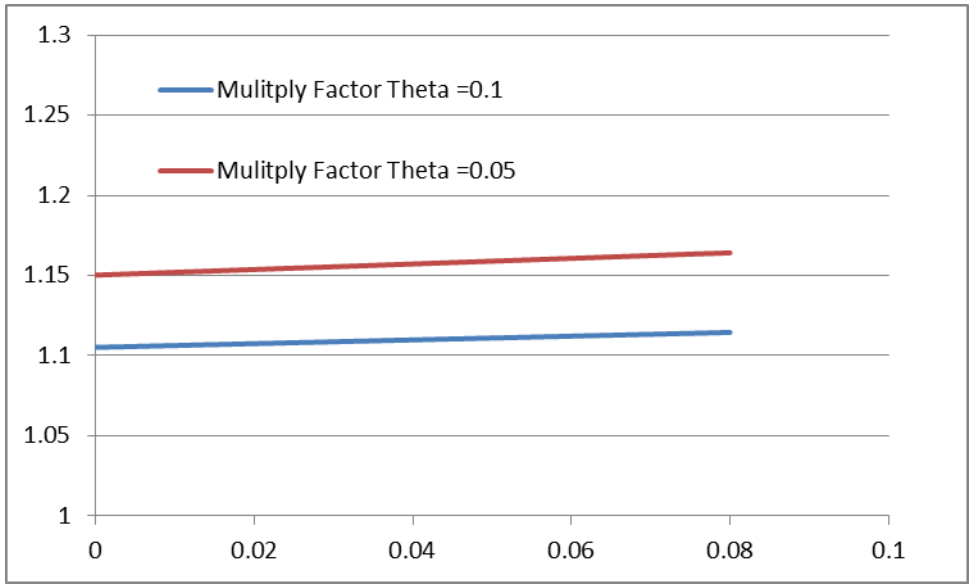
Approach 1 – The Aggregate Claim Model – Special Cases

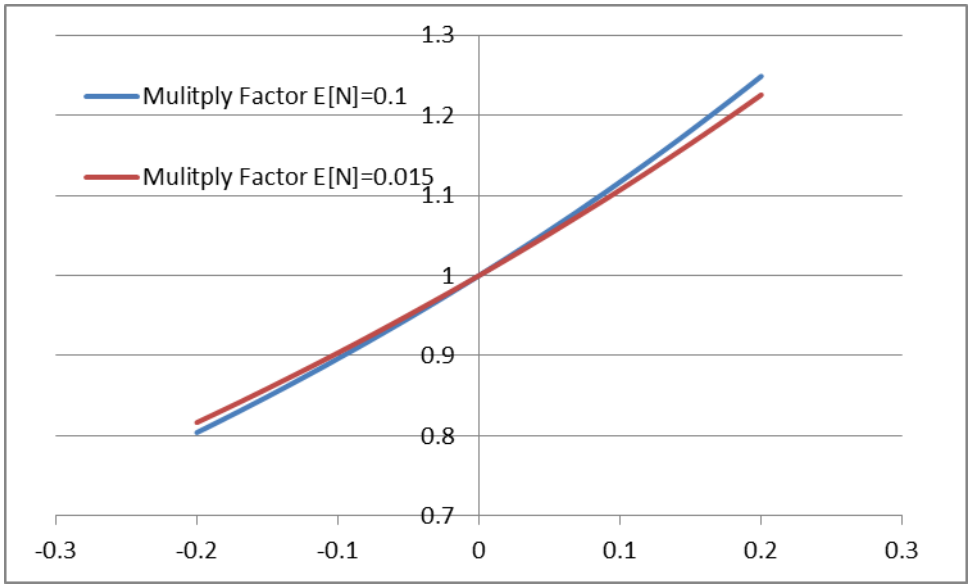
$$E[S] = E[N] * E[Y] * e^{E[N]*(e^{\theta}-1)+\theta}$$

θ	$e^{E[N]*(e^{\theta}-1)+\theta}$	Comments
=0	=1	Independent and dependent models are identical
>0	>1	A surcharge for the positive dependence between freq-sev
<0	<1	A discount for the negative dependence between freq-sev

Approach 1 – Opportunities for Improvement

❑ The Multiplicative Factor: $e^{E[N]*[(e^{\theta}-1)+\theta]}$





❑ Observations

- Given theta, the change of E[N] can not give a big enough variety of the multiplicative factor.
- The multiplicative factor is quasi-linear.

