Interpolation Along a Curve

Joseph Boor, FCAS, Ph.D., CERA
Actuary
The Florida Office of Insurance Regulation

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Why does Loss Development Factor interpolation matter?

- Have accident year development, need policy year development.
- Have triangle at regular twelve month December 31 periods, need to develop data at, say May 31.
- Some advisory organization benchmark data , has annual periods triangulated as of, say, March 31. Need to convert LDFs for use on standard December 31 data.

Why does Increased Limit Factor interpolation matter?

- Have existing increased limit factor table set up, want to expand the options available to policyholders.

Interpolation methods using basic algebra

- Linear interpolation.
- Geometric interpolation - fit growth curve to both points.
  - Remember paid loss correction from Berquist-Sherman.
- Take 1/LDF (=%incurred, %paid), and linearly interpolate 1/LDF, then invert.
- Geometric interpolation of 1/LDF.

Problems with basic algebra methods

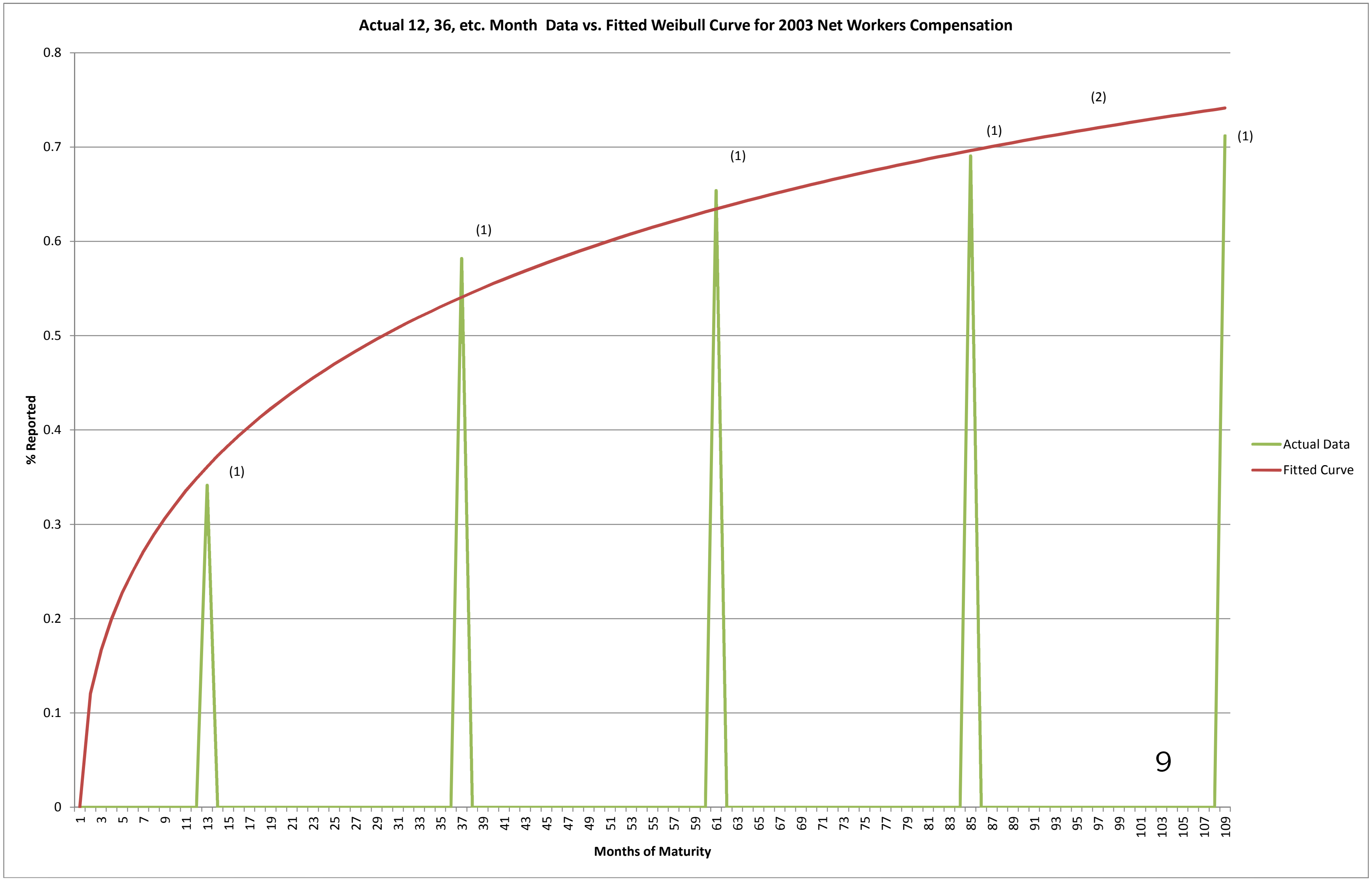
- Methods assume basic straight line or exponential curve
  - loss development incremental development is known to generally follow a “hump shaped” curve.
  - linear interpolation on ILFs fails the “Miccolis test”.

