RPM Workshop 4: Basic Ratemaking

Introduction to Credibility

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General Concept

- Principle 4 of the Statement of Principles Regarding Property and Casualty Ratemaking:
  - A rate cannot be "excessive, inadequate, or unfairly discriminatory"
    - — Excessive: Too high
    - — Inadequate: Too high
    - — Unfairly discriminatory: Allocation of overall rate to individuals is based on cost justification
- At various steps in the ratemaking process, the concept of credibility is introduced (state, class, segment, territory, etc)
- The credibility of data is commonly denoted by the letter "Z"
  - **$0 \le Z \le 1$**

Definitions of Credibility

- Common usage:
  - "Credibility" = the quality of being believed or trusted
  - Implies you are either credible or you are not
- In actuarial science:
  - Credibility is "a measure of the credence that…should be attached to a particular body of experience"

    -- L.H. Longley-Cook
  - Refers to the degree of believability of the data under analysis

    — A relative concept, not an absolute

Why Do We Need Credibility?

- Property / casualty insurance losses are inherently stochastic
  - Losses are fortuitous events

    — Any given insured may or may not have a claim in a given year

    — The size of the claim can vary significantly
- So how much can we believe our data? What other data can be used to aid in calculating the rate for an insured?
- Credibility is a balance of stability and responsiveness

History of Credibility in Ratemaking

- The CAS was founded in 1914, in part to help make rates for a new line of insurance – Workers Compensation – and credibility was born out the problem of how to blend new experience with initial pricing
- Early pioneers:
  - Mowbray (1914) -- how many trials/results need to be observed before I can believe my data?
  - Albert Whitney (1918) -- focus was on combining existing estimates and new data to derive new estimates:

    New Rate = Credibility x Observed Data + (1-Credibility) x Old Rate
  - Perryman (1932) -- how credible is my data if I have less than required for full credibility?

Methods of Incorporating Credibility

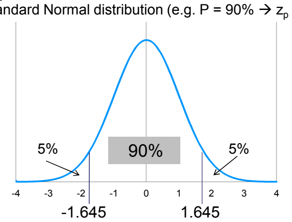
- Limited Fluctuation
  - Limit the effect that random fluctuations in the data can have on an estimate
    - — "Classical credibility"
- Least Squares
  - Make estimation errors as small as possible
    - — Greatest Accuracy
    - — Empirical Bayesian
    - — Bühlmann

Limited Fluctuation Credibility Description

- Goal: Determine how much data one needs before assigning it with full credibility (Z = 1)
  - Standard for full credibility
- Concepts:
  - Full credibility for estimating frequency
  - Full credibility for estimating severity
  - Full credibility for estimating pure premium
  - Amount of partial credibility when data is not fully credible
- Alternatively, the credibility (Z) of an estimate (T) is defined by the probability (P) that it is within a tolerance (k), of the true value

Limited Fluctuation – Meet the Variables

- T: Estimate → the data that we want to test for credibility (e.g. loss ratio)
- Z: Credibility, which is between 0 and 1
- k: Tolerance for error (e.g. the observation is within k = 2.5% of the mean)
- P: Probability that the observation is within k% of the mean. Calculated using the standard Normal distribution (e.g. P = 90% → z_p = 1.645)



Limited Fluctuation Derivation

- New estimate = (Credibility)*(Data) + (1-Credibility)*(Prior Estimate)

$$E2 = Z*T + (1-Z)*E1$$

Add and subtract Z*E[T] → $E2 = Z*T + Z*E[T] - Z*E[T] + (1-Z)*E1$

Regroup → $E2 = (1-Z)*E1 + Z*E(T) + Z*(T-E(T))$

Stability | Truth | Random Error

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Limited Fluctuation Formula for Z

- Probability that "Random Error" is "small" is P
  - For example, the probability {random error is less than 5%} is 90%

$$P[Z\,(T-E(T)) < kE(T)] = P$$

Isolate T → $P[T < E(T) + kE(T)/Z] = P$

Assuming T is Normally distributed, then…

Introduce mean and std dev. → $E(T) + kE(T)/Z = E(T) + z_p\sqrt{Var(T)}$

$$kE(T)/Z = z_p\sqrt{Var(T)}$$

$$Z = (kE(T)) / (z_p\sqrt{Var(T)})$$

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Limited Fluctuation Formula for Z – Frequency

