SYLLABUS OF BASIC EDUCATION
2018
Modern Actuarial Statistics-II – Exam MAS-II

This syllabus, posted January 17, 2017, is subject to further revisions until the changes for the Fall 2018 CAS Syllabus of Basic Education are approved by the Executive Council and announced in January 2018.

The syllabus for this four-hour exam is defined in the form of learning objectives, knowledge statements, and readings.

LEARNING OBJECTIVES set forth, usually in broad terms, what the candidate should be able to do in actual practice. Included in these learning objectives are certain methodologies that may not be possible to perform on an examination, such as applying the Metropolis-Hastings algorithm or building a decision tree, but that the candidate would still be expected to explain conceptually in the context of an examination.

KNOWLEDGE STATEMENTS identify some of the key terms, concepts, and methods that are associated with each learning objective. These knowledge statements are not intended to represent an exhaustive list of topics that may be tested, but they are illustrative of the scope of each learning objective.

READINGS support the learning objectives. It is intended that the readings, in conjunction with the material on earlier examinations, provide sufficient resources to allow the candidate to perform the learning objectives. Some readings are cited for more than one learning objective. The CAS Syllabus & Examination Committee emphasizes that candidates are expected to use the readings cited in this *Syllabus* as their primary study materials.

Thus, the learning objectives, knowledge statements, and readings complement each other. The learning objectives define the behaviors, the knowledge statements illustrate more fully the intended scope of the learning objectives, and the readings provide the source material to achieve the learning objectives. Learning objectives should not be seen as independent units, but as building blocks for the understanding and integration of important competencies that the candidate will be able to demonstrate.

Note that the range of weights shown should be viewed as a guideline only. There is no intent that they be strictly adhered to on any given examination—the actual weight may fall outside the published range on any particular examination.

The overall section weights should be viewed as having more significance than the weights for the individual learning objectives. Over a number of years of examinations, absent changes, it is likely that the average of the weights for each individual overall section will be in the vicinity of the guideline weights. For the weights of individual learning objectives, such convergence is less likely. On a given examination, in which it is very possible that not every individual learning objective will be tested, there will be more divergence of guideline weights and actual weights. Questions on a given learning objective may be drawn from any of the listed readings, or a combination of the readings. There may be no questions from one or more readings on a particular exam.

After each set of learning objectives, the readings are listed in abbreviated form. Complete text references are provided at the end of this exam syllabus.

Items marked with a bold **OP** (Online Publication) are available at no charge and may be downloaded from the CAS website.

Please check the "*Syllabus* Update" for this exam for any changes to this *Syllabus*.

A thorough knowledge of calculus and probability is assumed. Given the material covered on this exam, we assume that the candidate has knowledge of linear algebra concepts at the level commonly assumed as a prerequisite to taking an undergraduate level course in regression analysis. Candidates are expected to have mastered the concepts in Exam MAS-I. While some problems may have an insurance or risk management theme, no prior knowledge of insurance terminology is expected.

A variety of tables will be provided to the candidate with the exam. The tables include values for the standard normal distribution, abridged inventories of discrete and continuous probability distributions, Chi-square Distribution, t-Distribution, F-Distribution, Normal Distribution. Since they will be included with the examination, candidates will not be allowed to bring copies of the tables into the examination room.

A guessing adjustment will be used in grading this exam. Details are provided under "Guessing Adjustment" in the "Examination Rules-The Examination" section of the *Syllabus of Basic Education*.

A. Introduction to Credibility

Range of weight for Section A: 5-15 percent

Advances in statistical computing tools have now made it practical to include a form of credibility weighting when building regression type models. For example, what the statisticians call shrinkage in a Linear Mixed Effect Model is a form of least squares credibility weighting. These advanced techniques are covered extensively in Sections B and C. However, candidates should be familiar with the topics listed below as they can serve as a good introduction to those techniques and are still very much in practice today. Specifically, candidates should be familiar with limited fluctuation credibility and be able to calculate estimates using Bayesian credibility procedures. They should also be fluent with Bayesian and Bühlmann (least squares credibility) procedures both for discrete and continuous models.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. Understand the basic framework of credibility and be familiar with limited fluctuation credibility, including partial and full credibility 2. Understand the basic framework of Bühlmann credibility 3. Calculate different variance components for Bühlmann credibility 4. Calculate Bühlmann and Bühlmann-Straub credibility factor and estimates for frequency, severity, and aggregate loss 5. Understand the basic framework of Bayesian credibility 6. Calculate Bayes estimate/Bayesian premium 7. Bayesian versus Bühlmann credibility for conjugate distributions 8. Calculate credibility estimates using the Nonparametric empirical Bayes Method Range of weight for Learning Objectives A.1 through A.8 collectively: 5-15 percent	a. Limited fluctuation credibility, Partial and Full Credibility b. Conjugate priors, Poison/Gamma, Binomial/Beta, Normal/Normal c. Bühlmann Credibility Continuous d. Bühlmann Credibility Discrete e. Bayesian Analysis Discrete f. Bayesian Analysis Continuous g. Nonparametric Empirical Bayes

