Balancing robust statistics and data mining in ratemaking: Gradient Boosting Modeling

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Agenda

- Introduction to boosting methods
- Connection between boosting and statistical concepts (linear models, additive models, etc.)
- Gradient boosting trees in detail
- An application to auto insurance loss cost modeling
- Limitation of Gradient Boosting and proposed improvement - Direct Boosting
- Comparison of various modeling techniques
- Additional features of Boosting machines.

Non-life insurance ratemaking models: The two cultures

- **Data generating process in ratemaking models**

$$x \rightarrow \boxed{\text{nature}} \rightarrow y$$

  - x: driver, vehicle and policy characteristics.
  - y: claim frequency, claim severity, loss cost, etc.

- **The data modeling culture**

$$x \rightarrow \boxed{\text{Poisson, Gamma, Tweedie}} \rightarrow y$$

- **The algorithmic modeling culture**

$$x \rightarrow \boxed{\text{unknown}} \rightarrow y$$

  Algorithms (e.g., decision trees, NN, SVMs) operate on x to predict y

- **Objectives of statistical modeling**
  - *Accurate Prediction*
  - *Extract useful information*

Boosting methods: A compromise between both cultures

In particular, *Gradient Boosting Trees* provide ...

- Accuracy comparable to Neural Networks, SVMs and Random Forests
- Interpretable results
- 'Little' data pre-processing
- Detects and identifies important interactions
- Built-in feature selection
- Results invariant under order preserving transformations of variables
  - No need to ever consider functional form revision (log, sqrt, power)
- Applicable to a variety of response distributions (e.g., Poisson, Bernoulli, Gaussian, etc.)
- Not too much parameter tuning

Boosting framework

- Boosting idea
  - Based on "strength of weak learnability" principles
  - Example:

    IF `Gender=MALE` **AND** `Age<=25` **THEN** `claim_freq.=‘high’`
  - Simple or “weak" learners are not perfect!
  - Combination of weak learners $\Rightarrow$ increased accuracy
- Problems
  - What to use as the weak learner?
  - How to generate a sequence of weak learners?
  - How to combine them?

The predictive learning problem

Let $\mathbf{x} = \{x_1, \ldots, x_p\}$ be a vector of predictor variables, y be a target variable, and M a collection of instances $\{(y_i, \mathbf{x}_i)\ ;\ i = 1, \ldots, M\}$ of known $(y, \mathbf{x})$ values.

The objective is to learn a prediction function $\hat{f}(x) : \mathbf{x} \rightarrow y$ that minimizes the expectation of some loss function $L(y, f)$ over the joint distribution of all $(y, \mathbf{x})$-values

$$\hat{f}(\mathbf{x}) = \underset{f(\mathbf{x})}{\operatorname{argmin}}\ E_{y,\mathbf{x}} L(y, f(\mathbf{x}))$$

(e.g., $L(y, f(\mathbf{x}))$ = squared-error, absolute-error, exponential loss, etc.)

Boosting $\supseteq$ Additive Model $\supseteq$ Linear Model

$$\textbf{Linear Model}: E(y|\mathbf{x}) = f(\mathbf{x}) = \sum_{j=1}^{p} \beta_j x_j$$

$$\textbf{Additive Model}: E(y|\mathbf{x}) = f(\mathbf{x}) = \sum_{j=1}^{p} f_j(x_j)$$

$$\textbf{Boosting}: E(y|\mathbf{x}) = f(\mathbf{x}) = \sum_{t=1}^{T} \beta_t h(\mathbf{x}; \mathbf{a}_t)$$

where the functions $h(\mathbf{x}; \mathbf{a}_t)$ represent the weak learner, characterized by a set of parameters $\mathbf{a} = \{a_1, a_2, \ldots\}$.

Parameter estimation in Boosting amounts to solving

$$\min_{\{\beta_t, a_t\}_1^T} \sum_{i=1}^{M} L\left(y_i, \sum_{t=1}^{T} \beta_t h(\mathbf{x}_i; \mathbf{a}_t)\right)$$

Gradient boosting

- Friedman (2001) proposed a Gradient Boosting algorithm to solve the minimization problem above, which works well with a variety of different loss functions
- Models include regression (e.g., Gaussian, Poisson), outlier-resistant regression (Huber) and K-class classification, among others
- Trees are used as the weak learner
- Tree size is a parameter that determines the order of interaction
- Number of trees T in the sequence is chosen using a validation set (T too big will overfit).