- Assuming the insurance frequency process has a Poisson distribution, and ignoring severity:
  - Then E(T) = number of claims (N) and E(T) = Var(T), so:

$Z = (kE(T)) / (z_p\sqrt{Var(T)})$ becomes

$$Z = (kE(T)) / (z_p\sqrt{E(T)})$$

$$Z = (k\sqrt{E(T)}) / (z_p)$$

$$Z = (k\sqrt{N}) / (z_p)$$

Solving for N = Number of claims for full credibility (Z=1)

$$N = (z_p / k)^2$$

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Limited Fluctuation – Standards for Full Credibility

- Claim counts required for full credibility based on the previous derivation:
  - Remember, $N = (z_p / k)^2$

Number of Claims		k			
P	**z_p**	**2.5%**	**5.0%**	**7.5%**	**10.0%**
90.0%	1.645	4,326	**1,082**	481	291
95.0%	1.960	6,147	1,537	683	584
99.0%	2.576	10,623	2,656	1,180	664
99.99%	3.891	24,219	6,055	2,691	1,514

Limited Fluctuation – Example

- Calculate the expected loss ratio, given that the prior estimated loss ratio is 75%. Assume P=95% and k=10%.
  - Scenario 1:
    Data: Observed loss ratio = 67%, Claim count = 600
    - What is the standard for full credibility?
    - Does this data have full credibility?
    - What is the expected loss ratio?

Limited Fluctuation – Example (continued)

- Calculate the loss ratio, given that the prior estimated loss ratio is 75%. Assume P=95% and k=10%.
  - Scenario 2:
    Data: Observed loss ratio = 67%, Claim count = 400
    - Assuming Z = 0.72, what is the expected loss ratio?

Limited Fluctuation Formula for Z – Pure Premium

- Generalizing to apply to pure premium:
  - T = pure premium = frequency * severity = N * S
  - E(T) = E(N)*E(S) and Var(T) = E(N)*Var(S) + E(S)²*Var(N)

$Z = (kE(T)) / (z_p\sqrt{Var(T)})$

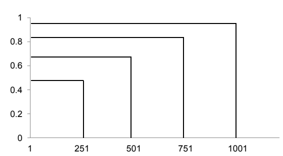
Reduces to, when solving for N = Number of claims for full credibility (Z=1)

$N = (z_p / k)^2 \times (Var(N)/E(N) + Var(S)/E(S)^2)$

Degree of confidence multiplier	Frequency distribution: tends to be close to 1 (equals 1 for Poisson)	Severity distribution: square of coefficient of variation (can be significant)

Limited Fluctuation – Partial Credibility

- Given a full credibility standard based on a number of claims N_F, what is the partial credibility of data based on a number of claims N that is less than N_F?
- Square root rule
  - **$Z = \sqrt{N / N_F}$**
- Calculate Z for N_F = 1,082 and N = 250, 500, 750, and 1,000. What do you notice?
- Exposures vs. Claims



Limited Fluctuation – Increasing Credibility

- Under the square root rule, credibility Z can be increased by
  - Getting more data (increasing N)
  - Accepting a greater margin of error (increasing k)
  - Conceding to smaller P = being less certain (decreasing z_p)

- Based on the formula

$Z = \sqrt{N / N_F}$

$Z = \sqrt{N/(z_p/k)^2}$

$Z = k^*\sqrt{N}/z_p$

Number of Claims	k			
P	**2.5%**	**5.0%**	**7.5%**	**10.0%**
90%	4,326	**1,082**	481	291
95%	6,147	1,537	683	584
99%	10,623	2,656	1,180	664
99.99%	24,219	6,055	2,691	1,514

Limited Fluctuation – Complement of Credibility

- Once the partial credibility Z has been determined, the complement (1-Z) must be applied to something else – the "complement of credibility"

If the data analyzed is…	*A good complement is...*
Pure premium for a class	**Pure premium for all classes**
Loss ratio for an individual risk	**Loss ratio for entire class**
Indicated rate change for a territory	**Indicated rate change for the entire state**
Indicated rate change for entire state	**Trend in loss ratio or the indication for the country**

Limited Fluctuation – Major Strength & Weaknesses

- The strength of limited fluctuation credibility is its simplicity
  - Thus its general acceptance and use
- Establishing a full credibility standard requires subjective selections regarding P and k
- Typical use of the formula based on the Poisson model is inappropriate for most applications
- Partial credibility formula – the square root rule – only holds for a normal approximation of the underlying distribution of the data. Insurance data tends to be skewed.
- Treats credibility as an intrinsic property of the data.