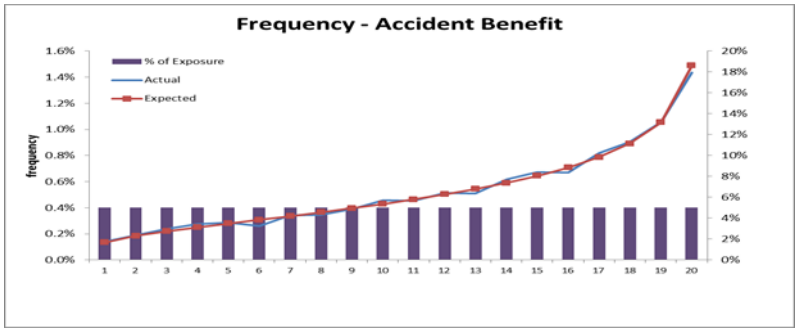
Approach 2 – Link Marginal Frequency and Marginal Severity Model through a Copula

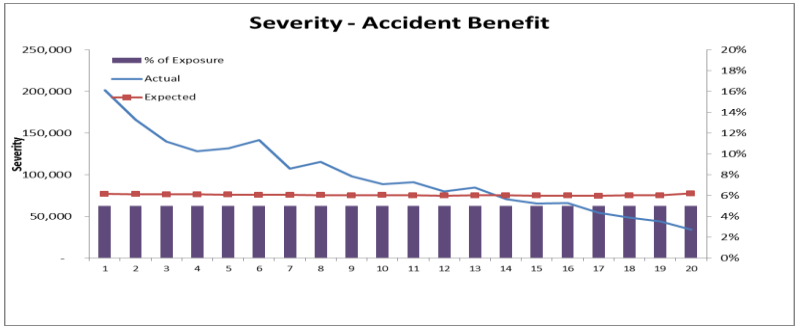
- **Copula concept**
  - Copula approach allows modelers to model the marginal distributions and the dependence structure separately
  - The dependence between underlying random variables is not influenced by the marginal behavior

- **Challenges:**
  - Complexity
  - Gaussian Copulas is not necessarily the most optimal choice

Approach 3 – Link Marginal Frequency and Marginal Severity Model through a Rotative Factor

- **A mathematically simplified but functionally strengthened approach**
  - Give the flexibility of bigger range of adjustment for correlation
  - The adjustment factor can be calculated easily from a curve fitting