Gradient boosting in detail

Algorithm 1 Gradient Boosting

Initialize $f_0(\mathbf{x})$ to be a constant, $f_0(\mathbf{x}) = \underset{\beta}{\operatorname{argmin}} \sum_{i=1}^{M} L(y_i, \beta)$

for $t = 1$ to T **do**

  Compute the negative gradient as the working response

$$r_i = -\left[\frac{\partial L(y_i, f(\mathbf{x}_i))}{\partial f(\mathbf{x}_i)}\right]_{f(\mathbf{x})=f_{t-1}(\mathbf{x})}, \quad i = \{1, \ldots, M\}$$

  Fit a regression tree to r_i by least-squares using the input $\mathbf{x}_i$ and get the estimate $\mathbf{a}_t$ of $\beta h(\mathbf{x}; \mathbf{a})$

  Get the estimate β_t by minimizing $L(y_i, f_{t-1}(\mathbf{x}_i) + \beta h(\mathbf{x}_i; \mathbf{a}_t))$

  Update $f_t(\mathbf{x}) = f_{t-1}(\mathbf{x}) + \beta_t h(\mathbf{x}; \mathbf{a}_t)$

end for

Output $\hat{f}(\mathbf{x}) = f_T(\mathbf{x})$

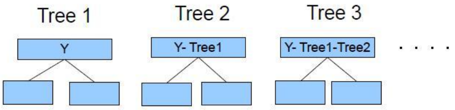
Gradient boosting for squared-error loss

- For squared-error loss, the gradient of L is just the usual residuals

$$L = (y_i - f(\mathbf{x}_i))^2$$

$$\frac{\partial L(y_i, f(\mathbf{x}_i))}{\partial f(\mathbf{x}_i)} = 2(y_i - f(\mathbf{x}_i)) = r_i$$

- In this case, the gradient boosting algorithm simply becomes



$$\hat{f}(\mathbf{x}) = Tree_1(\mathbf{x}) + Tree_2(\mathbf{x}) + \ldots + Tree_T(\mathbf{x})$$

Injecting randomness and shrinkage

Two additional ingredients to the boosting algorithm:

- **Shrinkage**
  - Scale the contribution of each tree by a factor $\tau \in (0,1]$. The update at each iteration is then

$$f_t(\mathbf{x}) = f_{t-1}(\mathbf{x}) + \tau.\beta_t h(\mathbf{x}; \mathbf{a}_t)$$

  - Low values of τ slow down the learning rate
  - Requires a higher number of trees in compensation
  - Accuracy is better
- **Randomness**
  - Sample the training data without replacement before fitting each tree – usually 1/2 size
  - $\uparrow$ Variance of the individual trees
  - $\downarrow$ Correlation between trees in the sequence
  - Net effect is a $\downarrow$ in the variance of the combined model.

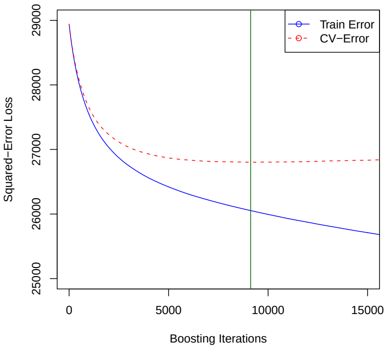
The Data

- Extracted from a major Canadian insurer
- Approx. 3.5 accident-years
- At-fault collision coverage
- Approx. 427,000 earned exposures (vehicle-years)
- Approx. 15,000 claims
- Data randomly partitioned into train (70%) and test (30%) data sets