Limited Fluctuation – Example 2

- Calculate the credibility-weighted loss ratio and indicated change, given that the expected loss ratio is 75%. Use the square root rule and when P= 90% and k = 2.5%.

Year	Loss Ratio	Claim Count
2006	67%	530
2007	77%	610
2008	79%	630
2009	77%	620
2010	86%	690

			Credibility	Cred-Wght Loss Ratio	Indicated Rate Chg
'08-'10	81%	1,940			
'06-'10	77%	3,080			

Limited Fluctuation – Example 3

- Given a current territory factor of 1.08, determine the indicated territory factor with 5 years of data. Use the square root rule and the limited fluctuation formula for pure premium. Assume a Poisson frequency distribution and severity coefficient of variation of 1.5.

Year	Territory Exposure	Territory Claim Count	Territory Loss Ratio	Statewide Loss Ratio
2006	3,000	330	125%	78%
2007	3,020	420	153%	83%
2008	3,030	630	269%	85%
2009	3,020	210	122%	79%
2010	3,050	190	108%	72%
'06-'10	15,120	1,780	162%	80%

Limited Fluctuation – Example 3 (continued)

$$N = (z_p / k)^2 * (Var(N)/E(N) + Var(S)/E(S)^2)$$

- Remember, with a Poisson distribution, Var(N) = E(N), so the second term is 1. The third term is the square of the coefficient of variation, which is 1.5^2. Now we just need to select the confidence levels.

- If we want to be within 5% of the true value 90% of the time, the value for $(z_p / k)^2$ is 1,082. Plugging into the formula:

$$N_{claims} = 1{,}082 * (1 + 1.5^2) = 3{,}516.5$$

- Assuming the 5-year statewide frequency is 0.2:

$$N_{exposures} = 3{,}516.5 / 0.2 = 17{,}582.5$$

Limited Fluctuation – Example 3 (continued)

- To show the impact of our selection of an exposure standard instead of a claims standard.

Year	Territory Exposure	Territory Claim Count	Exposure Credibility	Claim Credibility
2006	3,000	330	41.3%	30.6%
2007	3,020	420	41.4%	34.6%
2008	3,030	630	41.5%	42.3%
2009	3,020	210	41.4%	24.4%
2010	3,050	190	41.6%	23.2%
'06-'10	15,120	1,780	92.7%	71.1%

Using a claims standard of 3,517 and an exposure standard of 17,583

Limited Fluctuation – Example 3 (continued)

- Determine what the indicated territorial factor, assuming 15% for fixed expenses.

Year	Territory Loss Ratio	Territory Credibility	Statewide Loss Ratio	Cred Wght Loss Ratio
'06-'10	162%	92.7%	80%	156.0%

156.0% = 92.7% x 162% + 7.3% x 80%

The final indicated territorial factor is (156% / 80%)*0.85 + 0.15 = **1.81**

An alternative approach would be to calculate the indicated factor prior to applying credibility, and then credibility weight the current factor with the indicated factor.

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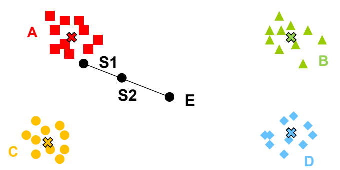
Least Squares Credibility Illustration

Steve Philbrick's target shooting example...



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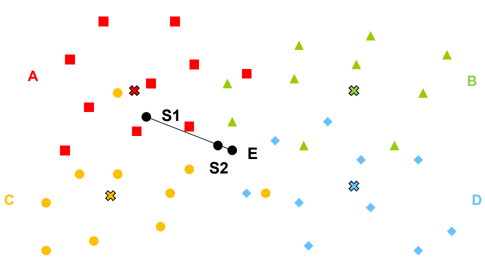
Least Squares Illustration (continued)

Which data exhibits more credibility?