READINGS

- NAM Chapters 6.1-6.3, 7.1-7.4, 8.1-8.2, and 9.1-9.2

B. Linear Mixed Models

Range of weight for Section B: 10-30 percent

This section covers linear models that use a form of credibility weighting for a designated subset of variables in the model called random effects. The candidates will be expected to understand the concepts of shrinkage for Linear Mixed Models as well as how to accommodate models with correlated observations or models where the variance is not a function of the mean. Mixed Models include both Gaussian (Linear Mixed Models) and non-Gaussian (Generalized Linear Mixed Models or Non-Linear Mixed Models) models. This section will only cover Linear Mixed Models. The candidate is expected to understand the linkage between shrinkage and credibility weighting, how to select the appropriate model to induce credibility weighting at the appropriate level when setting up the model structure, and how to account for correlation in the residuals.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. Understand the assumptions behind Linear Mixed Models and use that understanding to evaluate how to set up a Linear Mixed Effect Model design to best accomplish the goals of the modeling exercise Note: While OLS and GLM concepts will not be directly tested, the candidate is expected to have mastered those concepts from Exam MAS-I	a. Characteristics of Random and Fixed Effects explanatory variables for Linear Mixed Models • How to identify a Random Effect Variable • Interaction between fixed and random effect variables when calculating standard error of estimate b. Implications of Correlation Matrix choice by model form for Linear Mixed Models • Independence assumption for observations • Repeated Measures/Longitudinal Studies • Correlation forms for random vs. fixed effect variables • Hierarchical Model structure implementation of treatment and design structure • Explicitly model variance as a function of an explanatory variable

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
2. Understand the algorithms behind the numerical solutions for the Linear Mixed Model to enable interpretation of output from the statistical software employed in modeling to make appropriate choices when evaluating modeling options Note: While OLS and GLM concepts will not be directly tested, the candidate is expected to have mastered those concepts from Exam MAS-I	a. Restricted Maximum Likelihood b. Choice between Restricted Maximum Likelihood and Maximum Likelihood for Linear Mixed Models c. Estimable vs. Predictable Functions d. Best Linear Unbiased Predictor for a Linear Mixed Model and Interaction with Fixed Effects variables e. EM Algorithm f. Sandwich Estimator for Variance g. Credibility adjusted degrees of freedom for Linear Mixed Models (Saitherwaite / Kenward Rodgers) h. Conditional vs. Population Estimate for Mixed Models
3. Understand and be able to select the appropriate model structure and variable selection for a Linear Mixed Model given the behavior of the data set to be modeled by interpreting the model diagnostics and or summary statistics on the variables available in the model along with any graphs depicting how the dependent variable behaves as a function of possible explanatory variables Note: While OLS and GLM concepts will not be directly tested, the candidate is expected to have mastered those concepts from Exam MAS-I	a. Units of replication for Linear Mixed Models b. Blocking factors for Linear Mixed Models c. Estimable vs. Predictable Functions for Linear Mixed Models d. Interaction terms for Fixed Effects vs. Random Effect variables for Linear Mixed Models e. Model Selection for Linear Mixed Models when covariance structure changes f. Covariance structure selection for Linear Mixed Models g. Selection of fixed vs. random effect class for Mixed Effect explanatory variables h. Explicitly model variance for Linear Mixed Models i. Residual graphs evaluating normality and constant variable assumptions j. Goodness of fit statistics including: t-tests, F-tests, Chi-Square tests k. Know when nested model comparisons are appropriate l. Application of AIC & BIC relative measures of goodness of fit m. Application of Scatter Plots and Box Plots as an aide to model design

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
Range of weight for Learning Objectives B.1 through B.3 collectively: 10-30 percent	

READINGS

- West, Chapters 1-8

Chapters 1 and 2 contain an introduction to the modeling concepts underlying Linear Mixed Effect model. Chapters 3 through 8 contain examples that illustrate how to build a Linear Mixed Effect model to accommodate different circumstances. The chapters covering the examples include code from different software packages including SAS, SPSS, and HLM. Candidates should focus on understanding the design choices made in modeling, the output from those packages, and how that output was interpreted. Comments on how to make design choices and/or the type of hypothesis test to be employed at a given point in the modeling process expand on the introduction to modeling concepts covered in Chapters 1 and 2 and are a vital part of the reading from West.