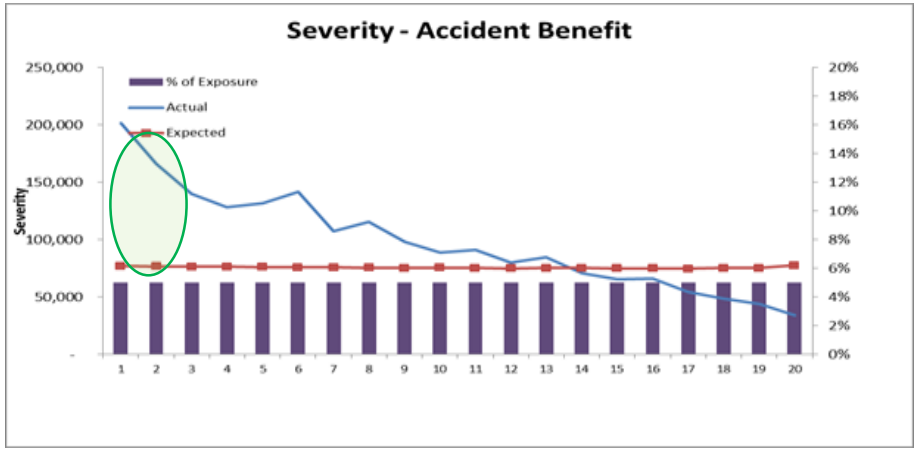


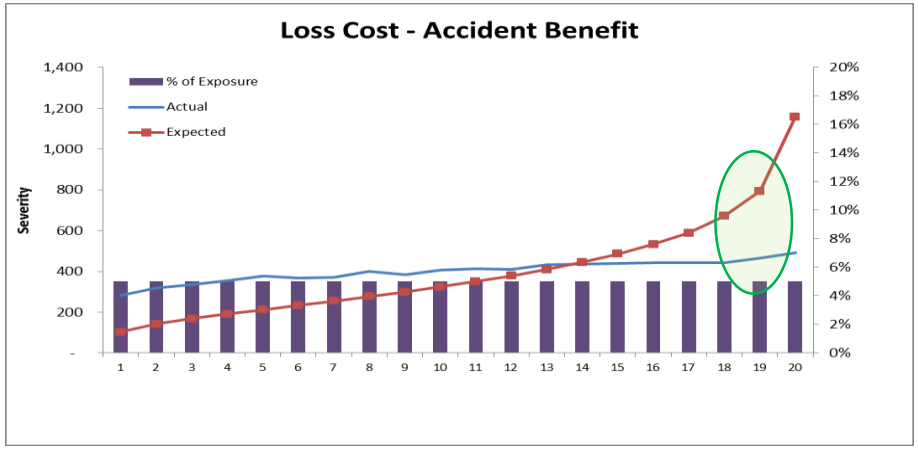


- Frequency model is robust. Frequency could be used to adjust for the severity/dependence, which is a similar concept as in Approach 1.
- Severity model doesn't fit the experience well. However, the deviation pattern is stable and obvious.
- A power/rotative factor is needed instead of a quasi-linear factor.
- The rotative factor is fitted from the difference between the actual severity and modeled severity by frequency. Here, it is $0.0178 * E[N]^{-0.774}$, which give more variety than the multiplicative factor in Approach 1.

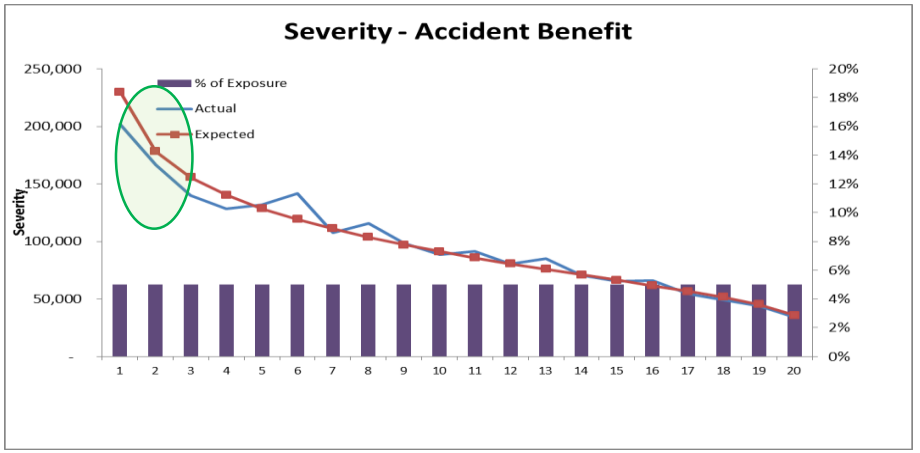
Approach 3 – Case Study Based on Accident Benefit