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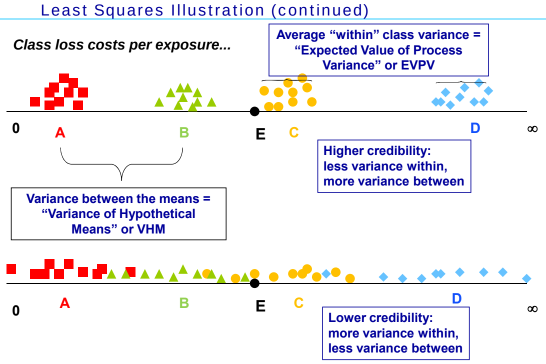
Least Squares Illustration (continued)



Least Squares – EVPV and VHM

- Assume we have 3 types of risk: low, medium, and high, which associated probabilities. Calculate the EVPV and VHM.

Risk	P(Claim)	P(Risk)	Variance	Mean^2
Low	20%	60%	0.16	0.04
Medium	30%	25%	0.21	0.09
High	40%	15%	0.24	0.16
Total	25.5%	100%	0.1845	0.0705

- EVPV: For binomial, variance = P(claim) x P(no claim)
  = (20%)(80%)(60%) + (30%)(70%)(25%) + (40%)(60%)(15%)
  = 0.1845
- VHM: Mean^2 – (Mean)^2
  = 0.0705 – $(0.255)^2$
  = 0.0055

Least Squares Derivation

- Similar to our limited fluctuation procedure:

E2 = w * T + (1 – w) * E1, where w = weight

- One method of weighting estimators is to have w be proportional to the reciprocal of the respective variances. So,

$$w = \frac{\frac{1}{(EVPV/n)}}{\frac{1}{(EVPV/n)} + \frac{1}{VHM}} \quad \text{and} \quad 1 - w = \frac{\frac{1}{VHM}}{\frac{1}{(EVPV/n)} + \frac{1}{VHM}}$$

- The denominator chosen to the weights add to 1. Next,

$$w = \frac{n}{(n + EVPV/VHM)} \quad \text{and} \quad 1 - w = 1 - \frac{n}{(n + EVPV/VHM)}$$

Least Squares Derivation (continued)

- Now, to simplify:

  w = n / (n + K)

  Z = n / (n + K), where K = EVPV / VHM

- This results in the minimum of squared errors
- Credibility Z can be increased by:
  - Getting more data (increasing n)
  - Getting less variance within classes (e.g., refining data categories) (decreasing EVPV)
  - Getting more variance between classes (increasing VHM)

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Least Squares – Example

- Assuming that you have the following book of business, calculate the EVPV, VHM, K, and Z. The prior estimate of the frequency is 0.517. With 4 years of observations and an observed frequency of 0.75, what is the estimated future frequency? Assume the claims are binomially distributed.

Risk	P(Claim)	P(Risk)
Low	40%	65%
Medium	70%	23%
High	80%	12%
Total	51.7%	100%

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Least Squares – Example (continued)

- To determine K, we use K = EVPV/VHM, which is
- Since we're told that we have 4 years of observations, n = 4. Therefore,
- The prior estimate of frequency is the same as the mean calculated before, 0.517, and the observed data results in a frequency of 0.75. This observed data as 31.9% credibility, so…

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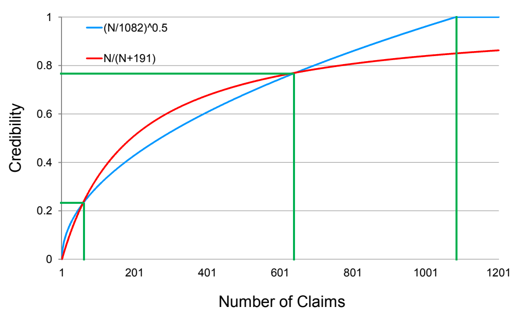
Least Squares – Strengths and Weaknesses

- The least squares credibility result is more intuitively appealing.
  - It is a relative concept
  - It is based on relative variances or volatility of the data
  - There is no such thing as full credibility
- Issues
  - Least squares credibility can be more difficult to apply. Practitioner needs to be able to identify variances.
  - The Credibility Parameter K, is a property of the *entire* set of data. So, for example, if a data set has a small, volatile class and a large, stable class, the credibility parameter of the two classes would be the same.
  - Assumes the complement of credibility is given to the overall mean, which may not be valid in real-world applications.

Comparing Limited Fluctuation and Least Squares



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