Overview of model candidate input variables

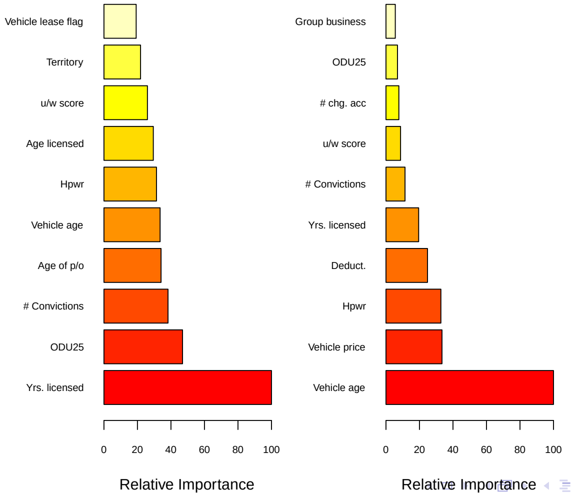
Driver	Accidents/convictions	Policy	Vehicle
Age of p/o	# at-fault accidents (1-3 yrs.)	Time on risk	Vehicle make
Yrs. Licensed	# at-fault accidents (4-6 yrs.)	Multi-vehicle flag	Vehicle new/used
Age Licensed	# Not-at-fault accidents (1-3 yrs.)	Deductible	Vehicle lease flag
License class	# Not-at-fault accidents (4-6 yrs.)	Billing type	hpwr
Gender	# driving convictions (1-3 yrs.)	Billing status	Vehicle age
Marital status	Examination costs (AB claims)	Territory	Vehicle price
Prior FA		occ. driver under 25	
u/w score		occ. driver over 25	
Insurance lapses		Group business	
Insurance suspensions		Business origin	
		Property flag	

- **Loss functions**
  - **Frequency model:** Bernoulli deviance
  - **Severity Model:** Squared-error loss
- **Shrinkage parameter** $\tau = 0.001$
- **Sub-sampling rate** $= 50\%$
- **Size of the individual trees**: started with single-split (no interactions), followed by (2-6)-way interactions.
- **Number of trees**: selected by cross-validation.

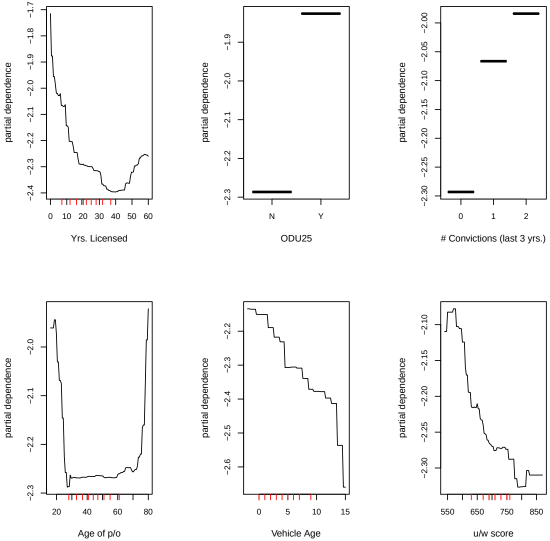


Relative importance of predictors

Frequency (*left*) and Severity (*right*).



Sample partial dependence plots – Frequency model

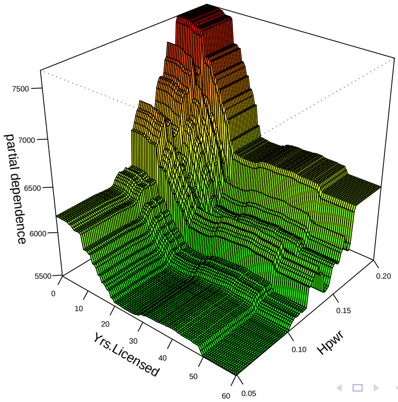


Inspecting interactions using Friedman's **H-stat**

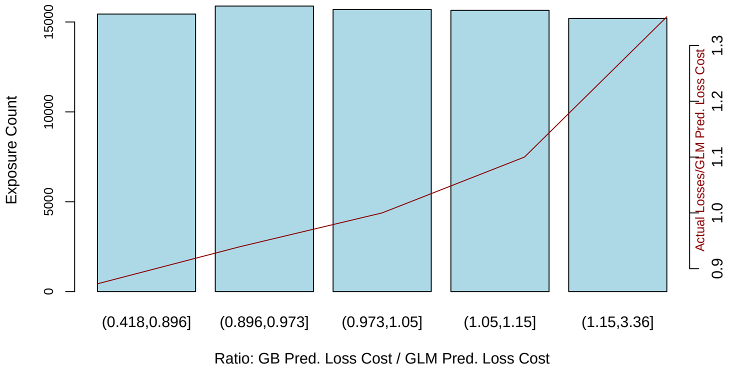
```
require(gbm)
n <- 50 # number of inputs
x <- 1:n
best.iter <- gbm.perf(gbm.model, plot.it = FALSE, method = "cv")
ans <- matrix(nrow = length(x), ncol = length(x))
for (i in 1:length(x)) {
    for (j in 1:length(x)) { if (i > j) {
      ans[i,j] <- interact.gbm(gbm.model,
                  data=mydata,
                  i.var =c(x[i],x[j]),
                  n.trees = best.iter)   }
                          }
                      }
```