Alternatives–Curve Fitting

- Alternative is to fit a curve to the data, then read interpolated values off the curve.
  - Generally "hump shaped" Weibull probability mass function curve for loss levels emerging at each point.
    - % incurred or %paid then follows corresponding CDF
  - Generally, two parameter Pareto used for distribution of losses by size underlying ILFs.
    - ILF for limit "L" is $\frac{\int_0^L xf(x)dx+L(1-F(L))}{\int_0^B xf(x)dx+B(1-F(B))}$, where "$B$" = basic limit

Alternatives–Curve Fitting

- Curve fitting approach has some strong advantages, but it may have problems ...



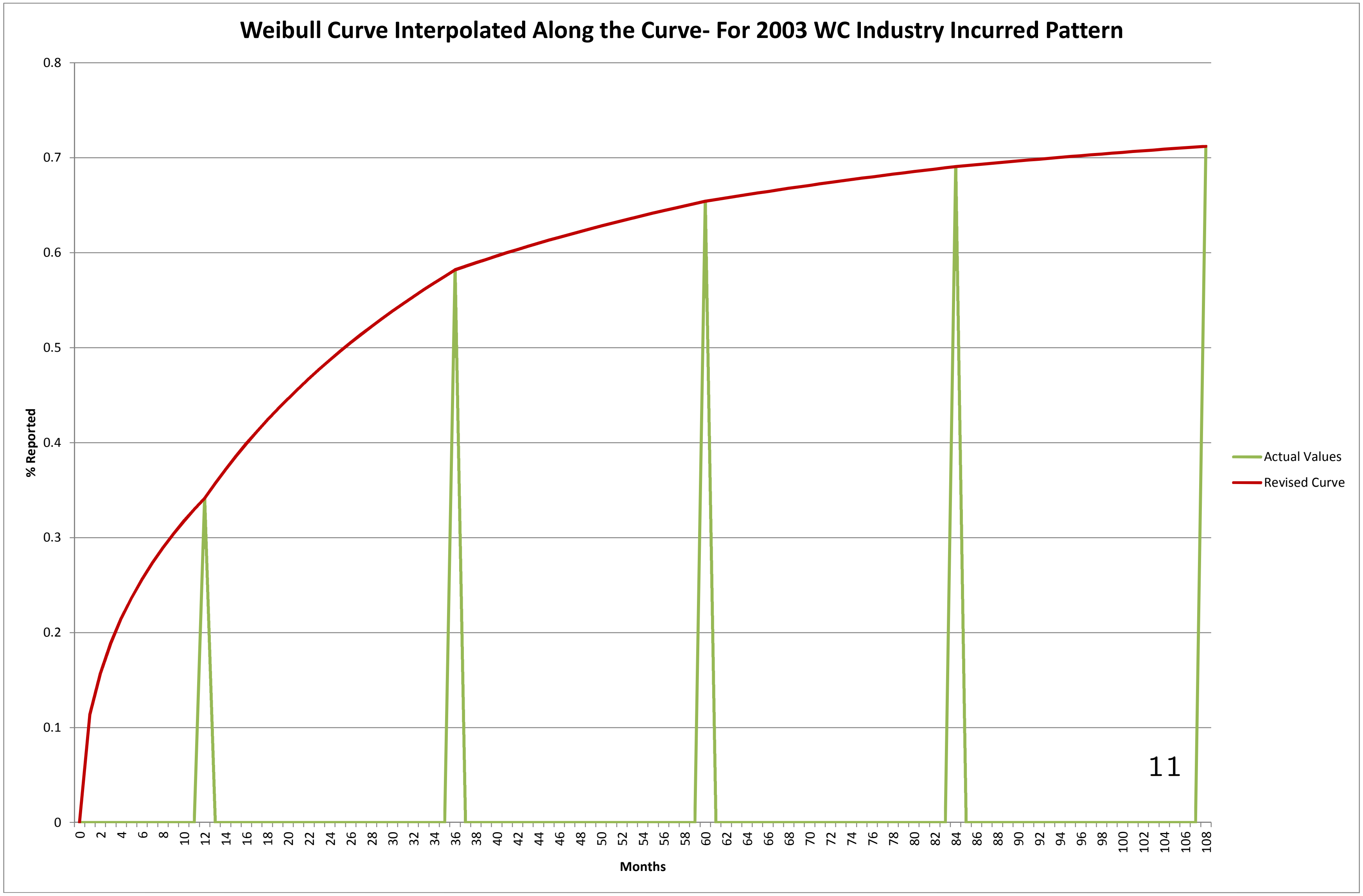
Actual 12, 36, etc. Month Data vs. Fitted Weibull Curve for 2003 Net Workers Compensation
% Reported
Months of Maturity
(1)
(1)
(1)
(1)
(2)
(1)
Actual Data
Fitted Curve