The format for questions on this exam will use the same format as the R code exhibits and graphs in the text to ensure consistency in question format across exam questions and over time. Exam questions from this Section may contain parameter tables and diagnostic tables or plots of the type shown in the text. Candidates should understand how to interpret these tables. Candidates who become familiar with a statistical language capable of generating this type of output, such as R, will have an easier time understanding and applying the concepts covered in the syllabus material. In particular, candidates that work with the R code examples in the West textbook, along with the datasets provided, will have a better grasp of the material than that obtained by simply reading the textbook. However, for exam questions from this Section, candidates will not be asked to write or interpret R code.

C. Bayesian Analysis and Markov Chain Monte Carlo

Range of weight for Section C: 50-70 percent

This section covers current techniques in use to apply Bayesian statistics to modeling problems where conjugate prior distributions do not apply. The candidate is expected to be able to apply Bayesian techniques, to understand a model, and to evaluate the resulting goodness of fit by interpreting the diagnostics that are described below. The candidate is also expected to understand why the Bayesian approach is different than the classical (frequentist) procedures.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. Understand basis and basics of Bayesian analysis and incorporate that understanding when interpreting model results • Difference between Bayesian and classical (frequentist) procedures • How probability is used as a measure of uncertainty • Use of simulation to create predictive distributions	a. Bayes' rule b. Subjectivity c. Likelihood d. Prior distribution e. Posterior distribution f. Posterior predictive distribution g. Explanatory variables
2. Evaluate the different options available when creating and using Bayesian models for a given modeling assignment. Understand how to set up a Bayesian MCMC model and evaluate how a given set of design choices affects the results of a model • Recognize benefits and limitations of different kinds of priors • Calculating posterior and posterior predictive distributions for single and multi-parameter models. • Hierarchical models • Linear Regression	a. Proper and improper priors b. Conjugate and non-conjugate priors c. Informative, non-informative, and weakly-informative priors d. Hyperpriors e. Exchangeability f. Sampling from posterior distribution g. Regularization

<table>
<tr><th>LEARNING OBJECTIVES</th><th>KNOWLEDGE STATEMENTS</th></tr>
<tr><td>3. Understand Bayesian computation, how Markov Chain Monte Carlo methods are used, and how to evaluate model performance. Evaluate when a given modeling approach should be used given modeling diagnostics on simulation performance. Interpret diagnostics on simulation performance
• Simulation and sampling
• Conditional sampling
• Convergence assessment
• Efficient samplers
• Hamiltonian Monte Carlo</td><td>a. Direct, rejection, and importance sampling
b. Markov chains
c. Gibbs sampler
d. Metropolis and Metropolis-Hastings algorithms
e. Warm-up / Burn-in
f. Convergence in parameter estimate measurement
g. Trace plot
h. Acceptance rate
i. Within-sequence correlation
j. Thinning
k. Effective number of draws
l. Potential scale reduction (R-Hat or Gelman-Rubin statistic)
m. Advantage of Hamiltonian Monte Carlo</td></tr>
<tr><td>4. Understand how to apply model checking, evaluation, comparison, and expansion techniques as an aid to interpreting and evaluating model diagnostics
• Know how to check model fit to data
• Understand limitations of various tests
• Understand measures of model predictive accuracy
• Understand information criteria, their uses and limitations (e.g. bias, dependence on prior, etc.)
• Compare models via predictive performance measures
• Understand how models can be expanded and what further checks may be needed

Range of weight for Learning Objectives C.1 through C.4 collectively: 50-70 percent</td><td>a. Sensitivity analysis
b. External validation
c. Posterior predictive checking
d. Residuals and residual plots
e. Predictive accuracy
f. Log predictive density
g. Out-of-sample predictive accuracy
h. Information criteria measures (AIC, DIC, WAIC)
i. Effective number of parameters
j. Cross-validation (LOO-CV)
k. Model expansion</td></tr>
</table>

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
READINGS	
• BDA, Chapters 1, 2, 3.1, 3.2, 5, 6, 7, 10, 11, 12.4, and 14 While candidates will not be tested on software code, it is in their best interest to have some hands-on experience with these techniques. As such, the interested candidate is directed to the R and Stan software tools, the 'rstan' package for R (which will install Stan), Appendix C of BDA, and https://github.com/avehtari/BDA_R_demos, which will have demonstrations for many of the examples in BDA. Exam questions will not be sourced directly from the Bibliographic note section at the end of each of the referenced chapters above.	