Interaction Matrix

	x_1	x_2	$\cdots$	x_n
x_1	*na*	*na*	$\cdots$	*na*
x_2	0.5	*na*	$\cdots$	*na*
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_n	0.9	0.8	$\cdots$	*na*



Prediction performance – Gradient Boosting vs. GLM



Improvement over GBM - Direct Boosting

- **GBM has quite a few advantages over other modeling techniques**
  - It is very intuitive - Aim to correct errors to maximum extend in each iteration
  - It is predictive - Empirical tests have shown that GBM is superior to other popular modeling techniques
  - It provides output with easy interpretation - The results can be visualized while NN, Gen Alogirthm cannot
- **But it does have some disadvantage as well ...**
  - It is not very fast - It can take 6 hours to model a data with 4 million entries
  - It is deficient in dataset with many zeros when using exponential form.
  - Some distributions are not easily available - E.g. Tweedie distribution

Improvement over GBM - Direct Boosting

- **What if ...**
  - there is a model that has all the advantages of GBM ...
  - but not the disadvantage?
  - Direct boosting may do the work.
- **DBM at a Glance**
  - It is a modified version of GBM
  - It is faster as it require few calculation at each iteration
  - The algorithm is more robust with data having many zeros
  - Tweedie distribution is incorporated

Direct Boosting in detail

- GBM first calculates :
  - The gradient for each observation
  - split the dataset into several groups with each group having max average difference in gradient
  - Obtain the group Loss function minimizer
  - Apply shrinkage factor
- DBM "thinks" the reverse. We first obtain the form of group loss function minimizer.
- Due to the shrinkage, we can apply taylor series to find the linear approximation of the minimzer. (Recall that $exp(x) \sim x$ when x is around 0)

Direct Boosting in detail

- This approximation is in general in summation term. E.g $\sum(y_i/f_i(x) - 1)/n$.
- Noting this, DBM calculation the summand at observation level. E.g $y_i/f_i(x) - 1$. We call this as pseudo minimizer
- Similar to GBM, DBM splits the dataset into several groups with each group having max average difference in pseudo minimizer
- Since the average is already the group loss function minimizer, the last step of GBM is not necessary.

Direct Boosting in detail

Algorithm 2 Direct Boosting for Tweedie Distribution

1: the Loss function to be negative of loglikelihood of Tweedie distribution with exponential form: $L(y, f(\mathbf{x})) = \sum \frac{y_i exp^{(1-p)f(\mathbf{x_i})}}{1-p} - \frac{exp^{(2-p)f(\mathbf{x_i})}}{2-p}$.

2: Calculate the Group loss minimizer, $h_i = ln(\frac{\sum y_i exp^{(1-p)f(\mathbf{x_i})}}{\sum exp^{(2-p)f(\mathbf{x_i})}})$.

3: Linear Approximation through Taylor's expansion, $h = \sum y_i exp^{(1-p)f(\mathbf{x_i})}/n - \sum exp^{(2-p)f(\mathbf{x_i})}/n$.

4: Pseudo loss minimizer $h = y_i exp^{(1-p)f(\mathbf{x_i})} - \sum exp^{(2-p)f(\mathbf{x_i})}$.