Correcting Fit Problems-Interpolation Along the Curve

- Stretch and rotate each segment of the fitted curve so that it hits the original data points exactly.

- if we have actual data points $d(t_0), d(t_1), d(t_2), \ldots, d(t_m)$ and a curve fitted to those points of $g(t)$, and we desire an estimate at $t^*, t_a < t^* < t_{a+1}, a \in 0, 1, 2, \ldots, m-1$, we take

$$\hat{d}(t^*) = d(t_a) + \frac{g(t^*) - g(t_a)}{g(t_{a+1}) - g(t_a)}[d(t_{a+1}) - d(t_a)].$$



Weibull Curve Interpolated Along the Curve- For 2003 WC Industry Incurred Pattern
% Reported
Months
Actual Values
Revised Curve

That is interpolating along the curve

Testing Against Alternatives

- vs. Straight linear interpolation, Straight Geometric Interpolation
- vs. Fitted Curve
  - for LDFs- Weibull regression
  - for ILFs - two parameter Pareto minimizing squared error in approximating known ILFs
- For LDFs - vs. linear, geometric interpolation of %paid/incurred

What is Weibull Regression?

- Weibull formula at time t for ILDFs is

$$1/ILDF(t) = \%Reported(t) = 1 - \exp(ct^b).$$

- So

$$1 - (1/ILDF(t)) = \%IBNR(t) = \exp(ct^b),$$

- or

$$\ln\left(\ln[1 - (1/ILDF(t))]\right) = \ln(c) + b\ln(t).$$

Weibull Regression-Part 2

- Given

$$\ln\left(\ln[1-(1/ILDF(t))]\right) = \ln(c) + b\ln(t),$$

- Just regress y values on left hand side to estimate Weibull parameters c and $\ln(b)$.