D. Statistical Learning

Range of weight for Section D: 5-15 percent

This section introduces candidates to a sample of foundational statistical learning techniques. Both supervised and unsupervised techniques are detailed in the readings and candidates should be able to distinguish between them. The supervised learning techniques are non-parametric in nature, meaning the model cannot easily be described in an equation form — they tend to model flexible, non-linear hypotheses well. The unsupervised learning techniques are useful for reducing the dimensions of the data to aid in the profiling of the data or to facilitate more efficient learning from the data. Candidates are expected to understand the mechanics of these algorithms and recognize their inherent strengths and weaknesses so as to be able to select the most appropriate procedure for the learning task at hand.

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
1. Understand and be able to explain how K-Nearest Neighbors (KNN) works in practice and its relationship with Bayes classifier.	a. Classification versus Regression for Supervised Learning b. Bayes classifier c. KNN decision boundary versus Bayes decision boundary
2. Understand and be able to explain the basics of decision trees, the purpose of tree pruning, and how extensions such as Bagging, Random Forest, and Boosting can improve the prediction accuracy of tree-based methods.	a. Recursive binary splitting for decision trees b. Pruning for decision trees c. Comparison of decision trees versus linear models d. Advantages and disadvantages of decision trees e. Bagging and OOB Error f. Similarity and differences between Bagging and Random Forest g. Sequential learning via Boosting h. Gini Index application for splitting
3. Understand and be able to explain the purpose of Principle Component Analysis (PCA) and be able to interpret related software outputs.	a. Loading vector and scores for principle component b. Effect of scaling on PCA c. Proportion of variance explained by PCA and scree plots d. Combining many dimensions (variables) into fewer e. Compare purpose of PCA to K-Means
4. Be familiar with clustering procedures and be able to interpret related software outputs.	d. K–Means clustering algorithm e. Agglomerative Hierarchical clustering algorithm f. Dendrogram g. Dissimilarity measure

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
Range of weight for Learning Objectives D.1 through D.4 collectively: 5-15 percent	

READINGS

- ISL, Chapters 1 (Background reading only), 2.2.3, 8, and 10. Exam questions will not be sourced directly from Chapter 1. Sections 2.3, 8.3, and 10.4 demonstrate the implementation of topics covered in these chapters with ‘R’. Examination questions will not test coding in R, but careful review of these sections will greatly help the candidates learn and understand these topics.

Complete Text References for Exam MAS-II

Text references are alphabetized by the citation column.

Citation	Abbreviation	Learning Objective	Source
Gelman, A., et al., *Bayesian Data Analysis*, 3rd Edition, Chapman and Hall/CRC Press, 2014.	BDA	C1-C4	**B**
James, G., et al., *An Introduction to Statistical Learning, with Application in R*, Springer, 2015.	ISL	D1-D4	**B**
Tse, Y., *Nonlife Actuarial Models, Theory Methods and Evaluation*, Cambridge University Press, 2009.	NAM	A1-A8	**B**
West, B. T.; Welsh, K. B.; and Galecki, A. T., *Linear Mixed Models: A Practical Guide Using Statistical Software*, 2nd Edition, Chapman and Hall/CRC Press, 2014.	West	B1-B3	**B**

Source Key

B	Book—may be purchased from the publisher or bookstore or borrowed from the CAS Library.
BO	Book (Optional)—may be purchased from the publisher or bookstore.
NEW	Indicates new or updated material.
OP	All text references marked as Online Publications will be available on a web page titled Complete Online Text References.
SK	Material included in the 2018 Study Kit.
SKU	Material included in both the 2018 CAS Study Kit and the 2018 Update to the 2017 Study Kit.

Items printed in **red** indicate an update, clarification, or change.

Publishers and Distributors

Contact information is furnished for those who wish to purchase the text references cited for this exam. Publishers and distributors are independent and listed for the convenience of candidates; inclusion does not constitute endorsement by the CAS.

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