5: **for** $t = 1$ to T **do**

6: Update $f_t(\mathbf{x}) = f_{t-1}(\mathbf{x}) + h_i$

7: **end for**

8: Output $\hat{f}(\mathbf{x}) = f_T(\mathbf{x})$

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Direct Boosting in detail - The predictive power: Retention modeling

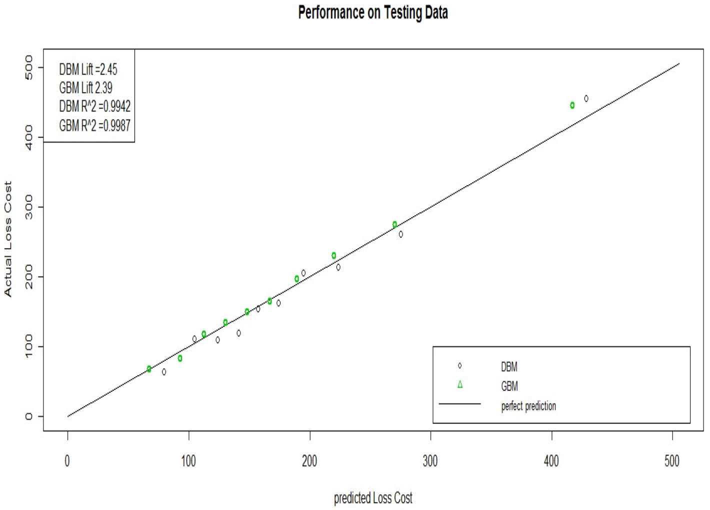
- The performance of various models are tested using same data and input varaibles.
- The model predicts the probability of churn (or renew). For predictive models, we have 40/30/30 for training/validation/testing.

Model	Lift (Top decile churn/average churn)	ROC Area
Decision Tree	2.6692	0.6981
GLM - Logistic	3.0332	0.7275
Support Vector Machines	3.0520	0.7312
Neural Net	3.0828	0.7293
GBM - Poisson	3.0879	0.7304
GBM - Logistic	3.1016	0.7330
DBM - Poisson	3.1306	0.7330

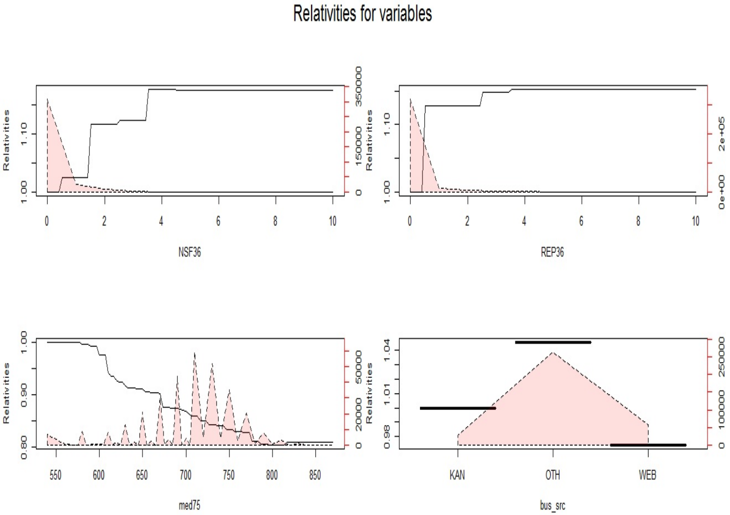
Direct Boosting in detail - The predictive power: Loss cost modeling

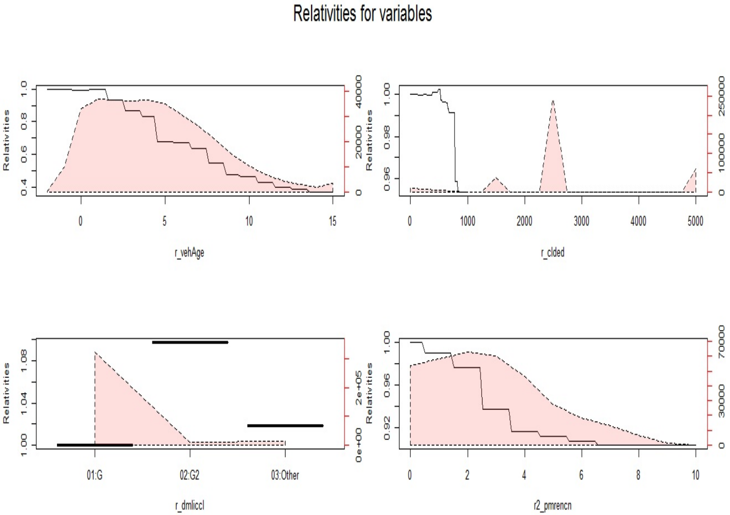
- Continuing the GBM vs GLM comparison for collison coverage, we compare the DBM performance against GBM.
- Since GBM does not work well in poisson and Tweedie,
  - We first need to model the frequency using logistic regression.
  - Gamma modeling in severity module then follows
  - Combine both to form the loss cost model.
  - relativities cannot be obtained as logistic regression is not in exponential form.
- On the contrary, DBM can model loss cost directly using Tweedie models.