- For paid LDFs (PLDFs) regression works the same

$$\ln\left(\ln[1-(1/PLDF(t))]\right) = \ln(c) + b\ln(t),$$

LDF Testing Data

- Paid and (where possible) Incurred Development per 2003 Sked P - 11 Common Lines.
- Sample of 10 Small Co. Paid and (where possible) Incurred Development per (2011) Sked P.
- Use even (0,24, 48, etc.) LDFs to project odd (12, 36, etc.) LDFs and vice versa.

Batting Averages of the Methods - NAIC Data

- Winning Percentage of Intermediate LDF Value Estimates from Various Interpolation Methods vs. Interpolation Along the Curve

- Percentage of the Tests in Which Each Method Was Superior to Interpolation Along the Curve

Curve Fit to:	Number of Curves Fit	Winning % of Interp. Along the Curve	Geometric Interpolation	Linear Interpolation	Linear %Pd or Incrrd Interpolation	Geometric %Pd or Incrrd Interpolation	Unadjusted Weibull	Number of Times Weibull Outside Range
Even Maturity Paid LDFs	11	73 %	6 %	6 %	11 %	0 %	18 %	1
Odd Maturity Paid LDFs	11	77 %	5 %	5 %	9 %	2 %	14 %	5
Even Maturity Incrd LDFs	10	58 %	13 %	10 %	15 %	3 %	30 %	1
Odd Maturity Incrd LDFs	10	68 %	18 %	15 %	25 %	5 %	18 %	3
Straight Average		69 %	10 %	9 %	15 %	3 %	20 %	

Relative Errors of the Various Methods - NAIC Data

- Geometric Average Ratio of Squared Error of Intermediate LDF Value Estimates from Various Interpolation Methods to Squared Error of Interpolation Along the Curve

- Ratios Capped at 5% and 2000%

Curve Fit to:	Number of Curves Fit	Geometric Interpolation	Linear Interpolation	Linear % Pd or Incrrd Interpolation	Geometric % Pd or Incrrd Interpolation	Unadjusted Weibull
Even Maturity Paid LDFs	11	1034 %	1183 %	568 %	1781 %	316 %
Odd Maturity Paid LDFs	11	1277 %	2278 %	548 %	2931 %	527 %
Even Maturity Incrd LDFs	10	801 %	864 %	655 %	1401 %	190 %
Odd Maturity Incrd LDFs	10	694 %	943 %	366 %	1904 %	386 %
Straight Average		935 %	1235 %	524 %	1947 %	336 %

Batting Averages of the Methods - Small Company Data

- Winning Percentage of Intermediate LDF Value Estimates from Various Interpolation Methods vs. Interpolation Along the Curve

- Percentage of the Tests in Which Each Method Was Superior to Interpolation Along the Curve

Curve Fit to:	Number of Curves Fit	Winning % of Interp. Along the Curve	Geometric Interpolation	Linear Interpolation	Linear %Pd or Incrrd Interpolation	Geometric %Pd or Incrrd Interpolation	Unadjusted Weibull	Number of Times Weibull Outside Range
Even Maturity Paid LDFs	10	38 %	27 %	23 %	33 %	7 %	43 %	3
Odd Maturity Paid LDFs	10	50 %	20 %	20 %	25 %	8 %	30 %	4
Even Maturity Incrd LDFs	7	43 %	38 %	33 %	29 %	19 %	36 %	3
Odd Maturity Incrd LDFs	8	28 %	28 %	28 %	34 %	9 %	38 %	5
Straight Average		40 %	28 %	26 %	30 %	11 %	36 %	

Relative Errors of the Various Methods - Small Company Data

- Geometric Average Ratio of Squared Error of Intermediate LDF Value Estimates from Various Interpolation Methods to Squared Error of Interpolation Along the Curve