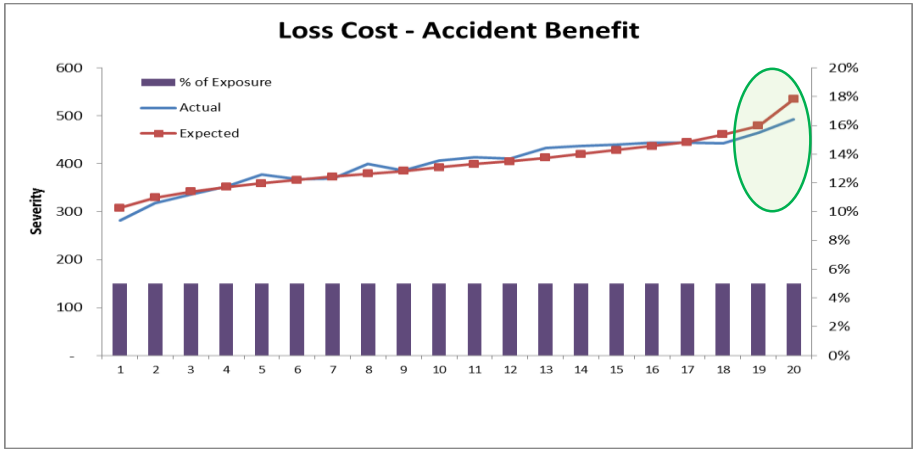
- **Before**



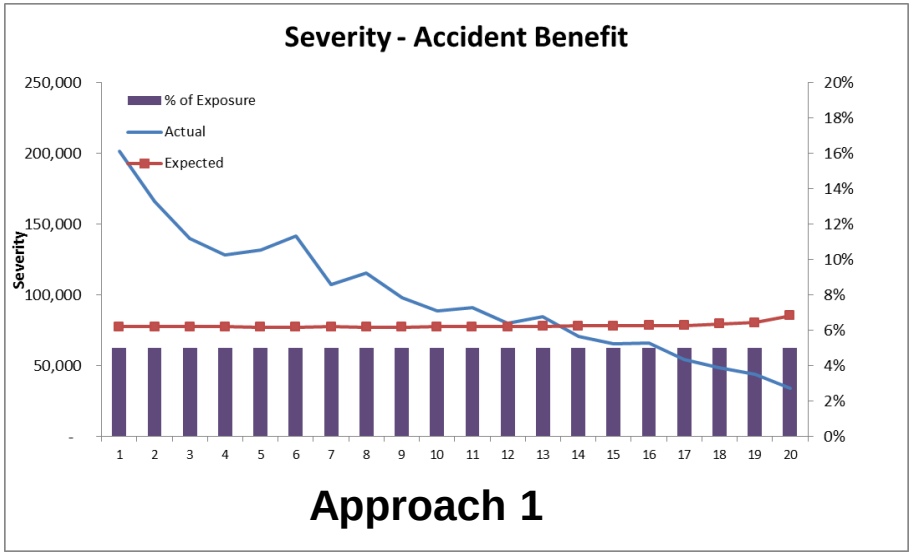


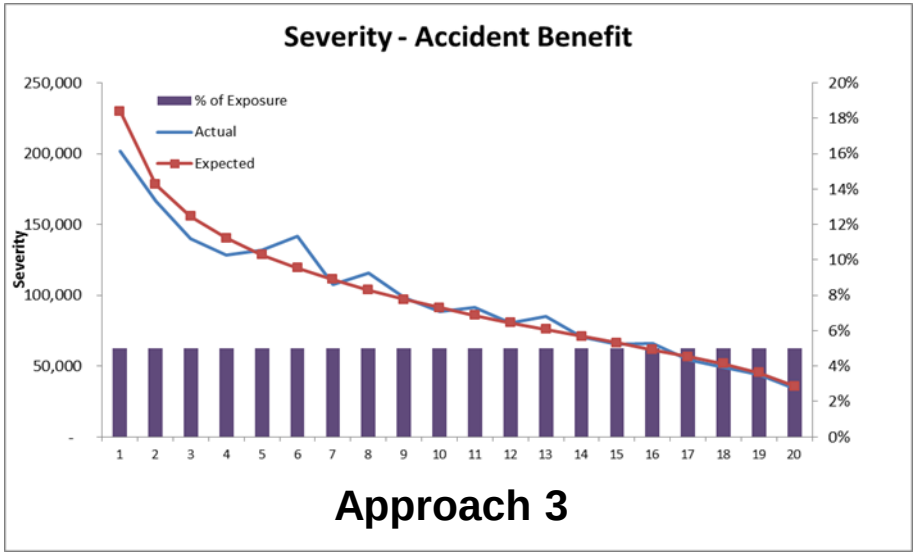
- **After**





Approach 1 vs. Approach 3





Cluster (Frequency)	Adj't Factor Approach 1 (a)	Adj't Factor Approach 3 (b)	Difference (c) = (b)/(a)-1
0.14%	1.00	2.84	1.84
0.23%	1.00	1.97	0.97
0.47%	1.00	1.12	0.13
0.70%	0.99	0.83	-0.17
1.34%	0.99	0.50	-0.49

Conclusion

From the segmentation perspective: correlation/dependence between Frequency and Severity is not negligible for certain coverages.

Dependence between frequency and severity is not always linear. However, there is a clear and stable pattern for most of the times.

Approach 3 provides a mathematically simple and practically robust method to reflect the dependence between frequency and severity to have more accurate segmentation.

Appendix 1 - Definitions and References

- **Definition of Coverages :**
  https://www.fsco.gov.on.ca/en/auto/brochures/Pages/brochure_autoins.aspx
- **Definition of correlation and dependence:**

  https://en.wikipedia.org/wiki/Correlation_and_dependence
- **Data: PY 2012- 2014 as of 12/31/2015 industry data is used. However, some transformation were applied for confidential reasons.**
- **References:**

[1] Krämer, N., Brechmann, E.C., Silvestrini, D., and Czado, C. (2013). Total loss estimation using copula-based regression models. Insurance: Mathematics and Economics, 53:829 - 839.

[2] Quijano-Xacur, O.A., and Garrido J. (2015). Generalised linear models for aggregate claims: To Tweedie or not? European Actuarial Journal, 5:181 - 202.

[3] Shi, P., Feng, X., and Ivantsova, A. (2015). Dependent frequency-severity modeling of insurance claims. Insurance: Mathematics and Economics, 64:417-428.

Questions?

Thank You

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