Direct Boosting vs Gradient Boosting



Direct Boosting - Relativities at a Glance

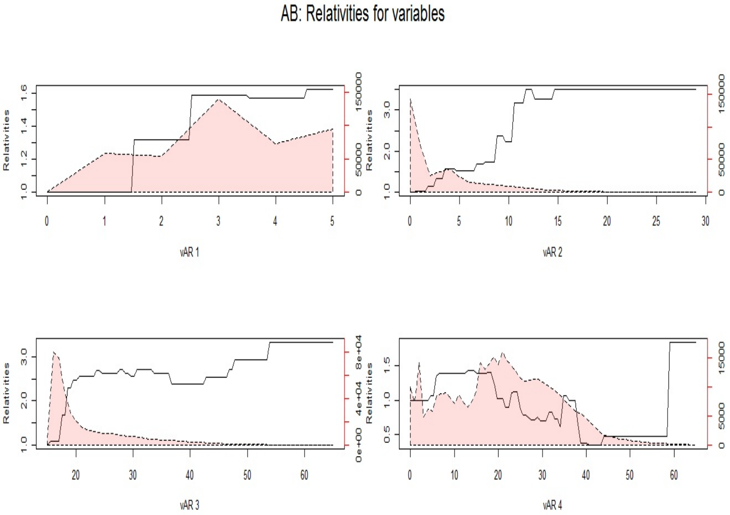



Relativities for variables
Relativities
r_vehAge
0
5
10
15
1.0
0.8
0.6
0.4
40000
20000
0
r_clded
0
1000
2000
3000
4000
5000
1.00
0.98
0.96
250000
100000
r_dmliccl
01:G
02:G2
03:Other
1.08
1.04
1.00
2e+05
0e+00
r2_pmrencn
0
2
4
6
8
10
0.96
0.92
70000
30000

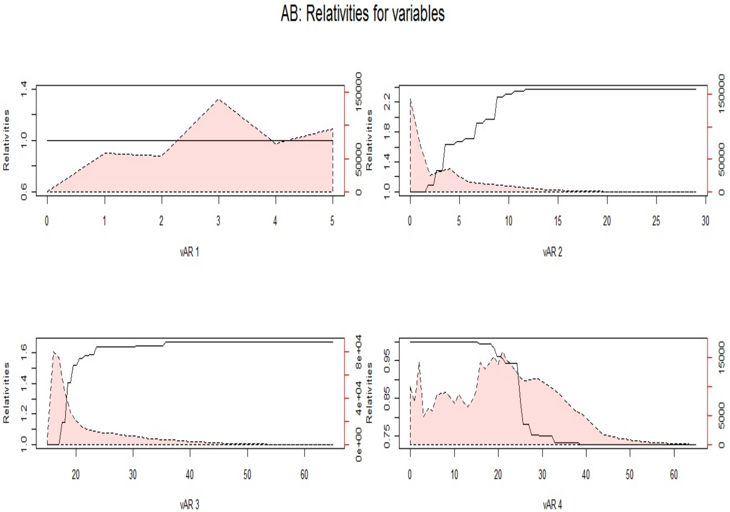
Direct Boosting in detail - Additional features

- With the above form DBM, is already more predictive than any other predictive models in all 6 of the datasets that we have tried. However, there are some more additional features that help make the model predictive.
- Monotonic constraint
  - In many occassions, some of the patterns are desirable. E.g, loss cost decreasing with years licensed.
  - This addtional feature tells the machine not to split the data in case of reversal.
  - The improvement is promising.

Monotonic Constraint



Monotonic Constraint



Direct Boosting in detail - Additional features

- Interaction constraint
  - The well promoted advantage of data mining techniques is to model any interaction to any degree
  - However, it can be a double-edged sword. It is most often that the interactions are generated from noise.
  - We are working towards the flexibility to allow users to select meaning intereaction.
  - An example is the model only fit 4 groups of intereaction, Group 1 - vehicle related, Group 2 - driver's related, Group 3 - Location related, Group 4 - User's specified.