- Ratios Capped at 5% and 2000%

Curve Fit to:	Number of Curves Fit	Geometric Interpolation	Linear Interpolation	Linear % Pd or Incrrd Interpolation	Geometric % Pd or Incrrd Interpolation	Unadjusted Weibull
Even Maturity Paid LDFs	10	365 %	419 %	232 %	943 %	147 %
Odd Maturity Paid LDFs	10	473 %	658 %	302 %	1362 %	265 %
Even Maturity Incrd LDFs	7	238 %	255 %	336 %	423 %	110 %
Odd Maturity Incrd LDFs	8	341 %	406 %	177 %	1453 %	201 %
Straight Average		355 %	429 %	253 %	985 %	176 %

Conclusions - LDF Testing

- Interpolating along the Weibull curve is the most accurate option.
- Benefits vs. straight Weibull curve are reduced when loss volume is thin.

Testing of Pareto Estimation of ILFs

- Got seven sets of excess factors from NCCI-converted them to ILFs
- High granularity of NCCI factors made accurate interpolation too easy.
- Selected key values \$25,000, \$50,000, \$75,000, \$100,000, \$150,000, etc.
- Fitted curve to odds to predict evens and vice versa.

Testing of Pareto Estimation of ILFs - Curve Fitting

- Fit curve to minimize sum of squared errors vs. real data of (α, T) Pareto distribution at the points you're fitting to, the L's.

$$ILF(L, \$250,000) = \frac{\alpha - \left(\frac{T}{L}\right)^{\alpha-1}}{\alpha - \left(\frac{T}{\$250{,}000}\right)^{\alpha-1}}.$$

- \$250,000 pre-chosen as basic limit (where ILF = 1.)

Batting Averages of the Methods - NCCI ILF-type Data

- Winning Percentage of Intermediate ILF Value Estimates from Various Interpolation Methods vs. Interpolation Along the Curve

	Fitted Curve	Interp. Along the Curve	Linear Interpolation	Geometric Interpolation
	12 %	81 %	7 %	0

Relative Errors of the Various Methods - NCCI ILF-type Data

- Geometric Average Ratio of Squared Errors Relative to ILF Interpolation Along the Curve

	Fitted Curve	Interp. Along the Curve	Linear Interpolation	Geometric Interpolation
Sq. Error Ratio	703 %	100 %	592 %	740 %

Conclusions - ILF Testing

- Interpolating along the curve is the most accurate option.

Last Step - Cubic Splines

- Alternate view from numerical analysts
  - Cubic Splines is interpolation method of choice
    - Cubic polynomial between each two data points
    - Hit each data point
    - First and second derivatives match at each data point
    - Second derivative zero at outer endpoints
  - Details and calculation spreadsheet in the paper.

Skip to Relative Errors of Cubic Splines - NAIC Data

- Geometric Average Ratio of Error of Intermediate LDF Value Estimates from Cubic Splines vs. Interpolation Along the Curve

- Ratios Capped at 2000% Above and 5% Below

Curve Fit to:	Number of Curves Fit	Error Ratio
Even Maturity Paid LDFs	11	188 %
Odd Maturity Paid LDFs	11	133 %
Even Maturity Incrd LDFs	10	271 %
Odd Maturity Incrd LDFs	10	152 %
Straight Average		178 %

Relative Errors of Cubic Splines - Small Company Data

- Geometric Average Ratio of Error of Intermediate LDF Value Estimates from Cubic Splines vs. Interpolation Along the Curve

- Ratios Capped at 2000% Above and 5% Below

Curve Fit to:	Number of Curves Fit	Error Ratio
Even Maturity Paid LDFs	10	274 %
Odd Maturity Paid LDFs	10	169 %
Even Maturity Incrd LDFs	7	201 %
Odd Maturity Incrd LDFs	8	142 %
Straight Average		193 %

Relative Errors of Cubic Splines - NCCI ILF-type Data

- Geometric average ratio of error of intermediate LDF value estimates from cubic splines vs. interpolation along the curve = 254%.

- Standard capping used

Summary

- As confirmed by testing, on average interpolation along the curve produces the most accurate estimates of all the methods reviewed.

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